Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/40-1.2.2.3-d+e-x^2-^m-a+b-x^2+c-x^4-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [413]. This is test number [40].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.76 (412)	0.24 (1)
Mathematica	98.55 (407)	1.45 (6)
Maple	96.61 (399)	3.39 (14)
Fricas	59.32 (245)	40.68 (168)
Giac	45.28 (187)	54.72 (226)
Sympy	45.04 (186)	54.96 (227)
Mupad	44.55 (184)	55.45 (229)
Maxima	27.36 (113)	72.64 (300)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

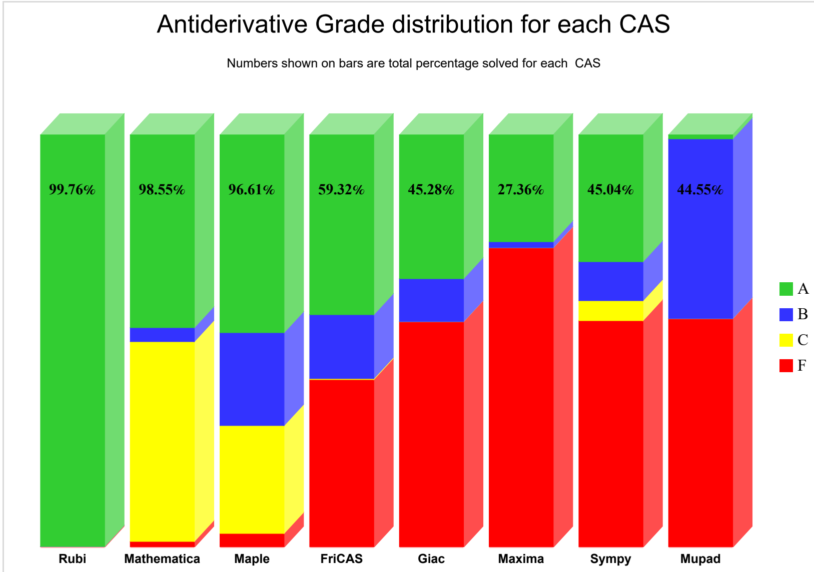
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

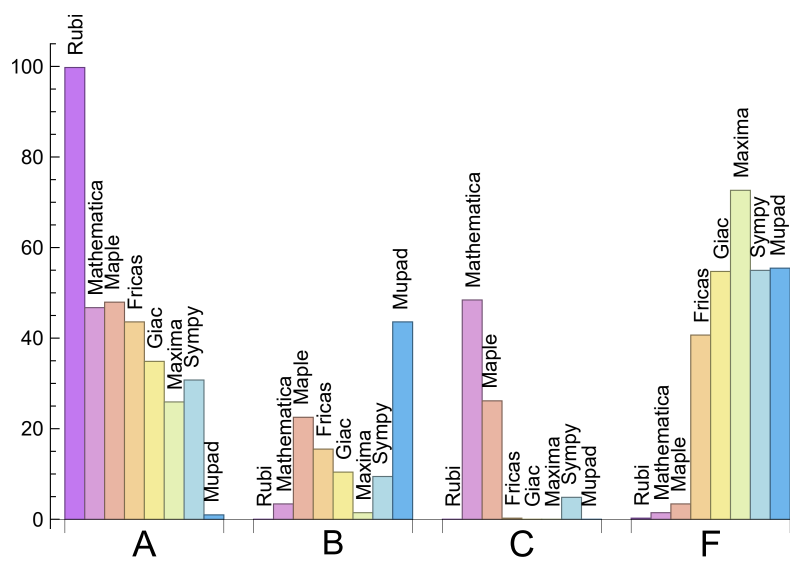
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.76	0.00	0.00	0.24
Maple	47.94	22.52	26.15	3.39
Mathematica	46.73	3.39	48.43	1.45
Fricas	43.58	15.50	0.24	40.68
Giac	34.87	10.41	0.00	54.72
Sympy	30.75	9.44	4.84	54.96
Maxima	25.91	1.45	0.00	72.64
Mupad	N/A	43.58	0.00	55.45

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	6	100.00 %	0.00 %	0.00 %
Maple	14	100.00 %	0.00 %	0.00 %
Fricas	168	39.29 %	9.52 %	51.19 %
Giac	226	95.13 %	0.00 %	4.87 %
Maxima	300	97.67 %	0.00 %	2.33 %
Sympy	227	83.26 %	14.10 %	2.64 %
Mupad	229	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

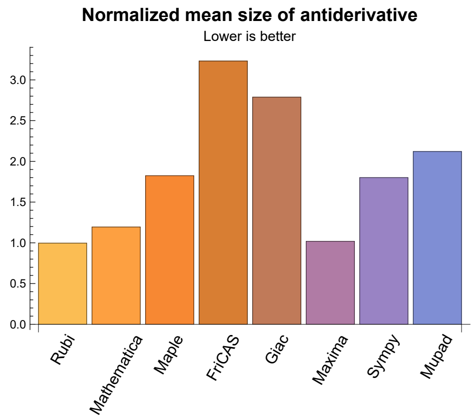
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

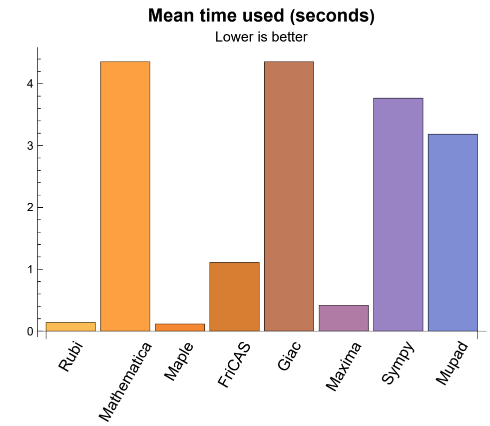
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.14	154.97	1.00	112.50	1.00
Mathematica	4.36	171.81	1.19	110.00	1.00
Maple	0.11	231.73	1.82	159.00	1.22
Maxima	0.42	114.59	1.02	74.00	0.95
Fricas	1.11	721.10	3.23	113.00	1.62
Sympy	3.77	175.18	1.80	81.00	1.08
Giac	4.36	970.98	2.79	79.00	1.00
Mupad	3.18	450.18	2.12	67.00	0.93

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{175, 399, 404, 405\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{113, 114, 115, 116, 118, 402, 403\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

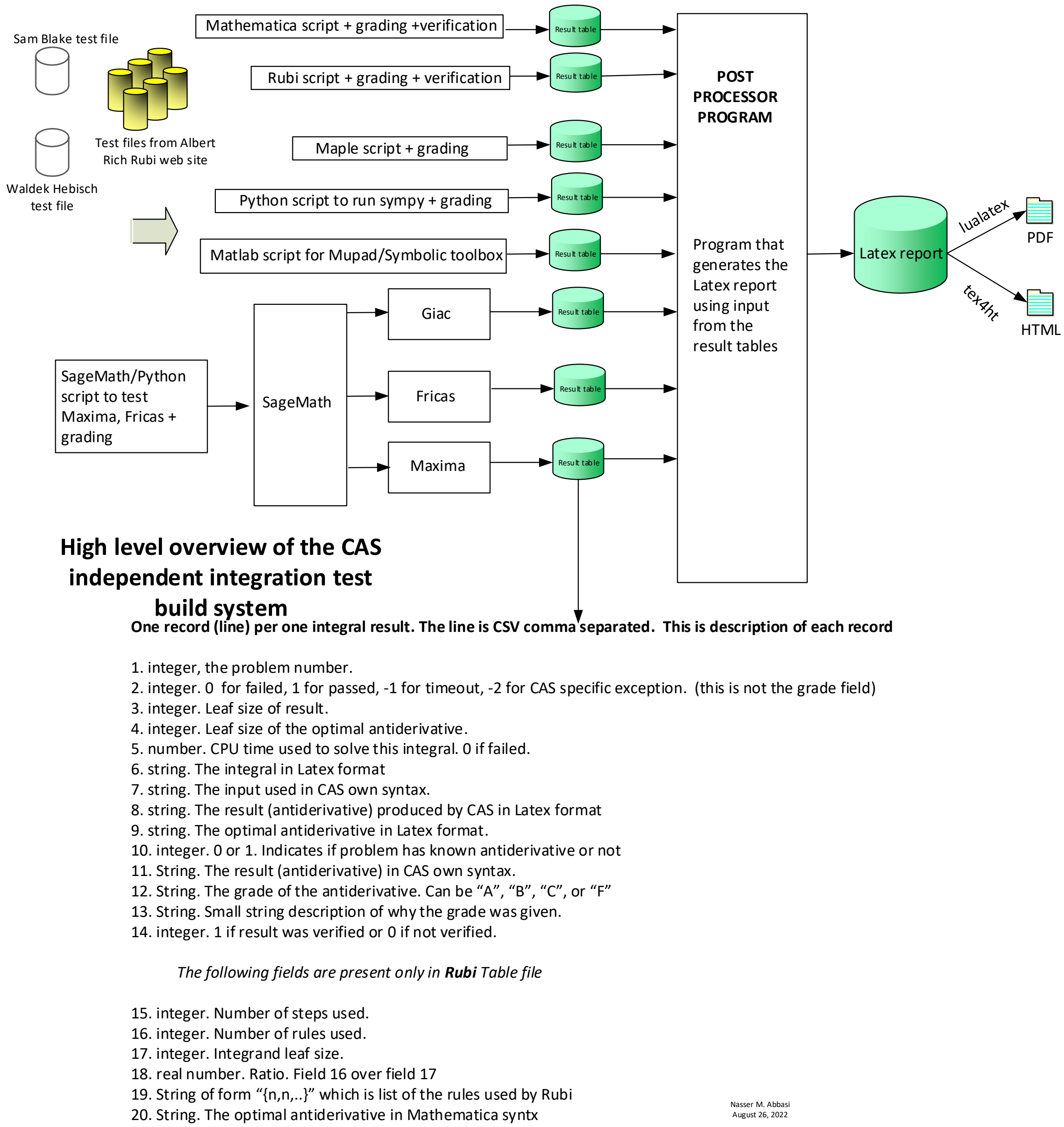
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

B grade: { }

C grade: { }

F grade: { 174 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 22, 24, 34, 35, 36, 37, 41, 42, 43, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 94, 95, 96, 97, 104, 105, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 170, 175, 176, 177, 178, 179, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 399, 400, 401, 402, 403, 404, 405, 408, 409, 410, 411 }

B grade: { 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 65, 80, 88 }

C grade: { 14, 15, 16, 17, 18, 19, 20, 21, 23, 25, 44, 45, 46, 48, 49, 73, 92, 93, 98, 99, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 199, 200, 201, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 406, 407, 412, 413 }

F grade: { 174, 180, 181, 186, 187, 188 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 7, 8, 13, 17, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 70, 71, 72, 73, 75, 76, 77, 82, 84, 85, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 158, 159, 160, 164, 165, 175, 182, 183, 184, 185, 189, 190, 191, 192, 193, 194, 199, 200, 201, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 316, 323, 324, 325, 331, 338, 347, 382, 383, 384, 386, 387, 388, 390, 391, 392, 393, 399, 404, 405, 407, 412, 413 }

B grade: { 5, 6, 9, 10, 11, 12, 14, 15, 16, 23, 25, 34, 35, 36, 39, 40, 53, 54, 67, 68, 69, 74, 78, 79, 80, 81, 83, 86, 91, 100, 101, 113, 114, 115, 116, 117, 118, 157, 161, 162, 163, 166, 167, 170, 172, 195, 196, 197, 198, 202, 203, 204, 208, 209, 210, 220, 221, 222, 223, 224, 282, 283, 284, 285, 317, 318, 319, 320, 321, 322, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 343, 344, 345, 346, 380, 381, 385, 389 }

C grade: { 18, 19, 20, 21, 22, 24, 150, 151, 152, 153, 154, 155, 156, 168, 169, 171, 173, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 394, 395, 396, 397, 398, 406, 408, 409, 410, 411 }

F grade: { 174, 176, 177, 178, 179, 180, 181, 186, 187, 188, 400, 401, 402, 403 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 37, 42, 43, 44, 47, 51, 52, 56, 57, 58, 61, 66, 72, 73, 74, 76, 77, 84, 85, 86, 89, 92, 93, 94, 96, 98, 99, 108, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 175, 189, 190, 191, 193, 194, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 276, 277, 278, 279, 280, 281, 283, 284, 285, 399, 404, 405 }

B grade: { 7, 65, 88, 95, 192, 282 }

C grade: { }

F grade: { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 45, 46, 48, 49, 50, 53, 54, 55, 59, 60, 62, 63, 64, 67, 68, 69, 70, 71, 75, 78, 79, 80, 81, 82, 83, 87, 90, 91, 97, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.5 FriCAS

A grade: { 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 96, 98, 99, 108, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 147, 175, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 214, 215, 216, 217, 218, 220, 221, 235, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 332, 333, 334, 335, 339, 340, 341, 367, 399, 404, 405 }

B grade: { 1, 2, 3, 4, 7, 22, 23, 65, 88, 94, 95, 97, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 137, 138, 139, 140, 142, 143, 144, 145, 146, 148, 149, 194, 195, 196, 197, 198, 211, 212, 213, 219, 222, 223, 224, 258, 259, 260, 263, 264, 265, 266, 267, 268, 270, 271, 272, 273, 408, 409, 410, 411 }

C grade: { 303 }

F grade: { 16, 17, 18, 19, 20, 21, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 243, 269, 274, 275, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 321, 322, 323, 329, 330, 331, 336, 337, 338, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 412, 413 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 100, 101, 106, 109, 120, 121, 122, 123, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 157, 158, 159, 164, 166, 189, 190, 191, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 258, 267, 279, 280 }

B grade: { 7, 14, 15, 23, 25, 38, 42, 65, 88, 95, 96, 104, 105, 124, 125, 126, 132, 133, 167, 192, 193, 194, 214, 215, 216, 217, 248, 249, 256, 257, 261, 262, 276, 277, 278, 281, 282, 283, 284 }

C grade: { 18, 19, 20, 21, 98, 99, 108, 150, 151, 152, 153, 168, 176, 177, 178, 179, 182, 183, 184, 185 }

F grade: { 22, 24, 102, 103, 107, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 142, 143, 148, 149, 154, 155, 156, 160, 161, 162, 163, 165, 169, 170, 171, 172, 173, 174, 175, 180, 181, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 259, 260, 263, 264, 265, 266, 268, 269, 270, 271, 272, 273, 274, 275, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.7 Giac

A grade: { 1, 2, 5, 6, 8, 11, 12, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 97, 98, 99, 102, 103, 108, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 175, 189, 190, 191, 192, 193, 194, 198, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 399, 404, 405 }

B grade: { 3, 4, 7, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 47, 50, 53, 65, 88, 95, 96, 100, 101, 104, 105, 195, 196, 197, 224, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275 }

C grade: { }

F grade: { 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 68, 80, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.8 Mupad

A grade: { 175, 399, 404, 405 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 179, 185, 189, 190, 191, 192, 193, 194, 214, 215, 216, 217, 218, 219, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 282, 283, 284, 285 }

C grade: { }

F grade: { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 113, 114, 115, 116, 117, 118, 119, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 276, 277, 278, 279, 280, 281, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 406, 407, 408, 409, 410, 411, 412, 413 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	183	206	221	767	109	241	599
N.S.	1	1.00	0.74	0.83	0.89	3.11	0.44	0.98	2.43
time (sec)	N/A	0.099	0.070	0.111	0.533	0.354	0.356	4.904	0.377

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	184	206	221	767	110	241	603
N.S.	1	1.00	0.74	0.83	0.89	3.11	0.45	0.98	2.44
time (sec)	N/A	0.087	0.026	0.143	0.516	0.355	0.358	5.022	0.257

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	95	104	109	755	110	230	579
N.S.	1	1.00	1.10	1.21	1.27	8.78	1.28	2.67	6.73
time (sec)	N/A	0.030	0.019	0.137	0.501	0.346	0.367	3.824	4.643

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	95	104	109	755	110	228	579
N.S.	1	1.00	1.10	1.21	1.27	8.78	1.28	2.65	6.73
time (sec)	N/A	0.026	0.013	0.142	0.499	0.349	0.368	3.665	4.578

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	33	140	39	33	41	52	29
N.S.	1	1.00	0.82	3.50	0.98	0.82	1.02	1.30	0.72
time (sec)	N/A	0.013	0.021	0.174	0.514	0.329	0.039	4.205	0.090

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	140	39	42	49	40	21
N.S.	1	1.00	0.86	2.75	0.76	0.82	0.96	0.78	0.41
time (sec)	N/A	0.014	0.013	0.109	0.512	0.348	0.034	3.699	4.433

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	32	13	25	29	32	29	12
N.S.	1	1.00	2.00	0.81	1.56	1.81	2.00	1.81	0.75
time (sec)	N/A	0.002	0.009	0.158	0.490	0.354	0.031	3.408	0.092

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	12	15	12	12
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	0.75	0.75
time (sec)	N/A	0.002	0.003	0.163	0.502	0.332	0.032	3.398	0.027

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	204	100	148	138	0	57
N.S.	1	1.00	0.80	2.72	1.33	1.97	1.84	0.00	0.76
time (sec)	N/A	0.024	0.012	0.163	0.496	0.354	0.171	0.000	4.793

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	91	204	70	151	131	0	43
N.S.	1	1.00	0.86	1.92	0.66	1.42	1.24	0.00	0.41
time (sec)	N/A	0.030	0.013	0.164	0.516	0.376	0.174	0.000	4.757

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	232	74	132	87	86	57
N.S.	1	1.00	0.80	3.09	0.99	1.76	1.16	1.15	0.76
time (sec)	N/A	0.030	0.019	0.153	0.502	0.374	0.083	3.881	4.406

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	75	232	60	138	80	91	41
N.S.	1	1.00	0.83	2.58	0.67	1.53	0.89	1.01	0.46
time (sec)	N/A	0.028	0.014	0.142	0.512	0.351	0.083	3.445	0.086

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	18	17	17	22	19	9
N.S.	1	1.00	1.92	1.38	1.31	1.31	1.69	1.46	0.69
time (sec)	N/A	0.004	0.004	0.165	0.520	0.328	0.058	3.211	0.040

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	45	100	0	20	70	0	-1
N.S.	1	1.00	2.81	6.25	0.00	1.25	4.38	0.00	-0.06
time (sec)	N/A	0.010	10.039	0.145	0.000	0.098	0.963	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	45	99	0	19	70	0	-1
N.S.	1	1.00	1.29	2.83	0.00	0.54	2.00	0.00	-0.03
time (sec)	N/A	0.020	10.025	0.132	0.000	0.091	0.959	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	74	107	0	0	61	0	-1
N.S.	1	1.00	1.72	2.49	0.00	0.00	1.42	0.00	-0.02
time (sec)	N/A	0.016	10.023	0.153	0.000	0.000	0.906	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	74	108	0	0	60	0	-1
N.S.	1	1.00	0.83	1.21	0.00	0.00	0.67	0.00	-0.01
time (sec)	N/A	0.030	10.030	0.151	0.000	0.000	0.915	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	47	120	0	0	66	0	-1
N.S.	1	1.00	0.53	1.35	0.00	0.00	0.74	0.00	-0.01
time (sec)	N/A	0.009	10.029	0.131	0.000	0.000	0.890	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	47	120	0	0	66	0	-1
N.S.	1	1.00	0.31	0.79	0.00	0.00	0.43	0.00	-0.01
time (sec)	N/A	0.020	10.028	0.141	0.000	0.000	0.883	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	122	0	0	70	0	-1
N.S.	1	1.00	0.84	1.36	0.00	0.00	0.78	0.00	-0.01
time (sec)	N/A	0.010	10.025	0.151	0.000	0.000	0.928	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	76	122	0	0	71	0	-1
N.S.	1	1.00	0.49	0.78	0.00	0.00	0.46	0.00	-0.01
time (sec)	N/A	0.023	10.026	0.144	0.000	0.000	0.889	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	15	0	31	0	0	-1
N.S.	1	1.00	1.00	1.50	0.00	3.10	0.00	0.00	-0.10
time (sec)	N/A	0.006	0.410	0.175	0.000	0.087	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	47	118	0	20	71	0	-1
N.S.	1	1.00	4.70	11.80	0.00	2.00	7.10	0.00	-0.10
time (sec)	N/A	0.011	10.024	0.149	0.000	0.085	0.932	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	30	0	0	-1
N.S.	1	1.00	1.04	1.22	0.00	1.30	0.00	0.00	-0.04
time (sec)	N/A	0.017	0.446	0.151	0.000	0.076	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	47	117	0	19	71	0	-1
N.S.	1	1.00	2.04	5.09	0.00	0.83	3.09	0.00	-0.04
time (sec)	N/A	0.021	10.021	0.140	0.000	0.080	0.953	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	181	71	0	174	122	1079	94
N.S.	1	1.00	2.21	0.87	0.00	2.12	1.49	13.16	1.15
time (sec)	N/A	0.072	0.071	0.039	0.000	0.337	0.290	5.329	4.433

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	181	71	0	174	122	173	98
N.S.	1	1.00	2.21	0.87	0.00	2.12	1.49	2.11	1.20
time (sec)	N/A	0.078	0.068	0.038	0.000	0.340	0.289	7.054	4.515

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	189	75	0	184	110	181	30
N.S.	1	1.00	2.42	0.96	0.00	2.36	1.41	2.32	0.38
time (sec)	N/A	0.067	0.065	0.036	0.000	0.343	0.295	6.752	0.128

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	189	75	0	184	121	181	88
N.S.	1	1.00	2.20	0.87	0.00	2.14	1.41	2.10	1.02
time (sec)	N/A	0.077	0.063	0.036	0.000	0.351	0.290	4.123	4.394

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	182	88	0	181	121	1077	99
N.S.	1	1.00	2.33	1.13	0.00	2.32	1.55	13.81	1.27
time (sec)	N/A	0.037	0.073	0.033	0.000	0.355	0.290	4.967	0.087

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	182	69	0	181	110	172	57
N.S.	1	1.00	2.33	0.88	0.00	2.32	1.41	2.21	0.73
time (sec)	N/A	0.032	0.074	0.033	0.000	0.340	0.298	4.627	4.436

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	181	112	184	29
N.S.	1	1.00	2.71	0.87	0.00	2.59	1.60	2.63	0.41
time (sec)	N/A	0.030	0.075	0.033	0.000	0.374	0.296	4.625	4.442

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	190	61	0	181	112	184	29
N.S.	1	1.00	2.71	0.87	0.00	2.59	1.60	2.63	0.41
time (sec)	N/A	0.032	0.075	0.033	0.000	0.334	0.301	4.177	0.110

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	250	291	0	251	158	250	129
N.S.	1	1.00	1.87	2.17	0.00	1.87	1.18	1.87	0.96
time (sec)	N/A	0.075	0.102	0.072	0.000	0.334	0.387	5.291	0.181

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	248	287	0	234	160	249	232
N.S.	1	1.00	1.91	2.21	0.00	1.80	1.23	1.92	1.78
time (sec)	N/A	0.112	0.074	0.044	0.000	0.362	0.384	4.828	4.522

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	248	287	0	234	160	249	232
N.S.	1	1.00	1.91	2.21	0.00	1.80	1.23	1.92	1.78
time (sec)	N/A	0.094	0.029	0.028	0.000	0.357	0.390	4.833	0.129

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	25	25	26	25	12
N.S.	1	1.00	1.00	0.90	0.86	0.86	0.90	0.86	0.41
time (sec)	N/A	0.017	0.013	0.029	0.278	0.350	0.216	3.356	4.410

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	138	52	0	164	117	51	55
N.S.	1	1.00	2.30	0.87	0.00	2.73	1.95	0.85	0.92
time (sec)	N/A	0.044	0.118	0.037	0.000	0.335	0.227	3.250	0.075

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	126	124	0	110	95	77	66
N.S.	1	1.00	2.03	2.00	0.00	1.77	1.53	1.24	1.06
time (sec)	N/A	0.039	0.036	0.044	0.000	0.327	0.187	3.948	4.385

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	134	124	0	120	83	77	24
N.S.	1	1.00	2.03	1.88	0.00	1.82	1.26	1.17	0.36
time (sec)	N/A	0.039	0.036	0.050	0.000	0.389	0.190	4.834	4.407

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	83	82	0	31	42	39	29
N.S.	1	1.00	1.84	1.82	0.00	0.69	0.93	0.87	0.64
time (sec)	N/A	0.042	0.047	0.052	0.000	0.353	0.039	3.864	0.087

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	12	11	19	22	11	19
N.S.	1	1.00	1.13	0.80	0.73	1.27	1.47	0.73	1.27
time (sec)	N/A	0.006	0.006	0.021	0.488	0.320	0.036	4.331	0.066

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	11	11	14	11	11
N.S.	1	1.00	1.00	0.86	0.79	0.79	1.00	0.79	0.79
time (sec)	N/A	0.004	0.003	0.010	0.528	0.354	0.029	3.252	0.026

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	97	34	33	33	44	33	29
N.S.	1	1.00	2.55	0.89	0.87	0.87	1.16	0.87	0.76
time (sec)	N/A	0.024	0.107	0.027	0.494	0.355	0.041	3.894	0.086

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	99	40	0	29	42	45	29
N.S.	1	1.00	2.06	0.83	0.00	0.60	0.88	0.94	0.60
time (sec)	N/A	0.025	0.057	0.038	0.000	0.331	0.040	3.553	4.391

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	97	40	0	33	44	52	29
N.S.	1	1.00	2.11	0.87	0.00	0.72	0.96	1.13	0.63
time (sec)	N/A	0.028	0.131	0.042	0.000	0.312	0.042	3.711	4.355

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	17	18	17	15	14	46	15
N.S.	1	1.00	0.81	0.86	0.81	0.71	0.67	2.19	0.71
time (sec)	N/A	0.009	0.004	0.155	0.533	0.313	0.029	3.943	4.288

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	101	40	0	31	42	52	29
N.S.	1	1.00	2.20	0.87	0.00	0.67	0.91	1.13	0.63
time (sec)	N/A	0.027	0.157	0.059	0.000	0.324	0.041	2.538	4.372

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	99	40	0	26	29	46	21
N.S.	1	1.00	2.25	0.91	0.00	0.59	0.66	1.05	0.48
time (sec)	N/A	0.023	0.060	0.043	0.000	0.313	0.037	4.024	0.057

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	14	20	0	15	12	42	15
N.S.	1	1.00	0.61	0.87	0.00	0.65	0.52	1.83	0.65
time (sec)	N/A	0.017	0.005	0.039	0.000	0.325	0.033	2.712	4.347

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	12	11	12	12	8	12	12
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.73	1.09	1.09
time (sec)	N/A	0.003	0.004	0.013	0.288	0.345	0.022	3.139	4.298

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	30	29	25	26	33	14
N.S.	1	1.00	0.74	0.77	0.74	0.64	0.67	0.85	0.36
time (sec)	N/A	0.011	0.004	0.020	0.280	0.361	0.035	4.037	0.297

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	82	0	47	46	77	20
N.S.	1	1.00	0.95	1.86	0.00	1.07	1.05	1.75	0.45
time (sec)	N/A	0.023	0.009	0.041	0.000	0.353	0.033	3.583	0.224

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	127	128	0	109	94	78	63
N.S.	1	1.00	1.92	1.94	0.00	1.65	1.42	1.18	0.95
time (sec)	N/A	0.019	0.041	0.046	0.000	0.332	0.186	4.596	0.068

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	84	82	0	28	39	39	30
N.S.	1	1.00	1.83	1.78	0.00	0.61	0.85	0.85	0.65
time (sec)	N/A	0.021	0.031	0.030	0.000	0.358	0.040	3.738	4.380

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	12	10	9	17	14	9	17
N.S.	1	1.00	1.33	1.11	1.00	1.89	1.56	1.00	1.89
time (sec)	N/A	0.006	0.005	0.020	0.514	0.331	0.035	3.119	4.363

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	11	11	7	11	11
N.S.	1	1.00	1.00	1.00	1.00	1.00	0.64	1.00	1.00
time (sec)	N/A	0.003	0.003	0.011	0.277	0.338	0.023	3.244	4.299

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	25	25	26	25	12
N.S.	1	1.00	1.00	0.90	0.86	0.86	0.90	0.86	0.41
time (sec)	N/A	0.010	0.004	0.016	0.283	0.336	0.033	4.009	0.063

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	34	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.68	0.40
time (sec)	N/A	0.015	0.008	0.024	0.000	0.320	0.033	2.724	4.369

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	43	46	41	20
N.S.	1	1.00	0.84	0.78	0.00	0.86	0.92	0.82	0.40
time (sec)	N/A	0.014	0.007	0.022	0.000	0.335	0.034	3.697	0.074

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	27	27	22	34	15
N.S.	1	1.00	1.00	0.90	0.87	0.87	0.71	1.10	0.48
time (sec)	N/A	0.010	0.003	0.142	0.287	0.323	0.031	2.582	0.068

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	41	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.82	0.40
time (sec)	N/A	0.015	0.009	0.031	0.000	0.351	0.034	3.837	4.350

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	40	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.80	0.40
time (sec)	N/A	0.018	0.009	0.022	0.000	0.365	0.034	4.409	0.068

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	42	39	0	45	46	41	20
N.S.	1	1.00	0.84	0.78	0.00	0.90	0.92	0.82	0.40
time (sec)	N/A	0.015	0.009	0.026	0.000	0.342	0.034	3.275	4.390

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	32	12	25	29	32	29	11
N.S.	1	1.00	2.29	0.86	1.79	2.07	2.29	2.07	0.79
time (sec)	N/A	0.004	0.005	0.010	0.503	0.322	0.029	3.169	4.328

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	30	29	27	29	33	15
N.S.	1	1.00	0.79	0.77	0.74	0.69	0.74	0.85	0.38
time (sec)	N/A	0.012	0.004	0.020	0.307	0.334	0.035	5.379	0.101

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	82	0	45	46	77	20
N.S.	1	1.00	0.88	1.71	0.00	0.94	0.96	1.60	0.42
time (sec)	N/A	0.027	0.012	0.030	0.000	0.320	0.035	3.367	0.127

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	124	124	0	101	88	0	73
N.S.	1	1.00	2.00	2.00	0.00	1.63	1.42	0.00	1.18
time (sec)	N/A	0.037	0.038	0.045	0.000	0.331	0.167	0.000	0.065

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	83	82	0	31	41	26	29
N.S.	1	1.00	1.69	1.67	0.00	0.63	0.84	0.53	0.59
time (sec)	N/A	0.060	0.081	0.053	0.000	0.346	0.037	3.876	0.083

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	81	70	0	31	41	26	29
N.S.	1	1.00	1.88	1.63	0.00	0.72	0.95	0.60	0.67
time (sec)	N/A	0.035	0.041	0.049	0.000	0.337	0.037	3.585	0.085

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	83	66	0	31	41	26	29
N.S.	1	1.00	1.69	1.35	0.00	0.63	0.84	0.53	0.59
time (sec)	N/A	0.042	0.047	0.044	0.000	0.392	0.038	3.696	4.391

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	2	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.001	0.002	0.010	0.506	0.328	0.025	4.261	4.332

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	99	34	33	31	41	26	29
N.S.	1	1.00	2.61	0.89	0.87	0.82	1.08	0.68	0.76
time (sec)	N/A	0.018	0.103	0.019	0.506	0.353	0.037	3.826	0.077

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	30	104	39	29	39	39	29
N.S.	1	1.00	0.86	2.97	1.11	0.83	1.11	1.11	0.83
time (sec)	N/A	0.012	0.009	0.152	0.497	0.361	0.034	4.334	4.368

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	12	20	0	7	7	30	7
N.S.	1	1.00	0.52	0.87	0.00	0.30	0.30	1.30	0.30
time (sec)	N/A	0.014	0.005	0.028	0.000	0.328	0.031	4.483	4.315

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	16	10	10	7	11	10
N.S.	1	1.00	0.91	1.45	0.91	0.91	0.64	1.00	0.91
time (sec)	N/A	0.002	0.003	0.012	0.282	0.344	0.020	3.608	4.341

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	29	22	21	21	19	43	12
N.S.	1	1.00	0.45	0.34	0.32	0.32	0.29	0.66	0.18
time (sec)	N/A	0.019	0.004	0.018	0.294	0.376	0.032	4.046	0.256

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	70	0	36	39	39	18
N.S.	1	1.00	0.93	1.63	0.00	0.84	0.91	0.91	0.42
time (sec)	N/A	0.023	0.007	0.039	0.000	0.346	0.032	3.081	4.395

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	82	0	39	39	39	18
N.S.	1	1.00	0.87	1.78	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.026	0.007	0.043	0.000	0.337	0.033	3.763	4.474

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	125	128	0	100	87	0	76
N.S.	1	1.00	2.02	2.06	0.00	1.61	1.40	0.00	1.23
time (sec)	N/A	0.018	0.040	0.033	0.000	0.348	0.173	0.000	4.338

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	87	82	0	31	42	26	31
N.S.	1	1.00	1.74	1.64	0.00	0.62	0.84	0.52	0.62
time (sec)	N/A	0.028	0.050	0.030	0.000	0.405	0.040	3.888	0.079

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	82	70	0	31	42	26	31
N.S.	1	1.00	1.86	1.59	0.00	0.70	0.95	0.59	0.70
time (sec)	N/A	0.019	0.029	0.033	0.000	0.328	0.038	3.250	0.079

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	10	66	0	13	10	26	13
N.S.	1	1.00	0.26	1.69	0.00	0.33	0.26	0.67	0.33
time (sec)	N/A	0.021	0.005	0.032	0.000	0.366	0.031	3.378	4.308

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	5	7	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.56	0.78	1.00
time (sec)	N/A	0.002	0.003	0.012	0.272	0.355	0.020	4.588	0.030

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	19	35	10
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.76	1.40	0.40
time (sec)	N/A	0.008	0.004	0.017	0.273	0.318	0.030	3.348	0.060

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	104	34	34	39	34	18
N.S.	1	1.00	0.87	2.26	0.74	0.74	0.85	0.74	0.39
time (sec)	N/A	0.012	0.007	0.148	0.501	0.321	0.029	3.279	0.060

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	0	39	39	39	18
N.S.	1	1.00	0.87	0.76	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.013	0.007	0.022	0.000	0.382	0.030	4.619	4.305

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	2
N.S.	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00
time (sec)	N/A	0.002	0.002	0.011	0.282	0.344	0.028	3.844	4.305

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	34	55	39	39	39	18
N.S.	1	1.00	1.05	0.89	1.45	1.03	1.03	1.03	0.47
time (sec)	N/A	0.019	0.007	0.017	0.530	0.400	0.033	3.807	0.112

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	40	70	0	39	39	39	18
N.S.	1	1.00	0.85	1.49	0.00	0.83	0.83	0.83	0.38
time (sec)	N/A	0.024	0.008	0.026	0.000	0.333	0.034	3.392	4.323

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	82	0	39	39	39	18
N.S.	1	1.00	0.87	1.78	0.00	0.85	0.85	0.85	0.39
time (sec)	N/A	0.024	0.007	0.025	0.000	0.376	0.033	3.109	4.388

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	99	34	33	33	46	33	29
N.S.	1	1.00	2.30	0.79	0.77	0.77	1.07	0.77	0.67
time (sec)	N/A	0.023	0.058	0.023	0.527	0.321	0.042	5.184	4.378

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	99	34	33	33	46	33	29
N.S.	1	1.00	2.30	0.79	0.77	0.77	1.07	0.77	0.67
time (sec)	N/A	0.022	0.017	0.018	0.506	0.351	0.045	5.402	0.002

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	23	34	22	25	17
N.S.	1	1.00	1.29	1.33	1.10	1.62	1.05	1.19	0.81
time (sec)	N/A	0.004	0.006	0.033	0.276	0.353	0.030	4.505	0.033

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	53	26	38	49	53	44	17
N.S.	1	1.00	1.89	0.93	1.36	1.75	1.89	1.57	0.61
time (sec)	N/A	0.008	0.011	0.024	0.565	0.344	0.311	6.420	4.385

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	72	45	54	58	474	60	290
N.S.	1	1.00	2.00	1.25	1.50	1.61	13.17	1.67	8.06
time (sec)	N/A	0.024	0.025	0.028	0.495	0.355	0.800	5.119	4.389

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	66	0	124	46	41	117
N.S.	1	1.00	0.99	0.89	0.00	1.68	0.62	0.55	1.58
time (sec)	N/A	0.030	0.032	0.035	0.000	0.453	0.076	7.920	0.108

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	97	78	69	69	740	69	827
N.S.	1	1.00	1.17	0.94	0.83	0.83	8.92	0.83	9.96
time (sec)	N/A	0.038	0.072	0.059	0.494	0.362	0.670	6.423	4.496

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	147	136	105	185	874	109	897
N.S.	1	1.00	1.24	1.14	0.88	1.55	7.34	0.92	7.54
time (sec)	N/A	0.060	0.146	0.074	0.513	0.356	0.989	7.113	4.495

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	111	371	0	4122	122	604	771
N.S.	1	1.00	0.47	1.59	0.00	17.62	0.52	2.58	3.29
time (sec)	N/A	0.161	0.065	0.086	0.000	0.577	0.702	6.973	4.490

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	165	605	0	5168	165	1112	1491
N.S.	1	1.00	0.52	1.91	0.00	16.35	0.52	3.52	4.72
time (sec)	N/A	0.205	0.124	0.212	0.000	0.764	0.953	7.941	4.499

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	53	145	0	251	0	122	121
N.S.	1	1.00	0.33	0.91	0.00	1.57	0.00	0.76	0.76
time (sec)	N/A	0.099	0.029	0.108	0.000	0.498	0.000	6.583	4.956

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	53	149	0	255	0	126	121
N.S.	1	1.00	0.31	0.87	0.00	1.48	0.00	0.73	0.70
time (sec)	N/A	0.092	0.022	0.104	0.000	0.394	0.000	6.657	4.953

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	137	136	0	517	1469	1501	1227
N.S.	1	1.00	0.86	0.85	0.00	3.23	9.18	9.38	7.67
time (sec)	N/A	0.085	0.047	0.067	0.000	0.376	1.410	5.603	1.067

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	136	132	0	513	1467	1501	1227
N.S.	1	1.00	0.85	0.82	0.00	3.21	9.17	9.38	7.67
time (sec)	N/A	0.070	0.033	0.056	0.000	0.384	1.436	5.181	5.246

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	115	90	0	512	27	0	133
N.S.	1	1.00	1.01	0.79	0.00	4.49	0.24	0.00	1.17
time (sec)	N/A	0.049	0.097	0.039	0.000	0.388	0.102	0.000	4.484

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	115	94	0	251	0	0	159
N.S.	1	1.00	0.94	0.77	0.00	2.06	0.00	0.00	1.30
time (sec)	N/A	0.051	0.084	0.064	0.000	0.339	0.000	0.000	5.060

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	115	87	88	264	143	92	133
N.S.	1	1.00	0.93	0.70	0.71	2.13	1.15	0.74	1.07
time (sec)	N/A	0.050	0.072	0.062	0.510	0.369	0.117	4.237	0.237

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	130	186	0	5405	172	0	1007
N.S.	1	1.00	0.96	1.37	0.00	39.74	1.26	0.00	7.40
time (sec)	N/A	0.079	0.090	0.035	0.000	2.013	1.027	0.000	4.587

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	138	190	0	1141	0	0	1155
N.S.	1	1.00	0.86	1.19	0.00	7.13	0.00	0.00	7.22
time (sec)	N/A	0.078	0.078	0.062	0.000	0.454	0.000	0.000	4.986

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	247	347	0	1457	0	0	2500
N.S.	1	1.00	0.60	0.84	0.00	3.52	0.00	0.00	6.04
time (sec)	N/A	0.305	0.126	0.079	0.000	0.643	0.000	0.000	5.219

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-2)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	163	279	0	1469	0	0	1575
N.S.	1	1.00	0.70	1.19	0.00	6.28	0.00	0.00	6.73
time (sec)	N/A	0.115	0.109	0.177	0.000	0.986	0.000	0.000	5.290

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	103	200	0	16	0	0	-1
N.S.	1	1.00	1.07	2.08	0.00	0.17	0.00	0.00	-0.01
time (sec)	N/A	0.087	10.132	0.081	0.000	0.096	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	19	113	0	18	0	0	-1
N.S.	1	1.00	0.76	4.52	0.00	0.72	0.00	0.00	-0.04
time (sec)	N/A	0.021	10.051	0.031	0.000	0.075	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	103	204	0	18	0	0	-1
N.S.	1	1.00	1.07	2.12	0.00	0.19	0.00	0.00	-0.01
time (sec)	N/A	0.119	10.092	0.079	0.000	0.075	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	107	204	0	18	0	0	-1
N.S.	1	1.00	1.16	2.22	0.00	0.20	0.00	0.00	-0.01
time (sec)	N/A	0.085	10.082	0.095	0.000	0.087	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	35	95	0	18	0	0	-1
N.S.	1	1.00	1.30	3.52	0.00	0.67	0.00	0.00	-0.04
time (sec)	N/A	0.025	10.064	0.037	0.000	0.080	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	107	204	0	18	0	0	-1
N.S.	1	1.00	1.16	2.22	0.00	0.20	0.00	0.00	-0.01
time (sec)	N/A	0.112	10.098	0.076	0.000	0.097	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	187	515	0	0	0	0	-1
N.S.	1	1.00	0.63	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.074	10.179	0.079	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	97	90	98	110	94	95
N.S.	1	1.00	1.00	0.92	0.85	0.92	1.04	0.89	0.90
time (sec)	N/A	0.054	0.014	0.127	0.288	0.329	0.015	3.782	4.350

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	72	69	75	78	71	71
N.S.	1	1.00	1.00	0.91	0.87	0.95	0.99	0.90	0.90
time (sec)	N/A	0.036	0.010	0.151	0.278	0.313	0.013	4.198	0.030

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	49	48	52	56	50	49
N.S.	1	1.00	1.00	0.88	0.86	0.93	1.00	0.89	0.88
time (sec)	N/A	0.020	0.007	0.156	0.278	0.386	0.009	5.041	0.024

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	28	29	29	28	26
N.S.	1	1.00	1.00	0.84	0.88	0.91	0.91	0.88	0.81
time (sec)	N/A	0.009	0.001	0.041	0.280	0.328	0.007	4.914	0.042

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	47	42	128	104	44	45
N.S.	1	1.00	1.00	0.85	0.76	2.33	1.89	0.80	0.82
time (sec)	N/A	0.022	0.024	0.182	0.500	0.341	0.155	3.226	0.069

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	70	62	215	138	62	68
N.S.	1	1.00	1.05	0.95	0.84	2.91	1.86	0.84	0.92
time (sec)	N/A	0.035	0.035	0.118	0.491	0.370	0.246	3.490	4.442

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	92	92	90	295	219	77	97
N.S.	1	1.00	0.99	0.99	0.97	3.17	2.35	0.83	1.04
time (sec)	N/A	0.046	0.042	0.169	0.495	0.325	0.371	3.961	4.481

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	113	113	122	407	204	100	129
N.S.	1	1.00	0.92	0.92	0.99	3.31	1.66	0.81	1.05
time (sec)	N/A	0.073	0.054	0.153	0.515	0.356	0.470	3.160	4.483

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	133	130	126	126	144	128	127
N.S.	1	1.00	1.00	0.98	0.95	0.95	1.08	0.96	0.95
time (sec)	N/A	0.070	0.013	0.161	0.287	0.380	0.017	3.302	0.058

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	97	90	89	89	104	91	89
N.S.	1	1.00	1.00	0.93	0.92	0.92	1.07	0.94	0.92
time (sec)	N/A	0.044	0.010	0.120	0.278	0.338	0.015	5.138	0.048

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	53	52	60	53	50
N.S.	1	1.00	1.00	0.85	0.88	0.87	1.00	0.88	0.83
time (sec)	N/A	0.018	0.002	0.148	0.283	0.330	0.010	4.973	0.026

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.006	0.001	0.135	0.277	0.313	0.007	6.379	0.028

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	97	104	103	257	236	105	141
N.S.	1	1.00	0.90	0.96	0.95	2.38	2.19	0.97	1.31
time (sec)	N/A	0.046	0.049	0.176	0.503	0.344	0.251	5.486	4.394

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	134	129	127	385	314	128	183
N.S.	1	1.00	1.02	0.98	0.97	2.94	2.40	0.98	1.40
time (sec)	N/A	0.122	0.072	0.160	0.506	0.352	0.456	5.331	4.399

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	154	152	152	509	257	145	164
N.S.	1	1.00	0.99	0.98	0.98	3.28	1.66	0.94	1.06
time (sec)	N/A	0.166	0.070	0.170	0.522	0.347	0.829	4.773	4.410

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	174	179	185	661	292	167	199
N.S.	1	1.00	0.95	0.97	1.01	3.59	1.59	0.91	1.08
time (sec)	N/A	0.193	0.088	0.190	0.507	0.368	1.894	4.618	4.486

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	200	212	221	804	335	198	240
N.S.	1	1.00	0.90	0.95	0.99	3.61	1.50	0.89	1.08
time (sec)	N/A	0.220	0.121	0.164	0.516	0.371	13.658	5.293	4.492

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	444	291	420	2728	500	498	2500
N.S.	1	1.00	1.02	0.67	0.96	6.24	1.14	1.14	5.72
time (sec)	N/A	0.290	0.218	0.156	0.513	1.614	8.938	7.946	5.081

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	360	254	336	2023	350	405	2712
N.S.	1	1.00	0.97	0.69	0.91	5.47	0.95	1.09	7.33
time (sec)	N/A	0.309	0.171	0.151	0.504	0.783	1.527	7.951	4.877

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	269	228	287	1415	238	318	1479
N.S.	1	1.00	0.91	0.77	0.97	4.76	0.80	1.07	4.98
time (sec)	N/A	0.188	0.164	0.120	0.511	0.407	0.789	8.576	4.794

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	183	206	225	739	109	245	599
N.S.	1	1.00	0.74	0.83	0.91	2.99	0.44	0.99	2.43
time (sec)	N/A	0.101	0.032	0.127	0.499	0.354	0.346	7.187	4.682

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	134	102	169	121	20	179	33
N.S.	1	1.00	0.72	0.55	0.91	0.65	0.11	0.97	0.18
time (sec)	N/A	0.072	0.012	0.131	0.512	0.338	0.060	5.944	4.409

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	234	253	266	3842	0	339	2500
N.S.	1	1.00	0.70	0.75	0.79	11.43	0.00	1.01	7.44
time (sec)	N/A	0.182	0.097	0.186	0.534	0.760	0.000	4.687	5.706

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	362	307	394	7933	0	517	2500
N.S.	1	1.00	0.80	0.68	0.87	17.51	0.00	1.14	5.52
time (sec)	N/A	0.248	0.297	0.189	0.522	9.592	0.000	5.614	6.548

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	371	284	294	2018	352	425	2560
N.S.	1	1.00	1.02	0.78	0.81	5.56	0.97	1.17	7.05
time (sec)	N/A	0.257	0.164	0.125	0.523	0.377	1.812	9.010	4.940

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	295	266	324	1531	275	350	1565
N.S.	1	1.00	0.85	0.76	0.93	4.39	0.79	1.00	4.48
time (sec)	N/A	0.197	0.108	0.156	0.521	0.387	1.070	4.302	4.786

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	267	245	258	842	136	273	637
N.S.	1	1.00	0.97	0.89	0.94	3.06	0.49	0.99	2.32
time (sec)	N/A	0.130	0.176	0.149	0.506	0.343	0.488	3.510	0.396

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	183	118	189	173	39	194	58
N.S.	1	1.00	0.91	0.58	0.94	0.86	0.19	0.96	0.29
time (sec)	N/A	0.086	0.066	0.142	0.511	0.336	0.134	3.969	0.084

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	429	334	489	9294	0	603	2500
N.S.	1	1.00	0.62	0.48	0.71	13.49	0.00	0.88	3.63
time (sec)	N/A	0.386	0.189	0.202	0.517	11.869	0.000	3.932	6.781

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	864	864	540	402	705	14534	0	855	2500
N.S.	1	1.00	0.62	0.47	0.82	16.82	0.00	0.99	2.89
time (sec)	N/A	0.585	0.366	0.252	0.542	97.755	0.000	3.295	8.330

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	386	203	506	0	0	214	0	-1
N.S.	1	0.99	0.52	1.30	0.00	0.00	0.55	0.00	-0.00
time (sec)	N/A	0.268	10.141	0.125	0.000	0.000	2.609	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	140	388	0	0	173	0	-1
N.S.	1	1.00	0.43	1.19	0.00	0.00	0.53	0.00	-0.00
time (sec)	N/A	0.182	10.106	0.145	0.000	0.000	2.040	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	120	266	0	0	124	0	-1
N.S.	1	1.00	0.45	1.01	0.00	0.00	0.47	0.00	-0.00
time (sec)	N/A	0.086	10.058	0.121	0.000	0.000	1.589	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	77	169	0	0	78	0	-1
N.S.	1	1.00	0.34	0.75	0.00	0.00	0.35	0.00	-0.00
time (sec)	N/A	0.045	10.038	0.109	0.000	0.000	0.945	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	95	107	0	0	0	0	-1
N.S.	1	1.00	0.28	0.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.186	10.109	0.158	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	522	556	0	0	0	0	-1
N.S.	1	1.00	0.90	0.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.480	10.380	0.147	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	729	729	332	1018	0	0	0	0	-1
N.S.	1	1.00	0.46	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.780	10.592	0.123	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	141	360	0	0	180	0	-1
N.S.	1	1.00	0.66	1.69	0.00	0.00	0.85	0.00	-0.00
time (sec)	N/A	0.169	10.110	0.133	0.000	0.000	2.211	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	121	246	0	0	129	0	-1
N.S.	1	1.00	0.75	1.52	0.00	0.00	0.80	0.00	-0.01
time (sec)	N/A	0.089	10.061	0.129	0.000	0.000	1.700	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	77	154	0	0	82	0	-1
N.S.	1	1.00	0.62	1.24	0.00	0.00	0.66	0.00	-0.01
time (sec)	N/A	0.052	10.038	0.108	0.000	0.000	0.989	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	91	97	0	0	0	0	-1
N.S.	1	1.00	1.26	1.35	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	10.099	0.159	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	508	523	0	0	0	0	-1
N.S.	1	1.00	1.70	1.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	10.515	0.122	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	321	961	0	0	0	0	-1
N.S.	1	1.00	0.76	2.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.460	10.670	0.119	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	458	1420	0	0	0	0	-1
N.S.	1	1.00	0.81	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.726	11.046	0.132	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	78	160	0	0	73	0	-1
N.S.	1	1.00	0.62	1.27	0.00	0.00	0.58	0.00	-0.01
time (sec)	N/A	0.051	10.031	0.116	0.000	0.000	1.053	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	92	99	0	0	0	0	-1
N.S.	1	1.00	1.26	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.024	10.094	0.128	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	86	158	0	0	70	0	-1
N.S.	1	1.00	1.59	2.93	0.00	0.00	1.30	0.00	-0.02
time (sec)	N/A	0.029	10.036	0.135	0.000	0.000	1.011	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	85	165	0	0	76	0	-1
N.S.	1	1.00	1.63	3.17	0.00	0.00	1.46	0.00	-0.02
time (sec)	N/A	0.031	10.033	0.126	0.000	0.000	1.008	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	80	175	0	0	83	0	-1
N.S.	1	1.00	0.34	0.74	0.00	0.00	0.35	0.00	-0.00
time (sec)	N/A	0.044	10.035	0.109	0.000	0.000	0.963	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	98	110	0	0	0	0	-1
N.S.	1	1.00	0.28	0.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	10.088	0.132	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	79	0	0	0	0	-1
N.S.	1	1.00	1.00	1.98	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	10.096	0.139	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	50	86	0	0	0	0	-1
N.S.	1	1.00	0.16	0.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	10.090	0.122	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	59	78	0	0	0	0	-1
N.S.	1	1.00	1.48	1.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	10.094	0.120	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	65	86	0	0	0	0	-1
N.S.	1	1.00	0.22	0.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.141	10.073	0.122	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.006	1.358	0.043	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.005	0.069	0.049	0.000	0.000	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	196	136	0	0	0	167	0	-1
N.S.	1	0.96	0.67	0.00	0.00	0.00	0.82	0.00	-0.00
time (sec)	N/A	0.151	0.588	0.041	0.000	0.000	65.260	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	142	106	0	0	0	119	0	-1
N.S.	1	0.95	0.71	0.00	0.00	0.00	0.79	0.00	-0.01
time (sec)	N/A	0.087	0.549	0.032	0.000	0.000	36.212	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	75	0	0	0	75	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.78	0.00	-0.01
time (sec)	N/A	0.032	0.443	0.020	0.000	0.000	18.209	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	34	0	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.77	0.00	0.93
time (sec)	N/A	0.007	0.060	0.018	0.000	0.000	3.825	0.000	4.364

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.613	0.038	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.581	0.044	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	103	86	75	0	0	129	0	-1
N.S.	1	0.95	0.80	0.69	0.00	0.00	1.19	0.00	-0.01
time (sec)	N/A	0.075	0.863	0.143	0.000	0.000	55.546	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	79	65	56	0	0	94	0	-1
N.S.	1	0.92	0.76	0.65	0.00	0.00	1.09	0.00	-0.01
time (sec)	N/A	0.045	0.782	0.132	0.000	0.000	30.554	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	0	0	61	0	-1
N.S.	1	1.00	1.00	0.88	0.00	0.00	1.45	0.00	-0.02
time (sec)	N/A	0.013	0.546	0.112	0.000	0.000	15.344	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	0	0	29	0	15
N.S.	1	1.00	1.00	0.94	0.00	0.00	1.61	0.00	0.83
time (sec)	N/A	0.003	0.037	0.116	0.000	0.000	3.232	0.000	0.070

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.783	0.037	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.988	0.043	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	1.117	0.053	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	42	56	109	75	53	42
N.S.	1	1.00	1.00	0.82	1.10	2.14	1.47	1.04	0.82
time (sec)	N/A	0.028	0.015	0.128	0.505	0.362	0.085	3.323	0.091

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	46	87	58	42	28
N.S.	1	1.00	1.00	0.82	1.21	2.29	1.53	1.11	0.74
time (sec)	N/A	0.021	0.012	0.123	0.523	0.366	0.077	4.195	0.055

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	38	69	34	26	21
N.S.	1	1.00	1.00	0.76	1.31	2.38	1.17	0.90	0.72
time (sec)	N/A	0.016	0.006	0.128	0.521	0.351	0.057	4.684	4.432

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	34	65	46	21	16
N.S.	1	1.00	1.00	0.67	1.42	2.71	1.92	0.88	0.67
time (sec)	N/A	0.007	0.003	0.116	0.518	0.330	0.052	4.042	0.058

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	65	54	68	200	226	56	74
N.S.	1	1.00	0.90	0.75	0.94	2.78	3.14	0.78	1.03
time (sec)	N/A	0.034	0.023	0.155	0.525	0.356	0.199	3.031	0.159

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	76	64	89	281	257	67	96
N.S.	1	1.00	0.85	0.72	1.00	3.16	2.89	0.75	1.08
time (sec)	N/A	0.052	0.039	0.152	0.524	0.375	0.273	2.629	0.163

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	70	1356	0	113	0	107	-1
N.S.	1	1.00	1.13	21.87	0.00	1.82	0.00	1.73	-0.02
time (sec)	N/A	0.029	0.067	0.244	0.000	0.356	0.000	2.838	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	50	818	0	79	0	81	-1
N.S.	1	1.00	1.32	21.53	0.00	2.08	0.00	2.13	-0.03
time (sec)	N/A	0.017	0.044	0.252	0.000	0.331	0.000	3.522	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	69	441	0	119	0	101	-1
N.S.	1	1.00	1.13	7.23	0.00	1.95	0.00	1.66	-0.02
time (sec)	N/A	0.025	0.090	0.244	0.000	0.342	0.000	3.210	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	865	0	151	0	114	-1
N.S.	1	1.00	1.00	10.81	0.00	1.89	0.00	1.42	-0.01
time (sec)	N/A	0.046	0.148	0.206	0.000	0.354	0.000	3.883	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	98	96	0	251	0	0	-1
N.S.	1	1.00	0.64	0.63	0.00	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.035	2.695	0.142	0.000	0.350	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	75	0	223	0	0	-1
N.S.	1	1.00	0.78	0.68	0.00	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.023	2.255	0.132	0.000	0.341	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	54	0	121	0	0	-1
N.S.	1	1.00	0.77	0.83	0.00	1.86	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.956	0.131	0.000	0.372	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	249	0	152	0	0	-1
N.S.	1	1.00	1.00	3.19	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.022	1.372	0.268	0.000	0.370	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	111	488	0	297	0	0	-1
N.S.	1	1.00	0.89	3.90	0.00	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.032	2.343	0.269	0.000	0.343	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	123	711	0	365	0	0	-1
N.S.	1	1.00	0.73	4.23	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.055	2.651	0.282	0.000	0.353	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	123	94	0	265	0	0	-1
N.S.	1	1.00	0.81	0.62	0.00	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.035	2.752	0.134	0.000	0.343	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	110	74	0	236	0	0	-1
N.S.	1	1.00	1.01	0.68	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.025	2.275	0.132	0.000	0.343	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	67	54	0	125	0	0	-1
N.S.	1	1.00	1.05	0.84	0.00	1.95	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.959	0.163	0.000	0.353	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	256	0	155	0	0	-1
N.S.	1	1.00	1.00	3.32	0.00	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.022	1.363	0.250	0.000	0.364	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	110	499	0	302	0	0	-1
N.S.	1	1.00	0.89	4.02	0.00	2.44	0.00	0.00	-0.01
time (sec)	N/A	0.032	2.376	0.265	0.000	0.352	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	122	728	0	376	0	0	-1
N.S.	1	1.00	0.73	4.36	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.055	2.651	0.276	0.000	0.342	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	38	25	0	73	0	0	-1
N.S.	1	1.00	1.27	0.83	0.00	2.43	0.00	0.00	-0.03
time (sec)	N/A	0.007	0.479	0.135	0.000	0.350	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	40	34	33	0	65	0	0	-1
N.S.	1	1.67	1.42	1.38	0.00	2.71	0.00	0.00	-0.04
time (sec)	N/A	0.008	0.478	0.138	0.000	0.355	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	72	71	59	0	137	0	0	-1
N.S.	1	0.99	0.97	0.81	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.076	3.542	0.142	0.000	0.366	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	121	134	0	440	345	151	182
N.S.	1	1.00	1.00	1.11	0.00	3.64	2.85	1.25	1.50
time (sec)	N/A	0.116	0.050	0.127	0.000	0.343	0.501	3.589	4.532

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	84	81	0	312	275	98	113
N.S.	1	1.00	0.98	0.94	0.00	3.63	3.20	1.14	1.31
time (sec)	N/A	0.074	0.031	0.151	0.000	0.348	0.372	2.724	4.522

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	51	0	215	212	58	52
N.S.	1	1.00	0.98	0.80	0.00	3.36	3.31	0.91	0.81
time (sec)	N/A	0.054	0.035	0.127	0.000	0.380	0.252	3.954	0.069

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	33	0	139	124	39	38
N.S.	1	1.00	0.98	0.67	0.00	2.84	2.53	0.80	0.78
time (sec)	N/A	0.021	0.008	0.028	0.000	0.339	0.100	3.207	4.491

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	133	109	0	919	0	147	2500
N.S.	1	1.00	0.98	0.80	0.00	6.76	0.00	1.08	18.38
time (sec)	N/A	0.122	0.130	0.176	0.000	0.465	0.000	3.195	5.403

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	177	174	0	1799	0	235	2500
N.S.	1	1.00	0.95	0.93	0.00	9.62	0.00	1.26	13.37
time (sec)	N/A	0.184	0.264	0.205	0.000	0.934	0.000	3.358	6.453

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	172	3792	0	564	0	167	-1
N.S.	1	1.00	1.24	27.28	0.00	4.06	0.00	1.20	-0.01
time (sec)	N/A	0.207	0.385	0.346	0.000	0.662	0.000	3.617	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	139	2436	0	489	0	129	-1
N.S.	1	1.00	1.29	22.56	0.00	4.53	0.00	1.19	-0.01
time (sec)	N/A	0.100	0.186	0.322	0.000	0.409	0.000	3.528	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	94	1425	0	437	0	88	-1
N.S.	1	1.00	1.24	18.75	0.00	5.75	0.00	1.16	-0.01
time (sec)	N/A	0.056	0.115	0.347	0.000	0.383	0.000	4.407	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	172	771	0	717	0	132	-1
N.S.	1	1.00	1.62	7.27	0.00	6.76	0.00	1.25	-0.01
time (sec)	N/A	0.093	0.258	0.327	0.000	0.459	0.000	3.851	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	174	1551	0	1085	0	331	-1
N.S.	1	1.00	1.17	10.41	0.00	7.28	0.00	2.22	-0.01
time (sec)	N/A	0.212	0.432	0.288	0.000	0.629	0.000	3.809	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	169	263	0	0	0	0	-1
N.S.	1	1.00	0.92	1.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	4.954	0.151	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	162	248	0	0	0	0	-1
N.S.	1	1.00	0.99	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	4.515	0.120	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	168	233	0	0	0	0	-1
N.S.	1	1.00	1.16	1.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	4.244	0.037	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	117	293	0	0	0	0	-1
N.S.	1	1.00	0.85	2.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	9.091	0.158	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	164	224	0	0	0	0	-1
N.S.	1	1.00	3.35	4.57	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.007	10.205	0.116	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	176	333	0	0	0	0	-1
N.S.	1	1.00	1.89	3.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.309	10.157	0.151	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	240	438	0	0	0	0	-1
N.S.	1	1.00	1.45	2.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.381	10.204	0.128	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	157	233	0	0	0	0	-1
N.S.	1	1.00	0.99	1.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	10.089	0.122	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	143	218	0	0	0	0	-1
N.S.	1	1.00	1.04	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	10.098	0.120	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	94	205	0	0	0	0	-1
N.S.	1	1.00	0.82	1.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.070	0.028	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	72	104	0	54	0	0	-1
N.S.	1	1.00	1.04	1.51	0.00	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.035	10.046	0.177	0.000	0.105	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	226	397	0	0	0	0	-1
N.S.	1	1.00	1.92	3.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	10.221	0.124	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	235	418	0	0	0	0	-1
N.S.	1	1.00	1.65	2.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	10.175	0.132	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	136	268	0	0	0	0	-1
N.S.	1	1.00	0.94	1.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	10.115	0.120	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	158	268	0	0	0	0	-1
N.S.	1	1.00	1.61	2.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	10.097	0.126	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	160	247	0	0	0	0	-1
N.S.	1	1.00	1.67	2.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.014	10.044	0.041	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	204	398	0	0	0	0	-1
N.S.	1	1.00	1.23	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	10.123	0.144	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	168	419	0	0	0	0	-1
N.S.	1	1.00	1.51	3.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	10.217	0.148	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	192	439	0	0	0	0	-1
N.S.	1	1.00	1.01	2.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.366	10.178	0.131	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	135	136	129	138	156	142	131
N.S.	1	1.00	1.00	1.01	0.96	1.02	1.16	1.05	0.97
time (sec)	N/A	0.086	0.025	0.148	0.286	0.335	0.018	5.176	0.063

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	103	99	106	112	108	101
N.S.	1	1.00	1.01	1.00	0.96	1.03	1.09	1.05	0.98
time (sec)	N/A	0.062	0.018	0.141	0.280	0.339	0.015	3.939	4.628

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	74	78	76	70
N.S.	1	1.00	1.00	0.96	0.95	1.01	1.07	1.04	0.96
time (sec)	N/A	0.038	0.013	0.136	0.289	0.336	0.013	3.776	4.587

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	39	42	39	43	38
N.S.	1	1.00	1.00	0.88	0.93	1.00	0.93	1.02	0.90
time (sec)	N/A	0.017	0.006	0.046	0.283	0.351	0.007	3.873	0.044

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	57	55	155	117	56	57
N.S.	1	1.00	0.98	0.86	0.83	2.35	1.77	0.85	0.86
time (sec)	N/A	0.028	0.034	0.127	0.488	0.345	0.218	4.062	0.085

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	88	79	74	266	153	75	77
N.S.	1	1.05	1.06	0.95	0.89	3.20	1.84	0.90	0.93
time (sec)	N/A	0.058	0.038	0.125	0.508	0.341	0.421	4.030	4.670

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	119	110	107	110	389	196	101	112
N.S.	1	1.03	0.96	0.93	0.96	3.38	1.70	0.88	0.97
time (sec)	N/A	0.073	0.066	0.136	0.527	0.348	0.785	4.742	4.847

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	154	142	130	147	535	241	134	144
N.S.	1	1.03	0.95	0.87	0.98	3.57	1.61	0.89	0.96
time (sec)	N/A	0.124	0.084	0.132	0.518	0.346	1.432	4.746	4.509

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	223	219	220	220	272	255	220
N.S.	1	1.00	1.00	0.98	0.99	0.99	1.22	1.14	0.99
time (sec)	N/A	0.133	0.054	0.140	0.298	0.341	0.025	3.830	4.484

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	156	155	151	157	192	181	148
N.S.	1	1.00	1.01	1.00	0.97	1.01	1.24	1.17	0.95
time (sec)	N/A	0.096	0.033	0.139	0.286	0.319	0.020	4.637	4.516

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	91	96	94	107	106	90
N.S.	1	1.00	1.00	0.95	1.00	0.98	1.11	1.10	0.94
time (sec)	N/A	0.045	0.015	0.084	0.313	0.373	0.015	4.189	0.038

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	45	41	48	43	42
N.S.	1	1.00	1.00	0.86	0.92	0.84	0.98	0.88	0.86
time (sec)	N/A	0.016	0.004	0.009	0.286	0.323	0.009	4.003	0.022

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	144	178	171	393	371	185	229
N.S.	1	1.00	1.01	1.24	1.20	2.75	2.59	1.29	1.60
time (sec)	N/A	0.085	0.043	0.146	0.512	0.353	0.542	3.163	4.469

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	183	213	197	595	484	207	293
N.S.	1	1.00	1.10	1.28	1.19	3.58	2.92	1.25	1.77
time (sec)	N/A	0.192	0.068	0.157	0.507	0.358	1.422	3.301	4.563

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	217	237	238	792	398	244	257
N.S.	1	1.00	1.08	1.18	1.18	3.94	1.98	1.21	1.28
time (sec)	N/A	0.264	0.071	0.153	0.512	0.349	10.482	4.732	0.118

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	267	285	288	1021	0	296	308
N.S.	1	1.00	1.07	1.14	1.15	4.08	0.00	1.18	1.23
time (sec)	N/A	0.337	0.095	0.138	0.528	0.350	0.000	4.422	4.599

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	345	347	352	1267	0	364	375
N.S.	1	1.00	1.09	1.09	1.11	4.00	0.00	1.15	1.18
time (sec)	N/A	0.398	0.142	0.147	0.519	0.357	0.000	3.522	4.574

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	88	79	74	266	153	75	77
N.S.	1	1.05	1.06	0.95	0.89	3.20	1.84	0.90	0.93
time (sec)	N/A	0.057	0.012	0.101	0.508	0.356	0.425	3.674	0.002

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	88	79	74	266	153	75	77
N.S.	1	1.00	1.06	0.95	0.89	3.20	1.84	0.90	0.93
time (sec)	N/A	0.056	0.011	0.128	0.503	0.370	0.428	4.030	0.115

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	570	621	0	15935	0	9285	2500
N.S.	1	1.00	1.24	1.35	0.00	34.72	0.00	20.23	5.45
time (sec)	N/A	1.002	0.436	0.125	0.000	31.228	0.000	6.679	9.313

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	402	402	0	9383	0	6407	2500
N.S.	1	1.00	1.27	1.27	0.00	29.69	0.00	20.28	7.91
time (sec)	N/A	0.504	0.354	0.148	0.000	6.083	0.000	4.714	7.290

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	269	249	0	4573	0	4107	2500
N.S.	1	1.00	1.13	1.05	0.00	19.21	0.00	17.26	10.50
time (sec)	N/A	0.424	0.206	0.112	0.000	1.105	0.000	4.243	6.484

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	172	164	0	1505	0	1402	2500
N.S.	1	1.00	0.99	0.94	0.00	8.65	0.00	8.06	14.37
time (sec)	N/A	0.147	0.092	0.035	0.000	0.505	0.000	5.222	5.382

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	117	0	613	87	1024	763
N.S.	1	1.00	0.86	0.78	0.00	4.09	0.58	6.83	5.09
time (sec)	N/A	0.073	0.055	0.018	0.000	0.468	0.652	4.578	0.514

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	274	215	0	15733	0	7650	2500
N.S.	1	1.00	1.08	0.85	0.00	61.94	0.00	30.12	9.84
time (sec)	N/A	0.403	0.180	0.199	0.000	12.981	0.000	6.115	9.446

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	354	345	0	0	0	2357	2500
N.S.	1	1.00	0.83	0.80	0.00	0.00	0.00	5.49	5.83
time (sec)	N/A	0.971	0.487	0.285	0.000	0.000	0.000	5.372	10.280

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	540	628	0	11934	0	8983	2500
N.S.	1	1.00	0.96	1.12	0.00	21.20	0.00	15.96	4.44
time (sec)	N/A	2.333	0.999	0.131	0.000	15.075	0.000	5.558	8.793

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	415	427	0	7250	0	6390	2500
N.S.	1	1.00	1.08	1.11	0.00	18.78	0.00	16.55	6.48
time (sec)	N/A	1.386	0.691	0.122	0.000	2.774	0.000	5.495	9.845

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	310	464	0	4539	0	4433	2500
N.S.	1	1.00	1.06	1.58	0.00	15.49	0.00	15.13	8.53
time (sec)	N/A	0.535	0.473	0.092	0.000	0.918	0.000	5.196	9.387

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	243	320	0	2309	0	2682	2500
N.S.	1	1.00	0.96	1.27	0.00	9.16	0.00	10.64	9.92
time (sec)	N/A	0.352	0.271	0.058	0.000	0.444	0.000	4.630	6.257

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	660	660	708	850	0	0	0	36949	2500
N.S.	1	1.00	1.07	1.29	0.00	0.00	0.00	55.98	3.79
time (sec)	N/A	1.867	1.737	0.421	0.000	0.000	0.000	12.936	16.455

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1077	1077	1020	1250	0	0	0	64316	2500
N.S.	1	1.00	0.95	1.16	0.00	0.00	0.00	59.72	2.32
time (sec)	N/A	8.621	3.556	0.641	0.000	0.000	0.000	16.315	17.810

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	181	277	258	180	505	180	-1
N.S.	1	1.00	0.84	1.29	1.20	0.84	2.35	0.84	-0.00
time (sec)	N/A	0.100	0.312	0.118	0.302	0.514	170.574	3.013	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	148	229	203	148	413	145	-1
N.S.	1	1.00	0.85	1.31	1.16	0.85	2.36	0.83	-0.01
time (sec)	N/A	0.076	0.222	0.112	0.297	0.382	34.018	4.077	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	112	181	148	115	272	106	-1
N.S.	1	1.00	0.85	1.37	1.12	0.87	2.06	0.80	-0.01
time (sec)	N/A	0.070	0.148	0.118	0.324	0.502	8.209	2.990	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	127	94	87	230	79	-1
N.S.	1	1.00	0.88	1.31	0.97	0.90	2.37	0.81	-0.01
time (sec)	N/A	0.039	0.089	0.123	0.291	0.371	3.967	3.663	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	87	117	94	128	134	80	-1
N.S.	1	1.04	0.98	1.31	1.06	1.44	1.51	0.90	-0.01
time (sec)	N/A	0.045	0.141	0.129	0.298	0.433	4.620	3.396	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	92	150	135	146	450	88	-1
N.S.	1	1.04	0.91	1.49	1.34	1.45	4.46	0.87	-0.01
time (sec)	N/A	0.049	0.163	0.115	0.308	0.447	7.081	3.494	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	69	232	176	99	639	75	133
N.S.	1	1.00	0.80	2.70	2.05	1.15	7.43	0.87	1.55
time (sec)	N/A	0.070	0.155	0.133	0.296	0.391	16.189	4.069	4.704

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	103	295	231	141	1989	113	154
N.S.	1	1.00	0.82	2.34	1.83	1.12	15.79	0.90	1.22
time (sec)	N/A	0.095	0.202	0.129	0.291	0.487	41.060	3.379	4.667

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	164	132	358	286	182	5187	148	189
N.S.	1	0.99	0.80	2.17	1.73	1.10	31.44	0.90	1.15
time (sec)	N/A	0.136	0.284	0.126	0.285	0.618	91.423	3.455	4.752

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	167	421	341	224	0	189	226
N.S.	1	1.00	0.80	2.00	1.62	1.07	0.00	0.90	1.08
time (sec)	N/A	0.144	0.455	0.114	0.300	0.728	0.000	3.683	4.760

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	119	172	0	0	0	0	-1
N.S.	1	1.00	0.62	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	6.614	0.121	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	114	155	0	0	0	0	-1
N.S.	1	1.00	0.68	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	5.490	0.111	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	109	137	0	0	0	0	-1
N.S.	1	1.00	0.73	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	4.562	0.036	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	102	121	0	0	0	0	-1
N.S.	1	1.00	0.72	0.86	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	3.677	0.028	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	232	90	138	0	0	0	0	-1
N.S.	1	1.30	0.51	0.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	9.672	0.122	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	208	162	0	0	0	0	-1
N.S.	1	1.00	1.00	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.084	10.102	0.131	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	174	186	0	0	0	0	-1
N.S.	1	1.00	0.73	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.382	10.157	0.126	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	129	206	0	0	0	0	-1
N.S.	1	1.00	0.59	0.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.080	10.063	0.130	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	124	189	0	0	0	0	-1
N.S.	1	1.00	0.63	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	8.065	0.124	0.000	0.000	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	119	172	0	0	0	0	-1
N.S.	1	1.00	0.66	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	6.606	0.039	0.000	0.000	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	114	155	0	0	0	0	-1
N.S.	1	1.00	0.66	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	4.923	0.033	0.000	0.000	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	148	170	0	0	0	0	-1
N.S.	1	1.00	0.71	0.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.130	10.118	0.124	0.000	0.000	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	333	213	177	0	0	0	0	-1
N.S.	1	1.50	0.96	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	10.115	0.128	0.000	0.000	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	288	174	186	0	0	0	0	-1
N.S.	1	1.25	0.75	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.417	10.148	0.125	0.000	0.000	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	106	138	0	0	0	0	-1
N.S.	1	1.00	0.68	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	10.062	0.118	0.000	0.000	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	104	121	0	0	0	0	-1
N.S.	1	1.00	0.73	0.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	10.073	0.109	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	69	106	0	0	0	0	-1
N.S.	1	1.00	0.57	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	10.045	0.023	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	46	0	10	0	0	-1
N.S.	1	1.00	1.04	0.96	0.00	0.21	0.00	0.00	-0.02
time (sec)	N/A	0.004	10.027	0.021	0.000	0.075	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	55	47	0	0	0	0	-1
N.S.	1	1.00	0.52	0.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	10.075	0.125	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	208	162	0	0	0	0	-1
N.S.	1	1.00	1.00	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.120	10.098	0.119	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	186	186	0	0	0	0	-1
N.S.	1	1.00	0.78	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.160	10.147	0.140	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	109	274	0	0	0	0	-1
N.S.	1	1.00	0.58	1.45	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	10.061	0.113	0.000	0.000	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	104	234	0	0	0	0	-1
N.S.	1	1.00	0.61	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	10.054	0.120	0.000	0.000	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	99	196	0	0	0	0	-1
N.S.	1	1.00	0.66	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	10.056	0.125	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	99	173	0	0	0	0	-1
N.S.	1	1.00	0.66	1.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	10.046	0.133	0.000	0.000	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	97	150	0	0	0	0	-1
N.S.	1	1.00	0.67	1.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	10.041	0.043	0.000	0.000	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	99	129	0	0	0	0	-1
N.S.	1	1.00	0.66	0.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	4.964	0.043	0.000	0.000	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	207	138	161	0	0	0	0	-1
N.S.	1	1.20	0.80	0.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	10.129	0.167	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	208	185	0	0	0	0	-1
N.S.	1	1.00	0.89	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	10.111	0.125	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	159	209	0	0	0	0	-1
N.S.	1	1.00	0.60	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.497	10.233	0.123	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	193	0	44	0	0	-1
N.S.	1	1.00	0.97	1.66	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.086	10.076	0.123	0.000	0.103	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	107	176	0	39	0	0	-1
N.S.	1	1.00	1.13	1.85	0.00	0.41	0.00	0.00	-0.01
time (sec)	N/A	0.055	8.009	0.122	0.000	0.080	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	159	0	34	0	0	-1
N.S.	1	1.00	1.38	2.15	0.00	0.46	0.00	0.00	-0.01
time (sec)	N/A	0.039	6.242	0.116	0.000	0.090	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	94	141	0	26	0	0	-1
N.S.	1	1.00	2.04	3.07	0.00	0.57	0.00	0.00	-0.02
time (sec)	N/A	0.029	4.984	0.034	0.000	0.098	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	90	125	0	22	0	0	-1
N.S.	1	1.00	2.05	2.84	0.00	0.50	0.00	0.00	-0.02
time (sec)	N/A	0.028	3.885	0.028	0.000	0.079	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	51	141	0	0	0	0	-1
N.S.	1	1.00	1.11	3.07	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	10.104	0.105	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	196	165	0	0	0	0	-1
N.S.	1	1.00	2.65	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	10.127	0.125	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	244	189	0	0	0	0	-1
N.S.	1	1.00	2.39	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.255	10.153	0.127	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	122	227	0	54	0	0	-1
N.S.	1	1.00	0.86	1.60	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.085	10.063	0.113	0.000	0.087	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	117	210	0	49	0	0	-1
N.S.	1	1.00	0.97	1.74	0.00	0.40	0.00	0.00	-0.01
time (sec)	N/A	0.069	10.059	0.121	0.000	0.082	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	112	193	0	44	0	0	-1
N.S.	1	1.00	1.12	1.93	0.00	0.44	0.00	0.00	-0.01
time (sec)	N/A	0.049	9.336	0.116	0.000	0.091	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	107	176	0	39	0	0	-1
N.S.	1	1.00	1.32	2.17	0.00	0.48	0.00	0.00	-0.01
time (sec)	N/A	0.037	7.350	0.043	0.000	0.078	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	102	159	0	34	0	0	-1
N.S.	1	1.00	1.38	2.15	0.00	0.46	0.00	0.00	-0.01
time (sec)	N/A	0.034	5.202	0.036	0.000	0.079	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	130	173	0	0	0	0	-1
N.S.	1	1.00	1.81	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	10.143	0.128	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	201	180	0	0	0	0	-1
N.S.	1	1.00	2.16	1.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	10.124	0.132	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	244	189	0	0	0	0	-1
N.S.	1	1.00	2.39	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.309	10.161	0.132	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	97	142	0	29	0	0	-1
N.S.	1	1.00	1.49	2.18	0.00	0.45	0.00	0.00	-0.02
time (sec)	N/A	0.048	10.072	0.124	0.000	0.074	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	92	125	0	24	0	0	-1
N.S.	1	1.00	2.00	2.72	0.00	0.52	0.00	0.00	-0.02
time (sec)	N/A	0.032	10.070	0.138	0.000	0.079	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	34	110	0	17	0	0	-1
N.S.	1	1.00	1.36	4.40	0.00	0.68	0.00	0.00	-0.04
time (sec)	N/A	0.023	10.054	0.025	0.000	0.101	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	47	0	8	0	0	-1
N.S.	1	1.00	1.90	4.70	0.00	0.80	0.00	0.00	-0.10
time (sec)	N/A	0.007	10.031	0.019	0.000	0.088	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	24	48	0	0	0	0	-1
N.S.	1	1.00	1.41	2.82	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.021	10.070	0.148	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	196	165	0	0	0	0	-1
N.S.	1	1.00	2.65	2.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	10.115	0.128	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	108	189	0	0	0	0	-1
N.S.	1	1.00	1.06	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	10.184	0.124	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	97	280	0	50	0	0	-1
N.S.	1	1.00	1.04	3.01	0.00	0.54	0.00	0.00	-0.01
time (sec)	N/A	0.065	10.076	0.127	0.000	0.124	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	92	240	0	45	0	0	-1
N.S.	1	1.00	1.24	3.24	0.00	0.61	0.00	0.00	-0.01
time (sec)	N/A	0.048	10.056	0.145	0.000	0.093	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	79	202	0	40	0	0	-1
N.S.	1	1.00	1.44	3.67	0.00	0.73	0.00	0.00	-0.02
time (sec)	N/A	0.033	10.061	0.119	0.000	0.078	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	79	179	0	0	0	0	-1
N.S.	1	1.00	1.44	3.25	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	10.064	0.120	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	79	156	0	0	0	0	-1
N.S.	1	1.00	1.44	2.84	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	10.065	0.039	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	79	133	0	0	0	0	-1
N.S.	1	1.00	1.44	2.42	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	5.890	0.035	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	101	164	0	0	0	0	-1
N.S.	1	1.00	1.40	2.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	10.149	0.129	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	196	188	0	0	0	0	-1
N.S.	1	1.00	1.96	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	10.127	0.135	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	244	212	0	0	0	0	-1
N.S.	1	1.00	1.91	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.362	10.163	0.130	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	354	292	0	0	0	0	-1
N.S.	1	1.00	1.46	1.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.094	5.692	0.192	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	349	275	0	0	0	0	-1
N.S.	1	1.00	1.58	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.073	5.239	0.129	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	343	258	0	0	0	0	-1
N.S.	1	1.00	1.73	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	4.860	0.133	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	338	240	0	0	0	0	-1
N.S.	1	1.00	1.91	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	4.409	0.057	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	331	224	0	0	0	0	-1
N.S.	1	1.00	1.96	1.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	3.677	0.028	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	283	386	0	0	0	0	-1
N.S.	1	1.00	0.88	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.103	9.367	0.169	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	481	410	0	0	0	0	-1
N.S.	1	1.00	1.69	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.095	10.421	0.134	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	308	434	0	0	0	0	-1
N.S.	1	1.00	0.99	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.433	10.322	0.128	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	364	326	0	0	0	0	-1
N.S.	1	1.00	1.36	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.107	7.864	0.125	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	358	309	0	0	0	0	-1
N.S.	1	1.00	1.45	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.091	7.263	0.128	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	354	292	0	0	0	0	-1
N.S.	1	1.00	1.57	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.062	6.776	0.124	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	349	275	0	0	0	0	-1
N.S.	1	1.00	1.69	1.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.049	6.012	0.046	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	343	258	0	0	0	0	-1
N.S.	1	1.00	1.73	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	4.831	0.032	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	477	418	0	0	0	0	-1
N.S.	1	1.00	1.68	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.156	10.424	0.128	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	372	309	425	0	0	0	0	-1
N.S.	1	1.22	1.01	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.347	10.262	0.132	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	309	434	0	0	0	0	-1
N.S.	1	1.00	0.70	0.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	10.309	0.144	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	337	241	0	0	0	0	-1
N.S.	1	1.00	1.80	1.29	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	10.268	0.128	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	331	224	0	0	0	0	-1
N.S.	1	1.00	1.95	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	10.244	0.128	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	214	209	0	0	0	0	-1
N.S.	1	1.00	1.42	1.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	10.105	0.039	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	142	85	0	38	0	0	-1
N.S.	1	1.00	2.22	1.33	0.00	0.59	0.00	0.00	-0.02
time (sec)	N/A	0.005	10.049	0.020	0.000	0.079	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	159	107	0	0	0	0	-1
N.S.	1	1.00	0.95	0.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	10.090	0.142	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	481	410	0	0	0	0	-1
N.S.	1	1.00	1.68	1.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.140	10.382	0.131	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	308	434	0	0	0	0	-1
N.S.	1	1.00	0.98	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	10.437	0.128	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	339	379	0	0	0	0	-1
N.S.	1	1.00	1.55	1.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.082	10.263	0.123	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	333	339	0	0	0	0	-1
N.S.	1	1.00	1.66	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.059	10.252	0.132	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	328	301	0	0	0	0	-1
N.S.	1	1.00	1.81	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	10.244	0.137	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	329	278	0	0	0	0	-1
N.S.	1	1.00	1.82	1.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	10.235	0.132	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	329	255	0	0	0	0	-1
N.S.	1	1.00	1.82	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	10.229	0.046	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	328	232	0	0	0	0	-1
N.S.	1	1.00	1.81	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	4.245	0.033	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	483	409	0	0	0	0	-1
N.S.	1	1.00	1.70	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.105	10.332	0.123	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	311	433	0	0	0	0	-1
N.S.	1	1.00	1.00	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.312	10.270	0.131	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	320	457	0	0	0	0	-1
N.S.	1	1.00	0.94	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.551	10.356	0.133	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	584	1186	0	0	0	0	-1
N.S.	1	1.00	1.25	2.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	11.782	0.151	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	488	756	0	0	0	0	-1
N.S.	1	1.00	1.37	2.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	10.986	0.116	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	0	0	0	-1
N.S.	1	1.00	1.07	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.054	10.143	0.025	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	214	200	0	0	0	0	-1
N.S.	1	1.00	0.53	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	10.143	0.122	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	1069	1279	0	0	0	0	-1
N.S.	1	1.00	1.49	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	11.087	0.121	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	596	1195	0	0	0	0	-1
N.S.	1	1.00	1.08	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.851	11.527	0.122	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	503	761	0	0	0	0	-1
N.S.	1	1.00	1.11	1.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.521	10.834	0.123	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	293	364	0	0	0	0	-1
N.S.	1	1.00	0.76	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.222	10.149	0.027	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	205	201	0	0	0	0	-1
N.S.	1	1.00	1.04	1.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	10.154	0.120	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	464	1293	0	0	0	0	-1
N.S.	1	1.00	0.65	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.656	13.344	0.129	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	304	355	0	0	0	0	-1
N.S.	1	1.00	0.63	0.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.303	10.180	0.030	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	216	198	0	0	0	0	-1
N.S.	1	1.00	1.06	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.124	10.141	0.116	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	295	357	0	0	0	0	-1
N.S.	1	1.00	1.01	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.059	10.180	0.033	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	207	199	0	0	0	0	-1
N.S.	1	1.00	0.50	0.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	10.156	0.111	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	154	380	0	0	0	0	-1
N.S.	1	1.00	0.67	1.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.099	10.123	0.122	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	127	235	0	0	0	0	-1
N.S.	1	1.00	0.76	1.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	10.089	0.131	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	73	108	0	0	0	0	-1
N.S.	1	1.00	0.60	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	10.045	0.032	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	59	55	0	0	0	0	-1
N.S.	1	1.00	0.48	0.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	10.084	0.119	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	399	175	443	0	0	0	0	-1
N.S.	1	1.26	0.55	1.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	10.317	0.117	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.007	0.092	0.046	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	373	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	0.549	0.033	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	345	303	0	0	0	0	0	-1
N.S.	1	0.96	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	0.466	0.027	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	232	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.131	0.405	0.010	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.102	0.006	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.008	0.378	0.032	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.007	0.351	0.038	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	446	446	270	251	0	0	0	0	-1
N.S.	1	1.00	0.61	0.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	10.657	0.131	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	719	247	0	0	0	0	-1
N.S.	1	1.00	3.30	1.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.174	11.001	0.142	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	77	327	0	323	0	0	-1
N.S.	1	1.00	1.18	5.03	0.00	4.97	0.00	0.00	-0.02
time (sec)	N/A	0.095	7.755	0.379	0.000	0.501	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	77	311	0	112	0	0	-1
N.S.	1	1.00	1.22	4.94	0.00	1.78	0.00	0.00	-0.02
time (sec)	N/A	0.089	7.726	0.390	0.000	0.582	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	81	336	0	328	0	0	-1
N.S.	1	1.00	1.12	4.67	0.00	4.56	0.00	0.00	-0.01
time (sec)	N/A	0.087	7.821	0.348	0.000	0.573	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	81	337	0	114	0	0	-1
N.S.	1	1.00	1.16	4.81	0.00	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.080	7.834	0.392	0.000	0.516	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	3652	437	0	0	0	0	-1
N.S.	1	1.00	6.52	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.399	17.345	0.117	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	3658	439	0	0	0	0	-1
N.S.	1	1.00	6.94	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.467	17.332	0.113	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [410] had the largest ratio of [46]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	6	1.00	17	0.353
2	A	9	6	1.00	18	0.333
3	A	3	3	1.00	18	0.167
4	A	3	3	1.00	19	0.158
5	A	5	3	1.00	17	0.176
6	A	3	2	1.00	17	0.118
7	A	2	2	1.00	17	0.118
8	A	2	2	1.00	17	0.118
9	A	5	3	1.00	27	0.111
10	A	3	2	1.00	28	0.071
11	A	5	3	1.00	21	0.143
12	A	3	2	1.00	22	0.091
13	A	3	3	1.00	15	0.200
14	A	2	2	1.00	22	0.091
15	A	5	5	1.00	23	0.217
16	A	3	3	1.00	21	0.143
17	A	6	6	1.00	22	0.273
18	A	1	1	1.00	22	0.045
19	A	3	3	1.00	21	0.143
20	A	1	1	1.00	23	0.043
21	A	3	3	1.00	22	0.136
22	A	1	1	1.00	28	0.036
23	A	2	2	1.00	24	0.083
24	A	4	4	1.00	28	0.143
25	A	5	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	5	3	1.00	26	0.115
27	A	5	3	1.00	26	0.115
28	A	5	3	1.00	27	0.111
29	A	5	3	1.00	27	0.111
30	A	3	2	1.00	27	0.074
31	A	3	2	1.00	27	0.074
32	A	3	2	1.00	28	0.071
33	A	3	2	1.00	28	0.071
34	A	3	2	1.00	30	0.067
35	A	5	3	1.00	29	0.103
36	A	6	4	1.00	29	0.138
37	A	3	2	1.00	32	0.062
38	A	5	3	1.00	31	0.097
39	A	5	3	1.00	22	0.136
40	A	5	3	1.00	23	0.130
41	A	3	2	1.00	22	0.091
42	A	3	2	1.00	22	0.091
43	A	3	3	1.00	22	0.136
44	A	5	3	1.00	22	0.136
45	A	5	3	1.00	22	0.136
46	A	5	3	1.00	20	0.150
47	A	5	3	1.00	17	0.176
48	A	5	3	1.00	22	0.136
49	A	5	3	1.00	22	0.136
50	A	5	3	1.00	22	0.136
51	A	2	2	1.00	22	0.091
52	A	7	3	1.00	22	0.136
53	A	5	3	1.00	22	0.136
54	A	3	2	1.00	22	0.091
55	A	3	2	1.00	22	0.091
56	A	3	2	1.00	22	0.091
57	A	2	2	1.00	22	0.091
58	A	3	2	1.00	22	0.091
59	A	3	2	1.00	22	0.091
60	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	17	0.118
62	A	3	2	1.00	22	0.091
63	A	3	2	1.00	22	0.091
64	A	3	2	1.00	22	0.091
65	A	3	3	1.00	22	0.136
66	A	7	3	1.00	22	0.136
67	A	5	3	1.00	22	0.136
68	A	5	3	1.00	18	0.167
69	A	3	2	1.00	18	0.111
70	A	3	2	1.00	18	0.111
71	A	3	2	1.00	18	0.111
72	A	2	2	1.00	18	0.111
73	A	5	3	1.00	16	0.188
74	A	5	3	1.00	13	0.231
75	A	5	3	1.00	18	0.167
76	A	2	2	1.00	18	0.111
77	A	7	3	1.00	18	0.167
78	A	5	3	1.00	18	0.167
79	A	5	3	1.00	18	0.167
80	A	3	2	1.00	20	0.100
81	A	3	2	1.00	20	0.100
82	A	3	2	1.00	20	0.100
83	A	3	2	1.00	20	0.100
84	A	2	2	1.00	20	0.100
85	A	3	2	1.00	18	0.111
86	A	3	2	1.00	15	0.133
87	A	3	2	1.00	20	0.100
88	A	3	3	1.00	20	0.150
89	A	5	3	1.00	20	0.150
90	A	5	3	1.00	20	0.150
91	A	5	3	1.00	20	0.150
92	A	5	3	1.00	22	0.136
93	A	5	3	1.00	22	0.136
94	A	3	3	1.00	20	0.150
95	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	2	1.00	22	0.091
97	A	3	2	1.00	18	0.111
98	A	9	5	1.00	18	0.278
99	A	10	6	1.00	18	0.333
100	A	9	5	1.00	18	0.278
101	A	10	6	1.00	18	0.333
102	A	9	5	1.00	29	0.172
103	A	9	5	1.00	26	0.192
104	A	9	5	1.00	24	0.208
105	A	9	5	1.00	22	0.227
106	A	9	5	1.00	25	0.200
107	A	9	5	1.00	31	0.161
108	A	9	5	1.00	32	0.156
109	A	9	5	1.00	23	0.217
110	A	9	5	1.00	25	0.200
111	A	9	5	1.00	29	0.172
112	A	9	5	1.00	32	0.156
113	A	4	4	1.00	22	0.182
114	A	5	5	1.00	24	0.208
115	A	4	4	1.00	24	0.167
116	A	4	4	1.00	24	0.167
117	A	4	4	1.00	24	0.167
118	A	4	4	1.00	24	0.167
119	A	3	3	1.00	39	0.077
120	A	2	1	1.00	17	0.059
121	A	2	1	1.00	17	0.059
122	A	2	1	1.00	17	0.059
123	A	2	1	1.00	15	0.067
124	A	3	2	1.00	17	0.118
125	A	3	3	1.00	17	0.176
126	A	3	3	1.00	17	0.176
127	A	4	4	1.00	17	0.235
128	A	2	1	1.00	19	0.053
129	A	2	1	1.00	19	0.053
130	A	2	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	1	1.00	9	0.111
132	A	3	2	1.00	19	0.105
133	A	4	3	1.00	19	0.158
134	A	5	4	1.00	19	0.210
135	A	5	5	1.00	19	0.263
136	A	5	5	1.00	19	0.263
137	A	11	7	1.00	19	0.368
138	A	11	7	1.00	19	0.368
139	A	11	7	1.00	19	0.368
140	A	9	6	1.00	17	0.353
141	A	9	6	1.00	9	0.667
142	A	12	8	1.00	19	0.421
143	A	14	9	1.00	19	0.474
144	A	11	8	1.00	19	0.421
145	A	11	8	1.00	19	0.421
146	A	10	7	1.00	17	0.412
147	A	10	7	1.00	9	0.778
148	A	22	9	1.00	19	0.474
149	A	24	10	1.00	19	0.526
150	A	6	5	0.99	21	0.238
151	A	5	5	1.00	21	0.238
152	A	4	4	1.00	21	0.190
153	A	3	3	1.00	19	0.158
154	A	3	3	1.00	21	0.143
155	A	6	6	1.00	21	0.286
156	A	7	7	1.00	21	0.333
157	A	8	8	1.00	22	0.364
158	A	7	7	1.00	22	0.318
159	A	6	6	1.00	20	0.300
160	A	2	2	1.00	22	0.091
161	A	10	10	1.00	22	0.454
162	A	11	11	1.00	22	0.500
163	A	12	11	1.00	22	0.500
164	A	6	6	1.00	21	0.286
165	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	3	1.00	29	0.103
167	A	3	3	1.00	29	0.103
168	A	3	3	1.00	22	0.136
169	A	3	3	1.00	24	0.125
170	A	2	2	1.00	21	0.095
171	A	3	3	1.00	21	0.143
172	A	1	1	1.00	22	0.045
173	A	3	3	1.00	21	0.143
174	F	0	0	N/A	0.	N/A
175	A	0	0	0.00	0	0.000
176	A	9	6	0.96	19	0.316
177	A	7	6	0.95	19	0.316
178	A	6	5	1.00	17	0.294
179	A	2	2	1.00	9	0.222
180	A	6	5	1.00	19	0.263
181	A	8	5	1.00	19	0.263
182	A	6	4	0.95	19	0.210
183	A	5	4	0.92	19	0.210
184	A	4	3	1.00	17	0.176
185	A	1	1	1.00	9	0.111
186	A	4	3	1.00	19	0.158
187	A	5	3	1.00	19	0.158
188	A	6	3	1.00	19	0.158
189	A	4	3	1.00	24	0.125
190	A	4	3	1.00	24	0.125
191	A	3	3	1.00	24	0.125
192	A	2	2	1.00	22	0.091
193	A	5	5	1.00	24	0.208
194	A	6	6	1.00	24	0.250
195	A	6	6	1.00	26	0.231
196	A	3	3	1.00	26	0.115
197	A	4	4	1.00	26	0.154
198	A	6	6	1.00	26	0.231
199	A	5	5	1.00	28	0.179
200	A	4	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	3	3	1.00	28	0.107
202	A	3	3	1.00	28	0.107
203	A	4	4	1.00	28	0.143
204	A	6	6	1.00	28	0.214
205	A	5	5	1.00	29	0.172
206	A	4	4	1.00	29	0.138
207	A	3	3	1.00	29	0.103
208	A	3	3	1.00	29	0.103
209	A	4	4	1.00	29	0.138
210	A	6	6	1.00	29	0.207
211	A	2	2	1.00	19	0.105
212	A	3	3	1.67	19	0.158
213	A	7	5	0.99	31	0.161
214	A	4	3	1.00	39	0.077
215	A	4	3	1.00	39	0.077
216	A	3	3	1.00	39	0.077
217	A	2	2	1.00	37	0.054
218	A	5	5	1.00	39	0.128
219	A	6	6	1.00	39	0.154
220	A	7	7	1.00	41	0.171
221	A	6	6	1.00	41	0.146
222	A	3	3	1.00	41	0.073
223	A	4	4	1.00	41	0.098
224	A	6	6	1.00	41	0.146
225	A	6	6	1.00	20	0.300
226	A	5	5	1.00	20	0.250
227	A	4	4	1.00	18	0.222
228	A	8	7	1.00	20	0.350
229	A	1	1	1.00	20	0.050
230	A	23	13	1.00	20	0.650
231	A	26	14	1.00	20	0.700
232	A	5	5	1.00	20	0.250
233	A	4	4	1.00	20	0.200
234	A	3	3	1.00	18	0.167
235	A	4	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	8	8	1.00	20	0.400
237	A	9	9	1.00	20	0.450
238	A	4	4	1.00	20	0.200
239	A	2	2	1.00	20	0.100
240	A	2	2	1.00	18	0.111
241	A	9	8	1.00	20	0.400
242	A	16	11	1.00	20	0.550
243	A	23	14	1.00	20	0.700
244	A	2	1	1.00	22	0.045
245	A	2	1	1.00	22	0.045
246	A	2	1	1.00	22	0.045
247	A	2	1	1.00	20	0.050
248	A	3	2	1.00	22	0.091
249	A	3	3	1.05	22	0.136
250	A	3	3	1.03	22	0.136
251	A	4	4	1.03	22	0.182
252	A	2	1	1.00	24	0.042
253	A	2	1	1.00	24	0.042
254	A	2	1	1.00	22	0.045
255	A	2	1	1.00	14	0.071
256	A	3	2	1.00	24	0.083
257	A	4	3	1.00	24	0.125
258	A	5	4	1.00	24	0.167
259	A	5	4	1.00	24	0.167
260	A	5	4	1.00	24	0.167
261	A	3	3	1.05	22	0.136
262	A	3	3	1.00	23	0.130
263	A	5	3	1.00	24	0.125
264	A	5	3	1.00	24	0.125
265	A	5	3	1.00	24	0.125
266	A	3	2	1.00	22	0.091
267	A	3	2	1.00	14	0.143
268	A	6	3	1.00	24	0.125
269	A	8	4	1.00	24	0.167
270	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
271	A	4	3	1.00	24	0.125
272	A	4	3	1.00	22	0.136
273	A	4	3	1.00	14	0.214
274	A	10	4	1.00	24	0.167
275	A	12	5	1.00	24	0.208
276	A	7	5	1.00	24	0.208
277	A	6	5	1.00	24	0.208
278	A	5	5	1.00	24	0.208
279	A	4	4	1.00	24	0.167
280	A	4	4	1.04	24	0.167
281	A	4	4	1.04	24	0.167
282	A	4	4	1.00	24	0.167
283	A	5	5	1.00	24	0.208
284	A	6	5	0.99	24	0.208
285	A	7	5	1.00	24	0.208
286	A	6	6	1.00	24	0.250
287	A	5	5	1.00	24	0.208
288	A	4	4	1.00	22	0.182
289	A	4	4	1.00	14	0.286
290	A	8	7	1.30	24	0.292
291	A	8	7	1.00	24	0.292
292	A	25	10	1.00	24	0.417
293	A	7	6	1.00	24	0.250
294	A	6	5	1.00	24	0.208
295	A	5	4	1.00	22	0.182
296	A	5	5	1.00	14	0.357
297	A	13	8	1.00	24	0.333
298	A	21	10	1.50	24	0.417
299	A	27	10	1.25	24	0.417
300	A	5	5	1.00	24	0.208
301	A	4	4	1.00	24	0.167
302	A	3	3	1.00	22	0.136
303	A	1	1	1.00	14	0.071
304	A	4	4	1.00	24	0.167
305	A	9	8	1.00	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	10	9	1.00	24	0.375
307	A	6	5	1.00	24	0.208
308	A	5	5	1.00	24	0.208
309	A	4	4	1.00	24	0.167
310	A	4	4	1.00	24	0.167
311	A	4	4	1.00	22	0.182
312	A	4	4	1.00	14	0.286
313	A	9	8	1.20	24	0.333
314	A	19	10	1.00	24	0.417
315	A	29	11	1.00	24	0.458
316	A	8	7	1.00	24	0.292
317	A	7	7	1.00	24	0.292
318	A	6	6	1.00	24	0.250
319	A	5	5	1.00	22	0.227
320	A	5	5	1.00	14	0.357
321	A	7	7	1.00	24	0.292
322	A	7	7	1.00	24	0.292
323	A	21	10	1.00	24	0.417
324	A	9	7	1.00	24	0.292
325	A	8	7	1.00	24	0.292
326	A	7	6	1.00	24	0.250
327	A	6	5	1.00	22	0.227
328	A	6	6	1.00	14	0.429
329	A	13	8	1.00	24	0.333
330	A	21	13	1.00	24	0.542
331	A	27	13	1.00	24	0.542
332	A	6	6	1.00	24	0.250
333	A	5	5	1.00	24	0.208
334	A	4	4	1.00	22	0.182
335	A	2	2	1.00	14	0.143
336	A	2	2	1.00	24	0.083
337	A	8	8	1.00	24	0.333
338	A	9	9	1.00	24	0.375
339	A	7	6	1.00	24	0.250
340	A	6	6	1.00	24	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	5	5	1.00	24	0.208
342	A	5	5	1.00	24	0.208
343	A	5	5	1.00	22	0.227
344	A	5	5	1.00	14	0.357
345	A	8	8	1.00	24	0.333
346	A	17	10	1.00	24	0.417
347	A	26	11	1.00	24	0.458
348	A	7	6	1.00	24	0.250
349	A	6	6	1.00	24	0.250
350	A	5	5	1.00	24	0.208
351	A	4	4	1.00	22	0.182
352	A	4	4	1.00	14	0.286
353	A	7	6	1.00	24	0.250
354	A	7	6	1.00	24	0.250
355	A	18	9	1.00	24	0.375
356	A	8	6	1.00	24	0.250
357	A	7	6	1.00	24	0.250
358	A	6	5	1.00	24	0.208
359	A	5	4	1.00	22	0.182
360	A	5	5	1.00	14	0.357
361	A	12	7	1.00	24	0.292
362	A	19	11	1.22	24	0.458
363	A	22	10	1.00	24	0.417
364	A	5	5	1.00	24	0.208
365	A	4	4	1.00	24	0.167
366	A	3	3	1.00	22	0.136
367	A	1	1	1.00	14	0.071
368	A	3	3	1.00	24	0.125
369	A	6	6	1.00	24	0.250
370	A	7	7	1.00	24	0.292
371	A	6	5	1.00	24	0.208
372	A	5	5	1.00	24	0.208
373	A	4	4	1.00	24	0.167
374	A	4	4	1.00	24	0.167
375	A	4	4	1.00	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	4	4	1.00	14	0.286
377	A	8	7	1.00	24	0.292
378	A	15	10	1.00	24	0.417
379	A	22	11	1.00	24	0.458
380	A	5	5	1.00	26	0.192
381	A	4	4	1.00	26	0.154
382	A	3	3	1.00	24	0.125
383	A	3	3	1.00	26	0.115
384	A	6	6	1.00	26	0.231
385	A	6	6	1.00	27	0.222
386	A	5	5	1.00	27	0.185
387	A	4	4	1.00	25	0.160
388	A	2	2	1.00	27	0.074
389	A	8	8	1.00	27	0.296
390	A	5	5	1.00	26	0.192
391	A	2	2	1.00	28	0.071
392	A	3	3	1.00	27	0.111
393	A	3	3	1.00	29	0.103
394	A	5	5	1.00	24	0.208
395	A	4	4	1.00	24	0.167
396	A	3	3	1.00	22	0.136
397	A	4	4	1.00	24	0.167
398	A	9	8	1.26	24	0.333
399	A	0	0	0.00	0	0.000
400	A	8	7	1.00	24	0.292
401	A	7	6	0.96	24	0.250
402	A	6	5	1.00	22	0.227
403	A	2	2	1.00	14	0.143
404	A	0	0	0.00	0	0.000
405	A	0	0	0.00	0	0.000
406	A	8	8	1.00	24	0.333
407	A	10	10	1.00	26	0.385
408	A	2	2	1.00	40	0.050
409	A	2	2	1.00	40	0.050
410	A	2	2	1.00	46	0.043

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	2	2	1.00	46	0.043
412	A	8	8	1.00	29	0.276
413	A	10	10	1.00	31	0.323

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{c+dx^2}{a+bx^4}\,dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(\right.}{4}$$

```
[Out] -1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(-d*a^(1/2)+c*b^(1/
2))/a^(3/4)/b^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^
(1/2))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arctan(-1+b^(1/4)
*x*2^(1/2)/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/4*arcta
n(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2
)
```

Rubi [A]

time = 0.10, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,d+\sqrt{b}\,c\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,d+\sqrt{b}\,c\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a + b*x^4),x]
```

```
[Out] -1/2*((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqr
t[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c - Sqrt[a]*d)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/
4)) + ((Sqrt[b]*c - Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rubi steps

$$\int \frac{c+dx^2}{a+bx^4}\,dx = \frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}\,\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}\,\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}$$

$$=\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b}-\frac{\left(\sqrt{b}\,c-\sqrt{\cdots}\right.}{\cdots}$$

$$=-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(\sqrt{a}+\sqrt{2}\cdots\right.}{4\sqrt{2}\,a^{3/4}b^{3/4}}$$

$$=-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{\cdots}\right.}{\cdots}$$

Mathematica [A]
time = 0.07, size = 183, normalized size = 0.74

$$\frac{-2\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\left(\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)-\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a + b*x^4),x]
```

```
[Out] (-2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[b]*c + Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c -
 Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^(3
/4))
```

Maple [A]
time = 0.11, size = 206, normalized size = 0.83

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(\_R^2 d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{d\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^
2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8*d/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^
(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arc
tan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Maxima [A]
time = 0.53, size = 221, normalized size = 0.89

$$\frac{\sqrt{2}\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 1/4*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)
*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*log(sqrt(b)*x^2 + s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(sqrt(b
)*c - sqrt(a)*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^
(3/4)*b^(3/4))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(166) = 332$.
time = 0.35, size = 767, normalized size = 3.11

$$-\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+2\,cd}{ab}}\log\left(-(b^{2}c^{4}-a^{2}d^{4})x+\left(a^{3}b^{2}d\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+ab^{2}c^{3}-a^{2}bcd^{2}\right)\sqrt{-\frac{ab\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+2\,cd}{ab}}\right)+\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+2\,cd}{ab}}\log\left(-(b^{2}c^{4}-a^{2}d^{4})x-\left(a^{3}b^{2}d\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+ab^{2}c^{3}-a^{2}bcd^{2}\right)\sqrt{-\frac{ab\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}+2\,cd}{ab}}\right)+\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-2\,cd}{ab}}\log\left(-(b^{2}c^{4}-a^{2}d^{4})x+\left(a^{3}b^{2}d\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-ab^{2}c^{3}+a^{2}bcd^{2}\right)\sqrt{\frac{ab\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-2\,cd}{ab}}\right)-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-2\,cd}{ab}}\log\left(-(b^{2}c^{4}-a^{2}d^{4})x-\left(a^{3}b^{2}d\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-ab^{2}c^{3}+a^{2}bcd^{2}\right)\sqrt{\frac{ab\sqrt{-\frac{b^{2}c^{4}-2\,abc^{2}d^{2}+a^{2}d^{4}}{a^{3}b^{3}}}-2\,cd}{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*
d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^
2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^
2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a
*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log
(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d
^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b
*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt(-(b^2*
c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a
^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - a*b^2*c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^
3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 +
a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - 2*c*d)/(a*b)))
```

Sympy [A]
time = 0.36, size = 109, normalized size = 0.44

$$\operatorname{RootSum}\left(256t^{4}a^{3}b^{3}+64t^{2}a^{2}b^{2}cd+a^{2}d^{4}+2abc^{2}d^{2}+b^{2}c^{4},\left(t\mapsto t\log\left(x+\frac{64t^{3}a^{3}b^{2}d+12ta^{2}bcd^{2}-4tab^{2}c^{3}}{a^{2}d^{4}-b^{2}c^{4}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c*
*2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d + 12*_t*a*
*2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Giac [A]
time = 4.90, size = 241, normalized size = 0.98

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c+\left(ab^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}-\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c-\left(ab^3\right)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b
^2*c + (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*log(
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^2*c - (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b^3)
```

Mupad [B]
time = 0.38, size = 599, normalized size = 2.43

$$-2\,\mathrm{atanh}\left(\frac{8b^3c^2x\sqrt{\frac{d^2\sqrt{-a^3b^3}}{16a^2b^3}-\frac{c^2\sqrt{-a^3b^3}}{16a^3b^2}-\frac{cd}{8ab}}}{2b^2c^2d-2abd^3+\frac{2bc^3\sqrt{-a^3b^3}}{a^2}-\frac{2cd^2\sqrt{-a^3b^3}}{a}}-\frac{8ab^2d^2x\sqrt{\frac{d^2\sqrt{-a^3b^3}}{16a^2b^3}-\frac{c^2\sqrt{-a^3b^3}}{16a^3b^2}-\frac{cd}{8ab}}}{2b^2c^2d-2abd^3+\frac{2bc^3\sqrt{-a^3b^3}}{a^2}-\frac{2cd^2\sqrt{-a^3b^3}}{a}}\right)\sqrt{-\frac{bc^2\sqrt{-a^3b^3}-ad^2\sqrt{-a^3b^3}+2a^2b^2cd}{16a^3b^3}}-2\,\mathrm{atanh}\left(\frac{8b^3c^2x\sqrt{\frac{c^2\sqrt{-a^3b^3}}{16a^3b^2}-\frac{cd}{8ab}-\frac{d^2\sqrt{-a^3b^3}}{16a^2b^3}}}{2b^2c^2d-2abd^3-\frac{2bc^3\sqrt{-a^3b^3}}{a^2}+\frac{2cd^2\sqrt{-a^3b^3}}{a}}-\frac{8ab^2d^2x\sqrt{\frac{c^2\sqrt{-a^3b^3}}{16a^3b^2}-\frac{cd}{8ab}-\frac{d^2\sqrt{-a^3b^3}}{16a^2b^3}}}{2b^2c^2d-2abd^3-\frac{2bc^3\sqrt{-a^3b^3}}{a^2}+\frac{2cd^2\sqrt{-a^3b^3}}{a}}\right)\sqrt{-\frac{ad^2\sqrt{-a^3b^3}-bc^2\sqrt{-a^3b^3}+2a^2b^2cd}{16a^3b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a + b*x^4),x)
```

```
[Out] - 2*atanh((8*b^3*c^2*x*((d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3) - (c^2*(-a^3*b^
3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (
2*b*c^3*(-a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^
2*x*((d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b
^2) - (c*d)/(8*a*b))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*b^3)^
(1/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a))*(-(b*c^2*(-a^3*b^3)^(1/2) - a*d
^2*(-a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) - 2*atanh((8*b^3*c
^2*x*((c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) - (d^2*(-a^3*b^3)
^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^3)
^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*x*((c^2*(-a^3*b^
3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) - (d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3
))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^3)^(1/2))/a^2 + (2*c*
d^2*(-a^3*b^3)^(1/2))/a))*(-(a*d^2*(-a^3*b^3)^(1/2) - b*c^2*(-a^3*b^3)^(1/2
) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

3.2 $\int \frac{c-dx^2}{a+bx^4} \, dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\log\left(\cdots\right.}{4\cdots}$$

```
[Out] 1/4*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(
3/4)*2^(1/2)+1/4*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))
/a^(3/4)/b^(3/4)*2^(1/2)-1/8*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1
/2))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*b^(1/4)*x
*2^(1/2)+a^(1/2)+x^2*b^(1/2))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{b}\,c-\sqrt{a}\,d\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{b}\,c-\sqrt{a}\,d\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\frac{\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{a}\,d+\sqrt{b}\,c\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)/(a + b*x^4),x]
```

```
[Out] -1/2*((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqr
t[2]*a^(3/4)*b^(3/4)) + ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(3/4)) - ((Sqrt[b]*c + Sqrt[a]*d)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/
4)) + ((Sqrt[b]*c + Sqrt[a]*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rubi steps

$$\int \frac{c-dx^2}{a+bx^4}\,dx = \frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}-d\right)\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx}{2b}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+d\right)\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx}{2b}$$

$$= \frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}-d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b}+\frac{\left(\frac{\sqrt{b}\,c}{\sqrt{a}}-d\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b}-\frac{\left(\sqrt{b}\,c+\cdots\right.}{\cdots}$$

$$= -\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\log\left(\sqrt{a}+\sqrt{2}\cdots\right.}{4\sqrt{2}\,a^{3/4}b^{3/\cdots}}$$

$$= -\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}b^{3/4}}-\cdots$$

Mathematica [A]
time = 0.03, size = 184, normalized size = 0.74

$$\frac{\left(-2\sqrt{b}\,c+2\sqrt{a}\,d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+2\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\left(\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)-\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)\right)}{4\sqrt{2}\,a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)/(a + b*x^4),x]
```

```
[Out] ((-2*Sqrt[b]*c + 2*Sqrt[a]*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*(
Sqrt[b]*c - Sqrt[a]*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c
 + Sqrt[a]*d)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] - Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(4*Sqrt[2]*a^(3/4)*b^
(3/4))
```

Maple [A]
time = 0.14, size = 206, normalized size = 0.83

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b+a\right)}\frac{\left(-\_R^2 d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{d\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c*(a/b)^(1/4)/a*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^
2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8*d/b/(a/b)^(1/4)*2^(1/2)*(ln((x^2-(a/b)^
(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arc
tan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Maxima [A]
time = 0.52, size = 221, normalized size = 0.89

$$\frac{\sqrt{2}\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\sqrt{2}\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(
2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)) + 1/4*sqrt(2)*(sqrt(b)*c - sqrt(a)*d)*arctan(1/2*sqrt(2)*(2*sqrt(b)
*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(b))*sqrt(b)) + 1/8*sqrt(2)*(sqrt(b)*c + sqrt(a)*d)*log(sqrt(b)*x^2 + s
qrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(3/4)*b^(3/4)) - 1/8*sqrt(2)*(sqrt(b
)*c + sqrt(a)*d)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^
(3/4)*b^(3/4))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(166) = 332$.
time = 0.35, size = 767, normalized size = 3.11

$$-\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)+\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x-\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}+2\,cd}{ab}}\right)+\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x+\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x-\left(a^3b^2d\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{-\frac{b^2c^4-2\,abc^2d^2+a^2d^4}{a^3b^3}}-2\,cd}{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d
)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*
c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt((a*b*
sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(
b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)
/(a^3*b^3)) + a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2
*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4
 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*
d^4)*x + (a^3*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) -
a*b^2*c^3 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^
4)/(a^3*b^3)) - 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*
d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3
*b^2*d*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 + a
^2*b*c*d^2)*sqrt(-(a*b*sqrt(-(b^2*c^4 - 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3))
 - 2*c*d)/(a*b)))
```

Sympy [A]
time = 0.36, size = 110, normalized size = 0.45

$$-\operatorname{RootSum}\left(256t^4a^3b^3-64t^2a^2b^2cd+a^2d^4+2abc^2d^2+b^2c^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^3b^2d-12ta^2bcd^2+4tab^2c^3}{a^2d^4-b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)/(b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d + a**2*d**4 + 2*a*b*c
**2*d**2 + b**2*c**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*b**2*d - 12*_t*a
**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Giac [A]
time = 5.02, size = 241, normalized size = 0.98

$$\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c-(ab^3)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3}+\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c+(ab^3)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}-\frac{\sqrt{2}\left((ab^3)^{\frac{1}{4}}b^2c+(ab^3)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{b}}\right)}{8ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b
^2*c - (a*b^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b
)^(1/4))/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c + (a*b^3)^(3/4)*d)*log(
x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) - 1/8*sqrt(2)*((a*b^3)^(1/
4)*b^2*c + (a*b^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a
*b^3)
```

Mupad [B]
time = 0.26, size = 603, normalized size = 2.44

$$2\,\mathrm{atanh}\left(\frac{8b^3c^2x\sqrt{\frac{cd}{8ab}-\frac{c^2\sqrt{-a^3b^3}}{16a^3b^2}+\frac{d^2\sqrt{-a^3b^3}}{16a^2b^3}}}{2b^2c^2d-2abd^3-\frac{2bc^3\sqrt{-a^3b^3}}{a^2}+\frac{2cd^2\sqrt{-a^3b^3}}{a}}-\frac{8ab^2d^2x\sqrt{\frac{cd}{8ab}-\frac{c^2\sqrt{-a^3b^3}}{16a^3b^2}+\frac{d^2\sqrt{-a^3b^3}}{16a^2b^3}}}{2b^2c^2d-2abd^3-\frac{2bc^3\sqrt{-a^3b^3}}{a^2}+\frac{2cd^2\sqrt{-a^3b^3}}{a}}\right)\sqrt{\frac{ad^2\sqrt{-a^3b^3}-bc^2\sqrt{-a^3b^3}+2a^2b^2cd}{16a^3b^3}}+2\,\mathrm{atanh}\left(\frac{8b^3c^2x\sqrt{\frac{cd}{8ab}+\frac{c^2\sqrt{-a^3b^3}}{16a^3b^2}-\frac{d^2\sqrt{-a^3b^3}}{16a^2b^3}}}{2b^2c^2d-2abd^3+\frac{2bc^3\sqrt{-a^3b^3}}{a^2}-\frac{2cd^2\sqrt{-a^3b^3}}{a}}-\frac{8ab^2d^2x\sqrt{\frac{cd}{8ab}+\frac{c^2\sqrt{-a^3b^3}}{16a^3b^2}-\frac{d^2\sqrt{-a^3b^3}}{16a^2b^3}}}{2b^2c^2d-2abd^3+\frac{2bc^3\sqrt{-a^3b^3}}{a^2}-\frac{2cd^2\sqrt{-a^3b^3}}{a}}\right)\sqrt{\frac{bc^2\sqrt{-a^3b^3}-ad^2\sqrt{-a^3b^3}+2a^2b^2cd}{16a^3b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)/(a + b*x^4),x)
```

```
[Out] 2*atanh((8*b^3*c^2*x*((c*d)/(8*a*b) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) +
 (d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*
b*c^3*(-a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*
x*((c*d)/(8*a*b) - (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(-a^3*b^3)^(1
/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 - (2*b*c^3*(-a^3*b^3)^(1
/2))/a^2 + (2*c*d^2*(-a^3*b^3)^(1/2))/a))*((a*d^2*(-a^3*b^3)^(1/2) - b*c^2*
(-a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) + 2*atanh((8*b^3*c^2*
x*((c*d)/(8*a*b) + (c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(-a^3*b^3)^(1
/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*b^3)^(1
/2))/a^2 - (2*c*d^2*(-a^3*b^3)^(1/2))/a) - (8*a*b^2*d^2*x*((c*d)/(8*a*b) +
(c^2*(-a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(-a^3*b^3)^(1/2))/(16*a^2*b^3))^
(1/2))/(2*b^2*c^2*d - 2*a*b*d^3 + (2*b*c^3*(-a^3*b^3)^(1/2))/a^2 - (2*c*d^2
*(-a^3*b^3)^(1/2))/a))*((b*c^2*(-a^3*b^3)^(1/2) - a*d^2*(-a^3*b^3)^(1/2) +
2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2)
```

3.3 $\int \frac{c+dx^2}{a-bx^4} \, dx$

Optimal. Leaf size=86

$$\frac{\left(\sqrt{b}\, c-\sqrt{a}\, d\right) \tan^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a}}\right)}{2 a^{3/4} b^{3/4}}+\frac{\left(\sqrt{b}\, c+\sqrt{a}\, d\right) \tanh^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a}}\right)}{2 a^{3/4} b^{3/4}}$$

```
[Out] 1/2*arctan(b^(1/4)*x/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*ar
ctanh(b^(1/4)*x/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1181, 211, 214}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a}}\right)\left(\sqrt{b}\, c-\sqrt{a}\, d\right)}{2 a^{3/4} b^{3/4}}+\frac{\left(\sqrt{a}\, d+\sqrt{b}\, c\right) \tanh^{-1}\left(\frac{\sqrt[4]{b}\, x}{\sqrt[4]{a}}\right)}{2 a^{3/4} b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c + Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\int \frac{c+dx^2}{a-bx^4}\,dx = \frac{1}{2}\left(-\frac{\sqrt{b}\,c}{\sqrt{a}}+d\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}-bx^2}\,dx+\frac{1}{2}\left(\frac{\sqrt{b}\,c}{\sqrt{a}}+d\right)\int \frac{1}{\sqrt{a}\,\sqrt{b}-bx^2}\,dx$$
$$= \frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Mathematica [A]
time = 0.02, size = 95, normalized size = 1.10

$$\frac{2\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c - Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c + Sqrt[
a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b^(
3/4))
```

Maple [A]
time = 0.14, size = 104, normalized size = 1.21

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(\_R^2 d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	36
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}-\frac{d\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(
1/4)))-1/4*d/b/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(
a/b)^(1/4))))
```

Maxima [A]

time = 0.50, size = 109, normalized size = 1.27

$$\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{2\,\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\,\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(b)*c - sqrt(a)*d)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(b)*c + sqrt(a)*d)*log((sqrt(b)
*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(58) = 116$.
time = 0.35, size = 755, normalized size = 8.78

$$\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\right)-\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x-\left(a^3b^2d\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-2cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-2cd}{ab}}\right)+\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-2cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x-\left(a^3b^2d\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-2cd}{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/
(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^
2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4
+ 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt
((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^2*c
^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*
b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 +
a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b
*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x +
 (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3
 + a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^
3)) - 2*c*d)/(a*b))) + 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d
^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt
((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)*
sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*
b)))
```

Sympy [A]
time = 0.37, size = 110, normalized size = 1.28

$$-\operatorname{RootSum}\left(256t^4a^3b^3-64t^2a^2b^2cd-a^2d^4+2abc^2d^2-b^2c^4,\left(t\mapsto t\log\left(x+\frac{-64t^3a^3b^2d+12ta^2bcd^2+4tab^2c^3}{a^2d^4-b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] -RootSum(256*_t**4*a**3*b**3 - 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c
**2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d + 12*_t*
a**2*b*c*d**2 + 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(58) = 116$.
time = 3.82, size = 230, normalized size = 2.67

$$-\frac{\sqrt{2}\left(b^{2}c+\sqrt{-ab}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c-\sqrt{-ab}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c-\sqrt{-ab}\,bd\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^{2}c-\sqrt{-ab}\,bd\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a
/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c - sqrt(-a*b)*b
*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^
(3/4) - 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*log(x^2 + sqrt(2)*x*(-a/b)^(1/
4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*log(
x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

Mupad [B]
time = 4.64, size = 579, normalized size = 6.73

$$2\,\mathrm{atanh}\left(\frac{8b^{3}c^{2}x\sqrt{\frac{cd}{8ab}-\frac{c^{2}\sqrt{a^{3}b^{3}}}{16a^{3}b^{2}}-\frac{d^{2}\sqrt{a^{3}b^{3}}}{16a^{2}b^{3}}}}{2b^{2}c^{2}d+2abd^{3}-\frac{2bc^{3}\sqrt{a^{3}b^{3}}}{a^{2}}-\frac{2cd^{2}\sqrt{a^{3}b^{3}}}{a}}+\frac{8ab^{2}d^{2}x\sqrt{\frac{cd}{8ab}-\frac{c^{2}\sqrt{a^{3}b^{3}}}{16a^{3}b^{2}}-\frac{d^{2}\sqrt{a^{3}b^{3}}}{16a^{2}b^{3}}}}{2b^{2}c^{2}d+2abd^{3}-\frac{2bc^{3}\sqrt{a^{3}b^{3}}}{a^{2}}-\frac{2cd^{2}\sqrt{a^{3}b^{3}}}{a}}\right)\sqrt{-\frac{ad^{2}\sqrt{a^{3}b^{3}}+bc^{2}\sqrt{a^{3}b^{3}}-2a^{2}b^{2}cd}{16a^{3}b^{3}}}+2\,\mathrm{atanh}\left(\frac{8b^{3}c^{2}x\sqrt{\frac{cd}{8ab}+\frac{c^{2}\sqrt{a^{3}b^{3}}}{16a^{3}b^{2}}+\frac{d^{2}\sqrt{a^{3}b^{3}}}{16a^{2}b^{3}}}}{2b^{2}c^{2}d+2abd^{3}+\frac{2bc^{3}\sqrt{a^{3}b^{3}}}{a^{2}}+\frac{2cd^{2}\sqrt{a^{3}b^{3}}}{a}}+\frac{8ab^{2}d^{2}x\sqrt{\frac{cd}{8ab}+\frac{c^{2}\sqrt{a^{3}b^{3}}}{16a^{3}b^{2}}+\frac{d^{2}\sqrt{a^{3}b^{3}}}{16a^{2}b^{3}}}}{2b^{2}c^{2}d+2abd^{3}+\frac{2bc^{3}\sqrt{a^{3}b^{3}}}{a^{2}}+\frac{2cd^{2}\sqrt{a^{3}b^{3}}}{a}}\right)\sqrt{\frac{ad^{2}\sqrt{a^{3}b^{3}}+bc^{2}\sqrt{a^{3}b^{3}}+2a^{2}b^{2}cd}{16a^{3}b^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(a - b*x^4),x)
```

```
[Out] 2*atanh((8*b^3*c^2*x*((c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) -
(d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*
c^3*(a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((
c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(a^3*b^3)^(1/2))/(
16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2))/a^
2 - (2*c*d^2*(a^3*b^3)^(1/2))/a))*(-(a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3
)^(1/2) - 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) + 2*atanh((8*b^3*c^2*x*((c*d)/
(8*a*b) + (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) + (d^2*(a^3*b^3)^(1/2))/(16*a^
2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (
2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((c*d)/(8*a*b) + (c^2*(a^3*b^3
)^(1/2))/(16*a^3*b^2) + (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c
^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2)
)/a))*((a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*
a^3*b^3))^(1/2)
```

3.4 $\int \frac{c-dx^2}{a-bx^4} \, dx$

Optimal. Leaf size=86

$$\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

```
[Out] 1/2*arctanh(b^(1/4)*x/a^(1/4))*(-d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)+1/2*a
rctan(b^(1/4)*x/a^(1/4))*(d*a^(1/2)+c*b^(1/2))/a^(3/4)/b^(3/4)
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1181, 211, 214}

$$\frac{\operatorname{ArcTan}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,d+\sqrt{b}\,c\right)}{2a^{3/4}b^{3/4}}+\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x^2)/(a - b*x^4),x]
```

```
[Out] ((Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4)) +
 ((Sqrt[b]*c - Sqrt[a]*d)*ArcTanh[(b^(1/4)*x)/a^(1/4)])/(2*a^(3/4)*b^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\int \frac{c-dx^2}{a-bx^4}\,dx = \frac{1}{2}\left(-\frac{\sqrt{b}\,c}{\sqrt{a}}-d\right)\int \frac{1}{-\sqrt{a}\,\sqrt{b}-bx^2}\,dx + \frac{1}{2}\left(\frac{\sqrt{b}\,c}{\sqrt{a}}-d\right)\int \frac{1}{\sqrt{a}\,\sqrt{b}-bx^2}\,dx$$
$$= \frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}} + \frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}}$$

Mathematica [A]
time = 0.01, size = 95, normalized size = 1.10

$$\frac{2\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)-\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\left(\log\left(\sqrt[4]{a}-\sqrt[4]{b}\,x\right)-\log\left(\sqrt[4]{a}+\sqrt[4]{b}\,x\right)\right)}{4a^{3/4}b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x^2)/(a - b*x^4),x]
```

```
[Out] (2*(Sqrt[b]*c + Sqrt[a]*d)*ArcTan[(b^(1/4)*x)/a^(1/4)] - (Sqrt[b]*c - Sqrt[
a]*d)*(Log[a^(1/4) - b^(1/4)*x] - Log[a^(1/4) + b^(1/4)*x]))/(4*a^(3/4)*b^(
3/4))
```

Maple [A]
time = 0.14, size = 104, normalized size = 1.21

method	result	size
risch	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^4 b-a\right)}\frac{\left(-\_R^2 d+c\right)\ln\left(x-\_R\right)}{\_R^3}}{4b}$	37
default	$\frac{c\left(\frac{a}{b}\right)^{\frac{1}{4}}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4a}+\frac{d\left(2\arctan\left(\frac{x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x^2+c)/(-b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*c*(a/b)^(1/4)/a*(ln((x+(a/b)^(1/4))/(x-(a/b)^(1/4)))+2*arctan(x/(a/b)^(
1/4)))+1/4*d/b/(a/b)^(1/4)*(2*arctan(x/(a/b)^(1/4))-ln((x+(a/b)^(1/4))/(x-(
a/b)^(1/4))))
```

Maxima [A]

time = 0.50, size = 109, normalized size = 1.27

$$\frac{\left(\sqrt{b}\,c+\sqrt{a}\,d\right)\arctan\left(\frac{\sqrt{b}\,x}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{2\,\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\left(\sqrt{b}\,c-\sqrt{a}\,d\right)\log\left(\frac{\sqrt{b}\,x-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,x+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{4\,\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(b)*c + sqrt(a)*d)*arctan(sqrt(b)*x/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a
)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*(sqrt(b)*c - sqrt(a)*d)*log((sqrt(b)
*x - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*x + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(b))*sqrt(b))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(58) = 116.
time = 0.35, size = 755, normalized size = 8.78

$$\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\right)-\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x-\left(a^3b^2d\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-ab^2c^3-a^2bcd^2\right)\sqrt{-\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+2cd}{ab}}\right)-\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-2cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x+\left(a^3b^2d\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-2cd}{ab}}\right)+\frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-2cd}{ab}}\log\left(-(b^2c^4-a^2d^4)x-\left(a^3b^2d\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}+ab^2c^3+a^2bcd^2\right)\sqrt{\frac{ab\sqrt{\frac{b^2c^4+2abc^2d^2+a^2d^4}{a^3b^3}}-2cd}{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)
/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d
^2 + a^2*d^4)/(a^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^
4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt(-(a*b*s
qrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))*log(-(b^
2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a
^3*b^3)) - a*b^2*c^3 - a^2*b*c*d^2)*sqrt(-(a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^
2 + a^2*d^4)/(a^3*b^3)) + 2*c*d)/(a*b))) - 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*
a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*
x + (a^3*b^2*d*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*
c^3 + a^2*b*c*d^2)*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*
b^3)) - 2*c*d)/(a*b))) + 1/4*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*
d^4)/(a^3*b^3)) - 2*c*d)/(a*b))*log(-(b^2*c^4 - a^2*d^4)*x - (a^3*b^2*d*sqr
t((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) + a*b^2*c^3 + a^2*b*c*d^2)
*sqrt((a*b*sqrt((b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/(a^3*b^3)) - 2*c*d)/(a*
b)))
```

Sympy [A]
time = 0.37, size = 110, normalized size = 1.28

$$\operatorname{RootSum}\left(256t^4a^3b^3+64t^2a^2b^2cd-a^2d^4+2abc^2d^2-b^2c^4,\left(t\mapsto t\log\left(x+\frac{-64t^3a^3b^2d-12ta^2bcd^2-4tab^2c^3}{a^2d^4-b^2c^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x**2+c)/(-b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**3 + 64*_t**2*a**2*b**2*c*d - a**2*d**4 + 2*a*b*c*
*2*d**2 - b**2*c**4, Lambda(_t, _t*log(x + (-64*_t**3*a**3*b**2*d - 12*_t*a
**2*b*c*d**2 - 4*_t*a*b**2*c**3)/(a**2*d**4 - b**2*c**4))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(58) = 116$.
time = 3.67, size = 228, normalized size = 2.65

$$-\frac{\sqrt{2}\left(b^{2}c-\sqrt{-ab}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c+\sqrt{-ab}\,bd\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\left(-ab^{3}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\left(b^{2}c+\sqrt{-ab}\,bd\right)\log\left(x^{2}+\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\left(b^{2}c+\sqrt{-ab}\,bd\right)\log\left(x^{2}-\sqrt{2}\,x\left(-\frac{a}{b}\right)^{\frac{1}{4}}+\sqrt{-\frac{a}{b}}\right)}{8\left(-ab^{3}\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x^2+c)/(-b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*(b^2*c - sqrt(-a*b)*b*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(-a
/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^(3/4) - 1/4*sqrt(2)*(b^2*c + sqrt(-a*b)*b
*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(-a/b)^(1/4))/(-a/b)^(1/4))/(-a*b^3)^
(3/4) - 1/8*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*log(x^2 + sqrt(2)*x*(-a/b)^(1/
4) + sqrt(-a/b))/(-a*b^3)^(3/4) + 1/8*sqrt(2)*(b^2*c + sqrt(-a*b)*b*d)*log(
x^2 - sqrt(2)*x*(-a/b)^(1/4) + sqrt(-a/b))/(-a*b^3)^(3/4)
```

Mupad [B]
time = 4.58, size = 579, normalized size = 6.73

$$-2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{-\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}+\frac{8\,a\,b^2\,d^2\,x\,\sqrt{-\frac{c\,d}{8\,a\,b}-\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3+\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}+\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\,\sqrt{-\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}+2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}-2\,\mathrm{atanh}\left(\frac{8\,b^3\,c^2\,x\,\sqrt{\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}+\frac{8\,a\,b^2\,d^2\,x\,\sqrt{\frac{c^2\,\sqrt{a^3\,b^3}}{16\,a^3\,b^2}-\frac{c\,d}{8\,a\,b}+\frac{d^2\,\sqrt{a^3\,b^3}}{16\,a^2\,b^3}}}{2\,b^2\,c^2\,d+2\,a\,b\,d^3-\frac{2\,b\,c^3\,\sqrt{a^3\,b^3}}{a^2}-\frac{2\,c\,d^2\,\sqrt{a^3\,b^3}}{a}}\right)\,\sqrt{\frac{a\,d^2\,\sqrt{a^3\,b^3}+b\,c^2\,\sqrt{a^3\,b^3}-2\,a^2\,b^2\,c\,d}{16\,a^3\,b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x^2)/(a - b*x^4),x)
```

```
[Out] - 2*atanh((8*b^3*c^2*x*(- (c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2
) - (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (
2*b*c^3*(a^3*b^3)^(1/2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*
x*(- (c*d)/(8*a*b) - (c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (d^2*(a^3*b^3)^(1
/2))/(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 + (2*b*c^3*(a^3*b^3)^(1/
2))/a^2 + (2*c*d^2*(a^3*b^3)^(1/2))/a))*(-(a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a
^3*b^3)^(1/2) + 2*a^2*b^2*c*d)/(16*a^3*b^3))^(1/2) - 2*atanh((8*b^3*c^2*x*(
(c^2*(a^3*b^3)^(1/2))/(16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)^(1/2))/
(16*a^2*b^3))^(1/2))/(2*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2))/a
^2 - (2*c*d^2*(a^3*b^3)^(1/2))/a) + (8*a*b^2*d^2*x*((c^2*(a^3*b^3)^(1/2))/(
16*a^3*b^2) - (c*d)/(8*a*b) + (d^2*(a^3*b^3)^(1/2))/(16*a^2*b^3))^(1/2))/(2
*b^2*c^2*d + 2*a*b*d^3 - (2*b*c^3*(a^3*b^3)^(1/2))/a^2 - (2*c*d^2*(a^3*b^3)
^(1/2))/a))*((a*d^2*(a^3*b^3)^(1/2) + b*c^2*(a^3*b^3)^(1/2) - 2*a^2*b^2*c*d
)/(16*a^3*b^3))^(1/2)
```

3.5 $\int \frac{2+3x^2}{4+9x^4}\,dx$

Optimal. Leaf size=40

$$-\frac{\tan^{-1}\left(1-\sqrt{3}\,x\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(1+\sqrt{3}\,x\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctan(-1+x*3^(1/2))*3^(1/2)+1/6*arctan(1+x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1176, 631, 210}

$$\frac{\text{ArcTan}\left(\sqrt{3}\,x+1\right)}{2\sqrt{3}}-\frac{\text{ArcTan}\left(1-\sqrt{3}\,x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] -1/2*ArcTan[1 - Sqrt[3]*x]/Sqrt[3] + ArcTan[1 + Sqrt[3]*x]/(2*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\int \frac{2+3x^2}{4+9x^4}\,dx = \frac{1}{6}\int \frac{1}{\frac{2}{3}-\frac{2x}{\sqrt{3}}+x^2}\,dx + \frac{1}{6}\int \frac{1}{\frac{2}{3}+\frac{2x}{\sqrt{3}}+x^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{3}\,x\right)}{2\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{3}\,x\right)}{2\sqrt{3}}$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{3}\,x\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(1+\sqrt{3}\,x\right)}{2\sqrt{3}}$$

Mathematica [**A**]
time = 0.02, size = 33, normalized size = 0.82

$$\frac{-\tan^{-1}\left(1-\sqrt{3}\,x\right)+\tan^{-1}\left(1+\sqrt{3}\,x\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] (-ArcTan[1 - Sqrt[3]*x] + ArcTan[1 + Sqrt[3]*x])/(2*Sqrt[3])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(27) = 54.
time = 0.17, size = 140, normalized size = 3.50

method	result
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{3x^3\sqrt{3}}{4}+\frac{x\sqrt{3}}{2}\right)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{x\sqrt{3}}{2}\right)}{6}$
default	$\frac{\sqrt{6}\,\sqrt{2}\,\left(\ln\left(\frac{x^2+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^2-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}+1\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}-1\right)\right)}{48}+\frac{\sqrt{6}\,\sqrt{2}\,\left(\ln\left(\frac{x^2-\sqrt{\cdots}}{x^2+\sqrt{\cdots}}\right.\right.}{\cdots}$
meijerg	$\frac{\sqrt{6}\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{3}\,(x^4)^{\frac{1}{4}}+\frac{3\sqrt{x^4}}{2}\right)}{2(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{3}\,(x^4)^{\frac{1}{4}}}\right)}{(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{3}\,(x^4)^{\frac{1}{4}}+\frac{3\sqrt{x^4}}{2}\right)}{2(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\text{a}\cdots}{\cdots}\right.}{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(9*x^4+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*6^(1/2)*2^(1/2)*(ln((x^2+1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2-1/3*6^(1/2)*x
*2^(1/2)+2/3))+2*arctan(1/2*6^(1/2)*x*2^(1/2)+1)+2*arctan(1/2*6^(1/2)*x*2^(
```

```
1/2)-1))+1/48*6^(1/2)*2^(1/2)*(ln((x^2-1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2+1/3*
6^(1/2)*x*2^(1/2)+2/3))+2*arctan(1/2*6^(1/2)*x*2^(1/2)+1)+2*arctan(1/2*6^(1
/2)*x*2^(1/2)-1))
```

Maxima [A]
time = 0.51, size = 39, normalized size = 0.98

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(3x+\sqrt{3}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(3x-\sqrt{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(3*x + sqrt(3))) + 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(3*x - sqrt(3)))
```

Fricas [A]
time = 0.33, size = 33, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{4}\sqrt{3}\left(3x^3+2x\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/4*sqrt(3)*(3*x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/2*sqrt
(3)*x)
```

Sympy [A]
time = 0.04, size = 41, normalized size = 1.02

$$\frac{\sqrt{3}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{2}\right)+2\operatorname{atan}\left(\frac{3\sqrt{3}x^3}{4}+\frac{\sqrt{3}x}{2}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(9*x**4+4),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/2) + 2*atan(3*sqrt(3)*x**3/4 + sqrt(3)*x/2))/12
```

Giac [A]
time = 4.21, size = 52, normalized size = 1.30

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{9}{8}\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2x+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{9}{8}\sqrt{2}\left(\frac{4}{9}\right)^{\frac{3}{4}}\left(2x-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x + sqrt(2)*(4/9)^(1/4))) + 1
/6*sqrt(3)*arctan(9/8*sqrt(2)*(4/9)^(3/4)*(2*x - sqrt(2)*(4/9)^(1/4)))
```

Mupad [B]
time = 0.09, size = 29, normalized size = 0.72

$$\frac{\sqrt{3}\,\left(\mathrm{atan}\left(\frac{3\,\sqrt{3}\,x^3}{4}+\frac{\sqrt{3}\,x}{2}\right)+\mathrm{atan}\left(\frac{\sqrt{3}\,x}{2}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(9*x^4 + 4),x)
```

```
[Out] (3^(1/2)*(atan((3^(1/2)*x)/2 + (3*3^(1/2)*x^3)/4) + atan((3^(1/2)*x)/2)))/6
```

3.6 $\int \frac{2-3x^2}{4+9x^4}\,dx$

Optimal. Leaf size=51

$$-\frac{\log\left(2-2\sqrt{3}\,x+3x^2\right)}{4\sqrt{3}}+\frac{\log\left(2+2\sqrt{3}\,x+3x^2\right)}{4\sqrt{3}}$$

```
[Out] -1/12*ln(2+3*x^2-2*x*3^(1/2))*3^(1/2)+1/12*ln(2+3*x^2+2*x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1179, 642}

$$\frac{\log\left(3x^2+2\sqrt{3}\,x+2\right)}{4\sqrt{3}}-\frac{\log\left(3x^2-2\sqrt{3}\,x+2\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] -1/4*Log[2 - 2*Sqrt[3]*x + 3*x^2]/Sqrt[3] + Log[2 + 2*Sqrt[3]*x + 3*x^2]/(4
*Sqrt[3])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{2-3x^2}{4+9x^4}\,dx &= -\frac{\int \frac{\frac{2}{\sqrt{3}}+2x}{-\frac{2}{3}-\frac{2x}{\sqrt{3}}-x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\frac{2}{\sqrt{3}}-2x}{-\frac{2}{3}+\frac{2x}{\sqrt{3}}-x^2}\,dx}{4\sqrt{3}}\\ &= -\frac{\log\left(2-2\sqrt{3}\,x+3x^2\right)}{4\sqrt{3}}+\frac{\log\left(2+2\sqrt{3}\,x+3x^2\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.86

$$\frac{-\log\left(-2+2\sqrt{3}\,x-3x^2\right)+\log\left(2+2\sqrt{3}\,x+3x^2\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)/(4 + 9*x^4),x]
```

```
[Out] (-Log[-2 + 2*Sqrt[3]*x - 3*x^2] + Log[2 + 2*Sqrt[3]*x + 3*x^2])/(4*Sqrt[3])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(39) = 78.
time = 0.11, size = 140, normalized size = 2.75

method	result
risch	$-\frac{\ln\left(2+3x^2-2x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(2+3x^2+2x\sqrt{3}\right)\sqrt{3}}{12}$
default	$\frac{\sqrt{6}\,\sqrt{2}\,\left(\ln\left(\frac{x^2+\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}{x^2-\frac{\sqrt{6}\,x\sqrt{2}}{3}+\frac{2}{3}}\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}+1\right)+2\arctan\left(\frac{\sqrt{6}\,x\sqrt{2}}{2}-1\right)\right)}{48}-\frac{\sqrt{6}\,\sqrt{2}\,\left(\ln\left(\frac{x^2-\sqrt{}}{x^2+\sqrt{}}\right.\right.}{}$
meijerg	$\frac{\sqrt{6}\left(-\frac{x\sqrt{2}\,\ln\left(1-\sqrt{3}\,(x^4)^{\frac{1}{4}}+\frac{3\sqrt{x^4}}{2}\right)}{2(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{3}\,(x^4)^{\frac{1}{4}}}\right)}{(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\ln\left(1+\sqrt{3}\,(x^4)^{\frac{1}{4}}+\frac{3\sqrt{x^4}}{2}\right)}{2(x^4)^{\frac{1}{4}}}+\frac{x\sqrt{2}\,\mathrm{a}}{}\right.}{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+2)/(9*x^4+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*6^(1/2)*2^(1/2)*(ln((x^2+1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2-1/3*6^(1/2)*x
*2^(1/2)+2/3))+2*arctan(1/2*6^(1/2)*x*2^(1/2)+1)+2*arctan(1/2*6^(1/2)*x*2^(
1/2)-1))-1/48*6^(1/2)*2^(1/2)*(ln((x^2-1/3*6^(1/2)*x*2^(1/2)+2/3)/(x^2+1/3*
6^(1/2)*x*2^(1/2)+2/3))+2*arctan(1/2*6^(1/2)*x*2^(1/2)+1)+2*arctan(1/2*6^(1
/2)*x*2^(1/2)-1))
```

Maxima [A]
time = 0.51, size = 39, normalized size = 0.76

$$\frac{1}{12}\sqrt{3}\,\log\left(3\,x^2+2\,\sqrt{3}\,x+2\right)-\frac{1}{12}\sqrt{3}\,\log\left(3\,x^2-2\,\sqrt{3}\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*log(3*x^2 + 2*sqrt(3)*x + 2) - 1/12*sqrt(3)*log(3*x^2 - 2*sqrt
(3)*x + 2)
```

Fricas [A]
time = 0.35, size = 42, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\,\log\left(\frac{9\,x^4+24\,x^2+4\,\sqrt{3}\,(3\,x^3+2\,x)+4}{9\,x^4+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*log((9*x^4 + 24*x^2 + 4*sqrt(3)*(3*x^3 + 2*x) + 4)/(9*x^4 + 4)
)
```

Sympy [A]
time = 0.03, size = 49, normalized size = 0.96

$$-\frac{\sqrt{3}\,\log\left(x^2-\frac{2\sqrt{3}\,x}{3}+\frac{2}{3}\right)}{12}+\frac{\sqrt{3}\,\log\left(x^2+\frac{2\sqrt{3}\,x}{3}+\frac{2}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+2)/(9*x**4+4),x)
[Out] -sqrt(3)*log(x**2 - 2*sqrt(3)*x/3 + 2/3)/12 + sqrt(3)*log(x**2 + 2*sqrt(3)*
x/3 + 2/3)/12
```

Giac [A]
time = 3.70, size = 40, normalized size = 0.78

$$\frac{1}{12}\sqrt{3}\,\log\left(x^2+\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x+\frac{2}{3}\right)-\frac{1}{12}\sqrt{3}\,\log\left(x^2-\sqrt{2}\left(\frac{4}{9}\right)^{\frac{1}{4}}x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(9*x^4+4),x, algorithm="giac")
[Out] 1/12*sqrt(3)*log(x^2 + sqrt(2)*(4/9)^(1/4)*x + 2/3) - 1/12*sqrt(3)*log(x^2
- sqrt(2)*(4/9)^(1/4)*x + 2/3)
```

Mupad [B]
time = 4.43, size = 21, normalized size = 0.41

$$\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\,x^2+2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 - 2)/(9*x^4 + 4),x)
[Out] (3^(1/2)*atanh((2*3^(1/2)*x)/(3*x^2 + 2)))/6
```

3.7 $\int \frac{2+3x^2}{4-9x^4}\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {26, 212}

$$\frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] ArcTanh[Sqrt[3/2]*x]/Sqrt[6]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{4-9x^4}\,dx &= \int \frac{1}{2-3x^2}\,dx\\ &= \frac{\tanh^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 2.00

$$\frac{-\log\left(\sqrt{6}-3x\right)+\log\left(\sqrt{6}+3x\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(4 - 9*x^4),x]
[Out] (-Log[Sqrt[6] - 3*x] + Log[Sqrt[6] + 3*x])/(2*Sqrt[6])
```

Maple [A]
time = 0.16, size = 13, normalized size = 0.81

method	result
default	$\frac{\operatorname{arctanh}\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
risch	$\frac{\sqrt{6}\,\ln\left(3x+\sqrt{6}\right)}{12}-\frac{\sqrt{6}\,\ln\left(3x-\sqrt{6}\right)}{12}$
meijerg	$-\frac{\sqrt{6}\,x\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{\sqrt{3}\,\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2}\right)-2\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2}\right)\right)}{24(x^4)^{\frac{1}{4}}}-\frac{\sqrt{6}\,x^3\left(\ln\left(1-\sqrt{3}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(-9*x^4+4),x,method=_RETURNVERBOSE)
[Out] 1/6*arctanh(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.
time = 0.49, size = 25, normalized size = 1.56

$$-\frac{1}{12}\sqrt{6}\log\left(\frac{3x-\sqrt{6}}{3x+\sqrt{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
[Out] -1/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 0.35, size = 29, normalized size = 1.81

$$\frac{1}{12}\sqrt{6}\log\left(\frac{3x^2+2\sqrt{6}\,x+2}{3x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(15) = 30$.
time = 0.03, size = 32, normalized size = 2.00

$$-\frac{\sqrt{6}\log\left(x-\frac{\sqrt{6}}{3}\right)}{12}+\frac{\sqrt{6}\log\left(x+\frac{\sqrt{6}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(-9*x**4+4),x)
```

```
[Out] -sqrt(6)*log(x - sqrt(6)/3)/12 + sqrt(6)*log(x + sqrt(6)/3)/12
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 3.41, size = 29, normalized size = 1.81

$$\frac{1}{12}\sqrt{6}\log\left(\left|x+\frac{1}{3}\sqrt{6}\right|\right)-\frac{1}{12}\sqrt{6}\log\left(\left|x-\frac{1}{3}\sqrt{6}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*log(abs(x + 1/3*sqrt(6))) - 1/12*sqrt(6)*log(abs(x - 1/3*sqrt(
6)))
```

Mupad [B]
time = 0.09, size = 12, normalized size = 0.75

$$\frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 + 2)/(9*x^4 - 4),x)
```

```
[Out] (6^(1/2)*atanh((6^(1/2)*x)/2))/6
```

3.8 $\int \frac{2-3x^2}{4-9x^4}\,dx$

Optimal. Leaf size=16

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {26, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x^2)/(4 - 9*x^4),x]
```

```
[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2-3x^2}{4-9x^4}\,dx &= \int \frac{1}{2+3x^2}\,dx\\ &= \frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}\,x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x^2)/(4 - 9*x^4),x]
[Out] ArcTan[Sqrt[3/2]*x]/Sqrt[6]
```

Maple [A]
time = 0.16, size = 13, normalized size = 0.81

method	result
default	$\frac{\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
risch	$\frac{\arctan\left(\frac{x\sqrt{6}}{2}\right)\sqrt{6}}{6}$
meijerg	$-\frac{\sqrt{6}\,x\left(\ln\left(1-\frac{\sqrt{3}\,\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2}\right)-\ln\left(1+\frac{\sqrt{3}\,\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2}\right)-2\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2}\right)\right)}{24(x^4)^{\frac{1}{4}}}+\frac{\sqrt{6}\,x^3\left(\ln\left(1-\frac{\sqrt{3}}{\ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+2)/(-9*x^4+4),x,method=_RETURNVERBOSE)
[Out] 1/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A]
time = 0.50, size = 12, normalized size = 0.75

$$\frac{1}{6}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="maxima")
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Fricas [A]
time = 0.33, size = 12, normalized size = 0.75

$$\frac{1}{6}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.94

$$\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+2)/(-9*x**4+4),x)
```

```
[Out] sqrt(6)*atan(sqrt(6)*x/2)/6
```

Giac [A]
time = 3.40, size = 12, normalized size = 0.75

$$\frac{1}{6}\,\sqrt{6}\,\arctan\left(\frac{1}{2}\,\sqrt{6}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+2)/(-9*x^4+4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*x)
```

Mupad [B]
time = 0.03, size = 12, normalized size = 0.75

$$\frac{\sqrt{6}\,\operatorname{atan}\left(\frac{\sqrt{6}\,x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 2)/(9*x^4 - 4),x)
```

```
[Out] (6^(1/2)*atan((6^(1/2)*x)/2))/6
```

3.9 $\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx$

Optimal. Leaf size=75

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

```
[Out] 1/2*b^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)*2^(1/2)+1/2*b^(1/4
)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1176, 631, 210}

$$\frac{\sqrt[4]{b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

```
[Out] -((b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))) + (b
^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(Sqrt[2]*a^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{a}\sqrt{b}+bx^2}{a+bx^4}\,dx = \frac{1}{2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx+\frac{1}{2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx$$
$$= \frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$
$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 0.80

$$\frac{\sqrt[4]{b}\left(-\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)+\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)\right)}{\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a]*Sqrt[b] + b*x^2)/(a + b*x^4),x]
```

```
[Out] (b^(1/4)*(-ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + ArcTan[1 + (Sqrt[2]*b^
(1/4)*x)/a^(1/4)]))/(Sqrt[2]*a^(1/4))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(51) = 102.
time = 0.16, size = 204, normalized size = 2.72

method	result
default	$\frac{\sqrt{b}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8\sqrt{a}}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))+1/8/(a/b)^(1/4)*2^(1/2)*(ln((x^
```

```
2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)
))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Maxima [A]
time = 0.50, size = 100, normalized size = 1.33

$$\frac{\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}+\frac{\sqrt{2}\,\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*sqrt(b)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 1/2*sqrt(2)*sqrt(b)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/sqrt(sqrt(a)*sqrt(b))
```

Fricas [A]
time = 0.35, size = 148, normalized size = 1.97

$$\left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4-4\,\sqrt{a}\,\sqrt{b}\,x^2+4\,\sqrt{\frac{1}{2}}\left(\sqrt{a}\,\sqrt{b}\,x^3-ax\right)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right),\ \sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}\,x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\right)+\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\frac{\sqrt{\frac{1}{2}}\left(\sqrt{a}\,\sqrt{b}\,x^3+ax\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*log((b*x^4 - 4*sqrt(a)*sqrt(b)*x^2 +
4*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a)) + a)/(b*x^4
+ a)), sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(sqrt(b)/sqrt
(a))) + sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(b)*x
^3 + a*x)*sqrt(sqrt(b)/sqrt(a))/a)]
```

Sympy [A]
time = 0.17, size = 138, normalized size = 1.84

$$-\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\,\sqrt{a}\,x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\,\sqrt{a}\,x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}-\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a
))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(-sqrt(b)/sqrt(a))*log
(sqrt(2)*sqrt(a)*x*sqrt(-sqrt(b)/sqrt(a))/sqrt(b) - sqrt(a)/sqrt(b) + x**2)
/4
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 4.79, size = 57, normalized size = 0.76

$$\frac{\sqrt{2}\,b^{1/4}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,b^{1/4}\,x}{2\,a^{1/4}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,b^{3/4}\,x^3}{2\,a^{3/4}}+\frac{\sqrt{2}\,b^{1/4}\,x}{2\,a^{1/4}}\right)\right)}{4\,a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + a^(1/2)*b^(1/2))/(a + b*x^4),x)
```

```
[Out] (2^(1/2)*b^(1/4)*(2*atan((2^(1/2)*b^(1/4)*x)/(2*a^(1/4))) + 2*atan((2^(1/2)
*b^(3/4)*x^3)/(2*a^(3/4)) + (2^(1/2)*b^(1/4)*x)/(2*a^(1/4)))))/(4*a^(1/4))
```

3.10 $\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx$

Optimal. Leaf size=106

$$-\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{2\sqrt{2}\sqrt[4]{a}}+\frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

```
[Out] -1/4*b^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/4)*2^(
1/2)+1/4*b^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/4)*
2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1179, 642\}$

$$\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{2\sqrt{2}\sqrt[4]{a}}-\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{2\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
```

```
[Out] -1/2*(b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(Sqrt
[2]*a^(1/4)) + (b^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x
^2])/(2*Sqrt[2]*a^(1/4))
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt{a}\sqrt{b}-bx^2}{a+bx^4}\,dx = -\frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}} - \frac{\sqrt[4]{b}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{2\sqrt{2}\sqrt[4]{a}}$$

$$= -\frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{2\sqrt{2}\sqrt[4]{a}} + \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}\right.}{2\sqrt{2}\sqrt[4]{a}}$$

Mathematica [**A**]
time = 0.01, size = 91, normalized size = 0.86

$$\frac{\sqrt[4]{b}\left(-\log\left(-\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x-\sqrt{b}x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)\right)}{2\sqrt{2}\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a]*Sqrt[b] - b*x^2)/(a + b*x^4),x]
[Out] (b^(1/4)*(-Log[-Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x - Sqrt[b]*x^2] + Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]))/(2*Sqrt[2]*a^(1/4))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(70) = 140$.
time = 0.16, size = 204, normalized size = 1.92

method	result
default	$\frac{\sqrt{b}\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{8\sqrt{a}} - \frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}x}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/8/a^(1/2)*b^(1/2)*(a/b)^(1/4)*2^(1/2)*(ln((x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b
)^(1/2))/(x^2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1
/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))-1/8/(a/b)^(1/4)*2^(1/2)*(ln((x^
2-(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*x*2^(1/2)+(a/b)^(1/2)
))+2*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x-1))
```

Maxima [**A**]
time = 0.52, size = 70, normalized size = 0.66

$$\frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}} - \frac{\sqrt{2}\,b^{\frac{1}{4}}\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*b^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/
a^(1/4) - 1/4*sqrt(2)*b^(1/4)*log(sqrt(b)*x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x +
 sqrt(a))/a^(1/4)
```

Fricas [A]
time = 0.38, size = 151, normalized size = 1.42

$$\left[\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{bx^4+4\sqrt{a}\sqrt{b}x^2+4\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3+ax\right)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}+a}{bx^4+a}\right), -\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\sqrt{\frac{1}{2}}x\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\right)+\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\arctan\left(\frac{\sqrt{\frac{1}{2}}\left(\sqrt{a}\sqrt{b}x^3-ax\right)\sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/2)*sqrt(sqrt(b)/sqrt(a))*log((b*x^4 + 4*sqrt(a)*sqrt(b)*x^2 + 4
*sqrt(1/2)*(sqrt(a)*sqrt(b)*x^3 + a*x)*sqrt(sqrt(b)/sqrt(a)) + a)/(b*x^4 +
a)), -sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*x*sqrt(-sqrt(b)/sqr
t(a))) + sqrt(1/2)*sqrt(-sqrt(b)/sqrt(a))*arctan(sqrt(1/2)*(sqrt(a)*sqrt(b)
*x^3 - a*x)*sqrt(-sqrt(b)/sqrt(a))/a)]
```

Sympy [A]
time = 0.17, size = 131, normalized size = 1.24

$$-\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(-\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\log\left(\frac{\sqrt{2}\sqrt{a}x\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}{\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a**(1/2)*b**(1/2))/(b*x**4+a),x)
```

```
[Out] -sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(-sqrt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))
/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4 + sqrt(2)*sqrt(sqrt(b)/sqrt(a))*log(sq
rt(2)*sqrt(a)*x*sqrt(sqrt(b)/sqrt(a))/sqrt(b) + sqrt(a)/sqrt(b) + x**2)/4
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a^(1/2)*b^(1/2))/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 4.76, size = 43, normalized size = 0.41

$$\frac{\sqrt{2}\, b^{1/4} \operatorname{atanh}\left(\frac{2\sqrt{2}\, a^{1/4}\, b^{11/4}\, x}{2\sqrt{a}\, b^{5/2}+2\, b^3\, x^2}\right)}{2\, a^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^2 - a^(1/2)*b^(1/2))/(a + b*x^4),x)
```

```
[Out] (2^(1/2)*b^(1/4)*atanh((2*2^(1/2)*a^(1/4)*b^(11/4)*x)/(2*a^(1/2)*b^(5/2) +
2*b^3*x^2)))/(2*a^(1/4))
```

3.11 $\int \frac{d+ex^2}{d^2+e^2x^4} \, dx$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

```
[Out] 1/2*arctan(-1+x*2^(1/2)*e^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)+1/2*arctan
(1+x*2^(1/2)*e^(1/2)/d^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1176, 631, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])) + ArcT
an[1 + (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]]/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^2}{d^2+e^2x^4}\,dx &= \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{e}}+x^2}\,dx}{2e}+\frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{e}}+x^2}\,dx}{2e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}\\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 60, normalized size = 0.80

$$\frac{-\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)+\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] (-ArcTan[1 - (Sqrt[2]*Sqrt[e]*x)/Sqrt[d]] + ArcTan[1 + (Sqrt[2]*Sqrt[e]*x)/
Sqrt[d]])/(Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(51) = 102$.
time = 0.15, size = 232, normalized size = 3.09

method	result
risch	$-\frac{\sqrt{2}\,\ln\left(-dex\sqrt{2}+e\,x^2\sqrt{-de}-d\sqrt{-de}\right)}{4\sqrt{-de}}+\frac{\sqrt{2}\,\ln\left(dex\sqrt{2}+e\,x^2\sqrt{-de}-d\sqrt{-de}\right)}{4\sqrt{-de}}$
default	$\frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}}{x^2-\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8d}+\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x}{x^2+\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(1/e^2*d^2)^(1/4)*2^(1/2)*(ln((x^2+(1/e^2*d^2)^(1/4)*x*2^(1/2)+(1/e^2
*d^2)^(1/2))/(x^2-(1/e^2*d^2)^(1/4)*x*2^(1/2)+(1/e^2*d^2)^(1/2)))+2*arctan(
2^(1/2)/(1/e^2*d^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/e^2*d^2)^(1/4)*x-1))+1/8
/e/(1/e^2*d^2)^(1/4)*2^(1/2)*(ln((x^2-(1/e^2*d^2)^(1/4)*x*2^(1/2)+(1/e^2*d^
2)^(1/2))/(x^2+(1/e^2*d^2)^(1/4)*x*2^(1/2)+(1/e^2*d^2)^(1/2)))+2*arctan(2^(
1/2)/(1/e^2*d^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/e^2*d^2)^(1/4)*x-1))
```

Maxima [A]
time = 0.50, size = 74, normalized size = 0.99

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,xe+\sqrt{2}\sqrt{d}\,e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{2}\right)}}{2\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\sqrt{d}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,xe-\sqrt{2}\sqrt{d}\,e^{\frac{1}{2}}\right)e^{\left(-\frac{1}{2}\right)}}{2\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e + sqrt(2)*sqrt(d)*e^(1/2))*e^(-1/2)/s
qrt(d))*e^(-1/2)/sqrt(d) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x*e - sqrt(2)*
sqrt(d)*e^(1/2))*e^(-1/2)/sqrt(d))*e^(-1/2)/sqrt(d)
```

Fricas [A]
time = 0.37, size = 132, normalized size = 1.76

$$\left[-\frac{\sqrt{2}\sqrt{-de}\,e^{(-1)}\log\left(\frac{x^4e^2-4\,dx^2e-2\sqrt{2}\left(x^3e-dx\right)\sqrt{-de}+d^2}{x^4e^2+d^2}\right)}{4\,d},\ \frac{\left(\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\,xe^{\frac{1}{2}}}{2\sqrt{d}}\right)e^{\frac{1}{2}}+\sqrt{2}\sqrt{d}\arctan\left(\frac{\sqrt{2}\left(x^3e+dx\right)e^{\frac{1}{2}}}{2\,d^{\frac{3}{2}}}\right)e^{\frac{1}{2}}\right)e^{(-1)}}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(2)*sqrt(-d*e)*e^(-1)*log((x^4*e^2 - 4*d*x^2*e - 2*sqrt(2)*(x^3*e
 - d*x)*sqrt(-d*e) + d^2)/(x^4*e^2 + d^2))/d, 1/2*(sqrt(2)*sqrt(d)*arctan(1
/2*sqrt(2)*x*e^(1/2)/sqrt(d))*e^(1/2) + sqrt(2)*sqrt(d)*arctan(1/2*sqrt(2)*
(x^3*e + d*x)*e^(1/2)/d^(3/2))*e^(1/2))*e^(-1)/d]
```

Sympy [A]
time = 0.08, size = 87, normalized size = 1.16

$$-\frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(-\sqrt{2}\,dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4}+\frac{\sqrt{2}\sqrt{-\frac{1}{de}}\log\left(\sqrt{2}\,dx\sqrt{-\frac{1}{de}}-\frac{d}{e}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(-1/(d*e))*log(-sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4 + s
qrt(2)*sqrt(-1/(d*e))*log(sqrt(2)*d*x*sqrt(-1/(d*e)) - d/e + x**2)/4
```

Giac [A]
time = 3.88, size = 86, normalized size = 1.15

$$\frac{\sqrt{2}\,\sqrt{-de}\,e^{(-1)}\log\left(\sqrt{2}\,(d^2)^{\frac{1}{4}}xe^{\left(-\frac{1}{2}\right)}+x^2+\sqrt{d^2}\,e^{(-1)}\right)}{4\,d}-\frac{\sqrt{2}\,\sqrt{-de}\,e^{(-1)}\log\left(-\sqrt{2}\,(d^2)^{\frac{1}{4}}xe^{\left(-\frac{1}{2}\right)}+x^2+\sqrt{d^2}\,e^{(-1)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*sqrt(-d*e)*e^(-1)*log(sqrt(2)*(d^2)^(1/4)*x*e^(-1/2) + x^2 + sq
rt(d^2)*e^(-1))/d - 1/4*sqrt(2)*sqrt(-d*e)*e^(-1)*log(-sqrt(2)*(d^2)^(1/4)*
x*e^(-1/2) + x^2 + sqrt(d^2)*e^(-1))/d
```

Mupad [B]
time = 4.41, size = 57, normalized size = 0.76

$$\frac{\sqrt{2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{2\,\sqrt{d}}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,e^{3/2}\,x^3}{2\,d^{3/2}}+\frac{\sqrt{2}\,\sqrt{e}\,x}{2\,\sqrt{d}}\right)\right)}{4\,\sqrt{d}\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(d^2 + e^2*x^4),x)
```

```
[Out] (2^(1/2)*(2*atan((2^(1/2)*e^(1/2)*x)/(2*d^(1/2))) + 2*atan((2^(1/2)*e^(3/2)
*x^3)/(2*d^(3/2)) + (2^(1/2)*e^(1/2)*x)/(2*d^(1/2)))))/(4*d^(1/2)*e^(1/2))
```

3.12 $\int \frac{d-ex^2}{d^2+e^2x^4}\,dx$

Optimal. Leaf size=90

$$-\frac{\log\left(d-\sqrt{2}\,\sqrt{d}\,\sqrt{e}\,x+ex^2\right)}{2\sqrt{2}\,\sqrt{d}\,\sqrt{e}}+\frac{\log\left(d+\sqrt{2}\,\sqrt{d}\,\sqrt{e}\,x+ex^2\right)}{2\sqrt{2}\,\sqrt{d}\,\sqrt{e}}$$

```
[Out] -1/4*ln(d+e*x^2-x*2^(1/2)*d^(1/2)*e^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)+1/4*ln(d
+e*x^2+x*2^(1/2)*d^(1/2)*e^(1/2))*2^(1/2)/d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1179, 642}

$$\frac{\log\left(\sqrt{2}\,\sqrt{d}\,\sqrt{e}\,x+d+ex^2\right)}{2\sqrt{2}\,\sqrt{d}\,\sqrt{e}}-\frac{\log\left(-\sqrt{2}\,\sqrt{d}\,\sqrt{e}\,x+d+ex^2\right)}{2\sqrt{2}\,\sqrt{d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] -1/2*Log[d - Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(Sqrt[2]*Sqrt[d]*Sqrt[e]) +
 Log[d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x + e*x^2]/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{d-ex^2}{d^2+e^2x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{e}}+2x}{-\frac{d}{e}-\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{e}}-x^2}\,dx}{2\sqrt{2}\sqrt{d}\sqrt{e}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt{d}}{\sqrt{e}}-2x}{-\frac{d}{e}+\frac{\sqrt{2}\sqrt{d}\,x}{\sqrt{e}}-x^2}\,dx}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

$$= -\frac{\log\left(d-\sqrt{2}\sqrt{d}\sqrt{e}\,x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}} + \frac{\log\left(d+\sqrt{2}\sqrt{d}\sqrt{e}\,x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Mathematica [**A**]
time = 0.01, size = 75, normalized size = 0.83

$$\frac{-\log\left(-d+\sqrt{2}\sqrt{d}\sqrt{e}\,x-ex^2\right)+\log\left(d+\sqrt{2}\sqrt{d}\sqrt{e}\,x+ex^2\right)}{2\sqrt{2}\sqrt{d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 + e^2*x^4),x]
```

```
[Out] (-Log[-d + Sqrt[2]*Sqrt[d]*Sqrt[e]*x - e*x^2] + Log[d + Sqrt[2]*Sqrt[d]*Sqr
t[e]*x + e*x^2])/(2*Sqrt[2]*Sqrt[d]*Sqrt[e])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(62) = 124$.
time = 0.14, size = 232, normalized size = 2.58

method	result
risch	$\frac{\sqrt{2}\,\ln\left(dex\sqrt{2}+e\,x^2\sqrt{de}+d\sqrt{de}\right)}{4\sqrt{de}} - \frac{\sqrt{2}\,\ln\left(-dex\sqrt{2}+e\,x^2\sqrt{de}+d\sqrt{de}\right)}{4\sqrt{de}}$
default	$\frac{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}}{x^2-\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{d^2}{e^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8d} - \frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x}{x^2+\left(\frac{d^2}{e^2}\right)^{\frac{1}{4}}x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(1/e^2*d^2)^(1/4)*2^(1/2)*(ln((x^2+(1/e^2*d^2)^(1/4)*x*2^(1/2)+(1/e^2
*d^2)^(1/2))/(x^2-(1/e^2*d^2)^(1/4)*x*2^(1/2)+(1/e^2*d^2)^(1/2)))+2*arctan(
2^(1/2)/(1/e^2*d^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/e^2*d^2)^(1/4)*x-1))-1/8
/e/(1/e^2*d^2)^(1/4)*2^(1/2)*(ln((x^2-(1/e^2*d^2)^(1/4)*x*2^(1/2)+(1/e^2*d^
```

```
2)^(1/2))/(x^2+(1/e^2*d^2)^(1/4)*x*2^(1/2)+(1/e^2*d^2)^(1/2)))+2*arctan(2^(
1/2)/(1/e^2*d^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/e^2*d^2)^(1/4)*x-1))
```

Maxima [A]
time = 0.51, size = 60, normalized size = 0.67

$$\frac{\sqrt{2}\,e^{\left(-\frac{1}{2}\right)}\log\left(x^2e+\sqrt{2}\,\sqrt{d}\,xe^{\frac{1}{2}}+d\right)}{4\,\sqrt{d}}-\frac{\sqrt{2}\,e^{\left(-\frac{1}{2}\right)}\log\left(x^2e-\sqrt{2}\,\sqrt{d}\,xe^{\frac{1}{2}}+d\right)}{4\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*e^(-1/2)*log(x^2*e + sqrt(2)*sqrt(d)*x*e^(1/2) + d)/sqrt(d) - 1
/4*sqrt(2)*e^(-1/2)*log(x^2*e - sqrt(2)*sqrt(d)*x*e^(1/2) + d)/sqrt(d)
```

Fricas [A]
time = 0.35, size = 138, normalized size = 1.53

$$\left[\frac{\sqrt{2}\,e^{\left(-\frac{1}{2}\right)}\log\left(\frac{x^4e^2+4\,dx^2e+2\,\sqrt{2}\,(x^3e+dx)\,\sqrt{d}\,e^{\frac{1}{2}}+d^2}{x^4e^2+d^2}\right)}{4\,\sqrt{d}},-\frac{\left(\sqrt{2}\,\sqrt{-de}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-de}\,x}{2\,d}\right)-\sqrt{2}\,\sqrt{-de}\,\arctan\left(\frac{\sqrt{2}\,(x^3e-dx)\,\sqrt{-de}}{2\,d^2}\right)\right)e^{(-1)}}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*e^(-1/2)*log((x^4*e^2 + 4*d*x^2*e + 2*sqrt(2)*(x^3*e + d*x)*sq
rt(d)*e^(1/2) + d^2)/(x^4*e^2 + d^2))/sqrt(d), -1/2*(sqrt(2)*sqrt(-d*e)*arc
tan(1/2*sqrt(2)*sqrt(-d*e)*x/d) - sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(x^
3*e - d*x)*sqrt(-d*e)/d^2))*e^(-1)/d]
```

Sympy [A]
time = 0.08, size = 80, normalized size = 0.89

$$-\frac{\sqrt{2}\,\sqrt{\frac{1}{de}}\log\left(-\sqrt{2}\,dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}+\frac{\sqrt{2}\,\sqrt{\frac{1}{de}}\log\left(\sqrt{2}\,dx\sqrt{\frac{1}{de}}+\frac{d}{e}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4+d**2),x)
```

```
[Out] -sqrt(2)*sqrt(1/(d*e))*log(-sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4 + sqr
t(2)*sqrt(1/(d*e))*log(sqrt(2)*d*x*sqrt(1/(d*e)) + d/e + x**2)/4
```

Giac [A]
time = 3.45, size = 91, normalized size = 1.01

$$\frac{\sqrt{2}\,\sqrt{-de}\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(d^2)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}+2\,x\right)e^{\frac{1}{2}}}{2\,(d^2)^{\frac{1}{4}}}\right)e^{(-1)}}{2\,d}+\frac{\sqrt{2}\,\sqrt{-de}\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(d^2)^{\frac{1}{4}}e^{\left(-\frac{1}{2}\right)}-2\,x\right)e^{\frac{1}{2}}}{2\,(d^2)^{\frac{1}{4}}}\right)e^{(-1)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*sqrt(-d*e)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) + 2
*x)*e^(1/2)/(d^2)^(1/4))*e^(-1)/d + 1/2*sqrt(2)*sqrt(-d*e)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(d^2)^(1/4)*e^(-1/2) - 2*x)*e^(1/2)/(d^2)^(1/4))*e^(-1)/d
```

Mupad [B]
time = 0.09, size = 41, normalized size = 0.46

$$\frac{\sqrt{2}\ \mathrm{atanh}\left(\frac{2\sqrt{2}\ \sqrt{d}\ e^{7/2}\,x}{2\,e^4\,x^2+2\,d\,e^3}\right)}{2\,\sqrt{d}\ \sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(d^2 + e^2*x^4),x)
```

```
[Out] (2^(1/2)*atanh((2*2^(1/2)*d^(1/2)*e^(7/2)*x)/(2*d*e^3 + 2*e^4*x^2)))/(2*d^(
1/2)*e^(1/2))
```

3.13 $\int \frac{5+2x^2}{-1+x^4}\,dx$

Optimal. Leaf size=13

$$-\frac{3}{2}\tan^{-1}(x)-\frac{7}{2}\tanh^{-1}(x)$$

```
[Out] -3/2*arctan(x)-7/2*arctanh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1181, 213, 209}

$$-\frac{3\text{ArcTan}(x)}{2}-\frac{7}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x^2)/(-1 + x^4),x]
```

```
[Out] (-3*ArcTan[x])/2 - (7*ArcTanh[x])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1181

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- c*(d/(2*q)), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\int \frac{5+2x^2}{-1+x^4}\,dx&=-\left(\frac{3}{2}\int \frac{1}{1+x^2}\,dx\right)+\frac{7}{2}\int \frac{1}{-1+x^2}\,dx\\&=-\frac{3}{2}\tan^{-1}(x)-\frac{7}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.92

$$-\frac{3}{2}\tan^{-1}(x)+\frac{7}{4}\log(1-x)-\frac{7}{4}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x^2)/(-1 + x^4),x]
[Out] (-3*ArcTan[x])/2 + (7*Log[1 - x])/4 - (7*Log[1 + x])/4
```

Maple [A]
time = 0.16, size = 18, normalized size = 1.38

method	result	size
default	$\frac{7\ln(-1+x)}{4}-\frac{7\ln(1+x)}{4}-\frac{3\arctan(x)}{2}$	18
risch	$\frac{7\ln(-1+x)}{4}-\frac{7\ln(1+x)}{4}-\frac{3\arctan(x)}{2}$	18
meijerg	$\frac{5x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{x^3\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{2(x^4)^{\frac{3}{4}}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+5)/(x^4-1),x,method=_RETURNVERBOSE)
[Out] 7/4*ln(-1+x)-7/4*ln(1+x)-3/2*arctan(x)
```

Maxima [A]
time = 0.52, size = 17, normalized size = 1.31

$$-\frac{3}{2}\arctan(x)-\frac{7}{4}\log(x+1)+\frac{7}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="maxima")
[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

Fricas [A]
time = 0.33, size = 17, normalized size = 1.31

$$-\frac{3}{2}\arctan(x)-\frac{7}{4}\log(x+1)+\frac{7}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="fricas")
```

```
[Out] -3/2*arctan(x) - 7/4*log(x + 1) + 7/4*log(x - 1)
```

Sympy [A]
time = 0.06, size = 22, normalized size = 1.69

$$\frac{7\log\left(x-1\right)}{4}-\frac{7\log\left(x+1\right)}{4}-\frac{3\,\mathrm{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+5)/(x**4-1),x)
[Out] 7*log(x - 1)/4 - 7*log(x + 1)/4 - 3*atan(x)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(9) = 18$.
time = 3.21, size = 19, normalized size = 1.46

$$-\frac{3}{2}\arctan\left(x\right)-\frac{7}{4}\log\left(\left|x+1\right|\right)+\frac{7}{4}\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5)/(x^4-1),x, algorithm="giac")
[Out] -3/2*arctan(x) - 7/4*log(abs(x + 1)) + 7/4*log(abs(x - 1))
```

Mupad [B]
time = 0.04, size = 9, normalized size = 0.69

$$-\frac{3\,\mathrm{atan}(x)}{2}-\frac{7\,\mathrm{atanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 5)/(x^4 - 1),x)
[Out] - (3*atan(x))/2 - (7*atanh(x))/2
```

3.14 $\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx$

Optimal. Leaf size=16

$$\frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}$$

```
[Out] EllipticE(x*b^(1/2),I)/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1213, 435}

$$\frac{E\left(\text{ArcSin}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx &= \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx\\ &= \frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 45, normalized size = 2.81

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)+\frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + (b*x^3*Hypergeometric2F1[1/2,
 3/4, 7/4, b^2*x^4])/3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(12) = 24.
time = 0.14, size = 100, normalized size = 6.25

method	result
meijerg	$\frac{b\,x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],b^2x^4\right)$
default	$-\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(\mathrm{EllipticF}\left(\sqrt{b}\,x,i\right)-\mathrm{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^2x^4+1}}+\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\mathrm{EllipticF}\left(\sqrt{\right.}\right.}{\sqrt{b}\,\sqrt{-b^2x^4+1}}$
elliptic	$-\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(\mathrm{EllipticF}\left(\sqrt{b}\,x,i\right)-\mathrm{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^2x^4+1}}+\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\mathrm{EllipticF}\left(\sqrt{\right.}\right.}{\sqrt{b}\,\sqrt{-b^2x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(-b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/2)/(-b^2*x^4+1)^(1/2)*(EllipticF(b
^(1/2)*x,I)-EllipticE(b^(1/2)*x,I))+1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1
/2)/(-b^2*x^4+1)^(1/2)*EllipticF(b^(1/2)*x,I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

Fricas [A]
time = 0.10, size = 20, normalized size = 1.25

$$-\frac{\sqrt{-b^2x^4+1}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-b^2*x^4 + 1)/(b*x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(12) = 24$.
time = 0.96, size = 70, normalized size = 4.38

$$\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(4
*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(2
*I*pi))/(4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{b\,x^2+1}{\sqrt{1-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + 1)/(1 - b^2*x^4)^(1/2),x)
[Out] int((b*x^2 + 1)/(1 - b^2*x^4)^(1/2), x)
```

3.15 $\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx$

Optimal. Leaf size=35

$$-\frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}+\frac{2F\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}$$

```
[Out] -EllipticE(x*b^(1/2),I)/b^(1/2)+2*EllipticF(x*b^(1/2),I)/b^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1213, 434, 435, 254, 227}

$$\frac{2F\left(\text{ArcSin}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}-\frac{E\left(\text{ArcSin}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] -(EllipticE[ArcSin[Sqrt[b]*x], -1]/Sqrt[b]) + (2*EllipticF[ArcSin[Sqrt[b]*x
], -1])/Sqrt[b]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx &= \int \frac{\sqrt{1-bx^2}}{\sqrt{1+bx^2}}\,dx\\
&= 2\int \frac{1}{\sqrt{1-bx^2}\,\sqrt{1+bx^2}}\,dx - \int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx\\
&= -\frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}} + 2\int \frac{1}{\sqrt{1-b^2x^4}}\,dx\\
&= -\frac{E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}} + \frac{2F\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 45, normalized size = 1.29

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)-\frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[1 - b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] - (b*x^3*Hypergeometric2F1[1/2,
 3/4, 7/4, b^2*x^4])/3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(27) = 54.
time = 0.13, size = 99, normalized size = 2.83

method	result

meijerg	$-\frac{b\,x^3\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3}+x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],b^2x^4\right)$
default	$\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(\text{EllipticF}\left(\sqrt{b}\,x,i\right)-\text{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^2x^4+1}}+\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\text{EllipticF}\left(\sqrt{b}\right.}{\sqrt{b}\,\sqrt{-b^2x^4+1}}$
elliptic	$\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\left(\text{EllipticF}\left(\sqrt{b}\,x,i\right)-\text{EllipticE}\left(\sqrt{b}\,x,i\right)\right)}{\sqrt{b}\,\sqrt{-b^2x^4+1}}+\frac{\sqrt{-b\,x^2+1}\,\sqrt{b\,x^2+1}\,\text{EllipticF}\left(\sqrt{b}\right.}{\sqrt{b}\,\sqrt{-b^2x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/2)/(-b^2*x^4+1)^(1/2)*(EllipticF(b^
(1/2)*x,I)-EllipticE(b^(1/2)*x,I))+1/b^(1/2)*(-b*x^2+1)^(1/2)*(b*x^2+1)^(1/
2)/(-b^2*x^4+1)^(1/2)*EllipticF(b^(1/2)*x,I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

Fricas [A]
time = 0.09, size = 19, normalized size = 0.54

$$\frac{\sqrt{-b^2x^4+1}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-b^2*x^4 + 1)/(b*x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(27) = 54.
time = 0.96, size = 70, normalized size = 2.00

$$-\frac{bx^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\,b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\,b^2x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(-b**2*x**4+1)**(1/2),x)
```

```
[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(2*I*pi))/(
4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(
2*I*pi))/(4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{b\,x^2-1}{\sqrt{1-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^2 - 1)/(1 - b^2*x^4)^(1/2),x)
```

```
[Out] -int((b*x^2 - 1)/(1 - b^2*x^4)^(1/2), x)
```

3.16 $\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{-1+b^2x^4}}$$

```
[Out] EllipticE(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1214, 1213, 435}

$$\frac{\sqrt{1-b^2x^4}\,E\left(\text{ArcSin}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] (Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b^2
*x^4])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1+bx^2}{\sqrt{-1+b^2x^4}}\,dx = \frac{\sqrt{1-b^2x^4}\int \frac{1+bx^2}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}}$$

$$= \frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx}{\sqrt{-1+b^2x^4}}$$

$$= \frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{-1+b^2x^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 74, normalized size = 1.72

$$\frac{\sqrt{1-b^2x^4}\left(3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)+bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)\right)}{3\sqrt{-1+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] (Sqrt[1 - b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b*x^3*H
ypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/(3*Sqrt[-1 + b^2*x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(35) = 70.
time = 0.15, size = 107, normalized size = 2.49

method	result
meijerg	$\frac{b\sqrt{-\mathrm{signum}\left(b^2x^4-1\right)}\,x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3\sqrt{\mathrm{signum}\left(b^2x^4-1\right)}}+\frac{\sqrt{-\mathrm{signum}\left(b^2x^4-1\right)}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],b^2x^4\right)}{\sqrt{\mathrm{signum}\left(b^2x^4-1\right)}}$
default	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-b},i\right)-\mathrm{EllipticE}\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\mathrm{EllipticF}}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$
elliptic	$\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-b},i\right)-\mathrm{EllipticE}\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\mathrm{EllipticF}}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*(EllipticF(
x*(-b)^(1/2),I)-EllipticE(x*(-b)^(1/2),I))+1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b
*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*EllipticF(x*(-b)^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 0.91, size = 61, normalized size = 1.42

$$-\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) -
I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{b\,x^2+1}{\sqrt{b^2\,x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + 1)/(b^2*x^4 - 1)^(1/2),x)
[Out] int((b*x^2 + 1)/(b^2*x^4 - 1)^(1/2), x)
```

3.17 $\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$-\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{-1+b^2x^4}}+\frac{2\sqrt{1-b^2x^4}\,F\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{-1+b^2x^4}}$$

```
[Out] -EllipticE(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)+2*Elli
pticF(x*b^(1/2),I)*(-b^2*x^4+1)^(1/2)/b^(1/2)/(b^2*x^4-1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1214, 1213, 434, 435, 254, 227}

$$\frac{2\sqrt{1-b^2x^4}\,F\left(\text{ArcSin}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{b^2x^4-1}}-\frac{\sqrt{1-b^2x^4}\,E\left(\text{ArcSin}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{b^2x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] -((Sqrt[1 - b^2*x^4]*EllipticE[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]*Sqrt[-1 + b
^2*x^4])) + (2*Sqrt[1 - b^2*x^4]*EllipticF[ArcSin[Sqrt[b]*x], -1])/(Sqrt[b]
*Sqrt[-1 + b^2*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-bx^2}{\sqrt{-1+b^2x^4}}\,dx &= \frac{\sqrt{1-b^2x^4}\int \frac{1-bx^2}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= \frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1-bx^2}}{\sqrt{1+bx^2}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= -\frac{\sqrt{1-b^2x^4}\int \frac{\sqrt{1+bx^2}}{\sqrt{1-bx^2}}\,dx}{\sqrt{-1+b^2x^4}}+\frac{\left(2\sqrt{1-b^2x^4}\right)\int \frac{1}{\sqrt{1-bx^2}\sqrt{1+bx^2}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= -\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{-1+b^2x^4}}+\frac{\left(2\sqrt{1-b^2x^4}\right)\int \frac{1}{\sqrt{1-b^2x^4}}\,dx}{\sqrt{-1+b^2x^4}}\\
&= -\frac{\sqrt{1-b^2x^4}\,E\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{-1+b^2x^4}}+\frac{2\sqrt{1-b^2x^4}\,F\left(\sin^{-1}\left(\sqrt{b}\,x\right)\middle|-1\right)}{\sqrt{b}\,\sqrt{-1+b^2x^4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 74, normalized size = 0.83

$$-\frac{\sqrt{1-b^2x^4}\left(-3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};b^2x^4\right)+bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};b^2x^4\right)\right)}{3\sqrt{-1+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[-1 + b^2*x^4],x]
```

```
[Out] -1/3*(Sqrt[1 - b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, b^2*x^4] + b
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, b^2*x^4]))/Sqrt[-1 + b^2*x^4]
```

Maple [A]
time = 0.15, size = 108, normalized size = 1.21

method	result
meijerg	$-\frac{b\sqrt{-\mathrm{signum}\left(b^2x^4-1\right)}\, x^3 \,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],b^2x^4\right)}{3\sqrt{\mathrm{signum}\left(b^2x^4-1\right)}}+\frac{\sqrt{-\mathrm{signum}\left(b^2x^4-1\right)}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],b^2x^4\right.}{\sqrt{\mathrm{signum}\left(b^2x^4-1\right)}}$
default	$-\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-b},i\right)-\mathrm{EllipticE}\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\mathrm{EllipticF}}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$
elliptic	$-\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-b},i\right)-\mathrm{EllipticE}\left(x\sqrt{-b},i\right)\right)}{\sqrt{-b}\,\sqrt{b^2x^4-1}}+\frac{\sqrt{b\,x^2+1}\,\sqrt{-b\,x^2+1}\,\mathrm{EllipticF}}{\sqrt{-b}\,\sqrt{b^2x^4-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*(EllipticF
(x*(-b)^(1/2),I)-EllipticE(x*(-b)^(1/2),I))+1/(-b)^(1/2)*(b*x^2+1)^(1/2)*(-
b*x^2+1)^(1/2)/(b^2*x^4-1)^(1/2)*EllipticF(x*(-b)^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 0.91, size = 60, normalized size = 0.67

$$\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(b**2*x**4-1)**(1/2),x)
```

```
[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4)/(4*gamma(7/4)) - I
*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4)/(4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{b\,x^2-1}{\sqrt{b^2\,x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^2 - 1)/(b^2*x^4 - 1)^(1/2),x)
```

```
[Out] -int((b*x^2 - 1)/(b^2*x^4 - 1)^(1/2), x)
```

3.18 $\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$-\frac{x\sqrt{1+b^2x^4}}{1+bx^2}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{1+b^2x^4}}$$

```
[Out] -x*(b^2*x^4+1)^(1/2)/(b*x^2+1)+(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2)
/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*(
(b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x^4+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1210}

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,E\left(2\text{ArcTan}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{b^2x^4+1}}-\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] -((x*Sqrt[1 + b^2*x^4])/(1 + b*x^2)) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 +
 b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx=-\frac{x\sqrt{1+b^2x^4}}{1+bx^2}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{1+b^2x^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 47, normalized size = 0.53

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)-\frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] - (b*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 120, normalized size = 1.35

method	result
meijerg	$-\frac{b\,x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-b^2x^4\right)$
default	$-\frac{i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{ib},i\right)-\mathrm{EllipticE}\left(x\sqrt{ib},i\right)\right)}{\sqrt{ib}\,\sqrt{b^2x^4+1}}+\frac{\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\mathrm{Ellip}}{\sqrt{ib}\,\sqrt{b^2x^4+1}}$
elliptic	$-\frac{i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{ib},i\right)-\mathrm{EllipticE}\left(x\sqrt{ib},i\right)\right)}{\sqrt{ib}\,\sqrt{b^2x^4+1}}+\frac{\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\mathrm{Ellip}}{\sqrt{ib}\,\sqrt{b^2x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I/(I*b)^(1/2)*(1-I*b*x^2)^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*(Ellip
ticF(x*(I*b)^(1/2),I)-EllipticE(x*(I*b)^(1/2),I))+1/(I*b)^(1/2)*(1-I*b*x^2)
^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*EllipticF(x*(I*b)^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.89, size = 66, normalized size = 0.74

$$-\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] -b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I*
pi))/(4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(b^2*x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{b\,x^2-1}{\sqrt{b^2\,x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^2 - 1)/(b^2*x^4 + 1)^(1/2),x)
```

```
[Out] -int((b*x^2 - 1)/(b^2*x^4 + 1)^(1/2), x)
```

3.19 $\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx$

Optimal. Leaf size=152

$$\frac{x\sqrt{1+b^2x^4}}{1+bx^2}-\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{1+b^2x^4}}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,F\left(2\tan^{-1}\left(\sqrt{b}\,x\right)\right.}{\sqrt{b}\,\sqrt{1+b^2x^4}}$$

```
[Out] x*(b^2*x^4+1)^(1/2)/(b*x^2+1)-(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2)/
cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*((
b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x^4+1)^(1/2)+(b*x^2+1)*(cos(2*ar
ctan(x*b^(1/2)))^2)^(1/2)/cos(2*arctan(x*b^(1/2)))*EllipticF(sin(2*arctan(x
*b^(1/2))),1/2*2^(1/2))*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(b^2*x^4+1)
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1212, 226, 1210}

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,F\left(2\text{ArcTan}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{b^2x^4+1}}-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,E\left(2\text{ArcTan}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{b^2x^4+1}}+\frac{x\sqrt{b^2x^4+1}}{bx^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] (x*Sqrt[1 + b^2*x^4])/(1 + b*x^2) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*
x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4]) +
((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt[b]*x
], 1/2])/(Sqrt[b]*Sqrt[1 + b^2*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1+bx^2}{\sqrt{1+b^2x^4}}\,dx = 2\int \frac{1}{\sqrt{1+b^2x^4}}\,dx - \int \frac{1-bx^2}{\sqrt{1+b^2x^4}}\,dx$$

$$= \frac{x\sqrt{1+b^2x^4}}{1+bx^2} - \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)\,|\,\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{1+b^2x^4}} + \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)}}}{\sqrt{b}\,\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 47, normalized size = 0.31

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)+\frac{1}{3}bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[1 + b^2*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + (b*x^3*Hypergeometric2F1[1
/2, 3/4, 7/4, -(b^2*x^4)])/3
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 120, normalized size = 0.79

method	result
meijerg	$\frac{b\,x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-b^2x^4\right)$
default	$\frac{i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{ib}\,,i\right)-\mathrm{EllipticE}\left(x\sqrt{ib}\,,i\right)\right)}{\sqrt{ib}\,\sqrt{b^2x^4+1}}+\frac{\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\mathrm{EllipticF}}{\sqrt{ib}\,\sqrt{b^2x^4+1}}$
elliptic	$\frac{i\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{ib}\,,i\right)-\mathrm{EllipticE}\left(x\sqrt{ib}\,,i\right)\right)}{\sqrt{ib}\,\sqrt{b^2x^4+1}}+\frac{\sqrt{-ib\,x^2+1}\,\sqrt{ib\,x^2+1}\,\mathrm{EllipticF}}{\sqrt{ib}\,\sqrt{b^2x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(b^2*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I/(I*b)^(1/2)*(1-I*b*x^2)^(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*(Ellipt
icF(x*(I*b)^(1/2),I)-EllipticE(x*(I*b)^(1/2),I))+1/(I*b)^(1/2)*(1-I*b*x^2)^
(1/2)*(1+I*b*x^2)^(1/2)/(b^2*x^4+1)^(1/2)*EllipticF(x*(I*b)^(1/2),I)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 0.88, size = 66, normalized size = 0.43

$$\frac{bx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(b**2*x**4+1)**(1/2),x)
```

```
[Out] b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4*g
amma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar(I*p
i))/(4*gamma(5/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(b^2*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(b^2*x^4 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2+1}{\sqrt{b^2\,x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + 1)/(b^2*x^4 + 1)^(1/2),x)
```

```
[Out] int((b*x^2 + 1)/(b^2*x^4 + 1)^(1/2), x)
```

3.20 $\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{x\sqrt{-1-b^2x^4}}{1+bx^2}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-1-b^2x^4}}$$

```
[Out] x*(-b^2*x^4-1)^(1/2)/(b*x^2+1)+(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2)
/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*(
(b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b^2*x^4-1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1210}

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,E\left(2\text{ArcTan}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-b^2x^4-1}}+\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] (x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b
*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx=\frac{x\sqrt{-1-b^2x^4}}{1+bx^2}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-1-b^2x^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 76, normalized size = 0.84

$$-\frac{\sqrt{1+b^2x^4}\left(-3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)+bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)\right)}{3\sqrt{-1-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] -1/3*(Sqrt[1 + b^2*x^4]*(-3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)]
+ b*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/Sqrt[-1 - b^2*x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 122, normalized size = 1.36

method	result
meijerg	$-\frac{b\sqrt{\operatorname{signum}\left(b^2x^4+1\right)}\,x^3\operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3\sqrt{-\operatorname{signum}\left(b^2x^4+1\right)}}+\frac{\sqrt{\operatorname{signum}\left(b^2x^4+1\right)}\,x\operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-b^2x^4\right)}{\sqrt{-\operatorname{signum}\left(b^2x^4+1\right)}}$
default	$\frac{i\sqrt{ibx^2+1}\,\sqrt{-ibx^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-ib},i\right)-\operatorname{EllipticE}\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+\frac{\sqrt{ibx^2+1}\,\sqrt{-ibx^2+1}\,\text{Elli}}{\sqrt{-ib}\,\sqrt{-b^2x^4-}}$
elliptic	$\frac{i\sqrt{ibx^2+1}\,\sqrt{-ibx^2+1}\,\left(\operatorname{EllipticF}\left(x\sqrt{-ib},i\right)-\operatorname{EllipticE}\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+\frac{\sqrt{ibx^2+1}\,\sqrt{-ibx^2+1}\,\text{Elli}}{\sqrt{-ib}\,\sqrt{-b^2x^4-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I/(-I*b)^(1/2)*(1+I*b*x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/2)*(Elli
pticF(x*(-I*b)^(1/2),I)-EllipticE(x*(-I*b)^(1/2),I))+1/(-I*b)^(1/2)*(1+I*b*
x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/2)*EllipticF(x*(-I*b)^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.93, size = 70, normalized size = 0.78

$$\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(4
*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_polar
(I*pi))/(4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^2 - 1)/sqrt(-b^2*x^4 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{b\,x^2-1}{\sqrt{-b^2\,x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^2 - 1)/(- b^2*x^4 - 1)^(1/2),x)
```

```
[Out] -int((b*x^2 - 1)/(- b^2*x^4 - 1)^(1/2), x)
```

3.21 $\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx$

Optimal. Leaf size=156

$$-\frac{x\sqrt{-1-b^2x^4}}{1+bx^2}-\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-1-b^2x^4}}+\frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,F\left(2\tan^{-1}\left(\sqrt{b}\,x\right.\right.}{\sqrt{b}\,\sqrt{-1-b^2x^4}}$$

```
[Out] -x*(-b^2*x^4-1)^(1/2)/(b*x^2+1)-(b*x^2+1)*(cos(2*arctan(x*b^(1/2)))^2)^(1/2
)/cos(2*arctan(x*b^(1/2)))*EllipticE(sin(2*arctan(x*b^(1/2))),1/2*2^(1/2))*
((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b^2*x^4-1)^(1/2)+(b*x^2+1)*(cos(2
*arctan(x*b^(1/2)))^2)^(1/2)/cos(2*arctan(x*b^(1/2)))*EllipticF(sin(2*arcta
n(x*b^(1/2))),1/2*2^(1/2))*((b^2*x^4+1)/(b*x^2+1)^2)^(1/2)/b^(1/2)/(-b^2*x^
4-1)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1212, 226, 1210}

$$\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,F\left(2\text{ArcTan}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-b^2x^4-1}}-\frac{(bx^2+1)\sqrt{\frac{b^2x^4+1}{(bx^2+1)^2}}\,E\left(2\text{ArcTan}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\,\sqrt{-b^2x^4-1}}-\frac{x\sqrt{-b^2x^4-1}}{bx^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] -((x*Sqrt[-1 - b^2*x^4])/(1 + b*x^2)) - ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1
+ b*x^2)^2]*EllipticE[2*ArcTan[Sqrt[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4
]) + ((1 + b*x^2)*Sqrt[(1 + b^2*x^4)/(1 + b*x^2)^2]*EllipticF[2*ArcTan[Sqrt
[b]*x], 1/2])/(Sqrt[b]*Sqrt[-1 - b^2*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1+bx^2}{\sqrt{-1-b^2x^4}}\,dx = 2\int \frac{1}{\sqrt{-1-b^2x^4}}\,dx - \int \frac{1-bx^2}{\sqrt{-1-b^2x^4}}\,dx$$

$$= -\frac{x\sqrt{-1-b^2x^4}}{1+bx^2} - \frac{(1+bx^2)\sqrt{\frac{1+b^2x^4}{(1+bx^2)^2}}\,E\left(2\tan^{-1}\left(\sqrt{b}\,x\right)|\frac{1}{2}\right)}{\sqrt{b}\sqrt{-1-b^2x^4}} + \frac{(1+bx^2)\sqrt{\frac{1}{(1}}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 76, normalized size = 0.49

$$\frac{\sqrt{1+b^2x^4}\left(3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-b^2x^4\right)+bx^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-b^2x^4\right)\right)}{3\sqrt{-1-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^2)/Sqrt[-1 - b^2*x^4],x]
```

```
[Out] (Sqrt[1 + b^2*x^4]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -(b^2*x^4)] + b*x^
3*Hypergeometric2F1[1/2, 3/4, 7/4, -(b^2*x^4)]))/(3*Sqrt[-1 - b^2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 122, normalized size = 0.78

method	result
meijerg	$\frac{b\sqrt{\mathrm{signum}\left(b^2x^4+1\right)}\,x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-b^2x^4\right)}{3\sqrt{-\mathrm{signum}\left(b^2x^4+1\right)}}+\frac{\sqrt{\mathrm{signum}\left(b^2x^4+1\right)}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-b^2x^4\right)}{\sqrt{-\mathrm{signum}\left(b^2x^4+1\right)}}$
default	$-\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-ib},i\right)-\mathrm{EllipticE}\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}}{\sqrt{-ib}\,\sqrt{-b^2x}}$
elliptic	$-\frac{i\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{-ib},i\right)-\mathrm{EllipticE}\left(x\sqrt{-ib},i\right)\right)}{\sqrt{-ib}\,\sqrt{-b^2x^4-1}}+\frac{\sqrt{ib\,x^2+1}\,\sqrt{-ib\,x^2+1}}{\sqrt{-ib}\,\sqrt{-b^2x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+1)/(-b^2*x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I/(-I*b)^(1/2)*(1+I*b*x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/2)*(Ell
ipticF(x*(-I*b)^(1/2),I)-EllipticE(x*(-I*b)^(1/2),I))+1/(-I*b)^(1/2)*(1+I*b
*x^2)^(1/2)*(1-I*b*x^2)^(1/2)/(-b^2*x^4-1)^(1/2)*EllipticF(x*(-I*b)^(1/2),I
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.89, size = 71, normalized size = 0.46

$$-\frac{ibx^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle| b^2x^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+1)/(-b**2*x**4-1)**(1/2),x)
```

```
[Out] -I*b*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b**2*x**4*exp_polar(I*pi))/(
4*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b**2*x**4*exp_pola
r(I*pi))/(4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+1)/(-b^2*x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 1)/sqrt(-b^2*x^4 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\,x^2+1}{\sqrt{-b^2\,x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2 + 1)/(- b^2*x^4 - 1)^(1/2),x)
```

```
[Out] int((b*x^2 + 1)/(- b^2*x^4 - 1)^(1/2), x)
```

3.22 $\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{E(\sin^{-1}(cx)|-1)}{c}$$

```
[Out] EllipticE(c*x,I)/c
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{435\}$

$$\frac{E(\text{ArcSin}(cx)|-1)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx = \frac{E(\sin^{-1}(cx)|-1)}{c}$$

Mathematica [A]
time = 0.41, size = 10, normalized size = 1.00

$$\frac{E(\sin^{-1}(cx)|-1)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + c^2*x^2]/Sqrt[1 - c^2*x^2],x]
```

```
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.18, size = 15, normalized size = 1.50

method	result
default	$\frac{\mathrm{EllipticE}(x\,\mathrm{csgn}(c)c,i)\mathrm{csgn}(c)}{c}$
elliptic	$\frac{\sqrt{-c^4x^4+1}\left(\frac{\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\sqrt{-c^4x^4+1}}-\frac{\sqrt{-c^2x^2+1}\sqrt{c^2x^2+1}\left(\mathrm{EllipticF}\left(x\sqrt{c^2},\ldots\right.\right.}{\sqrt{c^2}\sqrt{-c^4x^4+1}}\right.}{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticE(x*csgn(c)*c,I)*csgn(c)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(9) = 18.
time = 0.09, size = 31, normalized size = 3.10

$$-\frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c^2x^2+1}}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)**(1/2)/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(c**2*x**2 + 1)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)^(1/2)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 + 1)/sqrt(-c^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{\sqrt{c^2\,x^2+1}}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)^(1/2)/(1 - c^2*x^2)^(1/2),x)
[Out] int((c^2*x^2 + 1)^(1/2)/(1 - c^2*x^2)^(1/2), x)
```

3.23 $\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=10

$$\frac{E(\sin^{-1}(cx)|-1)}{c}$$

```
[Out] EllipticE(c*x,I)/c
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1213, 435}

$$\frac{E(\text{ArcSin}(cx)|-1)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]
[Out] EllipticE[ArcSin[c*x], -1]/c
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+c^2x^2}{\sqrt{1-c^4x^4}}\,dx &= \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx\\ &= \frac{E(\sin^{-1}(cx)|-1)}{c}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 47, normalized size = 4.70

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^4x^4\right)+\frac{1}{3}c^2x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] + (c^2*x^3*Hypergeometric2F1[1/
2, 3/4, 7/4, c^4*x^4])/3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(10) = 20$.
time = 0.15, size = 118, normalized size = 11.80

method	result
meijerg	$\frac{c^2x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],c^4x^4\right)}{3}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],c^4x^4\right)$
default	$\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}-\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{c^2},i\right)-\mathrm{EllipticE}\left(x\right.\right.}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}$
elliptic	$\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}-\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{c^2},i\right)-\mathrm{EllipticE}\left(x\right.\right.}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*Ellip
ticF(x*(c^2)^(1/2),I)-1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-
c^4*x^4+1)^(1/2)*(EllipticF(x*(c^2)^(1/2),I)-EllipticE(x*(c^2)^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.
time = 0.09, size = 20, normalized size = 2.00

$$-\frac{\sqrt{-c^4x^4+1}}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-c^4*x^4 + 1)/(c^2*x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(5) = 10$.
time = 0.93, size = 71, normalized size = 7.10

$$\frac{c^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi))
/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_pola
r(2*I*pi))/(4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*x^2 + 1)/sqrt(-c^4*x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{c^2\,x^2+1}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2 + 1)/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((c^2*x^2 + 1)/(1 - c^4*x^4)^(1/2), x)
```

3.24 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{E(\sin^{-1}(cx)|-1)}{c}+\frac{2F(\sin^{-1}(cx)|-1)}{c}$$

[Out] -EllipticE(c*x,I)/c+2*EllipticF(c*x,I)/c

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {434, 435, 254, 227}

$$\frac{2F(\text{ArcSin}(cx)|-1)}{c}-\frac{E(\text{ArcSin}(cx)|-1)}{c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]

[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx &= 2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}\,dx - \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{E(\sin^{-1}(cx)|-1)}{c} + 2\int \frac{1}{\sqrt{1-c^4x^4}}\,dx \\
&= -\frac{E(\sin^{-1}(cx)|-1)}{c} + \frac{2F(\sin^{-1}(cx)|-1)}{c}
\end{aligned}
$$

Mathematica [A]
time = 0.45, size = 24, normalized size = 1.04

$$\frac{E\left(\sin^{-1}\left(\sqrt{-c^2}\,x\right)\middle|-1\right)}{\sqrt{-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
[Out] EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 28, normalized size = 1.22

method	result
default	$\frac{(2\,\mathrm{EllipticF}(x\,\mathrm{csgn}(c)c,i)-\mathrm{EllipticE}(x\,\mathrm{csgn}(c)c,i))\mathrm{csgn}(c)}{c}$
elliptic	$\frac{\sqrt{-c^4x^4+1}\left(\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}+\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{c^2},\right.\right.}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}\right.}{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] (2*EllipticF(x*csgn(c)*c,I)-EllipticE(x*csgn(c)*c,I))*csgn(c)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Fricas [A]
time = 0.08, size = 30, normalized size = 1.30

$$\frac{\sqrt{c^2x^2+1}\,\sqrt{-c^2x^2+1}}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(c^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{\sqrt{c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-c^2\,x^2}}{\sqrt{c^2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2),x)
[Out] int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2), x)
```

3.25 $\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}} \, dx$

Optimal. Leaf size=23

$$-\frac{E(\sin^{-1}(cx)|-1)}{c}+\frac{2F(\sin^{-1}(cx)|-1)}{c}$$

```
[Out] -EllipticE(c*x,I)/c+2*EllipticF(c*x,I)/c
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1213, 434, 435, 254, 227}

$$\frac{2F(\text{ArcSin}(cx)|-1)}{c}-\frac{E(\text{ArcSin}(cx)|-1)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-c^2x^2}{\sqrt{1-c^4x^4}}\,dx &= \int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx\\ &= 2\int \frac{1}{\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}\,dx - \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx\\ &= -\frac{E(\sin^{-1}(cx)|-1)}{c} + 2\int \frac{1}{\sqrt{1-c^4x^4}}\,dx\\ &= -\frac{E(\sin^{-1}(cx)|-1)}{c} + \frac{2F(\sin^{-1}(cx)|-1)}{c}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 47, normalized size = 2.04

$$x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};c^4x^4\right) - \frac{1}{3}c^2x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};c^4x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - c^2*x^2)/Sqrt[1 - c^4*x^4],x]
```

```
[Out] x*Hypergeometric2F1[1/4, 1/2, 5/4, c^4*x^4] - (c^2*x^3*Hypergeometric2F1[1/
2, 3/4, 7/4, c^4*x^4])/3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(23) = 46.
time = 0.14, size = 117, normalized size = 5.09

method	result
meijerg	$-\frac{c^2x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],c^4x^4\right)}{3} + x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],c^4x^4\right)$
default	$\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}} + \frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{c^2},i\right)-\mathrm{EllipticE}\left(x\right.\right.}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}$

elliptic	$\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{c^2}\,,i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}+\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(\mathrm{EllipticF}\left(x\sqrt{c^2}\,,i\right)-\mathrm{EllipticE}\left(\right.\right.}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^4*x^4+1)^(1/2)*Ellip
ticF(x*(c^2)^(1/2),I)+1/(c^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/(-
c^4*x^4+1)^(1/2)*(EllipticF(x*(c^2)^(1/2),I)-EllipticE(x*(c^2)^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

Fricas [A]
time = 0.08, size = 19, normalized size = 0.83

$$\frac{\sqrt{-c^4x^4+1}}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-c^4*x^4 + 1)/(c^2*x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(14) = 28.
time = 0.95, size = 71, normalized size = 3.09

$$-\frac{c^2x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|c^4x^4e^{2i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] -c**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c**4*x**4*exp_polar(2*I*pi)
)/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c**4*x**4*exp_pol
ar(2*I*pi))/(4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(c^2*x^2 - 1)/sqrt(-c^4*x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{c^2\,x^2-1}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(c^2*x^2 - 1)/(1 - c^4*x^4)^(1/2),x)
[Out] -int((c^2*x^2 - 1)/(1 - c^4*x^4)^(1/2), x)
```

3.26 $\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx$

Optimal. Leaf size=82

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}+\frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

```
[Out] -arctan((-2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+arctan((2
*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2de-b}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2de-b}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[-b + 2*d*e] - 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]) + Arc
Tan[(Sqrt[-b + 2*d*e] + 2*e*x)/Sqrt[b + 2*d*e]]/Sqrt[b + 2*d*e]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{d+ex^2}{d^2+bx^2+e^2x^4}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{-b+2de}\,x}{e}+x^2}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{-b+2de}\,x}{e}+x^2}\,dx}{2e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{b+2de}{e^2}-x^2}\,dx, x, -\frac{\sqrt{-b+2de}}{e}+2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{b+2de}{e^2}-x^2}\,dx, x, \frac{\sqrt{-b}}{}\right.}{e}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}-2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}} + \frac{\tan^{-1}\left(\frac{\sqrt{-b+2de}+2ex}{\sqrt{b+2de}}\right)}{\sqrt{b+2de}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(82) = 164.
time = 0.07, size = 181, normalized size = 2.21

$$\frac{\frac{\left(-b+2de+\sqrt{b^2-4d^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}} + \frac{\left(b-2de+\sqrt{b^2-4d^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\,\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] (((-b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b
^2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] + ((b - 2*d*e + Sqrt[b^2
 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[
b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A]
time = 0.04, size = 71, normalized size = 0.87

method	result	si
default	$-\frac{\arctan\left(\frac{-2ex+\sqrt{2de-b}}{\sqrt{2de+b}}\right)}{\sqrt{2de+b}}+\frac{\arctan\left(\frac{2ex+\sqrt{2de-b}}{\sqrt{2de+b}}\right)}{\sqrt{2de+b}}$	71
risch	$-\frac{\ln\left(-e\,x^2\sqrt{-2de-b}+(2de+b)x+d\sqrt{-2de-b}\right)}{2\sqrt{-2de-b}}+\frac{\ln\left(-e\,x^2\sqrt{-2de-b}+(-2de-b)x+d\sqrt{-2de-b}\right)}{2\sqrt{-2de-b}}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan((-2*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+arctan((2
*e*x+(2*d*e-b)^(1/2))/(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/(x^4*e^2 + b*x^2 + d^2), x)
```

Fricas [A]
time = 0.34, size = 174, normalized size = 2.12

$$\left[-\frac{\sqrt{-2\,de-b}\,\log\left(\frac{x^4e^2-4\,dx^2e-bx^2+d^2-2\,(x^3e-dx)\sqrt{-2\,de-b}}{x^4e^2+bx^2+d^2}\right)}{2\,(2\,de+b)}, \frac{\sqrt{2\,de+b}\,\arctan\left(\frac{xe}{\sqrt{2\,de+b}}\right)+\sqrt{2\,de+b}\,\arctan\left(\frac{(x^3e^2+dxe+bx)\sqrt{2\,de+b}}{2\,d^2e+bd}\right)}{2\,de+b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e - b)*log((x^4*e^2 - 4*d*x^2*e - b*x^2 + d^2 - 2*(x^3*e -
d*x)*sqrt(-2*d*e - b))/(x^4*e^2 + b*x^2 + d^2))/(2*d*e + b), (sqrt(2*d*e +
b)*arctan(x*e/sqrt(2*d*e + b)) + sqrt(2*d*e + b)*arctan((x^3*e^2 + d*x*e +
b*x)*sqrt(2*d*e + b)/(2*d^2*e + b*d)))/(2*d*e + b)]
```

Sympy [A]
time = 0.29, size = 122, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b+2de}}-2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{b+2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b+2de}}+2de\sqrt{-\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(-1/(b + 2*d*e)) - 2*d*e*
sqrt(-1/(b + 2*d*e)))/e)/2 + sqrt(-1/(b + 2*d*e))*log(-d/e + x**2 + x*(b*sq
rt(-1/(b + 2*d*e)) + 2*d*e*sqrt(-1/(b + 2*d*e)))/e)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(81) = 162$.
time = 5.33, size = 1079, normalized size = 13.16

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")

[Out] (2*d^2*e^3 - b*d*e^2 + d*e^4)*sqrt(2*d*e + b)*arctan(2*sqrt(1/2)*x*e/sqrt(b + sqrt(-4*d^2*e^2 + b^2)))/(4*d^3*e^4 - b^2*d*e^2 + 2*d^2*e^5 + b*d*e^4) - 1/4*(8*sqrt(2*d*e^3 + b*e^2)*d^3*e^3 - 8*sqrt(-2*d*e^3 + b*e^2)*d^3*e^3 + 4*sqrt(2*d*e^3 + b*e^2)*b*d^2*e^2 - 4*sqrt(-2*d*e^3 + b*e^2)*b*d^2*e^2 - 2*sqrt(2*d*e^3 + b*e^2)*b^2*d*e + 2*sqrt(-2*d*e^3 + b*e^2)*b^2*d*e - 24*d^3*e^5 - 12*b*d^2*e^4 + 6*b^2*d*e^3 + 3*b^3*e^2 - 4*sqrt(-4*d^2*e^2 + b^2)*sqrt(2*d*e^3 + b*e^2)*d^2*e^2 + 4*sqrt(-4*d^2*e^2 + b^2)*sqrt(-2*d*e^3 + b*e^2)*d^2*e^2 - 4*sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*d^2*e^2 - sqrt(2*d*e^3 + b*e^2)*b^3 + sqrt(-2*d*e^3 + b*e^2)*b^3 + 4*sqrt(-4*d^2*e^2 + b^2)*d^2*e^4 + 4*sqrt(2*d*e^3 + b*e^2)*d^2*e^4 + 4*sqrt(-2*d*e^3 + b*e^2)*d^2*e^4 + 4*sqrt(2*d*e^3 + b*e^2)*b*d*e^3 - sqrt(-4*d^2*e^2 + b^2)*b^2*e^2 + sqrt(2*d*e^3 + b*e^2)*b^2*e^2 - sqrt(-2*d*e^3 + b*e^2)*b^2*e^2 - 6*sqrt(-4*d^2*e^2 + b^2)*sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*d*e + sqrt(-4*d^2*e^2 + b^2)*sqrt(2*d*e^3 + b*e^2)*b^2 - sqrt(-4*d^2*e^2 + b^2)*sqrt(-2*d*e^3 + b*e^2)*b^2 + sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*b^2 - 12*d^2*e^6 + 3*b^2*e^4 - 2*sqrt(-4*d^2*e^2 + b^2)*sqrt(2*d*e^3 + b*e^2)*d*e^3 + 2*sqrt(-4*d^2*e^2 + b^2)*sqrt(-2*d*e^3 + b*e^2)*d*e^3 - 2*sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*d*e^3 - 3*sqrt(-4*d^2*e^2 + b^2)*sqrt(2*d*e^3 + b*e^2)*b*e^2 + 3*sqrt(-4*d^2*e^2 + b^2)*sqrt(-2*d*e^3 + b*e^2)*b*e^2 - 3*sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*b*e^2 - 3*sqrt(-4*d^2*e^2 + b^2)*sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*b + 2*sqrt(-4*d^2*e^2 + b^2)*d*e^5 + 4*sqrt(-2*d*e^3 + b*e^2)*d*e^5 + 3*sqrt(-4*d^2*e^2 + b^2)*b*e^4 - 2*sqrt(2*d*e^3 + b*e^2)*b*e^4 + 2*sqrt(-2*d*e^3 + b*e^2)*b*e^4 - 3*sqrt(-4*d^2*e^2 + b^2)*sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*e^2)*arctan(2*sqrt(1/2)*x*e/sqrt(b - sqrt(-4*d^2*e^2 + b^2)))/(8*d^4*e^5 + 4*b*d^3*e^4 - 2*b^2*d^2*e^3 - b^3*d*e^2 + 4*b*d^2*e^5 + 2*b^2*d*e^4 - 2*d^2*e^7 - b*d*e^6)

Mupad [B]
time = 4.43, size = 94, normalized size = 1.15

$$\frac{\operatorname{atan}\left(\frac{e\,x}{\sqrt{b+2\,d\,e}}\right)+\operatorname{atan}\left(\frac{b^2\,x-\frac{x\,{\left(b+2\,d\,e\right)}^2}{2}+\frac{b\,x\,\left(b+2\,d\,e\right)}{2}+2\,b\,e^2\,x^3-e^2\,x^3\,\left(b+2\,d\,e\right)}{\left(b\,d-2\,d^2\,e\right)\,\sqrt{b+2\,d\,e}}\right)}{\sqrt{b+2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)/(b*x^2 + d^2 + e^2*x^4),x)

[Out] (atan((e*x)/(b + 2*d*e)^(1/2)) + atan((b^2*x - (x*(b + 2*d*e)^2)/2 + (b*x*(b + 2*d*e))/2 + 2*b*e^2*x^3 - e^2*x^3*(b + 2*d*e))/((b*d - 2*d^2*e)*(b + 2*d*e)^(1/2))))/(b + 2*d*e)^(1/2)

3.27 $\int \frac{d+ex^2}{d^2+fx^2+e^2x^4} \, dx$

Optimal. Leaf size=82

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

```
[Out] -arctan((-2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+arctan((2
*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2*d*e - f] - 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]) + ArcT
an[(Sqrt[2*d*e - f] + 2*e*x)/Sqrt[2*d*e + f]]/Sqrt[2*d*e + f]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
```

```
0]))
```

Rubi steps

$$\int \frac{d+ex^2}{d^2+fx^2+e^2x^4}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de-f}\,x}{e}+x^2}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de-f}\,x}{e}+x^2}\,dx}{2e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2}-x^2}\,dx, x, -\frac{\sqrt{2de-f}}{e}+2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de+f}{e^2}-x^2}\,dx, x, \frac{\sqrt{2de}}{e}\right.}{e}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2de-f}-2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}} + \frac{\tan^{-1}\left(\frac{\sqrt{2de-f}+2ex}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 181 vs. 2(82) = 164.
time = 0.07, size = 181, normalized size = 2.21

$$\frac{\frac{\left(2de-f+\sqrt{-4d^2e^2+f^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}} + \frac{\left(-2de+f+\sqrt{-4d^2e^2+f^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\,\sqrt{-4d^2e^2+f^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e - f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[-
4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e + f + Sqrt[
-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]])/
Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A]
time = 0.04, size = 71, normalized size = 0.87

method	result	siz
default	$-\frac{\arctan\left(\frac{-2ex+\sqrt{2de-f}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}+\frac{\arctan\left(\frac{2ex+\sqrt{2de-f}}{\sqrt{2de+f}}\right)}{\sqrt{2de+f}}$	71

risch	$-\frac{\ln\left(e\,x^{2}\sqrt{-2de-f}+(-2de-f)x-d\sqrt{-2de-f}\right)}{2\sqrt{-2de-f}}+\frac{\ln\left(e\,x^{2}\sqrt{-2de-f}+(2de+f)x-d\sqrt{-2de-f}\right)}{2\sqrt{-2de-f}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan((-2*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+arctan((2
*e*x+(2*d*e-f)^(1/2))/(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/(x^4*e^2 + f*x^2 + d^2), x)
```

Fricas [A]
time = 0.34, size = 174, normalized size = 2.12

$$\left[-\frac{\sqrt{-2\,de-f}\,\log\left(\frac{x^{4}e^{2}-4\,dx^{2}e-fx^{2}+d^{2}-2\,(x^{3}e-dx)\,\sqrt{-2\,de-f}}{x^{4}e^{2}+fx^{2}+d^{2}}\right)}{2\,(2\,de+f)},\ \frac{\sqrt{2\,de+f}\,\arctan\left(\frac{xe}{\sqrt{2\,de+f}}\right)+\sqrt{2\,de+f}\,\arctan\left(\frac{(x^{3}e^{2}+dxe+fx)\,\sqrt{2\,de+f}}{2\,d^{2}e+df}\right)}{2\,de+f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e - f)*log((x^4*e^2 - 4*d*x^2*e - f*x^2 + d^2 - 2*(x^3*e -
d*x)*sqrt(-2*d*e - f))/(x^4*e^2 + f*x^2 + d^2))/(2*d*e + f), (sqrt(2*d*e +
f)*arctan(x*e/sqrt(2*d*e + f)) + sqrt(2*d*e + f)*arctan((x^3*e^2 + d*x*e +
f*x)*sqrt(2*d*e + f)/(2*d^2*e + d*f)))/(2*d*e + f)]
```

Sympy [A]
time = 0.29, size = 122, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^{2}+\frac{x\left(-2de\sqrt{-\frac{1}{2de+f}}-f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{2de+f}}\log\left(-\frac{d}{e}+x^{2}+\frac{x\left(2de\sqrt{-\frac{1}{2de+f}}+f\sqrt{-\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e + f)) - f*
sqrt(-1/(2*d*e + f)))/e)/2 + sqrt(-1/(2*d*e + f))*log(-d/e + x**2 + x*(2*d*
e*sqrt(-1/(2*d*e + f)) + f*sqrt(-1/(2*d*e + f)))/e)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(81) = 162.
time = 7.05, size = 173, normalized size = 2.11

$$-\frac{(2\,d^2e^3 - dfe^2 + de^4)\sqrt{2\,de + f}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,xe}{\sqrt{f + \sqrt{-4\,d^2e^2 + f^2}}}\right)}{4\,d^3e^4 - df^2e^2 + 2\,d^2e^5 + dfe^4} - \frac{(2\,d^2e^3 - dfe^2 + de^4)\sqrt{2\,de + f}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,xe}{\sqrt{f - \sqrt{-4\,d^2e^2 + f^2}}}\right)}{4\,d^3e^4 - df^2e^2 + 2\,d^2e^5 + dfe^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")
```

```
[Out] -(2*d^2*e^3 - d*f*e^2 + d*e^4)*sqrt(2*d*e + f)*arctan(2*sqrt(1/2)*x*e/sqrt(
f + sqrt(-4*d^2*e^2 + f^2)))/(4*d^3*e^4 - d*f^2*e^2 + 2*d^2*e^5 + d*f*e^4)
- (2*d^2*e^3 - d*f*e^2 + d*e^4)*sqrt(2*d*e + f)*arctan(2*sqrt(1/2)*x*e/sqrt
(f - sqrt(-4*d^2*e^2 + f^2)))/(4*d^3*e^4 - d*f^2*e^2 + 2*d^2*e^5 + d*f*e^4)
```

Mupad [B]
time = 4.52, size = 98, normalized size = 1.20

$$\frac{\mathrm{atan}\left(\frac{f^2\,x-\frac{x\,(f+2\,d\,e)^2}{2}+\frac{f\,x\,(f+2\,d\,e)}{2}+2\,e^2\,f\,x^3-e^2\,x^3\,(f+2\,d\,e)}{(2\,d\,f-d\,(f+2\,d\,e))\,\sqrt{f+2\,d\,e}}\right)+\mathrm{atan}\left(\frac{e\,x}{\sqrt{f+2\,d\,e}}\right)}{\sqrt{f+2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(f*x^2 + d^2 + e^2*x^4),x)
```

```
[Out] (atan((f^2*x - (x*(f + 2*d*e)^2)/2 + (f*x*(f + 2*d*e))/2 + 2*e^2*f*x^3 - e^
2*x^3*(f + 2*d*e))/((2*d*f - d*(f + 2*d*e))*(f + 2*d*e)^(1/2))) + atan((e*x
)/(f + 2*d*e)^(1/2)))/(f + 2*d*e)^(1/2)
```

3.28 $\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

```
[Out] arctanh((-2*e*x+(2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2)-arctanh
((2*e*x+(2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2))/(-2*d*e+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1175, 632, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[b + 2*d*e] - 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e] - ArcTan
h[(Sqrt[b + 2*d*e] + 2*e*x)/Sqrt[b - 2*d*e]]/Sqrt[b - 2*d*e]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{d+ex^2}{d^2-bx^2+e^2x^4}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{b+2de}\,x}{e}+x^2}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{b+2de}\,x}{e}+x^2}\,dx}{2e}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2}-x^2}\,dx, x, -\frac{\sqrt{b+2de}}{e}+2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{\frac{b-2de}{e^2}-x^2}\,dx, x, \frac{\sqrt{b+2de}}{e}\right.}{e}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}-2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b+2de}+2ex}{\sqrt{b-2de}}\right)}{\sqrt{b-2de}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(78) = 156.
time = 0.06, size = 189, normalized size = 2.42

$$\frac{\frac{\left(b+2de+\sqrt{b^2-4d^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}} + \frac{\left(-b-2de+\sqrt{b^2-4d^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] (((b + 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b - Sqrt[b
^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]] + ((-b - 2*d*e + Sqrt[b
^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]]])/Sq
rt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A]
time = 0.04, size = 75, normalized size = 0.96

method	result	size
default	$\frac{\arctan\left(\frac{2ex+\sqrt{2de+b}}{\sqrt{2de-b}}\right)}{\sqrt{2de-b}} - \frac{\arctan\left(\frac{-2ex+\sqrt{2de+b}}{\sqrt{2de-b}}\right)}{\sqrt{2de-b}}$	75
risch	$\frac{\ln\left(e\,x^2\sqrt{-2de+b}+(2de-b)x-d\sqrt{-2de+b}\right)}{2\sqrt{-2de+b}} - \frac{\ln\left(e\,x^2\sqrt{-2de+b}+(-2de+b)x-d\sqrt{-2de+b}\right)}{2\sqrt{-2de+b}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2*d*e-b)^(1/2)*arctan((2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))-1/(2*d*e-
b)^(1/2)*arctan((-2*e*x+(2*d*e+b)^(1/2))/(2*d*e-b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/(x^4*e^2 - b*x^2 + d^2), x)
```

Fricas [A]
time = 0.34, size = 184, normalized size = 2.36

$$\left[-\frac{\sqrt{-2\,de+b}\log\left(\frac{x^4e^2-4\,dx^2e+bx^2+d^2-2\,(x^3e-dx)\sqrt{-2\,de+b}}{x^4e^2-bx^2+d^2}\right)}{2\,(2\,de-b)},\frac{\sqrt{2\,de-b}\arctan\left(\frac{xe}{\sqrt{2\,de-b}}\right)+\sqrt{2\,de-b}\arctan\left(\frac{(x^3e^2+dxe-bx)\sqrt{2\,de-b}}{2\,d^2e-bd}\right)}{2\,de-b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e + b)*log((x^4*e^2 - 4*d*x^2*e + b*x^2 + d^2 - 2*(x^3*e -
d*x)*sqrt(-2*d*e + b))/(x^4*e^2 - b*x^2 + d^2))/(2*d*e - b), (sqrt(2*d*e -
b)*arctan(x*e/sqrt(2*d*e - b)) + sqrt(2*d*e - b)*arctan((x^3*e^2 + d*x*e -
b*x)*sqrt(2*d*e - b)/(2*d^2*e - b*d)))/(2*d*e - b)]
```

Sympy [A]
time = 0.30, size = 110, normalized size = 1.41

$$\frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{\frac{1}{b-2de}}+2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}-\frac{\sqrt{\frac{1}{b-2de}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(b\sqrt{\frac{1}{b-2de}}-2de\sqrt{\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(-b*sqrt(1/(b - 2*d*e)) + 2*d*e*sqr
t(1/(b - 2*d*e)))/e)/2 - sqrt(1/(b - 2*d*e))*log(-d/e + x**2 + x*(b*sqrt(1/
(b - 2*d*e)) - 2*d*e*sqrt(1/(b - 2*d*e)))/e)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(77) = 154.

time = 6.75, size = 181, normalized size = 2.32

$$-\frac{\left(2\,d^2e^3+bde^2+de^4\right)\sqrt{2\,de-b}\ \arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{-\left(b+\sqrt{-4\,d^2e^2+b^2}\right)e^{(-2)}}}\right)}{4\,d^3e^4-b^2de^2+2\,d^2e^5-bde^4}-\frac{\left(2\,d^2e^3+bde^2+de^4\right)\sqrt{2\,de-b}\ \arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{-\left(b-\sqrt{-4\,d^2e^2+b^2}\right)e^{(-2)}}}\right)}{4\,d^3e^4-b^2de^2+2\,d^2e^5-bde^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")
```

```
[Out] -(2*d^2*e^3 + b*d*e^2 + d*e^4)*sqrt(2*d*e - b)*arctan(2*sqrt(1/2)*x/sqrt(-(
b + sqrt(-4*d^2*e^2 + b^2))*e^(-2)))/(4*d^3*e^4 - b^2*d*e^2 + 2*d^2*e^5 - b
*d*e^4) - (2*d^2*e^3 + b*d*e^2 + d*e^4)*sqrt(2*d*e - b)*arctan(2*sqrt(1/2)*
x/sqrt(-(b - sqrt(-4*d^2*e^2 + b^2))*e^(-2)))/(4*d^3*e^4 - b^2*d*e^2 + 2*d^
2*e^5 - b*d*e^4)
```

Mupad [B]
time = 0.13, size = 30, normalized size = 0.38

$$\frac{\mathrm{atanh}\left(\frac{x\,\sqrt{b-2\,d\,e}}{d-e\,x^2}\right)}{\sqrt{b-2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(d^2 - b*x^2 + e^2*x^4),x)
```

```
[Out] atanh((x*(b - 2*d*e)^(1/2))/(d - e*x^2))/(b - 2*d*e)^(1/2)
```

3.29 $\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx$

Optimal. Leaf size=86

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

```
[Out] -arctan((-2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+arctan((2
*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2*d*e + f] - 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]) + ArcT
an[(Sqrt[2*d*e + f] + 2*e*x)/Sqrt[2*d*e - f]]/Sqrt[2*d*e - f]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
```

```
0]))
```

Rubi steps

$$\int \frac{d+ex^2}{d^2-fx^2+e^2x^4}\,dx = \frac{\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2de+f}\,x}{e}+x^2}\,dx}{2e} + \frac{\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2de+f}\,x}{e}+x^2}\,dx}{2e}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2}-x^2}\,dx, x, -\frac{\sqrt{2de+f}}{e}+2x\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{2de-f}{e^2}-x^2}\,dx, x, \frac{\sqrt{2de}}{e}\right.}{e}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2de+f}-2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}} + \frac{\tan^{-1}\left(\frac{\sqrt{2de+f}+2ex}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(86) = 172.
time = 0.06, size = 189, normalized size = 2.20

$$\frac{\frac{\left(2de+f+\sqrt{-4d^2e^2+f^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}} + \frac{\left(-2de-f+\sqrt{-4d^2e^2+f^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\,\sqrt{-4d^2e^2+f^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f - Sqrt[
-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]] + ((-2*d*e - f + Sqr
t[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]]
])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A]
time = 0.04, size = 75, normalized size = 0.87

method	result	siz
default	$-\frac{\arctan\left(\frac{-2ex+\sqrt{2de+f}}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}+\frac{\arctan\left(\frac{2ex+\sqrt{2de+f}}{\sqrt{2de-f}}\right)}{\sqrt{2de-f}}$	75

risch	$-\frac{\ln\left(e\,x^2\sqrt{-2de+f}+(-2de+f)x-d\sqrt{-2de+f}\right)}{2\sqrt{-2de+f}}+\frac{\ln\left(e\,x^2\sqrt{-2de+f}+(2de-f)x-d\sqrt{-2de+f}\right)}{2\sqrt{-2de+f}}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan((-2*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+arctan((2
*e*x+(2*d*e+f)^(1/2))/(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/(x^4*e^2 - f*x^2 + d^2), x)
```

Fricas [A]
time = 0.35, size = 184, normalized size = 2.14

$$\left[-\frac{\sqrt{-2\,de+f}\,\log\left(\frac{x^4e^2-4\,dx^2e+fx^2+d^2-2\,(x^3e-dx)\,\sqrt{-2\,de+f}}{x^4e^2-fx^2+d^2}\right)}{2\,(2\,de-f)},\ \frac{\sqrt{2\,de-f}\,\arctan\left(\frac{xe}{\sqrt{2\,de-f}}\right)+\sqrt{2\,de-f}\,\arctan\left(\frac{(x^3e^2+dxe-fx)\,\sqrt{2\,de-f}}{2\,d^2e-df}\right)}{2\,de-f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*d*e + f)*log((x^4*e^2 - 4*d*x^2*e + f*x^2 + d^2 - 2*(x^3*e -
d*x)*sqrt(-2*d*e + f))/(x^4*e^2 - f*x^2 + d^2))/(2*d*e - f), (sqrt(2*d*e -
f)*arctan(x*e/sqrt(2*d*e - f)) + sqrt(2*d*e - f)*arctan((x^3*e^2 + d*x*e -
f*x)*sqrt(2*d*e - f)/(2*d^2*e - d*f)))/(2*d*e - f)]
```

Sympy [A]
time = 0.29, size = 121, normalized size = 1.41

$$-\frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{-\frac{1}{2de-f}}+f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}+\frac{\sqrt{-\frac{1}{2de-f}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{-\frac{1}{2de-f}}-f\sqrt{-\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
```

```
[Out] -sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(-2*d*e*sqrt(-1/(2*d*e - f)) + f*
sqrt(-1/(2*d*e - f)))/e)/2 + sqrt(-1/(2*d*e - f))*log(-d/e + x**2 + x*(2*d*
e*sqrt(-1/(2*d*e - f)) - f*sqrt(-1/(2*d*e - f)))/e)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(85) = 170.
time = 4.12, size = 181, normalized size = 2.10

$$-\frac{\left(2\,d^{2}e^{3}+dfe^{2}+de^{4}\right)\sqrt{2\,de-f}\,\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}\,x}{\sqrt{-\left(f+\sqrt{-4\,d^{2}e^{2}+f^{2}}\right)e^{(-2)}}}\right)}{4\,d^{3}e^{4}-df^{2}e^{2}+2\,d^{2}e^{5}-dfe^{4}}-\frac{\left(2\,d^{2}e^{3}+dfe^{2}+de^{4}\right)\sqrt{2\,de-f}\,\arctan\left(\frac{2\,\sqrt{\frac{1}{2}}\,x}{\sqrt{-\left(f-\sqrt{-4\,d^{2}e^{2}+f^{2}}\right)e^{(-2)}}}\right)}{4\,d^{3}e^{4}-df^{2}e^{2}+2\,d^{2}e^{5}-dfe^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")
```

```
[Out] -(2*d^2*e^3 + d*f*e^2 + d*e^4)*sqrt(2*d*e - f)*arctan(2*sqrt(1/2)*x/sqrt(-(
f + sqrt(-4*d^2*e^2 + f^2))*e^(-2)))/(4*d^3*e^4 - d*f^2*e^2 + 2*d^2*e^5 - d
*f*e^4) - (2*d^2*e^3 + d*f*e^2 + d*e^4)*sqrt(2*d*e - f)*arctan(2*sqrt(1/2)*
x/sqrt(-(f - sqrt(-4*d^2*e^2 + f^2))*e^(-2)))/(4*d^3*e^4 - d*f^2*e^2 + 2*d^
2*e^5 - d*f*e^4)
```

Mupad [B]
time = 4.39, size = 88, normalized size = 1.02

$$-\frac{\mathrm{atan}\left(\frac{e^2\,x^3\,\sqrt{2\,d\,e-f}-f\,x\,\sqrt{2\,d\,e-f}+d\,e\,x\,\sqrt{2\,d\,e-f}}{d\,\left(f-2\,d\,e\right)}\right)-\mathrm{atan}\left(\frac{e\,x}{\sqrt{2\,d\,e-f}}\right)}{\sqrt{2\,d\,e-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(d^2 - f*x^2 + e^2*x^4),x)
```

```
[Out] -(atan((e^2*x^3*(2*d*e - f)^(1/2) - f*x*(2*d*e - f)^(1/2) + d*e*x*(2*d*e -
f)^(1/2))/(d*(f - 2*d*e))) - atan((e*x)/(2*d*e - f)^(1/2)))/(2*d*e - f)^(1/
2)
```

3.30 $\int \frac{d-ex^2}{d^2+bx^2+e^2x^4} \, dx$

Optimal. Leaf size=78

$$-\frac{\log \left(d-\sqrt{-b+2de}\,x+ex^2\right)}{2\sqrt{-b+2de}}+\frac{\log \left(d+\sqrt{-b+2de}\,x+ex^2\right)}{2\sqrt{-b+2de}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e-b)^(1/2))/(2*d*e-b)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e-
b)^(1/2))/(2*d*e-b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1178, 642}

$$\frac{\log \left(x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}-\frac{\log \left(-x\sqrt{2de-b}+d+ex^2\right)}{2\sqrt{2de-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] -1/2*Log[d - Sqrt[-b + 2*d*e]*x + e*x^2]/Sqrt[-b + 2*d*e] + Log[d + Sqrt[-b
 + 2*d*e]*x + e*x^2]/(2*Sqrt[-b + 2*d*e])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d-ex^2}{d^2+bx^2+e^2x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{-b+2de}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{-b+2de}\,x}{e}-x^2}\,dx}{2\sqrt{-b+2de}} - \frac{\int \frac{\frac{\sqrt{-b+2de}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{-b+2de}\,x}{e}-x^2}\,dx}{2\sqrt{-b+2de}}$$
$$= -\frac{\log\left(d-\sqrt{-b+2de}\,x+ex^2\right)}{2\sqrt{-b+2de}} + \frac{\log\left(d+\sqrt{-b+2de}\,x+ex^2\right)}{2\sqrt{-b+2de}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(78) = 156.
time = 0.07, size = 182, normalized size = 2.33

$$\frac{\frac{\left(b+2de-\sqrt{b^2-4d^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b-\sqrt{b^2-4d^2e^2}}} - \frac{\left(b+2de+\sqrt{b^2-4d^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\,\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 + b*x^2 + e^2*x^4),x]
```

```
[Out] (((b + 2*d*e - Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b - Sqrt[b^
2 - 4*d^2*e^2]]])/Sqrt[b - Sqrt[b^2 - 4*d^2*e^2]] - ((b + 2*d*e + Sqrt[b^2
- 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[b + Sqrt[b^2 - 4*d^2*e^2]]])/Sqrt[b
 + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [**A**]
time = 0.03, size = 88, normalized size = 1.13

method	result	size
default	$\frac{\sqrt{2de-b}\,\ln\left(-e\,x^2+x\sqrt{2de-b}-d\right)}{-4de+2b}-\frac{\sqrt{2de-b}\,\ln\left(d+e\,x^2+x\sqrt{2de-b}\right)}{-4de+2b}$	88
risch	$-\frac{\ln\left(-e\,x^2\sqrt{2de-b}+(2de-b)x-d\sqrt{2de-b}\right)}{2\sqrt{2de-b}}+\frac{\ln\left(-e\,x^2\sqrt{2de-b}+(-2de+b)x-d\sqrt{2de-b}\right)}{2\sqrt{2de-b}}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-4*d*e+2*b)*(2*d*e-b)^(1/2)*ln(-e*x^2+x*(2*d*e-b)^(1/2)-d)-1/(-4*d*e+2*b
)*(2*d*e-b)^(1/2)*ln(d+e*x^2+x*(2*d*e-b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2*e - d)/(x^4*e^2 + b*x^2 + d^2), x)
```

Fricas [A]
time = 0.35, size = 181, normalized size = 2.32

$$\left[\frac{\log\left(\frac{x^4e^2+4\,dx^2e-bx^2+d^2+2\,(x^3e+dx)\,\sqrt{2\,de-b}}{x^4e^2+bx^2+d^2}\right)}{2\,\sqrt{2\,de-b}},\,-\frac{\sqrt{-2\,de+b}\,\arctan\left(\frac{\sqrt{-2\,de+b}\,xe}{2\,de-b}\right)-\sqrt{-2\,de+b}\,\arctan\left(\frac{(x^3e^2-dxe+bx)\,\sqrt{-2\,de+b}}{2\,d^2e-bd}\right)}{2\,de-b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((x^4*e^2 + 4*d*x^2*e - b*x^2 + d^2 + 2*(x^3*e + d*x)*sqrt(2*d*e -
b))/(x^4*e^2 + b*x^2 + d^2))/sqrt(2*d*e - b), -(sqrt(-2*d*e + b)*arctan(sqr
t(-2*d*e + b)*x*e/(2*d*e - b)) - sqrt(-2*d*e + b)*arctan((x^3*e^2 - d*x*e +
 b*x)*sqrt(-2*d*e + b)/(2*d^2*e - b*d)))/(2*d*e - b)]
```

Sympy [A]
time = 0.29, size = 121, normalized size = 1.55

$$\frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{-\frac{1}{b-2de}}+2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}-\frac{\sqrt{-\frac{1}{b-2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(b\sqrt{-\frac{1}{b-2de}}-2de\sqrt{-\frac{1}{b-2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4+b*x**2+d**2),x)
```

```
[Out] sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(-1/(b - 2*d*e)) + 2*d*e*sq
rt(-1/(b - 2*d*e)))/e)/2 - sqrt(-1/(b - 2*d*e))*log(d/e + x**2 + x*(b*sqrt(
-1/(b - 2*d*e)) - 2*d*e*sqrt(-1/(b - 2*d*e)))/e)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1077 vs. 2(72) = 144.
time = 4.97, size = 1077, normalized size = 13.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+b*x^2+d^2),x, algorithm="giac")
```

```
[Out] (2*d^2*e^3 + b*d*e^2 - d*e^4)*sqrt(-2*d*e + b)*arctan(2*sqrt(1/2)*x*e/sqrt(
b + sqrt(-4*d^2*e^2 + b^2)))/(4*d^3*e^4 - b^2*d*e^2 - 2*d^2*e^5 + b*d*e^4)
- 1/4*(8*sqrt(2*d*e^3 + b*e^2)*d^3*e^3 - 8*sqrt(-2*d*e^3 + b*e^2)*d^3*e^3 -
 4*sqrt(2*d*e^3 + b*e^2)*b*d^2*e^2 + 4*sqrt(-2*d*e^3 + b*e^2)*b*d^2*e^2 - 2
*sqrt(2*d*e^3 + b*e^2)*b^2*d*e + 2*sqrt(-2*d*e^3 + b*e^2)*b^2*d*e - 24*d^3*
e^5 + 12*b*d^2*e^4 + 6*b^2*d*e^3 - 3*b^3*e^2 + 4*sqrt(-4*d^2*e^2 + b^2)*sqr
t(2*d*e^3 + b*e^2)*d^2*e^2 - 4*sqrt(-4*d^2*e^2 + b^2)*sqrt(-2*d*e^3 + b*e^2
)*d^2*e^2 + 4*sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*d^2*e^2 + sqrt(2
*d*e^3 + b*e^2)*b^3 - sqrt(-2*d*e^3 + b*e^2)*b^3 - 4*sqrt(-4*d^2*e^2 + b^2)
*d^2*e^4 + 4*sqrt(2*d*e^3 + b*e^2)*d^2*e^4 + 4*sqrt(-2*d*e^3 + b*e^2)*d^2*e
^4 - 4*sqrt(-2*d*e^3 + b*e^2)*b*d*e^3 + sqrt(-4*d^2*e^2 + b^2)*b^2*e^2 - sq
rt(2*d*e^3 + b*e^2)*b^2*e^2 + sqrt(-2*d*e^3 + b*e^2)*b^2*e^2 - 6*sqrt(-4*d^
2*e^2 + b^2)*sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*d*e - sqrt(-4*d^2
*e^2 + b^2)*sqrt(2*d*e^3 + b*e^2)*b^2 + sqrt(-4*d^2*e^2 + b^2)*sqrt(-2*d*e^
3 + b*e^2)*b^2 - sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*b^2 + 12*d^2*
e^6 - 3*b^2*e^4 - 2*sqrt(-4*d^2*e^2 + b^2)*sqrt(2*d*e^3 + b*e^2)*d*e^3 + 2*
sqrt(-4*d^2*e^2 + b^2)*sqrt(-2*d*e^3 + b*e^2)*d*e^3 - 2*sqrt(2*d*e^3 + b*e^
2)*sqrt(-2*d*e^3 + b*e^2)*d*e^3 + 3*sqrt(-4*d^2*e^2 + b^2)*sqrt(2*d*e^3 + b
*e^2)*b*e^2 - 3*sqrt(-4*d^2*e^2 + b^2)*sqrt(-2*d*e^3 + b*e^2)*b*e^2 + 3*sqr
t(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*b*e^2 + 3*sqrt(-4*d^2*e^2 + b^2)*
sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*b + 2*sqrt(-4*d^2*e^2 + b^2)*d
*e^5 - 4*sqrt(2*d*e^3 + b*e^2)*d*e^5 - 3*sqrt(-4*d^2*e^2 + b^2)*b*e^4 + 2*s
qrt(2*d*e^3 + b*e^2)*b*e^4 - 2*sqrt(-2*d*e^3 + b*e^2)*b*e^4 + 3*sqrt(-4*d^2
*e^2 + b^2)*sqrt(2*d*e^3 + b*e^2)*sqrt(-2*d*e^3 + b*e^2)*e^2)*arctan(2*sqrt
(1/2)*x*e/sqrt(b - sqrt(-4*d^2*e^2 + b^2)))/(8*d^4*e^5 - 4*b*d^3*e^4 - 2*b^
2*d^2*e^3 + b^3*d*e^2 + 4*b*d^2*e^5 - 2*b^2*d*e^4 - 2*d^2*e^7 + b*d*e^6)
```

Mupad [B]
time = 0.09, size = 99, normalized size = 1.27

$$\frac{\mathrm{atan}\left(\frac{b\,x\,\left(b-2\,d\,e\right)+2\,b\,e^2\,x^3+4\,d^2\,e^2\,x-e^2\,x^3\,\left(b-2\,d\,e\right)+3\,d\,e\,x\,\left(b-2\,d\,e\right)}{\left(2\,e\,d^2+b\,d\right)\,\sqrt{b-2\,d\,e}}\right)-\mathrm{atan}\left(\frac{e\,x}{\sqrt{b-2\,d\,e}}\right)}{\sqrt{b-2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(b*x^2 + d^2 + e^2*x^4),x)
```

```
[Out] (atan((b*x*(b - 2*d*e) + 2*b*e^2*x^3 + 4*d^2*e^2*x - e^2*x^3*(b - 2*d*e) +
3*d*e*x*(b - 2*d*e))/((b*d + 2*d^2*e)*(b - 2*d*e)^(1/2))) - atan((e*x)/(b -
 2*d*e)^(1/2)))/(b - 2*d*e)^(1/2)
```

3.31 $\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx$

Optimal. Leaf size=78

$$-\frac{\log\left(d-\sqrt{2de-f}\,x+ex^2\right)}{2\sqrt{2de-f}}+\frac{\log\left(d+\sqrt{2de-f}\,x+ex^2\right)}{2\sqrt{2de-f}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e-f)^(1/2))/(2*d*e-f)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e-
f)^(1/2))/(2*d*e-f)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.074$, Rules used = {1178, 642}

$$\frac{\log\left(x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}-\frac{\log\left(-x\sqrt{2de-f}+d+ex^2\right)}{2\sqrt{2de-f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] -1/2*Log[d - Sqrt[2*d*e - f]*x + e*x^2]/Sqrt[2*d*e - f] + Log[d + Sqrt[2*d*
e - f]*x + e*x^2]/(2*Sqrt[2*d*e - f])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d-ex^2}{d^2+fx^2+e^2x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{2de-f}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{2de-f}\,x}{e}-x^2}\,dx}{2\sqrt{2de-f}} - \frac{\int \frac{\frac{\sqrt{2de-f}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{2de-f}\,x}{e}-x^2}\,dx}{2\sqrt{2de-f}}$$

$$= -\frac{\log\left(d-\sqrt{2de-f}\,x+ex^2\right)}{2\sqrt{2de-f}} + \frac{\log\left(d+\sqrt{2de-f}\,x+ex^2\right)}{2\sqrt{2de-f}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(78) = 156.
time = 0.07, size = 182, normalized size = 2.33

$$\frac{\frac{\left(2de+f-\sqrt{-4d^2e^2+f^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f-\sqrt{-4d^2e^2+f^2}}} - \frac{\left(2de+f+\sqrt{-4d^2e^2+f^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\sqrt{-4d^2e^2+f^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 + f*x^2 + e^2*x^4),x]
```

```
[Out] (((2*d*e + f - Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f - Sqrt[-
4*d^2*e^2 + f^2]]])/Sqrt[f - Sqrt[-4*d^2*e^2 + f^2]] - ((2*d*e + f + Sqrt[-
4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[f + Sqrt[-4*d^2*e^2 + f^2]]])/S
qrt[f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A]
time = 0.03, size = 69, normalized size = 0.88

method	result	size
default	$-\frac{\ln\left(-e\,x^2+x\sqrt{2de-f}-d\right)}{2\sqrt{2de-f}}+\frac{\ln\left(d+e\,x^2+x\sqrt{2de-f}\right)}{2\sqrt{2de-f}}$	69
risch	$\frac{\ln\left(\sqrt{2de-f}\,e\,x^2+(2de-f)x+\sqrt{2de-f}\,d\right)}{2\sqrt{2de-f}}-\frac{\ln\left(\sqrt{2de-f}\,e\,x^2+(-2de+f)x+\sqrt{2de-f}\,d\right)}{2\sqrt{2de-f}}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(2*d*e-f)^(1/2)*ln(-e*x^2+x*(2*d*e-f)^(1/2)-d)+1/2*ln(d+e*x^2+x*(2*d*e
-f)^(1/2))/(2*d*e-f)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2*e - d)/(x^4*e^2 + f*x^2 + d^2), x)
```

Fricas [A]
time = 0.34, size = 181, normalized size = 2.32

$$\left[\frac{\log\left(\frac{x^4e^2+4\,dx^2e-fx^2+d^2+2\,(x^3e+dx)\sqrt{2\,de-f}}{x^4e^2+fx^2+d^2}\right)}{2\,\sqrt{2\,de-f}},\,-\frac{\sqrt{-2\,de+f}\,\arctan\left(\frac{\sqrt{-2\,de+f}\,xe}{2\,de-f}\right)-\sqrt{-2\,de+f}\,\arctan\left(\frac{(x^3e^2-dxe+fx)\sqrt{-2\,de+f}}{2\,d^2e-df}\right)}{2\,de-f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((x^4*e^2 + 4*d*x^2*e - f*x^2 + d^2 + 2*(x^3*e + d*x)*sqrt(2*d*e -
f))/(x^4*e^2 + f*x^2 + d^2))/sqrt(2*d*e - f), -(sqrt(-2*d*e + f)*arctan(sqr
t(-2*d*e + f)*x*e/(2*d*e - f)) - sqrt(-2*d*e + f)*arctan((x^3*e^2 - d*x*e +
 f*x)*sqrt(-2*d*e + f)/(2*d^2*e - d*f)))/(2*d*e - f)]
```

Sympy [A]
time = 0.30, size = 110, normalized size = 1.41

$$-\frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{\frac{1}{2de-f}}+f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{2de-f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{\frac{1}{2de-f}}-f\sqrt{\frac{1}{2de-f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4+f*x**2+d**2),x)
```

```
[Out] -sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e - f)) + f*sqr
t(1/(2*d*e - f)))/e)/2 + sqrt(1/(2*d*e - f))*log(d/e + x**2 + x*(2*d*e*sqrt
(1/(2*d*e - f)) - f*sqrt(1/(2*d*e - f)))/e)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(72) = 144.
time = 4.63, size = 172, normalized size = 2.21

$$-\frac{(2\,d^2e^3+dfe^2-de^4)\sqrt{-2\,de+f}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,xe}{\sqrt{f+\sqrt{-4\,d^2e^2+f^2}}}\right)}{4\,d^3e^4-df^2e^2-2\,d^2e^5+dfe^4}+\frac{(2\,d^2e^3+dfe^2-de^4)\sqrt{-2\,de+f}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,xe}{\sqrt{f-\sqrt{-4\,d^2e^2+f^2}}}\right)}{4\,d^3e^4-df^2e^2-2\,d^2e^5+dfe^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4+f*x^2+d^2),x, algorithm="giac")
```

```
[Out] -(2*d^2*e^3 + d*f*e^2 - d*e^4)*sqrt(-2*d*e + f)*arctan(2*sqrt(1/2)*x*e/sqrt
(f + sqrt(-4*d^2*e^2 + f^2)))/(4*d^3*e^4 - d*f^2*e^2 - 2*d^2*e^5 + d*f*e^4)
 + (2*d^2*e^3 + d*f*e^2 - d*e^4)*sqrt(-2*d*e + f)*arctan(2*sqrt(1/2)*x*e/sq
rt(f - sqrt(-4*d^2*e^2 + f^2)))/(4*d^3*e^4 - d*f^2*e^2 - 2*d^2*e^5 + d*f*e^
4)
```

Mupad [B]
time = 4.44, size = 57, normalized size = 0.73

$$\frac{\mathrm{atan}\left(\frac{f\,x\,1\mathrm{i}-d\,e\,x\,2\mathrm{i}}{d\,\sqrt{2\,d\,e-f}+e\,x^2\,\sqrt{2\,d\,e-f}}\right)\,1\mathrm{i}}{\sqrt{2\,d\,e-f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(f*x^2 + d^2 + e^2*x^4),x)
```

```
[Out] (atan((f*x*1i - d*e*x*2i)/(d*(2*d*e - f)^(1/2) + e*x^2*(2*d*e - f)^(1/2)))*
1i)/(2*d*e - f)^(1/2)
```

3.32 $\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx$

Optimal. Leaf size=70

$$-\frac{\log\left(d-\sqrt{b+2de}\,x+ex^2\right)}{2\sqrt{b+2de}}+\frac{\log\left(d+\sqrt{b+2de}\,x+ex^2\right)}{2\sqrt{b+2de}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e+
b)^(1/2))/(2*d*e+b)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1178, 642}

$$\frac{\log\left(x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}-\frac{\log\left(-x\sqrt{b+2de}+d+ex^2\right)}{2\sqrt{b+2de}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] -1/2*Log[d - Sqrt[b + 2*d*e]*x + e*x^2]/Sqrt[b + 2*d*e] + Log[d + Sqrt[b +
2*d*e]*x + e*x^2]/(2*Sqrt[b + 2*d*e])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d-ex^2}{d^2-bx^2+e^2x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{b+2de}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{b+2de}\,x}{e}-x^2}\,dx}{2\sqrt{b+2de}} - \frac{\int \frac{\frac{\sqrt{b+2de}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{b+2de}\,x}{e}-x^2}\,dx}{2\sqrt{b+2de}}$$
$$= -\frac{\log\left(d-\sqrt{b+2de}\,x+ex^2\right)}{2\sqrt{b+2de}} + \frac{\log\left(d+\sqrt{b+2de}\,x+ex^2\right)}{2\sqrt{b+2de}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(70) = 140.
time = 0.07, size = 190, normalized size = 2.71

$$\frac{-\frac{\left(b-2de+\sqrt{b^2-4d^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b-\sqrt{b^2-4d^2e^2}}} + \frac{\left(b-2de-\sqrt{b^2-4d^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}\right)}{\sqrt{-b+\sqrt{b^2-4d^2e^2}}}}{\sqrt{2}\sqrt{b^2-4d^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x]
```

```
[Out] (-(((b - 2*d*e + Sqrt[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b - Sqrt
[b^2 - 4*d^2*e^2]]])/Sqrt[-b - Sqrt[b^2 - 4*d^2*e^2]]) + ((b - 2*d*e - Sqrt
[b^2 - 4*d^2*e^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]]])/
Sqrt[-b + Sqrt[b^2 - 4*d^2*e^2]])/(Sqrt[2]*Sqrt[b^2 - 4*d^2*e^2])
```

Maple [A]
time = 0.03, size = 61, normalized size = 0.87

method	result	size
default	$\frac{\ln\left(d+e\,x^2+x\sqrt{2de+b}\right)}{2\sqrt{2de+b}}-\frac{\ln\left(-e\,x^2+x\sqrt{2de+b}-d\right)}{2\sqrt{2de+b}}$	61
risch	$\frac{\ln\left(\sqrt{2de+b}\,e\,x^2+(2de+b)x+\sqrt{2de+b}\,d\right)}{2\sqrt{2de+b}}-\frac{\ln\left(\sqrt{2de+b}\,e\,x^2+(-2de-b)x+\sqrt{2de+b}\,d\right)}{2\sqrt{2de+b}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(d+e*x^2+x*(2*d*e+b)^(1/2))/(2*d*e+b)^(1/2)-1/2/(2*d*e+b)^(1/2)*ln(-e
*x^2+x*(2*d*e+b)^(1/2)-d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2*e - d)/(x^4*e^2 - b*x^2 + d^2), x)
```

Fricas [A]
time = 0.37, size = 181, normalized size = 2.59

$$\left[\frac{\log\left(\frac{x^4e^2+4\,dx^2e+bx^2+d^2+2\,(x^3e+dx)\sqrt{2\,de+b}}{x^4e^2-bx^2+d^2}\right)}{2\,\sqrt{2\,de+b}},\,-\frac{\sqrt{-2\,de-b}\,\arctan\left(\frac{\sqrt{-2\,de-b}\,xe}{2\,de+b}\right)-\sqrt{-2\,de-b}\,\arctan\left(\frac{(x^3e^2-dxe-bx)\sqrt{-2\,de-b}}{2\,d^2e+bd}\right)}{2\,de+b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((x^4*e^2 + 4*d*x^2*e + b*x^2 + d^2 + 2*(x^3*e + d*x)*sqrt(2*d*e +
b))/(x^4*e^2 - b*x^2 + d^2))/sqrt(2*d*e + b), -(sqrt(-2*d*e - b)*arctan(sqr
t(-2*d*e - b)*x*e/(2*d*e + b)) - sqrt(-2*d*e - b)*arctan((x^3*e^2 - d*x*e -
 b*x)*sqrt(-2*d*e - b)/(2*d^2*e + b*d)))/(2*d*e + b)]
```

Sympy [A]
time = 0.30, size = 112, normalized size = 1.60

$$-\frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-b\sqrt{\frac{1}{b+2de}}-2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{b+2de}}\log\left(\frac{d}{e}+x^2+\frac{x\left(b\sqrt{\frac{1}{b+2de}}+2de\sqrt{\frac{1}{b+2de}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4-b*x**2+d**2),x)
```

```
[Out] -sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(-b*sqrt(1/(b + 2*d*e)) - 2*d*e*sqr
t(1/(b + 2*d*e)))/e)/2 + sqrt(1/(b + 2*d*e))*log(d/e + x**2 + x*(b*sqrt(1/(
b + 2*d*e)) + 2*d*e*sqrt(1/(b + 2*d*e)))/e)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(64) = 128.
time = 4.63, size = 184, normalized size = 2.63

$$\frac{(2\,d^2e^3-bde^2-de^4)\sqrt{-2\,de-b}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{-\left(b+\sqrt{-4\,d^2e^2+b^2}\right)e^{(-2)}}}\right)}{4\,d^3e^4-b^2de^2-2\,d^2e^5-bde^4}-\frac{(2\,d^2e^3-bde^2-de^4)\sqrt{-2\,de-b}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{-\left(b-\sqrt{-4\,d^2e^2+b^2}\right)e^{(-2)}}}\right)}{4\,d^3e^4-b^2de^2-2\,d^2e^5-bde^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-b*x^2+d^2),x, algorithm="giac")
```

```
[Out] (2*d^2*e^3 - b*d*e^2 - d*e^4)*sqrt(-2*d*e - b)*arctan(2*sqrt(1/2)*x/sqrt(-(
b + sqrt(-4*d^2*e^2 + b^2))*e^(-2)))/(4*d^3*e^4 - b^2*d*e^2 - 2*d^2*e^5 - b
*d*e^4) - (2*d^2*e^3 - b*d*e^2 - d*e^4)*sqrt(-2*d*e - b)*arctan(2*sqrt(1/2)
*x/sqrt(-(b - sqrt(-4*d^2*e^2 + b^2))*e^(-2)))/(4*d^3*e^4 - b^2*d*e^2 - 2*d
^2*e^5 - b*d*e^4)
```

Mupad [B]
time = 4.44, size = 29, normalized size = 0.41

$$\frac{\operatorname{atanh}\left(\frac{x\sqrt{b+2\,d\,e}}{e\,x^2+d}\right)}{\sqrt{b+2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(d^2 - b*x^2 + e^2*x^4),x)
```

```
[Out] atanh((x*(b + 2*d*e)^(1/2))/(d + e*x^2))/(b + 2*d*e)^(1/2)
```

3.33 $\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx$

Optimal. Leaf size=70

$$-\frac{\log\left(d-\sqrt{2de+f}\,x+ex^2\right)}{2\sqrt{2de+f}}+\frac{\log\left(d+\sqrt{2de+f}\,x+ex^2\right)}{2\sqrt{2de+f}}$$

```
[Out] -1/2*ln(d+e*x^2-x*(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)+1/2*ln(d+e*x^2+x*(2*d*e+
f)^(1/2))/(2*d*e+f)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1178, 642}

$$\frac{\log\left(x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}-\frac{\log\left(-x\sqrt{2de+f}+d+ex^2\right)}{2\sqrt{2de+f}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] -1/2*Log[d - Sqrt[2*d*e + f]*x + e*x^2]/Sqrt[2*d*e + f] + Log[d + Sqrt[2*d*
e + f]*x + e*x^2]/(2*Sqrt[2*d*e + f])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d-ex^2}{d^2-fx^2+e^2x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{2de+f}}{e}+2x}{-\frac{d}{e}-\frac{\sqrt{2de+f}\,x}{e}-x^2}\,dx}{2\sqrt{2de+f}} - \frac{\int \frac{\frac{\sqrt{2de+f}}{e}-2x}{-\frac{d}{e}+\frac{\sqrt{2de+f}\,x}{e}-x^2}\,dx}{2\sqrt{2de+f}}$$
$$= -\frac{\log\left(d-\sqrt{2de+f}\,x+ex^2\right)}{2\sqrt{2de+f}} + \frac{\log\left(d+\sqrt{2de+f}\,x+ex^2\right)}{2\sqrt{2de+f}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(70) = 140.
time = 0.07, size = 190, normalized size = 2.71

$$\frac{-\frac{\left(-2de+f+\sqrt{-4d^2e^2+f^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f-\sqrt{-4d^2e^2+f^2}}}+\frac{\left(-2de+f-\sqrt{-4d^2e^2+f^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,ex}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}\right)}{\sqrt{-f+\sqrt{-4d^2e^2+f^2}}}}{\sqrt{2}\,\sqrt{-4d^2e^2+f^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x]
```

```
[Out] (-(((-2*d*e + f + Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f - Sq
rt[-4*d^2*e^2 + f^2]]])/Sqrt[-f - Sqrt[-4*d^2*e^2 + f^2]]) + ((-2*d*e + f -
 Sqrt[-4*d^2*e^2 + f^2])*ArcTan[(Sqrt[2]*e*x)/Sqrt[-f + Sqrt[-4*d^2*e^2 + f
^2]]])/Sqrt[-f + Sqrt[-4*d^2*e^2 + f^2]])/(Sqrt[2]*Sqrt[-4*d^2*e^2 + f^2])
```

Maple [A]
time = 0.03, size = 61, normalized size = 0.87

method	result	size
default	$\frac{\ln\left(d+e\,x^2+x\sqrt{2de+f}\right)}{2\sqrt{2de+f}}-\frac{\ln\left(-e\,x^2+x\sqrt{2de+f}-d\right)}{2\sqrt{2de+f}}$	61
risch	$\frac{\ln\left(\sqrt{2de+f}\,e\,x^2+(2de+f)x+\sqrt{2de+f}\,d\right)}{2\sqrt{2de+f}}-\frac{\ln\left(\sqrt{2de+f}\,e\,x^2+(-2de-f)x+\sqrt{2de+f}\,d\right)}{2\sqrt{2de+f}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(d+e*x^2+x*(2*d*e+f)^(1/2))/(2*d*e+f)^(1/2)-1/2/(2*d*e+f)^(1/2)*ln(-e
*x^2+x*(2*d*e+f)^(1/2)-d)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="maxima")
[Out] -integrate((x^2*e - d)/(x^4*e^2 - f*x^2 + d^2), x)
```

Fricas [A]
time = 0.33, size = 181, normalized size = 2.59

$$\left[\frac{\log\left(\frac{x^4e^2+4\,dx^2e+fx^2+d^2+2\,(x^3e+dx)\sqrt{2\,de+f}}{x^4e^2-fx^2+d^2}\right)}{2\,\sqrt{2\,de+f}}, -\frac{\sqrt{-2\,de-f}\,\arctan\left(\frac{\sqrt{-2\,de-f}\,xe}{2\,de+f}\right)-\sqrt{-2\,de-f}\,\arctan\left(\frac{(x^3e^2-dxe-fx)\sqrt{-2\,de-f}}{2\,d^2e+df}\right)}{2\,de+f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="fricas")
[Out] [1/2*log((x^4*e^2 + 4*d*x^2*e + f*x^2 + d^2 + 2*(x^3*e + d*x)*sqrt(2*d*e +
f))/(x^4*e^2 - f*x^2 + d^2))/sqrt(2*d*e + f), -(sqrt(-2*d*e - f)*arctan(sqr
t(-2*d*e - f)*x*e/(2*d*e + f)) - sqrt(-2*d*e - f)*arctan((x^3*e^2 - d*x*e -
 f*x)*sqrt(-2*d*e - f)/(2*d^2*e + d*f)))/(2*d*e + f)]
```

Sympy [A]
time = 0.30, size = 112, normalized size = 1.60

$$-\frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-2de\sqrt{\frac{1}{2de+f}}-f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}+\frac{\sqrt{\frac{1}{2de+f}}\log\left(\frac{d}{e}+x^2+\frac{x\left(2de\sqrt{\frac{1}{2de+f}}+f\sqrt{\frac{1}{2de+f}}\right)}{e}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(e**2*x**4-f*x**2+d**2),x)
[Out] -sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(-2*d*e*sqrt(1/(2*d*e + f)) - f*sqr
t(1/(2*d*e + f)))/e)/2 + sqrt(1/(2*d*e + f))*log(d/e + x**2 + x*(2*d*e*sqrt
(1/(2*d*e + f)) + f*sqrt(1/(2*d*e + f)))/e)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(64) = 128.
time = 4.18, size = 184, normalized size = 2.63

$$\frac{(2\,d^2e^3-dfe^2-de^4)\sqrt{-2\,de-f}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{-\left(f+\sqrt{-4\,d^2e^2+f^2}\right)e^{(-2)}}}\right)}{4\,d^3e^4-df^2e^2-2\,d^2e^5-dfe^4}-\frac{(2\,d^2e^3-dfe^2-de^4)\sqrt{-2\,de-f}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{-\left(f-\sqrt{-4\,d^2e^2+f^2}\right)e^{(-2)}}}\right)}{4\,d^3e^4-df^2e^2-2\,d^2e^5-dfe^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(e^2*x^4-f*x^2+d^2),x, algorithm="giac")
```

```
[Out] (2*d^2*e^3 - d*f*e^2 - d*e^4)*sqrt(-2*d*e - f)*arctan(2*sqrt(1/2)*x/sqrt(-(
f + sqrt(-4*d^2*e^2 + f^2))*e^(-2)))/(4*d^3*e^4 - d*f^2*e^2 - 2*d^2*e^5 - d
*f*e^4) - (2*d^2*e^3 - d*f*e^2 - d*e^4)*sqrt(-2*d*e - f)*arctan(2*sqrt(1/2)
*x/sqrt(-(f - sqrt(-4*d^2*e^2 + f^2))*e^(-2)))/(4*d^3*e^4 - d*f^2*e^2 - 2*d
^2*e^5 - d*f*e^4)
```

Mupad [B]
time = 0.11, size = 29, normalized size = 0.41

$$\frac{\operatorname{atanh}\left(\frac{x\,\sqrt{f+2\,d\,e}}{e\,x^2+d}\right)}{\sqrt{f+2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(d^2 - f*x^2 + e^2*x^4),x)
```

```
[Out] atanh((x*(f + 2*d*e)^(1/2))/(d + e*x^2))/(f + 2*d*e)^(1/2)
```

3.34 $\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$

Optimal. Leaf size=134

$$-\frac{e^{3/2}\log\left(\sqrt{c}\,d-\sqrt{e}\,\sqrt{2cd-be}\,x+\sqrt{c}\,ex^2\right)}{2\sqrt{c}\,\sqrt{2cd-be}}+\frac{e^{3/2}\log\left(\sqrt{c}\,d+\sqrt{e}\,\sqrt{2cd-be}\,x+\sqrt{c}\,ex^2\right)}{2\sqrt{c}\,\sqrt{2cd-be}}$$

```
[Out] -1/2*e^(3/2)*ln(d*c^(1/2)+e*x^2*c^(1/2)-x*e^(1/2)*(-b*e+2*c*d)^(1/2))/c^(1/
2)/(-b*e+2*c*d)^(1/2)+1/2*e^(3/2)*ln(d*c^(1/2)+e*x^2*c^(1/2)+x*e^(1/2)*(-b*
e+2*c*d)^(1/2))/c^(1/2)/(-b*e+2*c*d)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1178, 642}

$$\frac{e^{3/2}\log\left(\sqrt{e}\,x\sqrt{2cd-be}+\sqrt{c}\,d+\sqrt{c}\,ex^2\right)}{2\sqrt{c}\,\sqrt{2cd-be}}-\frac{e^{3/2}\log\left(-\sqrt{e}\,x\sqrt{2cd-be}+\sqrt{c}\,d+\sqrt{c}\,ex^2\right)}{2\sqrt{c}\,\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*(e^(3/2)*Log[Sqrt[c]*d - Sqrt[e]*Sqrt[2*c*d - b*e]*x + Sqrt[c]*e*x^2])
/(Sqrt[c]*Sqrt[2*c*d - b*e]) + (e^(3/2)*Log[Sqrt[c]*d + Sqrt[e]*Sqrt[2*c*d
- b*e]*x + Sqrt[c]*e*x^2])/(2*Sqrt[c]*Sqrt[2*c*d - b*e])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d-ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = -\frac{e^{3/2}\int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x}{-\frac{d}{e}-\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\sqrt{e}}-x^2}dx}{2\sqrt{c}\sqrt{2cd-be}} - \frac{e^{3/2}\int \frac{\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}-2x}{-\frac{d}{e}+\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\sqrt{e}}-x^2}dx}{2\sqrt{c}\sqrt{2cd-be}}$$

$$= -\frac{e^{3/2}\log\left(\sqrt{c}\,d-\sqrt{e}\sqrt{2cd-be}\,x+\sqrt{c}\,ex^2\right)}{2\sqrt{c}\sqrt{2cd-be}} + \frac{e^{3/2}\log\left(\sqrt{c}\,d+\sqrt{e}\sqrt{2cd-be}\right.}{2\sqrt{c}\sqrt{2cd-be}}$$

Mathematica [A]
time = 0.10, size = 250, normalized size = 1.87

$$\frac{e^{3/2}\left(-\frac{\left(-2cd-be+\sqrt{-4c^2d^2+b^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}\,x}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}-\frac{\left(2cd+be+\sqrt{-4c^2d^2+b^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}\,x}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-4c^2d^2+b^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*(-(((-2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sq
rt[c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-
4*c^2*d^2 + b^2*e^2]]) - ((2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[
b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*
e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(102) = 204$.
time = 0.07, size = 291, normalized size = 2.17

method	result
risch	$\frac{\sqrt{-c(eb-2cd)e}\,e\ln\left(-ce\,x^2+\sqrt{-c(eb-2cd)e}\,x-cd\right)}{2c(eb-2cd)} - \frac{\sqrt{-c(eb-2cd)e}\,e\ln\left(-ce\,x^2-\sqrt{-c(eb-2\right.}{2c(eb-2cd)}$

default	$4e^4c\left(-\frac{\left(e^2b+2cde-\sqrt{e^2\left(eb-2cd\right)\left(eb+2cd\right)}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cex\sqrt{2}}{\sqrt{\left(-e^2b+\sqrt{e^2\left(eb-2cd\right)\left(eb+2cd\right)}\right.}}\right.}{8\sqrt{e^2\left(eb-2cd\right)\left(eb+2cd\right)}\,ce^2\sqrt{\left(-e^2b+\sqrt{e^2\left(eb-2cd\right)\left(eb+2cd\right)}\right)c}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 4*e^4*c*(-1/8*(e^2*b+2*c*d*e-(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))/(e^2*(b*e
-2*c*d)*(b*e+2*c*d))^(1/2)/c/e^2*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*c
*d))^(1/2))*c)^(1/2)*arctanh(c*e*x*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2
*c*d))^(1/2))*c)^(1/2))+1/8*(-e^2*b-2*c*d*e-(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(
1/2))/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)/c/e^2*2^(1/2)/((e^2*b+(e^2*(b*e-2
*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/2)/((e^2*b+(e^2*(b*e-2
*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="maxima")
```

```
[Out] -integrate((x^2*e - d)/(c*x^4 + c*d^2*e^(-2) + b*x^2), x)
```

Fricas [A]
time = 0.33, size = 251, normalized size = 1.87

$$\left[\frac{e^{\frac{3}{2}}\log\left(\frac{4\,cdx^2e+cd^2+(cx^4-bx^2)e^2-\frac{2\left(bcx^3e^2-2\,c^2d^2x-\left(2\,c^2dx^3-bcdx\right)e\right)e^{\frac{1}{2}}}{\sqrt{2\,c^2d-bce}}}{cd^2+(cx^4+bx^2)e^2}\right)}{2\sqrt{2\,c^2d-bce}},\ -\sqrt{-\frac{e}{2\,c^2d-bce}}\arctan\left(cx\sqrt{-\frac{e}{2\,c^2d-bce}}\right)e+\sqrt{-\frac{e}{2\,c^2d-bce}}\arctan\left(-\frac{(cdx-(cx^3+bx)e)\sqrt{-\frac{e}{2\,c^2d-bce}}}{d}\right)e\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")
```

```
[Out] [1/2*e^(3/2)*log((4*c*d*x^2*e + c*d^2 + (c*x^4 - b*x^2)*e^2 - 2*(b*c*x^3*e^
2 - 2*c^2*d^2*x - (2*c^2*d*x^3 - b*c*d*x)*e)*e^(1/2)/sqrt(2*c^2*d - b*c*e))
/(c*d^2 + (c*x^4 + b*x^2)*e^2))/sqrt(2*c^2*d - b*c*e), -sqrt(-e/(2*c^2*d -
```

```
b*c*e))*arctan(c*x*sqrt(-e/(2*c^2*d - b*c*e))*e + sqrt(-e/(2*c^2*d - b*c*e
))*arctan(-(c*d*x - (c*x^3 + b*x)*e)*sqrt(-e/(2*c^2*d - b*c*e))/d)*e]
```

Sympy [A]
time = 0.39, size = 158, normalized size = 1.18

$$\frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be-2cd)}}+2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2}-\frac{\sqrt{-\frac{e^3}{c(be-2cd)}}\log\left(\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be-2cd)}}-2cd\sqrt{-\frac{e^3}{c(be-2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] sqrt(-e**3/(c*(b*e - 2*c*d)))*log(d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e -
 2*c*d))) + 2*c*d*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2 - sqrt(-e**3/(c*(b
*e - 2*c*d)))*log(d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e - 2*c*d))) - 2*c*d
*sqrt(-e**3/(c*(b*e - 2*c*d))))/e**2)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(104) = 208.
time = 5.29, size = 250, normalized size = 1.87

$$-\frac{(2c^2d^2e+bcde^2-c^2de^2)\sqrt{-2c^2de+bce^2}\arctan\left(\frac{2\sqrt{\frac{1}{2}}xe}{\sqrt{\frac{be^2+\sqrt{-4c^2d^2e^2+b^2e^4}}{c}}}\right)}{(4c^3d^3-2c^3d^2e-b^2cde^2+bc^2de^2)|c|}+\frac{(2c^2d^2e+bcde^2-c^2de^2)\sqrt{-2c^2de+bce^2}\arctan\left(\frac{2\sqrt{\frac{1}{2}}xe}{\sqrt{\frac{be^2-\sqrt{-4c^2d^2e^2+b^2e^4}}{c}}}\right)}{(4c^3d^3-2c^3d^2e-b^2cde^2+bc^2de^2)|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")
```

```
[Out] -(2*c^2*d^2*e + b*c*d*e^2 - c^2*d*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*arctan(2*
sqrt(1/2)*x*e/sqrt((b*e^2 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4))/c))/((4*c^3*d^3
 - 2*c^3*d^2*e - b^2*c*d*e^2 + b*c^2*d*e^2)*abs(c)) + (2*c^2*d^2*e + b*c*d*
e^2 - c^2*d*e^2)*sqrt(-2*c^2*d*e + b*c*e^2)*arctan(2*sqrt(1/2)*x*e/sqrt((b*
e^2 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4))/c))/((4*c^3*d^3 - 2*c^3*d^2*e - b^2*c
*d*e^2 + b*c^2*d*e^2)*abs(c))
```

Mupad [B]
time = 0.18, size = 129, normalized size = 0.96

$$-\frac{e^{3/2}\left(\mathrm{atan}\left(\frac{\sqrt{e}\,x\sqrt{b\,c\,e-2\,c^2\,d}}{b\,e-2\,c\,d}\right)+\mathrm{atan}\left(\frac{c\,e^{3/2}\,x^3\sqrt{b\,c\,e-2\,c^2\,d}+b\,e^{3/2}\,x\sqrt{b\,c\,e-2\,c^2\,d}-c\,d\sqrt{e}\,x\sqrt{b\,c\,e-2\,c^2\,d}}{d\,(2\,c^2\,d-b\,c\,e)}\right)\right)}{\sqrt{b\,c\,e-2\,c^2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - e*x^2)/(b*x^2 + c*x^4 + (c*d^2)/e^2),x)
```

```
[Out] -(e^(3/2)*(atan((e^(1/2)*x*(b*c*e - 2*c^2*d)^(1/2))/(b*e - 2*c*d)) + atan((
c*e^(3/2)*x^3*(b*c*e - 2*c^2*d)^(1/2) + b*e^(3/2)*x*(b*c*e - 2*c^2*d)^(1/2)
 - c*d*e^(1/2)*x*(b*c*e - 2*c^2*d)^(1/2))/(d*(2*c^2*d - b*c*e)))))/(b*c*e -
 2*c^2*d)^(1/2)
```

3.35 $\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$

Optimal. Leaf size=130

$$-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}$$

```
[Out] -e^(3/2)*arctan((-2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d)^(1/2))/(b*e+2*c*d)^(1/2)
)/c^(1/2)/(b*e+2*c*d)^(1/2)+e^(3/2)*arctan((2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d
)^(1/2))/(b*e+2*c*d)^(1/2))/c^(1/2)/(b*e+2*c*d)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1175, 632, 210}

$$\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}\,x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}-\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}\,x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*
e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] + 2*
Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx = \frac{e\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\,\sqrt{e}}+x^2}\,dx}{2c} + \frac{e\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\,\sqrt{e}}+x^2}\,dx}{2c}$$

$$= -\frac{e\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, -\frac{\sqrt{2cd-be}}{\sqrt{c}\,\sqrt{e}}+2x\right)}{c} - \frac{e\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, \frac{\sqrt{2c}}{\sqrt{c}}\right.}{c}$$

$$= -\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\,\sqrt{2cd+be}} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\,\sqrt{2cd+be}}$$

Mathematica [A]
time = 0.07, size = 248, normalized size = 1.91

$$\frac{e^{3/2}\left(\frac{\left(2cd-be+\sqrt{-4c^2d^2+b^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{e}\,x}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}+\frac{\left(-2cd+be+\sqrt{-4c^2d^2+b^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{e}\,x}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{2}\,\sqrt{c}\,\sqrt{-4c^2d^2+b^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/((c*d^2)/e^2 + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4*c
^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e
 + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(102) = 204.
time = 0.04, size = 287, normalized size = 2.21

method	result

risch	$\frac{\sqrt{-c\,(eb+2cd)\,e}\;e\ln\left(-ce\,x^2-\sqrt{-c\,(eb+2cd)\,e}\;x+cd\right)}{2c(eb+2cd)}-\frac{\sqrt{-c\,(eb+2cd)\,e}\;e\ln\left(-ce\,x^2+\sqrt{-c\,(eb+\right.}{2c(eb+2cd)}$
default	$4e^4c\left(-\frac{\left(-e^2b+2cde+\sqrt{e^2\,(eb-2cd)\,(eb+2cd)}\right)\sqrt{2}\;\operatorname{arctanh}\left(\frac{cex\sqrt{2}}{\sqrt{\left(-e^2b+\sqrt{e^2\,(eb-2cd)\,(eb+2cd}\right.}}\right.}{8ce^2\sqrt{e^2\,(eb-2cd)\,(eb+2cd)}\;\sqrt{\left(-e^2b+\sqrt{e^2\,(eb-2cd)\,(eb+2cd)}\right)c}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 4*e^4*c*(-1/8*(-e^2*b+2*c*d*e+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))/c/e^2/(e
^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*
c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+
2*c*d))^(1/2))*c)^(1/2))+1/8*(e^2*b-2*c*d*e+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(
1/2))/c/e^2/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((e^2*b+(e^2*(b*e-2
*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/2)/((e^2*b+(e^2*(b*e-2
*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/(c*x^4 + c*d^2*e^(-2) + b*x^2), x)
```

Fricas [A]
time = 0.36, size = 234, normalized size = 1.80

$$\left[\frac{1}{2}\sqrt{-\frac{e}{2\,c^2d+bce}}\,e\log\left(-\frac{4\,cdx^2e-cd^2-(cx^4-bx^2)e^2-2\,(bcx^3e^2-2\,c^2d^2x+(2\,c^2dx^3-bcdx)e)\sqrt{-\frac{e}{2\,c^2d+bce}}}{cd^2+(cx^4+bx^2)e^2}\right),\ \frac{\arctan\left(\frac{cxe^{\frac{1}{2}}}{\sqrt{2\,c^2d+bce}}\right)e^{\frac{3}{2}}}{\sqrt{2\,c^2d+bce}}+\frac{\arctan\left(\frac{(cdx+(cx^3+bx)e)e^{\frac{1}{2}}}{\sqrt{2\,c^2d+bce}\,d}\right)e^{\frac{3}{2}}}{\sqrt{2\,c^2d+bce}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-e/(2*c^2*d + b*c*e))*e*log(-(4*c*d*x^2*e - c*d^2 - (c*x^4 - b*x^
2)*e^2 - 2*(b*c*x^3*e^2 - 2*c^2*d^2*x + (2*c^2*d*x^3 - b*c*d*x)*e)*sqrt(-e/
```

```
(2*c^2*d + b*c*e)))/(c*d^2 + (c*x^4 + b*x^2)*e^2)), arctan(c*x*e^(1/2)/sqrt
(2*c^2*d + b*c*e))*e^(3/2)/sqrt(2*c^2*d + b*c*e) + arctan((c*d*x + (c*x^3 +
 b*x)*e)*e^(1/2)/(sqrt(2*c^2*d + b*c*e)*d))*e^(3/2)/sqrt(2*c^2*d + b*c*e)]
```

Sympy [A]
time = 0.38, size = 160, normalized size = 1.23

$$-\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(-be\sqrt{-\frac{e^3}{c(be+2cd)}}-2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}+\frac{\sqrt{-\frac{e^3}{c(be+2cd)}}\log\left(-\frac{d}{e}+x^2+\frac{x\left(be\sqrt{-\frac{e^3}{c(be+2cd)}}+2cd\sqrt{-\frac{e^3}{c(be+2cd)}}\right)}{e^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*d**2/e**2+b*x**2+c*x**4),x)
```

```
[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e
 + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(c*
(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) + 2*
c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(107) = 214.
time = 4.83, size = 249, normalized size = 1.92

$$\frac{(2c^2d^2e-bcde^2+c^2de^2)\sqrt{2c^2de+bce^2}\arctan\left(\frac{2\sqrt{\frac{1}{2}}xe}{\sqrt{\frac{be^2+\sqrt{-4c^2d^2e^2+b^2e^4}}{c}}}\right)}{(4c^3d^3+2c^3d^2e-b^2cde^2+bc^2de^2)|c|}+\frac{(2c^2d^2e-bcde^2+c^2de^2)\sqrt{2c^2de+bce^2}\arctan\left(\frac{2\sqrt{\frac{1}{2}}xe}{\sqrt{\frac{be^2-\sqrt{-4c^2d^2e^2+b^2e^4}}{c}}}\right)}{(4c^3d^3+2c^3d^2e-b^2cde^2+bc^2de^2)|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*d^2/e^2+b*x^2+c*x^4),x, algorithm="giac")
```

```
[Out] (2*c^2*d^2*e - b*c*d*e^2 + c^2*d*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*arctan(2*sq
rt(1/2)*x*e/sqrt((b*e^2 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4))/c))/((4*c^3*d^3 +
 2*c^3*d^2*e - b^2*c*d*e^2 + b*c^2*d*e^2)*abs(c)) + (2*c^2*d^2*e - b*c*d*e^
2 + c^2*d*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*arctan(2*sqrt(1/2)*x*e/sqrt((b*e^2
 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4))/c))/((4*c^3*d^3 + 2*c^3*d^2*e - b^2*c*d*
e^2 + b*c^2*d*e^2)*abs(c))
```

Mupad [B]
time = 4.52, size = 232, normalized size = 1.78

$$\frac{e^{3/2}\left(\mathrm{atan}\left(\frac{c\sqrt{e}\,x}{\sqrt{c\,(b\,e+2\,c\,d)}}\right)-\mathrm{atan}\left(\frac{(2\,d\,c^2+b\,e\,c)\left(x\left(\frac{\sqrt{e}\,\left(c\,d\,e^7-\frac{4\,c^3\,d^2\,e^7}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}+\frac{e^{3/2}\,(2\,c^2\,d\,e^6-b\,c\,e^7)}{c\,d\,\sqrt{2\,d\,c^2+b\,e\,c}\,(b\,e-2\,c\,d)}\right)+\frac{\sqrt{e}\,x^3\,\left(c\,e^8-\frac{2\,b\,c^2\,e^9}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,(b\,e+2\,c\,d)}\,(b\,e-2\,c\,d)}\right)}{c\,e^7}\right)\right)}{\sqrt{2\,d\,c^2+b\,e\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(b*x^2 + c*x^4 + (c*d^2)/e^2),x)
```

```
[Out] (e^(3/2)*(atan((c*e^(1/2)*x)/(c*(b*e + 2*c*d))^(1/2)) - atan(((2*c^2*d + b*
c*e)*(x*((e^(1/2)*(c*d*e^7 - (4*c^3*d^2*e^7)/(2*c^2*d + b*c*e)))/(d*(c*(b*e
 + 2*c*d))^(1/2)*(b*e - 2*c*d)) + (e^(3/2)*(2*c^2*d*e^6 - b*c*e^7))/(c*d*(2
*c^2*d + b*c*e)^(1/2)*(b*e - 2*c*d))) + (e^(1/2)*x^3*(c*e^8 - (2*b*c^2*e^9)
/(2*c^2*d + b*c*e)))/(d*(c*(b*e + 2*c*d))^(1/2)*(b*e - 2*c*d))))/(c*e^7))))
/(2*c^2*d + b*c*e)^(1/2)
```

3.36 $\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx$

Optimal. Leaf size=130

$$-\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\,\sqrt{2cd+be}}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\,\sqrt{2cd+be}}$$

```
[Out] -e^(3/2)*arctan((-2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d)^(1/2))/(b*e+2*c*d)^(1/2)
)/c^(1/2)/(b*e+2*c*d)^(1/2)+e^(3/2)*arctan((2*x*c^(1/2)*e^(1/2)+(-b*e+2*c*d
)^(1/2))/(b*e+2*c*d)^(1/2))/c^(1/2)/(b*e+2*c*d)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2015, 1175, 632, 210}

$$\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\,\sqrt{be+2cd}}-\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\,\sqrt{e}\,x}{\sqrt{be+2cd}}\right)}{\sqrt{c}\,\sqrt{be+2cd}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
```

```
[Out] -((e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] - 2*Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*
e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])) + (e^(3/2)*ArcTan[(Sqrt[2*c*d - b*e] + 2*
Sqrt[c]*Sqrt[e]*x)/Sqrt[2*c*d + b*e]])/(Sqrt[c]*Sqrt[2*c*d + b*e])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 2015

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rubi steps

$$\int \frac{d+ex^2}{bx^2+c\left(\frac{d^2}{e^2}+x^4\right)}\,dx = \int \frac{d+ex^2}{\frac{cd^2}{e^2}+bx^2+cx^4}\,dx$$

$$= \frac{e\int \frac{1}{\frac{d}{e}-\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c} + \frac{e\int \frac{1}{\frac{d}{e}+\frac{\sqrt{2cd-be}\,x}{\sqrt{c}\sqrt{e}}+x^2}\,dx}{2c}$$

$$= -\frac{e\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, -\frac{\sqrt{2cd-be}}{\sqrt{c}\sqrt{e}}+2x\right)}{c} - \frac{e\text{Subst}\left(\int \frac{1}{-\frac{b}{c}-\frac{2d}{e}-x^2}\,dx, x, \ldots\right)}{c}$$

$$= -\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}-2\sqrt{c}\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-be}+2\sqrt{c}\sqrt{e}\,x}{\sqrt{2cd+be}}\right)}{\sqrt{c}\sqrt{2cd+be}}$$

Mathematica [A]
time = 0.03, size = 248, normalized size = 1.91

$$\frac{e^{3/2}\left(\frac{\left(2cd-be+\sqrt{-4c^2d^2+b^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}\,x}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be-\sqrt{-4c^2d^2+b^2e^2}}}+\frac{\left(-2cd+be+\sqrt{-4c^2d^2+b^2e^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{e}\,x}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{be+\sqrt{-4c^2d^2+b^2e^2}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{-4c^2d^2+b^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(b*x^2 + c*(d^2/e^2 + x^4)),x]
[Out] (e^(3/2)*(((2*c*d - b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[e]*x)/Sqrt[b*e - Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e - Sqrt[-4*c
^2*d^2 + b^2*e^2]] + ((-2*c*d + b*e + Sqrt[-4*c^2*d^2 + b^2*e^2])*ArcTan[(S
qrt[2]*Sqrt[c]*Sqrt[e]*x)/Sqrt[b*e + Sqrt[-4*c^2*d^2 + b^2*e^2]]])/Sqrt[b*e
```

```
 + Sqrt[-4*c^2*d^2 + b^2*e^2]]))/(Sqrt[2]*Sqrt[c]*Sqrt[-4*c^2*d^2 + b^2*e^2
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(102) = 204$.
time = 0.03, size = 287, normalized size = 2.21

method	result
risch	$\frac{\sqrt{-c(eb+2cd)e}\, e \ln\left(-ce\,x^2-\sqrt{-c(eb+2cd)e}\, x+cd\right)}{2c(eb+2cd)} - \frac{\sqrt{-c(eb+2cd)e}\, e \ln\left(-ce\,x^2+\sqrt{-c(eb+2\right.}}{2c(eb+2cd)}$
default	$4e^4c\left(-\frac{\left(-e^2b+2cde+\sqrt{e^2(eb-2cd)(eb+2cd)}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cex\sqrt{2}}{\sqrt{\left(-e^2b+\sqrt{e^2(eb-2cd)(eb+2cd)}\right.}}\right.}{8ce^2\sqrt{e^2(eb-2cd)(eb+2cd)}\sqrt{\left(-e^2b+\sqrt{e^2(eb-2cd)(eb+2cd)}\right)c}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(b*x^2+c*(1/e^2*d^2+x^4)),x,method=_RETURNVERBOSE)
```

```
[Out] 4*e^4*c*(-1/8*(-e^2*b+2*c*d*e+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2))/c/e^2/(e
^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+2*
c*d))^(1/2))*c)^(1/2)*arctanh(c*e*x*2^(1/2)/((-e^2*b+(e^2*(b*e-2*c*d)*(b*e+
2*c*d))^(1/2))*c)^(1/2))+1/8*(e^2*b-2*c*d*e+(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(
1/2))/c/e^2/(e^2*(b*e-2*c*d)*(b*e+2*c*d))^(1/2)*2^(1/2)/((e^2*b+(e^2*(b*e-2
*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)*arctan(c*e*x*2^(1/2)/((e^2*b+(e^2*(b*e-2
*c*d)*(b*e+2*c*d))^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/(b*x^2 + (x^4 + d^2*e^(-2))*c), x)
```

Fricas [A]
time = 0.36, size = 234, normalized size = 1.80

$$\left[\frac{1}{2}\sqrt{-\frac{e}{2\,c^2d+bce}}\,e\log\left(-\frac{4\,cdx^2e-cd^2-(cx^4-bx^2)e^2-2\,(bcx^3e^2-2\,c^2d^2x+(2\,c^2dx^3-bcdx)e)\sqrt{-\frac{e}{2\,c^2d+bce}}}{cd^2+(cx^4+bx^2)e^2}\right), \frac{\arctan\left(\frac{cxe^{\frac{1}{2}}}{\sqrt{2\,c^2d+bce}}\right)e^{\frac{3}{2}}}{\sqrt{2\,c^2d+bce}}+\frac{\arctan\left(\frac{(cdx+(cx^3+bx)e)e^{\frac{1}{2}}}{\sqrt{2\,c^2d+bce}\,d}\right)e^{\frac{3}{2}}}{\sqrt{2\,c^2d+bce}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-e/(2*c^2*d + b*c*e))*e*log(-(4*c*d*x^2*e - c*d^2 - (c*x^4 - b*x^
2)*e^2 - 2*(b*c*x^3*e^2 - 2*c^2*d^2*x + (2*c^2*d*x^3 - b*c*d*x)*e)*sqrt(-e/
(2*c^2*d + b*c*e)))/(c*d^2 + (c*x^4 + b*x^2)*e^2)), arctan(c*x*e^(1/2)/sqrt
(2*c^2*d + b*c*e))*e^(3/2)/sqrt(2*c^2*d + b*c*e) + arctan((c*d*x + (c*x^3 +
 b*x)*e)*e^(1/2)/(sqrt(2*c^2*d + b*c*e)*d))*e^(3/2)/sqrt(2*c^2*d + b*c*e)]
```

Sympy [A]
time = 0.39, size = 160, normalized size = 1.23

$$-\frac{\sqrt{-\frac{e^{3}}{c\left(be+2cd\right)}}\log\left(-\frac{d}{e}+x^{2}+\frac{x\left(-be\sqrt{-\frac{e^{3}}{c\left(be+2cd\right)}}-2cd\sqrt{-\frac{e^{3}}{c\left(be+2cd\right)}}\right)}{e^{2}}\right)}{2}+\frac{\sqrt{-\frac{e^{3}}{c\left(be+2cd\right)}}\log\left(-\frac{d}{e}+x^{2}+\frac{x\left(be\sqrt{-\frac{e^{3}}{c\left(be+2cd\right)}}+2cd\sqrt{-\frac{e^{3}}{c\left(be+2cd\right)}}\right)}{e^{2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(b*x**2+c*(d**2/e**2+x**4)),x)
```

```
[Out] -sqrt(-e**3/(c*(b*e + 2*c*d)))*log(-d/e + x**2 + x*(-b*e*sqrt(-e**3/(c*(b*e
 + 2*c*d))) - 2*c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2 + sqrt(-e**3/(c*
(b*e + 2*c*d)))*log(-d/e + x**2 + x*(b*e*sqrt(-e**3/(c*(b*e + 2*c*d))) + 2*
c*d*sqrt(-e**3/(c*(b*e + 2*c*d))))/e**2)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(107) = 214.
time = 4.83, size = 249, normalized size = 1.92

$$\frac{\left(2\,c^{2}d^{2}e-bcde^{2}+c^{2}de^{2}\right)\sqrt{2\,c^{2}de+bce^{2}}\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,xe}{\sqrt{\frac{be^{2}+\sqrt{-4\,c^{2}d^{2}e^{2}+b^{2}e^{4}}}{c}}}\right)}{\left(4\,c^{3}d^{3}+2\,c^{3}d^{2}e-b^{2}cde^{2}+bc^{2}de^{2}\right)|c|}+\frac{\left(2\,c^{2}d^{2}e-bcde^{2}+c^{2}de^{2}\right)\sqrt{2\,c^{2}de+bce^{2}}\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,xe}{\sqrt{\frac{be^{2}-\sqrt{-4\,c^{2}d^{2}e^{2}+b^{2}e^{4}}}{c}}}\right)}{\left(4\,c^{3}d^{3}+2\,c^{3}d^{2}e-b^{2}cde^{2}+bc^{2}de^{2}\right)|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b*x^2+c*(d^2/e^2+x^4)),x, algorithm="giac")
```

```
[Out] (2*c^2*d^2*e - b*c*d*e^2 + c^2*d*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*arctan(2*sq
rt(1/2)*x*e/sqrt((b*e^2 + sqrt(-4*c^2*d^2*e^2 + b^2*e^4))/c))/((4*c^3*d^3 +
 2*c^3*d^2*e - b^2*c*d*e^2 + b*c^2*d*e^2)*abs(c)) + (2*c^2*d^2*e - b*c*d*e^
2 + c^2*d*e^2)*sqrt(2*c^2*d*e + b*c*e^2)*arctan(2*sqrt(1/2)*x*e/sqrt((b*e^2
 - sqrt(-4*c^2*d^2*e^2 + b^2*e^4))/c))/((4*c^3*d^3 + 2*c^3*d^2*e - b^2*c*d*
e^2 + b*c^2*d*e^2)*abs(c))
```

Mupad [B]
time = 0.13, size = 232, normalized size = 1.78

$$\frac{e^{3/2}\left(\mathrm{atan}\left(\frac{c\,\sqrt{e}\,x}{\sqrt{c\,\left(b\,e+2\,c\,d\right)}}\right)-\mathrm{atan}\left(\frac{\left(2\,d\,c^2+b\,e\,c\right)\,\left(x\,\left(\frac{\sqrt{e}\,\left(c\,d\,e^7-\frac{4\,c^3\,d^2\,e^7}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,\left(b\,e+2\,c\,d\right)}\,\left(b\,e-2\,c\,d\right)}+\frac{e^{3/2}\,\left(2\,c^2\,d\,e^6-b\,c\,e^7\right)}{c\,d\,\sqrt{2\,d\,c^2+b\,e\,c}\,\left(b\,e-2\,c\,d\right)}\right)+\frac{\sqrt{e}\,x^3\,\left(c\,e^8-\frac{2\,b\,c^2\,e^9}{2\,d\,c^2+b\,e\,c}\right)}{d\,\sqrt{c\,\left(b\,e+2\,c\,d\right)}\,\left(b\,e-2\,c\,d\right)}\right)}{c\,e^7}\right)\right)}{\sqrt{2\,d\,c^2+b\,e\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(b*x^2 + c*(x^4 + d^2/e^2)),x)
```

```
[Out] (e^(3/2)*(atan((c*e^(1/2)*x)/(c*(b*e + 2*c*d))^(1/2)) - atan(((2*c^2*d + b*
c*e)*(x*((e^(1/2)*(c*d*e^7 - (4*c^3*d^2*e^7)/(2*c^2*d + b*c*e)))/(d*(c*(b*e
 + 2*c*d))^(1/2)*(b*e - 2*c*d)) + (e^(3/2)*(2*c^2*d*e^6 - b*c*e^7))/(c*d*(2
*c^2*d + b*c*e)^(1/2)*(b*e - 2*c*d))) + (e^(1/2)*x^3*(c*e^8 - (2*b*c^2*e^9)
/(2*c^2*d + b*c*e)))/(d*(c*(b*e + 2*c*d))^(1/2)*(b*e - 2*c*d))))/(c*e^7))))
/(2*c^2*d + b*c*e)^(1/2)
```

3.37 $\int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4} \, dx$

Optimal. Leaf size=29

$$-\frac{1}{2}\log\left(a-x+bx^2\right)+\frac{1}{2}\log\left(a+x+bx^2\right)$$

```
[Out] -1/2*ln(b*x^2+a-x)+1/2*ln(b*x^2+a+x)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1178, 642}

$$\frac{1}{2}\log\left(a+bx^2+x\right)-\frac{1}{2}\log\left(a+bx^2-x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
```

```
[Out] -1/2*Log[a - x + b*x^2] + Log[a + x + b*x^2]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{a-bx^2}{a^2+(-1+2ab)x^2+b^2x^4} \, dx &= -\left(\frac{1}{2}\int \frac{\frac{1}{b}+2x}{-\frac{a}{b}-\frac{x}{b}-x^2} \, dx\right)-\frac{1}{2}\int \frac{\frac{1}{b}-2x}{-\frac{a}{b}+\frac{x}{b}-x^2} \, dx \\ &= -\frac{1}{2}\log\left(a-x+bx^2\right)+\frac{1}{2}\log\left(a+x+bx^2\right) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$-\frac{1}{2}\log\left(a-x+bx^2\right)+\frac{1}{2}\log\left(a+x+bx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
[Out] -1/2*Log[a - x + b*x^2] + Log[a + x + b*x^2]/2
```

Maple [A]
time = 0.03, size = 26, normalized size = 0.90

method	result	size
default	$-\frac{\ln\left(b\,x^{2}+a-x\right)}{2}+\frac{\ln\left(b\,x^{2}+a+x\right)}{2}$	26
norman	$-\frac{\ln\left(b\,x^{2}+a-x\right)}{2}+\frac{\ln\left(b\,x^{2}+a+x\right)}{2}$	26
risch	$-\frac{\ln\left(b\,x^{2}+a-x\right)}{2}+\frac{\ln\left(b\,x^{2}+a+x\right)}{2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x,method=_RETURNVERBOSE)
[Out] -1/2*ln(b*x^2+a-x)+1/2*ln(b*x^2+a+x)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.86

$$\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

Fricas [A]
time = 0.35, size = 25, normalized size = 0.86

$$\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

Sympy [**A**]
time = 0.22, size = 26, normalized size = 0.90

$$-\frac{\log\left(\frac{a}{b}+x^2-\frac{x}{b}\right)}{2}+\frac{\log\left(\frac{a}{b}+x^2+\frac{x}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
[Out] -log(a/b + x**2 - x/b)/2 + log(a/b + x**2 + x/b)/2
```

Giac [**A**]
time = 3.36, size = 25, normalized size = 0.86

$$\frac{1}{2}\log\left(bx^2+a+x\right)-\frac{1}{2}\log\left(bx^2+a-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")
[Out] 1/2*log(b*x^2 + a + x) - 1/2*log(b*x^2 + a - x)
```

Mupad [**B**]
time = 4.41, size = 12, normalized size = 0.41

$$\operatorname{atanh}\left(\frac{x}{b\,x^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)/(x^2*(2*a*b - 1) + a^2 + b^2*x^4),x)
[Out] atanh(x/(a + b*x^2))
```

3.38 $\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{1+2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

[Out] arctanh((-2*b*x+1)/(-4*a*b+1)^(1/2))/(-4*a*b+1)^(1/2)-arctanh((2*b*x+1)/(-4*a*b+1)^(1/2))/(-4*a*b+1)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1175, 632, 212}

$$\frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{2bx+1}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]

[Out] ArcTanh[(1 - 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b] - ArcTanh[(1 + 2*b*x)/Sqrt[1 - 4*a*b]]/Sqrt[1 - 4*a*b]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
```

```
0]))
```

Rubi steps

$$\int \frac{a+bx^2}{a^2+(-1+2ab)x^2+b^2x^4}\,dx = \frac{\int \frac{1}{\frac{a}{b}-\frac{x}{b}+x^2}\,dx}{2b}+\frac{\int \frac{1}{\frac{a}{b}+\frac{x}{b}+x^2}\,dx}{2b}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2}-x^2}\,dx, x, -\frac{1}{b}+2x\right)}{b}-\frac{\text{Subst}\left(\int \frac{1}{\frac{1-4ab}{b^2}-x^2}\,dx, x, \frac{1}{b}+2x\right)}{b}$$
$$= \frac{\tanh^{-1}\left(\frac{1-2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}-\frac{\tanh^{-1}\left(\frac{1+2bx}{\sqrt{1-4ab}}\right)}{\sqrt{1-4ab}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 138 vs. 2(60) = 120.
time = 0.12, size = 138, normalized size = 2.30

$$\frac{\frac{\left(1+\sqrt{1-4ab}\right)\tan^{-1}\left(\frac{bx}{\sqrt{-\frac{1}{2}+ab-\frac{1}{2}\sqrt{1-4ab}}}\right)}{\sqrt{-1+2ab-\sqrt{1-4ab}}}+\frac{\left(-1+\sqrt{1-4ab}\right)\tan^{-1}\left(\frac{\sqrt{2}\,bx}{\sqrt{-1+2ab+\sqrt{1-4ab}}}\right)}{\sqrt{-1+2ab+\sqrt{1-4ab}}}}{\sqrt{2-8ab}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(a^2 + (-1 + 2*a*b)*x^2 + b^2*x^4),x]
[Out] (((1 + Sqrt[1 - 4*a*b])*ArcTan[(b*x)/Sqrt[-1/2 + a*b - Sqrt[1 - 4*a*b]/2]])
/Sqrt[-1 + 2*a*b - Sqrt[1 - 4*a*b]] + ((-1 + Sqrt[1 - 4*a*b])*ArcTan[(Sqrt[
2]*b*x)/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a*b]]])/Sqrt[-1 + 2*a*b + Sqrt[1 - 4*a
*b]])/Sqrt[2 - 8*a*b]
```

Maple [A]
time = 0.04, size = 52, normalized size = 0.87

method	result	siz
default	$\frac{\arctan\left(\frac{2bx+1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}+\frac{\arctan\left(\frac{2bx-1}{\sqrt{4ab-1}}\right)}{\sqrt{4ab-1}}$	52

risch	$-\frac{\ln\left(b\,x^{2}\sqrt{-4ab+1}+(-4ab+1)x-a\sqrt{-4ab+1}\right)}{2\sqrt{-4ab+1}}+\frac{\ln\left(b\,x^{2}\sqrt{-4ab+1}+x(4ab-1)-a\sqrt{-4ab+1}\right)}{2\sqrt{-4ab+1}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(4*a*b-1)^(1/2)*arctan((2*b*x+1)/(4*a*b-1)^(1/2))+1/(4*a*b-1)^(1/2)*arcta
n((2*b*x-1)/(4*a*b-1)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*b-0.25>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.34, size = 164, normalized size = 2.73

$$\left[-\frac{\sqrt{-4\,ab+1}\,\log\left(\frac{b^{2}x^{4}-(6\,ab-1)x^{2}+a^{2}-2\,(bx^{3}-ax)\sqrt{-4\,ab+1}}{b^{2}x^{4}+(2\,ab-1)x^{2}+a^{2}}\right)}{2\,(4\,ab-1)},\ \frac{\sqrt{4\,ab-1}\,\arctan\left(\frac{bx}{\sqrt{4\,ab-1}}\right)+\sqrt{4\,ab-1}\,\arctan\left(\frac{(b^{2}x^{3}+(3\,ab-1)x)\sqrt{4\,ab-1}}{4\,a^{2}b-a}\right)}{4\,ab-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-4*a*b + 1)*log((b^2*x^4 - (6*a*b - 1)*x^2 + a^2 - 2*(b*x^3 - a*
x)*sqrt(-4*a*b + 1))/(b^2*x^4 + (2*a*b - 1)*x^2 + a^2))/(4*a*b - 1), (sqrt(
4*a*b - 1)*arctan(b*x/sqrt(4*a*b - 1)) + sqrt(4*a*b - 1)*arctan((b^2*x^3 +
(3*a*b - 1)*x)*sqrt(4*a*b - 1)/(4*a^2*b - a)))/(4*a*b - 1)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(56) = 112$.
time = 0.23, size = 117, normalized size = 1.95

$$-\frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^{2}+\frac{x\left(-4ab\sqrt{-\frac{1}{4ab-1}}+\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}+\frac{\sqrt{-\frac{1}{4ab-1}}\log\left(-\frac{a}{b}+x^{2}+\frac{x\left(4ab\sqrt{-\frac{1}{4ab-1}}-\sqrt{-\frac{1}{4ab-1}}\right)}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(a**2+(2*a*b-1)*x**2+b**2*x**4),x)
```

```
[Out] -sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(-4*a*b*sqrt(-1/(4*a*b - 1)) + sq
rt(-1/(4*a*b - 1)))/b)/2 + sqrt(-1/(4*a*b - 1))*log(-a/b + x**2 + x*(4*a*b*
sqrt(-1/(4*a*b - 1)) - sqrt(-1/(4*a*b - 1)))/b)/2
```

Giac [A]
time = 3.25, size = 51, normalized size = 0.85

$$\frac{\arctan\left(\frac{2\,bx+1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}}+\frac{\arctan\left(\frac{2\,bx-1}{\sqrt{4\,ab-1}}\right)}{\sqrt{4\,ab-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(a^2+(2*a*b-1)*x^2+b^2*x^4),x, algorithm="giac")
```

```
[Out] arctan((2*b*x + 1)/sqrt(4*a*b - 1))/sqrt(4*a*b - 1) + arctan((2*b*x - 1)/sq
rt(4*a*b - 1))/sqrt(4*a*b - 1)
```

Mupad [B]
time = 0.07, size = 55, normalized size = 0.92

$$\frac{\operatorname{atan}\left(\frac{b\,x}{\sqrt{4\,a\,b-1}}\right)+\operatorname{atan}\left(\frac{\frac{3\,x\,(4\,a\,b-1)}{4}-\frac{x}{4}+b^2\,x^3}{a\,\sqrt{4\,a\,b-1}}\right)}{\sqrt{4\,a\,b-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2*(2*a*b - 1) + a^2 + b^2*x^4),x)
```

```
[Out] (atan((b*x)/(4*a*b - 1)^(1/2)) + atan(((3*x*(4*a*b - 1))/4 - x/4 + b^2*x^3)
/(a*(4*a*b - 1)^(1/2))))/(4*a*b - 1)^(1/2)
```

3.39 $\int \frac{1+2x^2}{1+bx^2+4x^4} \, dx$

Optimal. Leaf size=62

$$-\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}$$

```
[Out] -arctan((-4*x+(4-b)^(1/2))/(4+b)^(1/2))/(4+b)^(1/2)+arctan((4*x+(4-b)^(1/2)
)/(4+b)^(1/2))/(4+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{4-b}+4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}-\frac{\text{ArcTan}\left(\frac{\sqrt{4-b}-4x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[4 - b] - 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]) + ArcTan[(Sqrt[4 - b
] + 4*x)/Sqrt[4 + b]]/Sqrt[4 + b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+2x^2}{1+bx^2+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{1}{2}\sqrt{4-b}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{1}{2}\sqrt{4-b}\,x+x^2}\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b)-x^2}\,dx,x,-\frac{\sqrt{4-b}}{2}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4-b)}\right.$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{4-b}-4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{4-b}+4x}{\sqrt{4+b}}\right)}{\sqrt{4+b}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 126 vs. 2(62) = 124.
time = 0.04, size = 126, normalized size = 2.03

$$\frac{\frac{\left(4-b+\sqrt{-16+b^2}\right)\tan^{-1}\left(\frac{2\sqrt{2}\,x}{\sqrt{b-\sqrt{-16+b^2}}}\right)}{\sqrt{b-\sqrt{-16+b^2}}}+\frac{\left(-4+b+\sqrt{-16+b^2}\right)\tan^{-1}\left(\frac{2\sqrt{2}\,x}{\sqrt{b+\sqrt{-16+b^2}}}\right)}{\sqrt{b+\sqrt{-16+b^2}}}}{\sqrt{2}\,\sqrt{-16+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] (((4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]]
)/Sqrt[b - Sqrt[-16 + b^2]] + ((-4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]
*x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-1
6 + b^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(50) = 100.
time = 0.04, size = 124, normalized size = 2.00

method	result
risch	$-\frac{\ln\left(-2x^2\sqrt{-4-b}+x(4+b)+\sqrt{-4-b}\right)}{2\sqrt{-4-b}}+\frac{\ln\left(-2x^2\sqrt{-4-b}+(-4-b)x+\sqrt{-4-b}\right)}{2\sqrt{-4-b}}$
default	$\frac{\left(4+\sqrt{(b-4)(4+b)}-b\right)\arctan\left(\frac{4x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)}{\sqrt{(b-4)(4+b)}\,\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}+\frac{\left(-4+\sqrt{(b-4)(4+b)}+b\right)\text{arctan}}{\sqrt{(b-4)(4+b)}\,\sqrt{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] (4+((b-4)*(4+b))^(1/2)-b)/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)+2*b)^
(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))+(-4+((b-4)*(4+b))^(1/2
)+b)/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((
b-4)*(4+b))^(1/2)+2*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + b*x^2 + 1), x)
```

Fricas [A]
time = 0.33, size = 110, normalized size = 1.77

$$\left[-\frac{\sqrt{-b-4}\log\left(\frac{4\,x^4-(b+8)x^2-2\,(2\,x^3-x)\sqrt{-b-4}+1}{4\,x^4+bx^2+1}\right)}{2\,(b+4)}, \frac{\sqrt{b+4}\arctan\left(\frac{4\,x^3+(b+2)x}{\sqrt{b+4}}\right)+\sqrt{b+4}\arctan\left(\frac{2\,x}{\sqrt{b+4}}\right)}{b+4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b - 4)*log((4*x^4 - (b + 8)*x^2 - 2*(2*x^3 - x)*sqrt(-b - 4) +
1)/(4*x^4 + b*x^2 + 1))/(b + 4), (sqrt(b + 4)*arctan((4*x^3 + (b + 2)*x)/sq
rt(b + 4)) + sqrt(b + 4)*arctan(2*x/sqrt(b + 4)))/(b + 4)]
```

Sympy [A]
time = 0.19, size = 95, normalized size = 1.53

$$-\frac{\sqrt{-\frac{1}{b+4}}\log\left(x^2+x\left(-\frac{b\sqrt{-\frac{1}{b+4}}}{2}-2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2}+\frac{\sqrt{-\frac{1}{b+4}}\log\left(x^2+x\left(\frac{b\sqrt{-\frac{1}{b+4}}}{2}+2\sqrt{-\frac{1}{b+4}}\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] -sqrt(-1/(b + 4))*log(x**2 + x*(-b*sqrt(-1/(b + 4))/2 - 2*sqrt(-1/(b + 4)))
 - 1/2)/2 + sqrt(-1/(b + 4))*log(x**2 + x*(b*sqrt(-1/(b + 4))/2 + 2*sqrt(-1
/(b + 4))) - 1/2)/2
```

Giac [A]
time = 3.95, size = 77, normalized size = 1.24

$$\frac{\sqrt{b+4}\,(b-5)\arctan\left(\frac{4\sqrt{\frac{1}{2}}\,x}{\sqrt{b+\sqrt{b^2-16}}}\right)}{b^2-b-20}+\frac{\sqrt{b+4}\,(b-5)\arctan\left(\frac{4\sqrt{\frac{1}{2}}\,x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{b^2-b-20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] sqrt(b + 4)*(b - 5)*arctan(4*sqrt(1/2)*x/sqrt(b + sqrt(b^2 - 16)))/(b^2 - b
 - 20) + sqrt(b + 4)*(b - 5)*arctan(4*sqrt(1/2)*x/sqrt(b - sqrt(b^2 - 16)))
/(b^2 - b - 20)
```

Mupad [B]
time = 4.39, size = 66, normalized size = 1.06

$$-\frac{\mathrm{atan}\left(\frac{-b^3\,x-4\,b^2\,x^3-2\,b^2\,x+16\,b\,x+64\,x^3+32\,x}{\left(b^2-16\right)\,\sqrt{b+4}}\right)-\mathrm{atan}\left(\frac{2\,x}{\sqrt{b+4}}\right)}{\sqrt{b+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(b*x^2 + 4*x^4 + 1),x)
```

```
[Out] -(atan((32*x + 16*b*x - 2*b^2*x - b^3*x + 64*x^3 - 4*b^2*x^3)/((b^2 - 16)*(
b + 4)^(1/2))) - atan((2*x)/(b + 4)^(1/2)))/(b + 4)^(1/2)
```

3.40 $\int \frac{1+2x^2}{1-bx^2+4x^4} \, dx$

Optimal. Leaf size=66

$$-\frac{\tan^{-1}\left(\frac{\sqrt{4+b}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}+\frac{\tan^{-1}\left(\frac{\sqrt{4+b}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

```
[Out] -arctan((-4*x+(4+b)^(1/2))/(4-b)^(1/2))/(4-b)^(1/2)+arctan((4*x+(4+b)^(1/2)
)/(4-b)^(1/2))/(4-b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b+4}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}-\frac{\text{ArcTan}\left(\frac{\sqrt{b+4}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[4 + b] - 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]) + ArcTan[(Sqrt[4 + b
] + 4*x)/Sqrt[4 - b]]/Sqrt[4 - b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+2x^2}{1-bx^2+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{1}{2}\sqrt{4+b}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{1}{2}\sqrt{4+b}\,x+x^2}\,dx$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b)-x^2}\,dx, x, -\frac{\sqrt{4+b}}{2}+2x\right)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{4}(-4+b)\cdots}\right.$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{4+b}-4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}} + \frac{\tan^{-1}\left(\frac{\sqrt{4+b}+4x}{\sqrt{4-b}}\right)}{\sqrt{4-b}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 134 vs. 2(66) = 132.
time = 0.04, size = 134, normalized size = 2.03

$$\frac{\frac{\left(4+b+\sqrt{-16+b^2}\right)\tan^{-1}\left(\frac{2\sqrt{2}\,x}{\sqrt{-b-\sqrt{-16+b^2}}}\right)}{\sqrt{-b-\sqrt{-16+b^2}}} + \frac{\left(-4-b+\sqrt{-16+b^2}\right)\tan^{-1}\left(\frac{2\sqrt{2}\,x}{\sqrt{-b+\sqrt{-16+b^2}}}\right)}{\sqrt{-b+\sqrt{-16+b^2}}}}{\sqrt{2}\sqrt{-16+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - b*x^2 + 4*x^4),x]
```

```
[Out] (((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[-b - Sqrt[-16 + b^2]]
])/Sqrt[-b - Sqrt[-16 + b^2]] + ((-4 - b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[
2]*x)/Sqrt[-b + Sqrt[-16 + b^2]]])/Sqrt[-b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqr
t[-16 + b^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(54) = 108.
time = 0.05, size = 124, normalized size = 1.88

method	result
risch	$\frac{\ln\left(2x^2\sqrt{b-4}+(4-b)x-\sqrt{b-4}\right)}{2\sqrt{b-4}} - \frac{\ln\left(2x^2\sqrt{b-4}+x(b-4)-\sqrt{b-4}\right)}{2\sqrt{b-4}}$
default	$\frac{\left(-4+\sqrt{(b-4)(4+b)}-b\right)\arctan\left(\frac{4x}{\sqrt{2\sqrt{(b-4)(4+b)}-2b}}\right)}{\sqrt{(b-4)(4+b)}\sqrt{2\sqrt{(b-4)(4+b)}-2b}} + \frac{\left(4+\sqrt{(b-4)(4+b)}+b\right)\arctan\left(\cdots\right.}{\sqrt{(b-4)(4+b)}\sqrt{-2\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-4+((b-4)*(4+b))^(1/2)-b)/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)-2*b)^
(1/2)*arctan(4*x/(2*((b-4)*(4+b))^(1/2)-2*b)^(1/2))+(4+((b-4)*(4+b))^(1/2)+
b)/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)-2*b)^(1/2)*arctan(4*x/(-2*((
b-4)*(4+b))^(1/2)-2*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - b*x^2 + 1), x)
```

Fricas [A]
time = 0.39, size = 120, normalized size = 1.82

$$\left[\frac{\log\left(\frac{4\,x^4+(b-8)x^2-2\,(2\,x^3-x)\,\sqrt{b-4}+1}{4\,x^4-bx^2+1}\right)}{2\,\sqrt{b-4}}, \frac{\sqrt{-b+4}\,\arctan\left(\frac{(4\,x^3-(b-2)x)\,\sqrt{-b+4}}{b-4}\right)+\sqrt{-b+4}\,\arctan\left(\frac{2\,\sqrt{-b+4}\,x}{b-4}\right)}{b-4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="fricas")
```

```
[Out] [1/2*log((4*x^4 + (b - 8)*x^2 - 2*(2*x^3 - x)*sqrt(b - 4) + 1)/(4*x^4 - b*x
^2 + 1))/sqrt(b - 4), (sqrt(-b + 4)*arctan((4*x^3 - (b - 2)*x)*sqrt(-b + 4)
/(b - 4)) + sqrt(-b + 4)*arctan(2*sqrt(-b + 4)*x/(b - 4)))/(b - 4)]
```

Sympy [A]
time = 0.19, size = 83, normalized size = 1.26

$$\frac{\sqrt{\frac{1}{b-4}}\log\left(x^2+x\left(-\frac{b\sqrt{\frac{1}{b-4}}}{2}+2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2}-\frac{\sqrt{\frac{1}{b-4}}\log\left(x^2+x\left(\frac{b\sqrt{\frac{1}{b-4}}}{2}-2\sqrt{\frac{1}{b-4}}\right)-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-b*x**2+1),x)
```

```
[Out] sqrt(1/(b - 4))*log(x**2 + x*(-b*sqrt(1/(b - 4))/2 + 2*sqrt(1/(b - 4))) - 1
/2)/2 - sqrt(1/(b - 4))*log(x**2 + x*(b*sqrt(1/(b - 4))/2 - 2*sqrt(1/(b - 4
))) - 1/2)/2
```

Giac [A]
time = 4.83, size = 77, normalized size = 1.17

$$-\frac{(b+5)\sqrt{-b+4}\arctan\left(\frac{x}{\sqrt{-\frac{1}{8}b+\frac{1}{8}\sqrt{b^2-16}}}\right)}{b^2+b-20}-\frac{(b+5)\sqrt{-b+4}\arctan\left(\frac{x}{\sqrt{-\frac{1}{8}b-\frac{1}{8}\sqrt{b^2-16}}}\right)}{b^2+b-20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-b*x^2+1),x, algorithm="giac")
```

```
[Out] -(b + 5)*sqrt(-b + 4)*arctan(x/sqrt(-1/8*b + 1/8*sqrt(b^2 - 16)))/(b^2 + b
- 20) - (b + 5)*sqrt(-b + 4)*arctan(x/sqrt(-1/8*b - 1/8*sqrt(b^2 - 16)))/(b
^2 + b - 20)
```

Mupad [B]
time = 4.41, size = 24, normalized size = 0.36

$$-\frac{\operatorname{atanh}\left(\frac{x\sqrt{b-4}}{2x^2-1}\right)}{\sqrt{b-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - b*x^2 + 1),x)
```

```
[Out] -atanh((x*(b - 4)^(1/2))/(2*x^2 - 1))/(b - 4)^(1/2)
```

3.41 $\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

```
[Out] 1/10*arctan(2*x/(1/2*10^(1/2)-1/2*2^(1/2)))*10^(1/2)+1/10*arctan(2*x/(1/2*1
0^(1/2)+1/2*2^(1/2)))*10^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1177, 209}

$$\frac{\text{ArcTan}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}}+\frac{\text{ArcTan}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[10] + ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/
Sqrt[10]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1+2x^2}{1+6x^2+4x^4}\,dx = \frac{1}{5}\left(5-\sqrt{5}\right)\int \frac{1}{3-\sqrt{5}+4x^2}\,dx + \frac{1}{5}\left(5+\sqrt{5}\right)\int \frac{1}{3+\sqrt{5}+4x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{10}} + \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{10}}$$

Mathematica [**A**]
time = 0.05, size = 83, normalized size = 1.84

$$\frac{\left(-1+\sqrt{5}\right)\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{2\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{2\sqrt{5\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
[Out] ((-1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]])/(2*Sqrt[5*(3 - Sqrt[5])])
+ ((1 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(2*Sqrt[5*(3 + Sqrt[5])])
```

Maple [**A**]
time = 0.05, size = 82, normalized size = 1.82

method	result	size
risch	$\frac{\sqrt{10}\,\arctan\left(\frac{\sqrt{10}\,x}{5}\right)}{10}+\frac{\sqrt{10}\,\arctan\left(\frac{2\sqrt{10}\,x^3}{5}+\frac{4\sqrt{10}\,x}{5}\right)}{10}$	35
default	$\frac{2\left(\sqrt{5}-1\right)\sqrt{5}\,\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}+\frac{2\sqrt{5}\,\left(\sqrt{5}+1\right)\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+6*x^2+1),x,method=_RETURNVERBOSE)
[Out] 2/5*(5^(1/2)-1)*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2^(
1/2)))+2/5*5^(1/2)*(5^(1/2)+1)/(2*10^(1/2)+2*2^(1/2))*arctan(8*x/(2*10^(1/2
)+2*2^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + 6*x^2 + 1), x)
```

Fricas [A]
time = 0.35, size = 31, normalized size = 0.69

$$\frac{1}{10}\sqrt{10}\arctan\left(\frac{2}{5}\sqrt{10}\left(x^3+2x\right)\right)+\frac{1}{10}\sqrt{10}\arctan\left(\frac{1}{5}\sqrt{10}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*arctan(2/5*sqrt(10)*(x^3 + 2*x)) + 1/10*sqrt(10)*arctan(1/5*s
qrt(10)*x)
```

Sympy [A]
time = 0.04, size = 42, normalized size = 0.93

$$\frac{\sqrt{10}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{10}\,x}{5}\right)+2\,\mathrm{atan}\left(\frac{2\sqrt{10}\,x^3}{5}+\frac{4\sqrt{10}\,x}{5}\right)\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+6*x**2+1),x)
```

```
[Out] sqrt(10)*(2*atan(sqrt(10)*x/5) + 2*atan(2*sqrt(10)*x**3/5 + 4*sqrt(10)*x/5)
)/20
```

Giac [A]
time = 3.86, size = 39, normalized size = 0.87

$$\frac{1}{10}\sqrt{10}\arctan\left(\frac{4x}{\sqrt{10}+\sqrt{2}}\right)+\frac{1}{10}\sqrt{10}\arctan\left(\frac{4x}{\sqrt{10}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(10)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/10*sqrt(10)*arctan(4*x/(
sqrt(10) - sqrt(2)))
```

Mupad [B]
time = 0.09, size = 29, normalized size = 0.64

$$\frac{\sqrt{10}\,\left(\mathrm{atan}\left(\frac{2\sqrt{10}\,x^3}{5}+\frac{4\sqrt{10}\,x}{5}\right)+\mathrm{atan}\left(\frac{\sqrt{10}\,x}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(6*x^2 + 4*x^4 + 1),x)
```

```
[Out] (10^(1/2)*(atan((4*10^(1/2)*x)/5 + (2*10^(1/2)*x^3)/5) + atan((10^(1/2)*x)/
5)))/10
```

3.42 $\int \frac{1+2x^2}{1+5x^2+4x^4} \, dx$

Optimal. Leaf size=15

$$\frac{1}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}(2x)$$

```
[Out] 1/3*arctan(x)+1/3*arctan(2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1177, 209}

$$\frac{\text{ArcTan}(x)}{3}+\frac{1}{3}\text{ArcTan}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[x]/3 + ArcTan[2*x]/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+2x^2}{1+5x^2+4x^4}\,dx &= \frac{2}{3}\int \frac{1}{1+4x^2}\,dx+\frac{4}{3}\int \frac{1}{4+4x^2}\,dx\\ &= \frac{1}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}(2x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.13

$$-\frac{1}{3}\tan^{-1}\left(\frac{3x}{-1+2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] -1/3*ArcTan[(3*x)/(-1 + 2*x^2)]
```

Maple [A]
time = 0.02, size = 12, normalized size = 0.80

method	result	size
default	$\frac{\arctan(x)}{3}+\frac{\arctan(2x)}{3}$	12
risch	$\frac{\arctan\left(\frac{4}{3}x^3+\frac{7}{3}x\right)}{3}+\frac{\arctan\left(\frac{2x}{3}\right)}{3}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+5*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arctan(x)+1/3*arctan(2*x)
```

Maxima [A]
time = 0.49, size = 11, normalized size = 0.73

$$\frac{1}{3}\arctan\left(2\,x\right)+\frac{1}{3}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(2*x) + 1/3*arctan(x)
```

Fricas [A]
time = 0.32, size = 19, normalized size = 1.27

$$\frac{1}{3}\arctan\left(\frac{4}{3}\,x^3+\frac{7}{3}\,x\right)+\frac{1}{3}\arctan\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(4/3*x^3 + 7/3*x) + 1/3*arctan(2/3*x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(10) = 20$.
time = 0.04, size = 22, normalized size = 1.47

$$\frac{\operatorname{atan}\left(\frac{2x}{3}\right)}{3}+\frac{\operatorname{atan}\left(\frac{4x^3}{3}+\frac{7x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+5*x**2+1),x)
```

```
[Out] atan(2*x/3)/3 + atan(4*x**3/3 + 7*x/3)/3
```

Giac [A]
time = 4.33, size = 11, normalized size = 0.73

$$\frac{1}{3}\,\arctan\left(2\,x\right)+\frac{1}{3}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*arctan(2*x) + 1/3*arctan(x)
```

Mupad [B]
time = 0.07, size = 19, normalized size = 1.27

$$\frac{\operatorname{atan}\left(\frac{2\,x}{3}\right)}{3}+\frac{\operatorname{atan}\left(\frac{4\,x^3}{3}+\frac{7\,x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(5*x^2 + 4*x^4 + 1),x)
```

```
[Out] atan((2*x)/3)/3 + atan((7*x)/3 + (4*x^3)/3)/3
```

3.43 $\int \frac{1+2x^2}{1+4x^2+4x^4} \, dx$

Optimal. Leaf size=14

$$\frac{\tan^{-1}\left(\sqrt{2}\, x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {28, 21, 209}

$$\frac{\text{ArcTan}\left(\sqrt{2}\, x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2x^2}{1+4x^2+4x^4}\,dx = 4\int \frac{1+2x^2}{(2+4x^2)^2}\,dx$$
$$= \int \frac{1}{1+2x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.00

$$\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.86

method	result	size
default	$\frac{\arctan\left(\sqrt{2}\,x\right)\sqrt{2}}{2}$	12
risch	$\frac{\arctan\left(\sqrt{2}\,x\right)\sqrt{2}}{2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(2^(1/2)*x)*2^(1/2)
```

Maxima [A]
time = 0.53, size = 11, normalized size = 0.79

$$\frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Fricas [A]
time = 0.35, size = 11, normalized size = 0.79

$$\frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 1.00

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+4*x**2+1),x)
[Out] sqrt(2)*atan(sqrt(2)*x)/2
```

Giac [A]
time = 3.25, size = 11, normalized size = 0.79

$$\frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Mupad [B]
time = 0.03, size = 11, normalized size = 0.79

$$\frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^2 + 4*x^4 + 1),x)
[Out] (2^(1/2)*atan(2^(1/2)*x))/2
```

3.44 $\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx$

Optimal. Leaf size=38

$$-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\frac{1+4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -1/7*arctan(1/7*(1-4*x)*7^(1/2))*7^(1/2)+1/7*arctan(1/7*(1+4*x)*7^(1/2))*7^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{4x+1}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\text{ArcTan}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(1 - 4*x)/Sqrt[7]]/Sqrt[7]) + ArcTan[(1 + 4*x)/Sqrt[7]]/Sqrt[7]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+2x^2}{1+3x^2+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{x}{2}+x^2}\,dx + \frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{x}{2}+x^2}\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{7}{4}-x^2}\,dx, x, -\frac{1}{2}+2x\right)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{7}{4}-x^2}\,dx, x, \frac{1}{2}+2x\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\frac{1+4x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 97, normalized size = 2.55

$$\frac{\left(-i+\sqrt{7}\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(3-i\sqrt{7}\right)}}\right)}{\sqrt{42-14i\sqrt{7}}} + \frac{\left(i+\sqrt{7}\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(3+i\sqrt{7}\right)}}\right)}{\sqrt{42+14i\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] ((-I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 - I*Sqrt[7])/2]])/Sqrt[42 - (14*I)*Sqr
t[7]] + ((I + Sqrt[7])*ArcTan[(2*x)/Sqrt[(3 + I*Sqrt[7])/2]])/Sqrt[42 + (14
*I)*Sqrt[7]]
```

Maple [A]
time = 0.03, size = 34, normalized size = 0.89

method	result	size
default	$\frac{\arctan\left(\frac{(1+4x)\sqrt{7}}{7}\right)\sqrt{7}}{7}+\frac{\sqrt{7}\,\arctan\left(\frac{(4x-1)\sqrt{7}}{7}\right)}{7}$	34
risch	$\frac{\sqrt{7}\,\arctan\left(\frac{2x\sqrt{7}}{7}\right)}{7}+\frac{\sqrt{7}\,\arctan\left(\frac{4x^3\sqrt{7}}{7}+\frac{5x\sqrt{7}}{7}\right)}{7}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*arctan(1/7*(1+4*x)*7^(1/2))*7^(1/2)+1/7*7^(1/2)*arctan(1/7*(4*x-1)*7^(1
/2))
```

Maxima [A]
time = 0.49, size = 33, normalized size = 0.87

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(4\,x+1)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(4\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)*
(4*x - 1))
```

Fricas [A]
time = 0.35, size = 33, normalized size = 0.87

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(4\,x^3+5\,x)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{2}{7}\sqrt{7}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x^3 + 5*x)) + 1/7*sqrt(7)*arctan(2/7*sqrt
(7)*x)
```

Sympy [A]
time = 0.04, size = 44, normalized size = 1.16

$$\frac{\sqrt{7}\cdot\left(2\,\mathrm{atan}\left(\frac{2\sqrt{7}\,x}{7}\right)+2\,\mathrm{atan}\left(\frac{4\sqrt{7}\,x^3}{7}+\frac{5\sqrt{7}\,x}{7}\right)\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+3*x**2+1),x)
```

```
[Out] sqrt(7)*(2*atan(2*sqrt(7)*x/7) + 2*atan(4*sqrt(7)*x**3/7 + 5*sqrt(7)*x/7))/
14
```

Giac [A]
time = 3.89, size = 33, normalized size = 0.87

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(4\,x+1)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(4\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 1)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7)*
(4*x - 1))
```

Mupad [B]
time = 0.09, size = 29, normalized size = 0.76

$$\frac{\sqrt{7}\,\left(\mathrm{atan}\left(\frac{4\,\sqrt{7}\,x^3}{7}+\frac{5\,\sqrt{7}\,x}{7}\right)+\mathrm{atan}\left(\frac{2\,\sqrt{7}\,x}{7}\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(3*x^2 + 4*x^4 + 1),x)
```

```
[Out] (7^(1/2)*(atan((5*7^(1/2)*x)/7 + (4*7^(1/2)*x^3)/7) + atan((2*7^(1/2)*x)/7)
))/7
```

3.45 $\int \frac{1+2x^2}{1+2x^2+4x^4} \, dx$

Optimal. Leaf size=48

$$-\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}\,x}{\sqrt{3}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{1+2\sqrt{2}\,x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[Out] -1/6*arctan(1/3*(1-2*x*2^(1/2))*3^(1/2))*6^(1/2)+1/6*arctan(1/3*(1+2*x*2^(1
/2))*3^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt{2}\,x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\text{ArcTan}\left(\frac{1-2\sqrt{2}\,x}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(1 - 2*Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTan[(1 + 2*Sqrt[2]*x)/Sqr
t[3]]/Sqrt[6]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+2x^2}{1+2x^2+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{x}{\sqrt{2}}+x^2}\,dx + \frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{x}{\sqrt{2}}+x^2}\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-x^2}\,dx, x, -\frac{1}{\sqrt{2}}+2x\right)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-x^2}\,dx, x, \frac{1}{\sqrt{2}}+\right.$$
$$= -\frac{\tan^{-1}\left(\frac{1-2\sqrt{2}\,x}{\sqrt{3}}\right)}{\sqrt{6}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt{2}\,x}{\sqrt{3}}\right)}{\sqrt{6}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 99, normalized size = 2.06

$$\frac{\left(-i+\sqrt{3}\right)\tan^{-1}\left(\frac{2x}{\sqrt{1-i\sqrt{3}}}\right)}{2\sqrt{3\left(1-i\sqrt{3}\right)}} + \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{2x}{\sqrt{1+i\sqrt{3}}}\right)}{2\sqrt{3\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] ((-I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 - I*Sqrt[3]]])/(2*Sqrt[3*(1 - I*Sqrt[3]
)]) + ((I + Sqrt[3])*ArcTan[(2*x)/Sqrt[1 + I*Sqrt[3]]])/(2*Sqrt[3*(1 + I*Sq
rt[3])])
```

Maple [A]
time = 0.04, size = 40, normalized size = 0.83

method	result	size
risch	$\frac{\sqrt{6}\,\arctan\left(\frac{x\sqrt{6}}{3}\right)}{6}+\frac{\sqrt{6}\,\arctan\left(\frac{2x^3\sqrt{6}}{3}+\frac{2x\sqrt{6}}{3}\right)}{6}$	35
default	$\frac{\sqrt{6}\,\arctan\left(\frac{\left(4x-\sqrt{2}\right)\sqrt{6}}{6}\right)}{6}+\frac{\sqrt{6}\,\arctan\left(\frac{\left(4x+\sqrt{2}\right)\sqrt{6}}{6}\right)}{6}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*6^(1/2)*arctan(1/6*(4*x-2^(1/2))*6^(1/2))+1/6*6^(1/2)*arctan(1/6*(4*x+2
^(1/2))*6^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="maxima")
[Out] integrate((2*x^2 + 1)/(4*x^4 + 2*x^2 + 1), x)
```

Fricas [A]
time = 0.33, size = 29, normalized size = 0.60

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{2}{3}\sqrt{6}\left(x^{3}+x\right)\right)+\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{3}\sqrt{6}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="fricas")
[Out] 1/6*sqrt(6)*arctan(2/3*sqrt(6)*(x^3 + x)) + 1/6*sqrt(6)*arctan(1/3*sqrt(6)*
x)
```

Sympy [A]
time = 0.04, size = 42, normalized size = 0.88

$$\frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}\right)+2\,\mathrm{atan}\left(\frac{2\sqrt{6}\,x^{3}}{3}+\frac{2\sqrt{6}\,x}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+2*x**2+1),x)
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3) + 2*atan(2*sqrt(6)*x**3/3 + 2*sqrt(6)*x/3))/12
```

Giac [A]
time = 3.55, size = 45, normalized size = 0.94

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{4}{3}\sqrt{3}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x+\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{6}\sqrt{6}\arctan\left(\frac{4}{3}\sqrt{3}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x-\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*arctan(4/3*sqrt(3)*(1/4)^(3/4)*(2*x + (1/4)^(1/4))) + 1/6*sqrt(
6)*arctan(4/3*sqrt(3)*(1/4)^(3/4)*(2*x - (1/4)^(1/4)))
```

Mupad [B]
time = 4.39, size = 29, normalized size = 0.60

$$\frac{\sqrt{6}\,\left(\mathrm{atan}\left(\frac{2\,\sqrt{6}\,x^3}{3}+\frac{2\,\sqrt{6}\,x}{3}\right)+\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(2*x^2 + 4*x^4 + 1),x)
```

```
[Out] (6^(1/2)*(atan((2*6^(1/2)*x)/3 + (2*6^(1/2)*x^3)/3) + atan((6^(1/2)*x)/3)))
/6
```

3.46 $\int \frac{1+2x^2}{1+x^2+4x^4}\,dx$

Optimal. Leaf size=46

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}+4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -1/5*arctan(1/5*(-4*x+3^(1/2))*5^(1/2))*5^(1/2)+1/5*arctan(1/5*(4*x+3^(1/2)
)*5^(1/2))*5^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{4x+\sqrt{3}}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\text{ArcTan}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[3] - 4*x)/Sqrt[5]]/Sqrt[5]) + ArcTan[(Sqrt[3] + 4*x)/Sqrt[5]
]/Sqrt[5]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+2x^2}{1+x^2+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{3}\,x}{2}+x^2}\,dx + \frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{3}\,x}{2}+x^2}\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{5}{4}-x^2}\,dx, x, -\frac{\sqrt{3}}{2}+2x\right)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{5}{4}-x^2}\,dx, x, \frac{\sqrt{3}}{2}+\right.$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3}-4x}{\sqrt{5}}\right)}{\sqrt{5}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}+4x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 97, normalized size = 2.11

$$\frac{\left(-3i+\sqrt{15}\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{15}\right)}}\right)}{\sqrt{30-30i\sqrt{15}}} + \frac{\left(3i+\sqrt{15}\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{15}\right)}}\right)}{\sqrt{30+30i\sqrt{15}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] ((-3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 - I*Sqrt[15])/2]])/Sqrt[30 - (30*I)
*Sqrt[15]] + ((3*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(1 + I*Sqrt[15])/2]])/Sqrt
[30 + (30*I)*Sqrt[15]]
```

Maple [A]
time = 0.04, size = 40, normalized size = 0.87

method	result	size
risch	$\frac{\sqrt{5}\,\arctan\left(\frac{2x\sqrt{5}}{5}\right)}{5} + \frac{\sqrt{5}\,\arctan\left(\frac{4x^3\sqrt{5}}{5}+\frac{3x\sqrt{5}}{5}\right)}{5}$	35
default	$\frac{\arctan\left(\frac{\left(4x+\sqrt{3}\right)\sqrt{5}}{5}\right)\sqrt{5}}{5} + \frac{\sqrt{5}\,\arctan\left(\frac{\left(4x-\sqrt{3}\right)\sqrt{5}}{5}\right)}{5}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*arctan(1/5*(4*x+3^(1/2))*5^(1/2))*5^(1/2)+1/5*5^(1/2)*arctan(1/5*(4*x-3
^(1/2))*5^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 + x^2 + 1), x)
```

Fricas [A]
time = 0.31, size = 33, normalized size = 0.72

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\left(4x^{3}+3x\right)\right)+\frac{1}{5}\sqrt{5}\arctan\left(\frac{2}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(4*x^3 + 3*x)) + 1/5*sqrt(5)*arctan(2/5*sqrt
(5)*x)
```

Sympy [A]
time = 0.04, size = 44, normalized size = 0.96

$$\frac{\sqrt{5}\cdot\left(2\,\mathrm{atan}\left(\frac{2\sqrt{5}x}{5}\right)+2\,\mathrm{atan}\left(\frac{4\sqrt{5}x^{3}}{5}+\frac{3\sqrt{5}x}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+x**2+1),x)
```

```
[Out] sqrt(5)*(2*atan(2*sqrt(5)*x/5) + 2*atan(4*sqrt(5)*x**3/5 + 3*sqrt(5)*x/5))/
10
```

Giac [A]
time = 3.71, size = 52, normalized size = 1.13

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{2}{5}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x+\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{5}\sqrt{5}\arctan\left(\frac{2}{5}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x-\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*arctan(2/5*sqrt(10)*(1/4)^(3/4)*(4*x + sqrt(6)*(1/4)^(1/4))) +
1/5*sqrt(5)*arctan(2/5*sqrt(10)*(1/4)^(3/4)*(4*x - sqrt(6)*(1/4)^(1/4)))
```

Mupad [B]
time = 4.36, size = 29, normalized size = 0.63

$$\frac{\sqrt{5}\,\left(\mathrm{atan}\left(\frac{4\,\sqrt{5}\,x^3}{5}+\frac{3\,\sqrt{5}\,x}{5}\right)+\mathrm{atan}\left(\frac{2\,\sqrt{5}\,x}{5}\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(x^2 + 4*x^4 + 1),x)
```

```
[Out] (5^(1/2)*(atan((3*5^(1/2)*x)/5 + (4*5^(1/2)*x^3)/5) + atan((2*5^(1/2)*x)/5)
))/5
```

3.47 $\int \frac{1+2x^2}{1+4x^4} \, dx$

Optimal. Leaf size=21

$$-\frac{1}{2}\tan^{-1}(1-2x)+\frac{1}{2}\tan^{-1}(1+2x)$$

```
[Out] 1/2*arctan(-1+2*x)+1/2*arctan(1+2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1176, 631, 210}

$$\frac{1}{2}\text{ArcTan}(2x+1)-\frac{1}{2}\text{ArcTan}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -1/2*ArcTan[1 - 2*x] + ArcTan[1 + 2*x]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\int \frac{1+2x^2}{1+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{\frac{1}{2}+x+x^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-2x\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+2x\right)$$
$$= -\frac{1}{2}\tan^{-1}(1-2x) + \frac{1}{2}\tan^{-1}(1+2x)$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 0.81

$$-\frac{1}{2}\tan^{-1}\left(\frac{2x}{-1+2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 + 4*x^4),x]
[Out] -1/2*ArcTan[(2*x)/(-1 + 2*x^2)]
```

Maple [A]
time = 0.16, size = 18, normalized size = 0.86

method	result
risch	$\frac{\arctan(x)}{2}+\frac{\arctan(2x^3+x)}{2}$
default	$\frac{\arctan(2x-1)}{2}+\frac{\arctan(2x+1)}{2}$
meijerg	$\frac{\sqrt{2}\left(\frac{x^3\sqrt{2}\,\ln\left(1-2\left(x^4\right)^{\frac{1}{4}}+2\sqrt{x^4}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\left(x^4\right)^{\frac{1}{4}}}{1-\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+2\left(x^4\right)^{\frac{1}{4}}+2\sqrt{x^4}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\left(x^4\right)}{1+\left(x^4\right.}\right.}{\left(x^4\right)^{\frac{3}{4}}}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(2*x-1)+1/2*arctan(2*x+1)
```

Maxima [A]
time = 0.53, size = 17, normalized size = 0.81

$$\frac{1}{2}\arctan\left(2\,x+1\right)+\frac{1}{2}\arctan\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(2*x + 1) + 1/2*arctan(2*x - 1)
```

Fricas [**A**]
time = 0.31, size = 15, normalized size = 0.71

$$\frac{1}{2}\arctan\left(2\,x^{3}+x\right)+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(2*x^3 + x) + 1/2*arctan(x)
```

Sympy [**A**]
time = 0.03, size = 14, normalized size = 0.67

$$\frac{\operatorname{atan}\left(x\right)}{2}+\frac{\operatorname{atan}\left(2x^{3}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4+1),x)
```

```
[Out] atan(x)/2 + atan(2*x**3 + x)/2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(17) = 34.
time = 3.94, size = 46, normalized size = 2.19

$$\frac{1}{2}\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{2}\arctan\left(2\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(2*sqrt(2)*(1/4)^(3/4)*(2*x + sqrt(2)*(1/4)^(1/4))) + 1/2*arctan(
2*sqrt(2)*(1/4)^(3/4)*(2*x - sqrt(2)*(1/4)^(1/4)))
```

Mupad [**B**]
time = 4.29, size = 15, normalized size = 0.71

$$\frac{\mathrm{atan}(2\,x^{3}+x)}{2}+\frac{\mathrm{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 + 1),x)
```

```
[Out] atan(x + 2*x^3)/2 + atan(x)/2
```

3.48 $\int \frac{1+2x^2}{1-x^2+4x^4}\,dx$

Optimal. Leaf size=46

$$-\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{5}+4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctan(1/3*(-4*x+5^(1/2))*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(4*x+5^(1/2)
)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{4x+\sqrt{5}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[5] - 4*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(Sqrt[5] + 4*x)/Sqrt[3]
]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+2x^2}{1-x^2+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}\,x}{2}+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}\,x}{2}+x^2}\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx, x, -\frac{\sqrt{5}}{2}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx, x, \frac{\sqrt{5}}{2}+2x\right)$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{5}-4x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{5}+4x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 101, normalized size = 2.20

$$\frac{\left(-5i+\sqrt{15}\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1-i\sqrt{15}\right)}}+\frac{\left(5i+\sqrt{15}\right)\tan^{-1}\left(\frac{2x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{15}\right)}}\right)}{\sqrt{30\left(-1+i\sqrt{15}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] ((-5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 - I*Sqrt[15])/2]])/Sqrt[30*(-1 - I
*Sqrt[15])] + ((5*I + Sqrt[15])*ArcTan[(2*x)/Sqrt[(-1 + I*Sqrt[15])/2]])/Sq
rt[30*(-1 + I*Sqrt[15])]
```

Maple [A]
time = 0.06, size = 40, normalized size = 0.87

method	result	size
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{2x\sqrt{3}}{3}\right)}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{4x^3\sqrt{3}}{3}+\frac{x\sqrt{3}}{3}\right)}{3}$	35
default	$\frac{\arctan\left(\frac{\left(4x+\sqrt{5}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(4x-\sqrt{5}\right)\sqrt{3}}{3}\right)}{3}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arctan(1/3*(4*x+5^(1/2))*3^(1/2))*3^(1/2)+1/3*3^(1/2)*arctan(1/3*(4*x-5
^(1/2))*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="maxima")
[Out] integrate((2*x^2 + 1)/(4*x^4 - x^2 + 1), x)
```

Fricas [A]
time = 0.32, size = 31, normalized size = 0.67

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(4x^{3}+x\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(4*x^3 + x)) + 1/3*sqrt(3)*arctan(2/3*sqrt(3
)*x)
```

Sympy [A]
time = 0.04, size = 42, normalized size = 0.91

$$\frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}\right)+2\,\mathrm{atan}\left(\frac{4\sqrt{3}x^{3}}{3}+\frac{\sqrt{3}x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-x**2+1),x)
[Out] sqrt(3)*(2*atan(2*sqrt(3)*x/3) + 2*atan(4*sqrt(3)*x**3/3 + sqrt(3)*x/3))/6
```

Giac [A]
time = 2.54, size = 52, normalized size = 1.13

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{6}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x+\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{6}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4x-\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(2/3*sqrt(6)*(1/4)^(3/4)*(4*x + sqrt(10)*(1/4)^(1/4))) +
1/3*sqrt(3)*arctan(2/3*sqrt(6)*(1/4)^(3/4)*(4*x - sqrt(10)*(1/4)^(1/4)))
```

Mupad [B]
time = 4.37, size = 29, normalized size = 0.63

$$\frac{\sqrt{3}\,\left(\mathrm{atan}\left(\frac{4\,\sqrt{3}\,x^3}{3}+\frac{\sqrt{3}\,x}{3}\right)+\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - x^2 + 1),x)
```

```
[Out] (3^(1/2)*(atan((3^(1/2)*x)/3 + (4*3^(1/2)*x^3)/3) + atan((2*3^(1/2)*x)/3)))
/3
```

3.49 $\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx$

Optimal. Leaf size=44

$$-\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{3}+2\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(2*x*2^(1/2)-3^(1/2))*2^(1/2)+1/2*arctan(2*x*2^(1/2)+3^(1/2))*2^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(2\sqrt{2}\,x+\sqrt{3}\right)}{\sqrt{2}}-\frac{\text{ArcTan}\left(\sqrt{3}-2\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTan[Sqrt[3] - 2*Sqrt[2]*x]/Sqrt[2]) + ArcTan[Sqrt[3] + 2*Sqrt[2]*x]/Sq
rt[2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+2x^2}{1-2x^2+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-\sqrt{\frac{3}{2}}\,x+x^2}\,dx+\frac{1}{4}\int \frac{1}{\frac{1}{2}+\sqrt{\frac{3}{2}}\,x+x^2}\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-x^2}\,dx, x, -\sqrt{\frac{3}{2}}+2x\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-x^2}\,dx, x, \sqrt{\frac{3}{2}}+\right.$$
$$= -\frac{\tan^{-1}\left(\sqrt{3}-2\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{3}+2\sqrt{2}\,x\right)}{\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 99, normalized size = 2.25

$$\frac{\left(-3i+\sqrt{3}\right)\tan^{-1}\left(\frac{2x}{\sqrt{-1-i\sqrt{3}}}\right)}{2\sqrt{3\left(-1-i\sqrt{3}\right)}}+\frac{\left(3i+\sqrt{3}\right)\tan^{-1}\left(\frac{2x}{\sqrt{-1+i\sqrt{3}}}\right)}{2\sqrt{3\left(-1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] ((-3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 - I*Sqrt[3]]])/(2*Sqrt[3*(-1 - I*Sqr
t[3])]) + ((3*I + Sqrt[3])*ArcTan[(2*x)/Sqrt[-1 + I*Sqrt[3]]])/(2*Sqrt[3*(-
1 + I*Sqrt[3])])
```

Maple [A]
time = 0.04, size = 40, normalized size = 0.91

method	result	size
risch	$\frac{\arctan\left(\sqrt{2}\,x\right)\sqrt{2}}{2}+\frac{\sqrt{2}\,\arctan\left(2\sqrt{2}\,x^3\right)}{2}$	27
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\left(4x+\sqrt{6}\right)\sqrt{2}}{2}\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\frac{\left(4x-\sqrt{6}\right)\sqrt{2}}{2}\right)}{2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*arctan(1/2*(4*x+6^(1/2))*2^(1/2))+1/2*2^(1/2)*arctan(1/2*(4*x-6
^(1/2))*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1), x)
```

Fricas [A]
time = 0.31, size = 26, normalized size = 0.59

$$\frac{1}{2}\sqrt{2}\arctan\left(2\sqrt{2}\,x^3\right)+\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(2*sqrt(2)*x^3) + 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Sympy [A]
time = 0.04, size = 29, normalized size = 0.66

$$\frac{\sqrt{2}\cdot\left(2\operatorname{atan}\left(\sqrt{2}\,x\right)+2\operatorname{atan}\left(2\sqrt{2}\,x^3\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-2*x**2+1),x)
```

```
[Out] sqrt(2)*(2*atan(sqrt(2)*x) + 2*atan(2*sqrt(2)*x**3))/4
```

Giac [A]
time = 4.02, size = 46, normalized size = 1.05

$$\frac{1}{2}\sqrt{2}\arctan\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x+\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(4\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(2\,x-\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(4*(1/4)^(3/4)*(2*x + sqrt(3)*(1/4)^(1/4))) + 1/2*sqrt(2)
*arctan(4*(1/4)^(3/4)*(2*x - sqrt(3)*(1/4)^(1/4)))
```

Mupad [B]
time = 0.06, size = 21, normalized size = 0.48

$$\frac{\sqrt{2}\,\left(\mathrm{atan}\left(\sqrt{2}\,x\right)+\mathrm{atan}\left(2\,\sqrt{2}\,x^3\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - 2*x^2 + 1),x)
[Out] (2^(1/2)*(atan(2^(1/2)*x) + atan(2*2^(1/2)*x^3)))/2
```

3.50 $\int \frac{1+2x^2}{1-3x^2+4x^4} \, dx$

Optimal. Leaf size=23

$$-\tan^{-1}\left(\sqrt{7}-4x\right)+\tan^{-1}\left(\sqrt{7}+4x\right)$$

```
[Out] arctan(4*x-7^(1/2))+arctan(4*x+7^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 210}

$$\text{ArcTan}\left(4x+\sqrt{7}\right)-\text{ArcTan}\left(\sqrt{7}-4x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] -ArcTan[Sqrt[7] - 4*x] + ArcTan[Sqrt[7] + 4*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+2x^2}{1-3x^2+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{\sqrt{7}\,x}{2}+x^2}\,dx + \frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{\sqrt{7}\,x}{2}+x^2}\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{4}-x^2}\,dx, x, -\frac{\sqrt{7}}{2}+2x\right)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{-\frac{1}{4}-x^2}\,dx, x, \frac{\sqrt{7}}{2}+2$$
$$= -\tan^{-1}\left(\sqrt{7}-4x\right) + \tan^{-1}\left(\sqrt{7}+4x\right)$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 0.61

$$-\tan^{-1}\left(\frac{x}{-1+2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
[Out] -ArcTan[x/(-1 + 2*x^2)]
```

Maple [A]
time = 0.04, size = 20, normalized size = 0.87

method	result	size
risch	$\arctan\left(4x^3-x\right)+\arctan\left(2x\right)$	16
default	$\arctan\left(4x-\sqrt{7}\right)+\arctan\left(4x+\sqrt{7}\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-3*x^2+1),x,method=_RETURNVERBOSE)
[Out] arctan(4*x-7^(1/2))+arctan(4*x+7^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="maxima")
[Out] integrate((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1), x)
```

Fricas [A]
time = 0.33, size = 15, normalized size = 0.65

$$\arctan\left(4\,x^3 - x\right) + \arctan\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(4*x^3 - x) + arctan(2*x)
```

Sympy [A]
time = 0.03, size = 12, normalized size = 0.52

$$\operatorname{atan}\left(2x\right) + \operatorname{atan}\left(4x^3 - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-3*x**2+1),x)
```

```
[Out] atan(2*x) + atan(4*x**3 - x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(19) = 38.
time = 2.71, size = 42, normalized size = 1.83

$$\arctan\left(2\,\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x + \sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right) + \arctan\left(2\,\sqrt{2}\left(\frac{1}{4}\right)^{\frac{3}{4}}\left(4\,x - \sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] arctan(2*sqrt(2)*(1/4)^(3/4)*(4*x + sqrt(14)*(1/4)^(1/4))) + arctan(2*sqrt(
2)*(1/4)^(3/4)*(4*x - sqrt(14)*(1/4)^(1/4)))
```

Mupad [B]
time = 4.35, size = 15, normalized size = 0.65

$$\operatorname{atan}(2\,x) - \operatorname{atan}\left(x - 4\,x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - 3*x^2 + 1),x)
```

```
[Out] atan(2*x) - atan(x - 4*x^3)
```

3.51 $\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx$

Optimal. Leaf size=11

$$\frac{x}{1-2x^2}$$

[Out] x/(-2*x^2+1)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {28, 391}

$$\frac{x}{1-2x^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]

[Out] x/(1 - 2*x^2)

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^2}{1-4x^2+4x^4}\,dx &= 4\int \frac{1+2x^2}{\left(-2+4x^2\right)^2}\,dx\\ &= \frac{x}{1-2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.09

$$-\frac{x}{-1+2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] -(x/(-1 + 2*x^2))
```

Maple [A]
time = 0.01, size = 11, normalized size = 1.00

method	result	size
default	$-\frac{x}{2\left(x^{2}-\frac{1}{2}\right)}$	11
risch	$-\frac{x}{2\left(x^{2}-\frac{1}{2}\right)}$	11
gosper	$-\frac{x}{2x^{2}-1}$	13
norman	$-\frac{x}{2x^{2}-1}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-4*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x/(x^2-1/2)
```

Maxima [A]
time = 0.29, size = 12, normalized size = 1.09

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="maxima")
```

```
[Out] -x/(2*x^2 - 1)
```

Fricas [A]
time = 0.35, size = 12, normalized size = 1.09

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] -x/(2*x^2 - 1)
```

Sympy [A]
time = 0.02, size = 8, normalized size = 0.73

$$-\frac{x}{2x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-4*x**2+1),x)
```

```
[Out] -x/(2*x**2 - 1)
```

Giac [A]
time = 3.14, size = 12, normalized size = 1.09

$$-\frac{x}{2\,x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")
```

```
[Out] -x/(2*x^2 - 1)
```

Mupad [B]
time = 4.30, size = 12, normalized size = 1.09

$$-\frac{x}{2\,\left(x^2-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - 4*x^2 + 1),x)
```

```
[Out] -x/(2*(x^2 - 1/2))
```

3.52 $\int \frac{1+2x^2}{1-5x^2+4x^4} \, dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(1+x)+\frac{1}{2}\log(1+2x)$$

```
[Out] -1/2*ln(1-2*x)+1/2*ln(1-x)-1/2*ln(1+x)+1/2*ln(1+2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 630, 31}

$$-\frac{1}{2}\log(1-2x)+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - 2*x] + Log[1 - x]/2 - Log[1 + x]/2 + Log[1 + 2*x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+2x^2}{1-5x^2+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-\frac{3x}{2}+x^2}\,dx + \frac{1}{4}\int \frac{1}{\frac{1}{2}+\frac{3x}{2}+x^2}\,dx$$
$$= \frac{1}{2}\int \frac{1}{-1+x}\,dx - \frac{1}{2}\int \frac{1}{-\frac{1}{2}+x}\,dx + \frac{1}{2}\int \frac{1}{\frac{1}{2}+x}\,dx - \frac{1}{2}\int \frac{1}{1+x}\,dx$$
$$= -\frac{1}{2}\log(1-2x) + \frac{1}{2}\log(1-x) - \frac{1}{2}\log(1+x) + \frac{1}{2}\log(1+2x)$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 0.74

$$-\frac{1}{2}\log\left(1-x-2x^2\right) + \frac{1}{2}\log\left(1+x-2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
[Out] -1/2*Log[1 - x - 2*x^2] + Log[1 + x - 2*x^2]/2
```

Maple [A]
time = 0.02, size = 30, normalized size = 0.77

method	result	size
risch	$-\frac{\ln(2x^2+x-1)}{2}+\frac{\ln(2x^2-x-1)}{2}$	26
default	$\frac{\ln(2x+1)}{2}+\frac{\ln(-1+x)}{2}-\frac{\ln(2x-1)}{2}-\frac{\ln(1+x)}{2}$	30
norman	$\frac{\ln(2x+1)}{2}+\frac{\ln(-1+x)}{2}-\frac{\ln(2x-1)}{2}-\frac{\ln(1+x)}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-5*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(2*x+1)+1/2*ln(-1+x)-1/2*ln(2*x-1)-1/2*ln(1+x)
```

Maxima [A]
time = 0.28, size = 29, normalized size = 0.74

$$\frac{1}{2}\log\left(2\,x+1\right) - \frac{1}{2}\log\left(2\,x-1\right) - \frac{1}{2}\log\left(x+1\right) + \frac{1}{2}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="maxima")
[Out] 1/2*log(2*x + 1) - 1/2*log(2*x - 1) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

Fricas [A]
time = 0.36, size = 25, normalized size = 0.64

$$-\frac{1}{2}\,\log\left(2\,x^{2}+x-1\right)+\frac{1}{2}\,\log\left(2\,x^{2}-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="fricas")
[Out] -1/2*log(2*x^2 + x - 1) + 1/2*log(2*x^2 - x - 1)
```

Sympy [A]
time = 0.03, size = 26, normalized size = 0.67

$$\frac{\log\left(x^{2}-\frac{x}{2}-\frac{1}{2}\right)}{2}-\frac{\log\left(x^{2}+\frac{x}{2}-\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-5*x**2+1),x)
[Out] log(x**2 - x/2 - 1/2)/2 - log(x**2 + x/2 - 1/2)/2
```

Giac [A]
time = 4.04, size = 33, normalized size = 0.85

$$\frac{1}{2}\,\log\left(|2\,x+1|\right)-\frac{1}{2}\,\log\left(|2\,x-1|\right)-\frac{1}{2}\,\log\left(|x+1|\right)+\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
[Out] 1/2*log(abs(2*x + 1)) - 1/2*log(abs(2*x - 1)) - 1/2*log(abs(x + 1)) + 1/2*l
og(abs(x - 1))
```

Mupad [B]
time = 0.30, size = 14, normalized size = 0.36

$$-\mathrm{atanh}\left(\frac{x}{2\,x^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - 5*x^2 + 1),x)
[Out] -atanh(x/(2*x^2 - 1))
```

3.53 $\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx$

Optimal. Leaf size=44

$$\frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{5}+2\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(2*x*2^(1/2)-5^(1/2))*2^(1/2)-1/2*arctanh(2*x*2^(1/2)+5^(1/2))*
2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 212}

$$\frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(2\sqrt{2}\,x+\sqrt{5}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTanh[Sqrt[5] - 2*Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[5] + 2*Sqrt[2]*x]/Sqr
t[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+2x^2}{1-6x^2+4x^4}\,dx = \frac{1}{4}\int \frac{1}{\frac{1}{2}-\sqrt{\frac{5}{2}}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{\frac{1}{2}+\sqrt{\frac{5}{2}}\,x+x^2}\,dx$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{2}-x^2}\,dx, x, -\sqrt{\frac{5}{2}}+2x\right)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{2}-x^2}\,dx, x, \sqrt{\frac{5}{2}}+2x\right)$$
$$= \frac{\tanh^{-1}\left(\sqrt{5}-2\sqrt{2}\,x\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\sqrt{5}+2\sqrt{2}\,x\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.95

$$\frac{\log\left(1+\sqrt{2}\,x-2x^2\right)-\log\left(-1+\sqrt{2}\,x+2x^2\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
[Out] (Log[1 + Sqrt[2]*x - 2*x^2] - Log[-1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(35) = 70$.
time = 0.04, size = 82, normalized size = 1.86

method	result	size
risch	$\frac{\sqrt{2}\,\ln\left(-\sqrt{2}\,x+2x^2-1\right)}{4} - \frac{\sqrt{2}\,\ln\left(\sqrt{2}\,x+2x^2-1\right)}{4}$	39
default	$-\frac{2\left(-5+\sqrt{5}\right)\sqrt{5}\,\text{arctanh}\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)} - \frac{2\left(5+\sqrt{5}\right)\sqrt{5}\,\text{arctanh}\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)/(4*x^4-6*x^2+1),x,method=_RETURNVERBOSE)
[Out] -2/5*(-5+5^(1/2))*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctanh(8*x/(2*10^(1/2)-2*
2^(1/2)))-2/5*(5+5^(1/2))*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctanh(8*x/(2*10^
(1/2)+2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1), x)
```

Fricas [A]
time = 0.35, size = 47, normalized size = 1.07

$$\frac{1}{4}\sqrt{2}\log\left(\frac{4x^4 - 2x^2 - 2\sqrt{2}(2x^3 - x) + 1}{4x^4 - 6x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((4*x^4 - 2*x^2 - 2*sqrt(2)*(2*x^3 - x) + 1)/(4*x^4 - 6*x^2
+ 1))
```

Sympy [A]
time = 0.03, size = 46, normalized size = 1.05

$$\frac{\sqrt{2}\log\left(x^2 - \frac{\sqrt{2}x}{2} - \frac{1}{2}\right)}{4} - \frac{\sqrt{2}\log\left(x^2 + \frac{\sqrt{2}x}{2} - \frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x/2 - 1/2)/4 - sqrt(2)*log(x**2 + sqrt(2)*x/2 -
1/2)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(35) = 70.
time = 3.58, size = 77, normalized size = 1.75

$$-\frac{1}{4}\sqrt{2}\log\left(\left|x + \frac{1}{4}\sqrt{10} + \frac{1}{4}\sqrt{2}\right|\right) + \frac{1}{4}\sqrt{2}\log\left(\left|x + \frac{1}{4}\sqrt{10} - \frac{1}{4}\sqrt{2}\right|\right) - \frac{1}{4}\sqrt{2}\log\left(\left|x - \frac{1}{4}\sqrt{10} + \frac{1}{4}\sqrt{2}\right|\right) + \frac{1}{4}\sqrt{2}\log\left(\left|x - \frac{1}{4}\sqrt{10} - \frac{1}{4}\sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/4*sqrt(2)*log(abs
(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10) +
1/4*sqrt(2))) + 1/4*sqrt(2)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2)))
```

Mupad [B]
time = 0.22, size = 20, normalized size = 0.45

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2\,x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)/(4*x^4 - 6*x^2 + 1),x)
[Out] -(2^(1/2)*atanh((2^(1/2)*x)/(2*x^2 - 1)))/2
```

3.54 $\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx$

Optimal. Leaf size=66

$$-\frac{\log\left(1-\sqrt{4-b}\,x+2x^2\right)}{2\sqrt{4-b}}+\frac{\log\left(1+\sqrt{4-b}\,x+2x^2\right)}{2\sqrt{4-b}}$$

```
[Out] -1/2*ln(1+2*x^2-x*(4-b)^(1/2))/(4-b)^(1/2)+1/2*ln(1+2*x^2+x*(4-b)^(1/2))/(4
-b)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1178, 642}

$$\frac{\log\left(\sqrt{4-b}\,x+2x^2+1\right)}{2\sqrt{4-b}}-\frac{\log\left(-\sqrt{4-b}\,x+2x^2+1\right)}{2\sqrt{4-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[4 - b]*x + 2*x^2]/Sqrt[4 - b] + Log[1 + Sqrt[4 - b]*x + 2
*x^2]/(2*Sqrt[4 - b])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-2x^2}{1+bx^2+4x^4}\,dx = -\frac{\int \frac{\frac{\sqrt{4-b}}{2}+2x}{-\frac{1}{2}-\frac{1}{2}\sqrt{4-b}\,x-x^2}\,dx}{2\sqrt{4-b}} - \frac{\int \frac{\frac{\sqrt{4-b}}{2}-2x}{-\frac{1}{2}+\frac{1}{2}\sqrt{4-b}\,x-x^2}\,dx}{2\sqrt{4-b}}$$
$$= -\frac{\log\left(1-\sqrt{4-b}\,x+2x^2\right)}{2\sqrt{4-b}} + \frac{\log\left(1+\sqrt{4-b}\,x+2x^2\right)}{2\sqrt{4-b}}$$

Mathematica [A]
time = 0.04, size = 127, normalized size = 1.92

$$\frac{\frac{\left(4+b-\sqrt{-16+b^2}\right)\tan^{-1}\left(\frac{2\sqrt{2}\,x}{\sqrt{b-\sqrt{-16+b^2}}}\right)}{\sqrt{b-\sqrt{-16+b^2}}} - \frac{\left(4+b+\sqrt{-16+b^2}\right)\tan^{-1}\left(\frac{2\sqrt{2}\,x}{\sqrt{b+\sqrt{-16+b^2}}}\right)}{\sqrt{b+\sqrt{-16+b^2}}}}{\sqrt{2}\,\sqrt{-16+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + b*x^2 + 4*x^4),x]
```

```
[Out] (((4 + b - Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*x)/Sqrt[b - Sqrt[-16 + b^2]]]
)/Sqrt[b - Sqrt[-16 + b^2]] - ((4 + b + Sqrt[-16 + b^2])*ArcTan[(2*Sqrt[2]*
x)/Sqrt[b + Sqrt[-16 + b^2]]])/Sqrt[b + Sqrt[-16 + b^2]])/(Sqrt[2]*Sqrt[-16
 + b^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(54) = 108$.
time = 0.05, size = 128, normalized size = 1.94

method	result
risch	$-\frac{\ln\left(-2x^2\sqrt{4-b}+(4-b)x-\sqrt{4-b}\right)}{2\sqrt{4-b}}+\frac{\ln\left(-2x^2\sqrt{4-b}+x(b-4)-\sqrt{4-b}\right)}{2\sqrt{4-b}}$
default	$\frac{\left(4-\sqrt{(b-4)(4+b)}+b\right)\arctan\left(\frac{4x}{\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}\right)}{\sqrt{(b-4)(4+b)}\,\sqrt{-2\sqrt{(b-4)(4+b)}+2b}}+\frac{\left(-4-\sqrt{(b-4)(4+b)}-b\right)\text{arctan}}{\sqrt{(b-4)(4+b)}\,\sqrt{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] (4-((b-4)*(4+b))^(1/2)+b)/((b-4)*(4+b))^(1/2)/(-2*((b-4)*(4+b))^(1/2)+2*b)^
(1/2)*arctan(4*x/(-2*((b-4)*(4+b))^(1/2)+2*b)^(1/2))+(-4-((b-4)*(4+b))^(1/2
)-b)/((b-4)*(4+b))^(1/2)/(2*((b-4)*(4+b))^(1/2)+2*b)^(1/2)*arctan(4*x/(2*((
b-4)*(4+b))^(1/2)+2*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 + b*x^2 + 1), x)
```

Fricas [A]
time = 0.33, size = 109, normalized size = 1.65

$$\left[-\frac{\sqrt{-b+4}\,\log\left(\frac{4\,x^4-(b-8)x^2+2\,(2\,x^3+x)\sqrt{-b+4}+1}{4\,x^4+bx^2+1}\right)}{2\,(b-4)},\ \frac{\sqrt{b-4}\,\arctan\left(\frac{4\,x^3+(b-2)x}{\sqrt{b-4}}\right)-\sqrt{b-4}\,\arctan\left(\frac{2\,x}{\sqrt{b-4}}\right)}{b-4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b + 4)*log((4*x^4 - (b - 8)*x^2 + 2*(2*x^3 + x)*sqrt(-b + 4) +
1)/(4*x^4 + b*x^2 + 1))/(b - 4), (sqrt(b - 4)*arctan((4*x^3 + (b - 2)*x)/sq
rt(b - 4)) - sqrt(b - 4)*arctan(2*x/sqrt(b - 4)))/(b - 4)]
```

Sympy [A]
time = 0.19, size = 94, normalized size = 1.42

$$\frac{\sqrt{-\frac{1}{b-4}}\,\log\left(x^2+x\left(-\frac{b\sqrt{-\frac{1}{b-4}}}{2}+2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2}-\frac{\sqrt{-\frac{1}{b-4}}\,\log\left(x^2+x\left(\frac{b\sqrt{-\frac{1}{b-4}}}{2}-2\sqrt{-\frac{1}{b-4}}\right)+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+b*x**2+1),x)
```

```
[Out] sqrt(-1/(b - 4))*log(x**2 + x*(-b*sqrt(-1/(b - 4))/2 + 2*sqrt(-1/(b - 4)))
+ 1/2)/2 - sqrt(-1/(b - 4))*log(x**2 + x*(b*sqrt(-1/(b - 4))/2 - 2*sqrt(-1/
(b - 4))) + 1/2)/2
```

Giac [A]
time = 4.60, size = 78, normalized size = 1.18

$$-\frac{(b+3)\sqrt{b-4}\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{b+\sqrt{b^2-16}}}\right)}{b^2-b-12}+\frac{(b+3)\sqrt{b-4}\arctan\left(\frac{4\sqrt{\frac{1}{2}}x}{\sqrt{b-\sqrt{b^2-16}}}\right)}{b^2-b-12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] -(b + 3)*sqrt(b - 4)*arctan(4*sqrt(1/2)*x/sqrt(b + sqrt(b^2 - 16)))/(b^2 -
b - 12) + (b + 3)*sqrt(b - 4)*arctan(4*sqrt(1/2)*x/sqrt(b - sqrt(b^2 - 16))
)/(b^2 - b - 12)
```

Mupad [B]
time = 0.07, size = 63, normalized size = 0.95

$$-\frac{\operatorname{atan}\left(\frac{2\,x}{\sqrt{b-4}}\right)-\operatorname{atan}\left(\frac{b^3\,x+4\,b^2\,x^3-2\,b^2\,x-16\,b\,x-64\,x^3+32\,x}{{\left(b-4\right)}^{3/2}\,\left(b+4\right)}\right)}{\sqrt{b-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(b*x^2 + 4*x^4 + 1),x)
```

```
[Out] -(atan((2*x)/(b - 4)^(1/2)) - atan((32*x - 16*b*x - 2*b^2*x + b^3*x - 64*x^
3 + 4*b^2*x^3)/((b - 4)^(3/2)*(b + 4))))/(b - 4)^(1/2)
```

3.55 $\int \frac{1-2x^2}{1+6x^2+4x^4} \, dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(2*x/(1/2*10^(1/2)-1/2*2^(1/2)))*2^(1/2)-1/2*arctan(2*x/(1/2*10^(
1/2)+1/2*2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1177, 209}

$$\frac{\text{ArcTan}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]/Sqrt[2] - ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]]/S
qrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-2x^2}{1+6x^2+4x^4}\,dx = \left(-1-\sqrt{5}\right)\int \frac{1}{3+\sqrt{5}+4x^2}\,dx + \left(-1+\sqrt{5}\right)\int \frac{1}{3-\sqrt{5}+4x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.03, size = 84, normalized size = 1.83

$$\frac{-\left(\left(-5+\sqrt{5}\right)\sqrt{3+\sqrt{5}}\tan^{-1}\left(\frac{2x}{\sqrt{3-\sqrt{5}}}\right)\right)-\sqrt{3-\sqrt{5}}\left(5+\sqrt{5}\right)\tan^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{5}}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 6*x^2 + 4*x^4),x]
```

```
[Out] (-((-5 + Sqrt[5])*Sqrt[3 + Sqrt[5]]*ArcTan[(2*x)/Sqrt[3 - Sqrt[5]]]) - Sqrt
[3 - Sqrt[5]]*(5 + Sqrt[5])*ArcTan[(2*x)/Sqrt[3 + Sqrt[5]]])/(4*Sqrt[5])
```

Maple [A]
time = 0.03, size = 82, normalized size = 1.78

method	result	size
risch	$\frac{\sqrt{2}\,\arctan\left(2\sqrt{2}\,x^3+2\sqrt{2}\,x\right)}{2}-\frac{\arctan\left(\sqrt{2}\,x\right)\sqrt{2}}{2}$	34
default	$-\frac{2\left(-5+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}-\frac{2\left(5+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+6*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(-5+5^(1/2))*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctan(8*x/(2*10^(1/2)-2*2
^(1/2)))-2/5*(5+5^(1/2))*5^(1/2)/(2*10^(1/2)+2*2^(1/2))*arctan(8*x/(2*10^(1
/2)+2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="maxima")
[Out] -integrate((2*x^2 - 1)/(4*x^4 + 6*x^2 + 1), x)
```

Fricas [A]
time = 0.36, size = 28, normalized size = 0.61

$$\frac{1}{2}\sqrt{2}\arctan\left(2\sqrt{2}\left(x^3+x\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(2*sqrt(2)*(x^3 + x)) - 1/2*sqrt(2)*arctan(sqrt(2)*x)
```

Sympy [A]
time = 0.04, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2}\cdot\left(2\operatorname{atan}\left(\sqrt{2}\,x\right)-2\operatorname{atan}\left(2\sqrt{2}\,x^3+2\sqrt{2}\,x\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+6*x**2+1),x)
[Out] -sqrt(2)*(2*atan(sqrt(2)*x) - 2*atan(2*sqrt(2)*x**3 + 2*sqrt(2)*x))/4
```

Giac [A]
time = 3.74, size = 39, normalized size = 0.85

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{4\,x}{\sqrt{10}+\sqrt{2}}\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{4\,x}{\sqrt{10}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+6*x^2+1),x, algorithm="giac")
[Out] -1/2*sqrt(2)*arctan(4*x/(sqrt(10) + sqrt(2))) + 1/2*sqrt(2)*arctan(4*x/(sqr
t(10) - sqrt(2)))
```

Mupad [B]
time = 4.38, size = 30, normalized size = 0.65

$$\frac{\sqrt{2}\,\left(\mathrm{atan}\left(2\,\sqrt{2}\,x^3+2\,\sqrt{2}\,x\right)-\mathrm{atan}\left(\sqrt{2}\,x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(6*x^2 + 4*x^4 + 1),x)
[Out] (2^(1/2)*(atan(2*2^(1/2)*x + 2*2^(1/2)*x^3) - atan(2^(1/2)*x)))/2
```

3.56 $\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx$

Optimal. Leaf size=9

$$-\tan^{-1}(x)+\tan^{-1}(2x)$$

```
[Out] -arctan(x)+arctan(2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1177, 209}

$$\text{ArcTan}(2x)-\text{ArcTan}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
```

```
[Out] -ArcTan[x] + ArcTan[2*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+5x^2+4x^4}\,dx &= 2\int \frac{1}{1+4x^2}\,dx-4\int \frac{1}{4+4x^2}\,dx\\ &= -\tan^{-1}(x)+\tan^{-1}(2x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.33

$$\tan^{-1}\left(\frac{x}{1+2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 5*x^2 + 4*x^4),x]
[Out] ArcTan[x/(1 + 2*x^2)]
```

Maple [A]
time = 0.02, size = 10, normalized size = 1.11

method	result	size
default	$-\arctan(x)+\arctan(2x)$	10
risch	$-\arctan(2x)+\arctan(4x^3+3x)$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+5*x^2+1),x,method=_RETURNVERBOSE)
[Out] -arctan(x)+arctan(2*x)
```

Maxima [A]
time = 0.51, size = 9, normalized size = 1.00

$$\arctan(2\,x)-\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="maxima")
[Out] arctan(2*x) - arctan(x)
```

Fricas [A]
time = 0.33, size = 17, normalized size = 1.89

$$\arctan\left(4\,x^3+3\,x\right)-\arctan(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="fricas")
[Out] arctan(4*x^3 + 3*x) - arctan(2*x)
```

Sympy [A]
time = 0.03, size = 14, normalized size = 1.56

$$-\operatorname{atan}(2x)+\operatorname{atan}\left(4x^3+3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+5*x**2+1),x)
```

```
[Out] -atan(2*x) + atan(4*x**3 + 3*x)
```

Giac [A]
time = 3.12, size = 9, normalized size = 1.00

$$\arctan(2x) - \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] arctan(2*x) - arctan(x)
```

Mupad [B]
time = 4.36, size = 17, normalized size = 1.89

$$\mathrm{atan}\left(4x^3 + 3x\right) - \mathrm{atan}(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(5*x^2 + 4*x^4 + 1),x)
```

```
[Out] atan(3*x + 4*x^3) - atan(2*x)
```

3.57 $\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx$

Optimal. Leaf size=11

$$\frac{x}{1+2x^2}$$

```
[Out] x/(2*x^2+1)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {28, 391}

$$\frac{x}{2x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 + 2*x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+4x^2+4x^4}\,dx &= 4\int \frac{1-2x^2}{(2+4x^2)^2}\,dx\\ &= \frac{x}{1+2x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{x}{1+2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 4*x^2 + 4*x^4),x]
```

```
[Out] x/(1 + 2*x^2)
```

Maple [A]
time = 0.01, size = 11, normalized size = 1.00

method	result	size
default	$\frac{x}{2x^2+1}$	11
risch	$\frac{x}{2x^2+1}$	11
gosper	$\frac{x}{2x^2+1}$	12
norman	$\frac{x}{2x^2+1}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/(x^2+1/2)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 1.00

$$\frac{x}{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] x/(2*x^2 + 1)
```

Fricas [A]
time = 0.34, size = 11, normalized size = 1.00

$$\frac{x}{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] x/(2*x^2 + 1)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.64

$$\frac{x}{2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+4*x**2+1),x)
```

```
[Out] x/(2*x**2 + 1)
```

Giac [A]
time = 3.24, size = 11, normalized size = 1.00

$$\frac{x}{2\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] x/(2*x^2 + 1)
```

Mupad [B]
time = 4.30, size = 11, normalized size = 1.00

$$\frac{x}{2\,\left(x^2+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^2 + 4*x^4 + 1),x)
```

```
[Out] x/(2*(x^2 + 1/2))
```

3.58 $\int \frac{1-2x^2}{1+3x^2+4x^4} \, dx$

Optimal. Leaf size=29

$$-\frac{1}{2}\log\left(1-x+2x^2\right)+\frac{1}{2}\log\left(1+x+2x^2\right)$$

```
[Out] -1/2*ln(2*x^2-x+1)+1/2*ln(2*x^2+x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1178, 642}

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - x + 2*x^2] + Log[1 + x + 2*x^2]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+3x^2+4x^4} \, dx &= -\left(\frac{1}{2}\int \frac{\frac{1}{2}+2x}{-\frac{1}{2}-\frac{x}{2}-x^2} \, dx\right)-\frac{1}{2}\int \frac{\frac{1}{2}-2x}{-\frac{1}{2}+\frac{x}{2}-x^2} \, dx\\ &= -\frac{1}{2}\log\left(1-x+2x^2\right)+\frac{1}{2}\log\left(1+x+2x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 1.00

$$-\frac{1}{2}\log\left(1-x+2x^2\right)+\frac{1}{2}\log\left(1+x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 3*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - x + 2*x^2] + Log[1 + x + 2*x^2]/2
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.90

method	result	size
default	$-\frac{\ln(2x^2-x+1)}{2}+\frac{\ln(2x^2+x+1)}{2}$	26
norman	$-\frac{\ln(2x^2-x+1)}{2}+\frac{\ln(2x^2+x+1)}{2}$	26
risch	$-\frac{\ln(2x^2-x+1)}{2}+\frac{\ln(2x^2+x+1)}{2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(2*x^2-x+1)+1/2*ln(2*x^2+x+1)
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.86

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

Fricas [A]
time = 0.34, size = 25, normalized size = 0.86

$$\frac{1}{2}\log\left(2x^2+x+1\right)-\frac{1}{2}\log\left(2x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

Sympy [A]
time = 0.03, size = 26, normalized size = 0.90

$$-\frac{\log\left(x^2-\frac{x}{2}+\frac{1}{2}\right)}{2}+\frac{\log\left(x^2+\frac{x}{2}+\frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+3*x**2+1),x)
```

```
[Out] -log(x**2 - x/2 + 1/2)/2 + log(x**2 + x/2 + 1/2)/2
```

Giac [A]
time = 4.01, size = 25, normalized size = 0.86

$$\frac{1}{2}\log\left(2\,x^2+x+1\right)-\frac{1}{2}\log\left(2\,x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(2*x^2 + x + 1) - 1/2*log(2*x^2 - x + 1)
```

Mupad [B]
time = 0.06, size = 12, normalized size = 0.41

$$\operatorname{atanh}\left(\frac{x}{2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(3*x^2 + 4*x^4 + 1),x)
```

```
[Out] atanh(x/(2*x^2 + 1))
```

3.59 $\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$-\frac{\log\left(1-\sqrt{2}\,x+2x^2\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+2x^2\right)}{2\sqrt{2}}$$

```
[Out] -1/4*ln(1+2*x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+2*x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1178, 642}

$$\frac{\log\left(2x^2+\sqrt{2}\,x+1\right)}{2\sqrt{2}}-\frac{\log\left(2x^2-\sqrt{2}\,x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[2]*x + 2*x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x + 2*x^2]/(2*Sqr
t[2])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+2x^2+4x^4}\,dx &= -\frac{\int \frac{\frac{1}{\sqrt{2}}+2x}{-\frac{1}{2}-\frac{x}{\sqrt{2}}-x^2}\,dx}{2\sqrt{2}}-\frac{\int \frac{\frac{1}{\sqrt{2}}-2x}{-\frac{1}{2}+\frac{x}{\sqrt{2}}-x^2}\,dx}{2\sqrt{2}}\\ &= -\frac{\log\left(1-\sqrt{2}\,x+2x^2\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+2x^2\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.84

$$\frac{-\log\left(-1+\sqrt{2}\,x-2x^2\right)+\log\left(1+\sqrt{2}\,x+2x^2\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 2*x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[2]*x - 2*x^2] + Log[1 + Sqrt[2]*x + 2*x^2])/(2*Sqrt[2])
```

Maple [A]
time = 0.02, size = 39, normalized size = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-\sqrt{2}\,x\right)\sqrt{2}}{4}+\frac{\ln\left(1+2x^2+\sqrt{2}\,x\right)\sqrt{2}}{4}$	39
risch	$-\frac{\ln\left(1+2x^2-\sqrt{2}\,x\right)\sqrt{2}}{4}+\frac{\ln\left(1+2x^2+\sqrt{2}\,x\right)\sqrt{2}}{4}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(1+2*x^2-2^(1/2)*x)*2^(1/2)+1/4*ln(1+2*x^2+2^(1/2)*x)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="maxima")
[Out] -integrate((2*x^2 - 1)/(4*x^4 + 2*x^2 + 1), x)
```

Fricas [A]
time = 0.32, size = 45, normalized size = 0.90

$$\frac{1}{4}\,\sqrt{2}\,\log\left(\frac{4\,x^4+6\,x^2+2\,\sqrt{2}\,(2\,x^3+x)+1}{4\,x^4+2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((4*x^4 + 6*x^2 + 2*sqrt(2)*(2*x^3 + x) + 1)/(4*x^4 + 2*x^2
+ 1))
```

Sympy [A]
time = 0.03, size = 46, normalized size = 0.92

$$-\frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}\,x}{2}+\frac{1}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}\,x}{2}+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+2*x**2+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x/2 + 1/2)/4 + sqrt(2)*log(x**2 + sqrt(2)*x/2 +
 1/2)/4
```

Giac [A]
time = 2.72, size = 34, normalized size = 0.68

$$\frac{1}{4}\sqrt{2}\log\left(x^2+\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(x^2 + (1/4)^(1/4)*x + 1/2) - 1/4*sqrt(2)*log(x^2 - (1/4)^(1
/4)*x + 1/2)
```

Mupad [B]
time = 4.37, size = 20, normalized size = 0.40

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{2\,x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(2*x^2 + 4*x^4 + 1),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*x)/(2*x^2 + 1)))/2
```

3.60 $\int \frac{1-2x^2}{1+x^2+4x^4} dx$

Optimal. Leaf size=50

$$-\frac{\log\left(1-\sqrt{3}\,x+2x^2\right)}{2\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+2x^2\right)}{2\sqrt{3}}$$

```
[Out] -1/6*ln(1+2*x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+2*x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1178, 642}

$$\frac{\log\left(2x^2+\sqrt{3}\,x+1\right)}{2\sqrt{3}}-\frac{\log\left(2x^2-\sqrt{3}\,x+1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[3]*x + 2*x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + 2*x^2]/(2*Sqr
t[3])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-2x^2}{1+x^2+4x^4}\,dx &= -\frac{\int \frac{\frac{\sqrt{3}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{3}\,x}{2}-x^2}\,dx}{2\sqrt{3}}-\frac{\int \frac{\frac{\sqrt{3}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{3}\,x}{2}-x^2}\,dx}{2\sqrt{3}}\\
&= -\frac{\log\left(1-\sqrt{3}\,x+2x^2\right)}{2\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+2x^2\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.84

$$\frac{-\log\left(-1+\sqrt{3}\,x-2x^2\right)+\log\left(1+\sqrt{3}\,x+2x^2\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[3]*x - 2*x^2] + Log[1 + Sqrt[3]*x + 2*x^2])/(2*Sqrt[3])
```

Maple [A]
time = 0.02, size = 39, normalized size = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+2x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+2x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(1+2*x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+2*x^2+x*3^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="maxima")
[Out] -integrate((2*x^2 - 1)/(4*x^4 + x^2 + 1), x)
```

Fricas [A]
time = 0.33, size = 43, normalized size = 0.86

$$\frac{1}{6}\,\sqrt{3}\,\log\left(\frac{4\,x^4+7\,x^2+2\,\sqrt{3}\,(2\,x^3+x)+1}{4\,x^4+x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((4*x^4 + 7*x^2 + 2*sqrt(3)*(2*x^3 + x) + 1)/(4*x^4 + x^2 +
1))
```

Sympy [A]
time = 0.03, size = 46, normalized size = 0.92

$$-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x/2 + 1/2)/6 + sqrt(3)*log(x**2 + sqrt(3)*x/2 +
 1/2)/6
```

Giac [A]
time = 3.70, size = 41, normalized size = 0.82

$$\frac{1}{6}\sqrt{3}\log\left(x^2+\frac{1}{2}\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{6}\sqrt{3}\log\left(x^2-\frac{1}{2}\sqrt{6}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(x^2 + 1/2*sqrt(6)*(1/4)^(1/4)*x + 1/2) - 1/6*sqrt(3)*log(x^
2 - 1/2*sqrt(6)*(1/4)^(1/4)*x + 1/2)
```

Mupad [B]
time = 0.07, size = 20, normalized size = 0.40

$$\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{2\,x^2+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(x^2 + 4*x^4 + 1),x)
```

```
[Out] (3^(1/2)*atanh((3^(1/2)*x)/(2*x^2 + 1)))/3
```

3.61 $\int \frac{1-2x^2}{1+4x^4} \, dx$

Optimal. Leaf size=31

$$-\frac{1}{4}\log\left(1-2x+2x^2\right)+\frac{1}{4}\log\left(1+2x+2x^2\right)$$

```
[Out] -1/4*ln(2*x^2-2*x+1)+1/4*ln(2*x^2+2*x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1179, 642}

$$\frac{1}{4}\log\left(2x^2+2x+1\right)-\frac{1}{4}\log\left(2x^2-2x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 + 4*x^4),x]
```

```
[Out] -1/4*Log[1 - 2*x + 2*x^2] + Log[1 + 2*x + 2*x^2]/4
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1+4x^4}\,dx &= -\left(\frac{1}{4}\int \frac{1+2x}{-\frac{1}{2}-x-x^2}\,dx\right)-\frac{1}{4}\int \frac{1-2x}{-\frac{1}{2}+x-x^2}\,dx\\ &= -\frac{1}{4}\log\left(1-2x+2x^2\right)+\frac{1}{4}\log\left(1+2x+2x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.00

$$-\frac{1}{4}\log\left(1-2x+2x^2\right)+\frac{1}{4}\log\left(1+2x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 + 4*x^4),x]
[Out] -1/4*Log[1 - 2*x + 2*x^2] + Log[1 + 2*x + 2*x^2]/4
```

Maple [A]
time = 0.14, size = 28, normalized size = 0.90

method	result
default	$-\frac{\ln(2x^2-2x+1)}{4}+\frac{\ln(2x^2+2x+1)}{4}$
norman	$-\frac{\ln(2x^2-2x+1)}{4}+\frac{\ln(2x^2+2x+1)}{4}$
risch	$-\frac{\ln(2x^2-2x+1)}{4}+\frac{\ln(2x^2+2x+1)}{4}$
meijerg	$-\frac{\sqrt{2}\left(\frac{x^3\sqrt{2}\,\ln\left(1-2\left(x^4\right)^{\frac{1}{4}}+2\sqrt{x^4}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\left(x^4\right)^{\frac{1}{4}}}{1-\left(x^4\right)^{\frac{1}{4}}}\right)}{\left(x^4\right)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+2\left(x^4\right)^{\frac{1}{4}}+2\sqrt{x^4}\right)}{2\left(x^4\right)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{(x}{1+(}\right.}{\left(x^4\right)^{\frac{3}{4}}}}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*ln(2*x^2-2*x+1)+1/4*ln(2*x^2+2*x+1)
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.87

$$\frac{1}{4}\,\log\left(2\,x^2+2\,x+1\right)-\frac{1}{4}\,\log\left(2\,x^2-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="maxima")
[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

Fricas [A]
time = 0.32, size = 27, normalized size = 0.87

$$\frac{1}{4}\,\log\left(2\,x^2+2\,x+1\right)-\frac{1}{4}\,\log\left(2\,x^2-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="fricas")
[Out] 1/4*log(2*x^2 + 2*x + 1) - 1/4*log(2*x^2 - 2*x + 1)
```

Sympy [A]
time = 0.03, size = 22, normalized size = 0.71

$$-\frac{\log\left(x^2-x+\frac{1}{2}\right)}{4}+\frac{\log\left(x^2+x+\frac{1}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4+1),x)
```

```
[Out] -log(x**2 - x + 1/2)/4 + log(x**2 + x + 1/2)/4
```

Giac [A]
time = 2.58, size = 34, normalized size = 1.10

$$\frac{1}{4}\log\left(x^2+\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{4}\log\left(x^2-\sqrt{2}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*log(x^2 + sqrt(2)*(1/4)^(1/4)*x + 1/2) - 1/4*log(x^2 - sqrt(2)*(1/4)^(1
/4)*x + 1/2)
```

Mupad [B]
time = 0.07, size = 15, normalized size = 0.48

$$\frac{\operatorname{atanh}\left(\frac{2\,x}{2\,x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 + 1),x)
```

```
[Out] atanh((2*x)/(2*x^2 + 1))/2
```

3.62 $\int \frac{1-2x^2}{1-x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$-\frac{\log\left(1-\sqrt{5}\,x+2x^2\right)}{2\sqrt{5}}+\frac{\log\left(1+\sqrt{5}\,x+2x^2\right)}{2\sqrt{5}}$$

```
[Out] -1/10*ln(1+2*x^2-x*5^(1/2))*5^(1/2)+1/10*ln(1+2*x^2+x*5^(1/2))*5^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1178, 642}

$$\frac{\log\left(2x^2+\sqrt{5}\,x+1\right)}{2\sqrt{5}}-\frac{\log\left(2x^2-\sqrt{5}\,x+1\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[5]*x + 2*x^2]/Sqrt[5] + Log[1 + Sqrt[5]*x + 2*x^2]/(2*Sqr
t[5])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1-x^2+4x^4}\,dx &= -\frac{\int \frac{\frac{\sqrt{5}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{5}\,x}{2}-x^2}\,dx}{2\sqrt{5}}-\frac{\int \frac{\frac{\sqrt{5}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{5}\,x}{2}-x^2}\,dx}{2\sqrt{5}}\\ &= -\frac{\log\left(1-\sqrt{5}\,x+2x^2\right)}{2\sqrt{5}}+\frac{\log\left(1+\sqrt{5}\,x+2x^2\right)}{2\sqrt{5}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.84

$$\frac{-\log\left(-1+\sqrt{5}\,x-2x^2\right)+\log\left(1+\sqrt{5}\,x+2x^2\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[5]*x - 2*x^2] + Log[1 + Sqrt[5]*x + 2*x^2])/(2*Sqrt[5])
```

Maple [A]
time = 0.03, size = 39, normalized size = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(1+2x^2+x\sqrt{5}\right)\sqrt{5}}{10}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(1+2x^2+x\sqrt{5}\right)\sqrt{5}}{10}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/10*ln(1+2*x^2-x*5^(1/2))*5^(1/2)+1/10*ln(1+2*x^2+x*5^(1/2))*5^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="maxima")
[Out] -integrate((2*x^2 - 1)/(4*x^4 - x^2 + 1), x)
```

Fricas [A]
time = 0.35, size = 45, normalized size = 0.90

$$\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{4\,x^4+9\,x^2+2\,\sqrt{5}\,(2\,x^3+x)+1}{4\,x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*log((4*x^4 + 9*x^2 + 2*sqrt(5)*(2*x^3 + x) + 1)/(4*x^4 - x^2 +
 1))
```

Sympy [A]
time = 0.03, size = 46, normalized size = 0.92

$$-\frac{\sqrt{5}\log\left(x^2-\frac{\sqrt{5}\,x}{2}+\frac{1}{2}\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\frac{\sqrt{5}\,x}{2}+\frac{1}{2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-x**2+1),x)
```

```
[Out] -sqrt(5)*log(x**2 - sqrt(5)*x/2 + 1/2)/10 + sqrt(5)*log(x**2 + sqrt(5)*x/2
+ 1/2)/10
```

Giac [A]
time = 3.84, size = 41, normalized size = 0.82

$$\frac{1}{10}\sqrt{5}\log\left(x^2+\frac{1}{2}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{10}\sqrt{5}\log\left(x^2-\frac{1}{2}\sqrt{10}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*log(x^2 + 1/2*sqrt(10)*(1/4)^(1/4)*x + 1/2) - 1/10*sqrt(5)*log
(x^2 - 1/2*sqrt(10)*(1/4)^(1/4)*x + 1/2)
```

Mupad [B]
time = 4.35, size = 20, normalized size = 0.40

$$\frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,x}{2\,x^2+1}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - x^2 + 1),x)
```

```
[Out] (5^(1/2)*atanh((5^(1/2)*x)/(2*x^2 + 1)))/5
```

3.63 $\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$-\frac{\log\left(1-\sqrt{6}\,x+2x^2\right)}{2\sqrt{6}}+\frac{\log\left(1+\sqrt{6}\,x+2x^2\right)}{2\sqrt{6}}$$

```
[Out] -1/12*ln(1+2*x^2-x*6^(1/2))*6^(1/2)+1/12*ln(1+2*x^2+x*6^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1178, 642}

$$\frac{\log\left(2x^2+\sqrt{6}\,x+1\right)}{2\sqrt{6}}-\frac{\log\left(2x^2-\sqrt{6}\,x+1\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[6]*x + 2*x^2]/Sqrt[6] + Log[1 + Sqrt[6]*x + 2*x^2]/(2*Sqr
t[6])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-2x^2}{1-2x^2+4x^4}\,dx = -\frac{\int \frac{\sqrt{\frac{3}{2}}+2x}{-\frac{1}{2}-\sqrt{\frac{3}{2}}\,x-x^2}\,dx}{2\sqrt{6}} - \frac{\int \frac{\sqrt{\frac{3}{2}}-2x}{-\frac{1}{2}+\sqrt{\frac{3}{2}}\,x-x^2}\,dx}{2\sqrt{6}}$$
$$= -\frac{\log\left(1-\sqrt{6}\,x+2x^2\right)}{2\sqrt{6}} + \frac{\log\left(1+\sqrt{6}\,x+2x^2\right)}{2\sqrt{6}}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.84

$$\frac{-\log\left(-1+\sqrt{6}\,x-2x^2\right)+\log\left(1+\sqrt{6}\,x+2x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 2*x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[6]*x - 2*x^2] + Log[1 + Sqrt[6]*x + 2*x^2])/(2*Sqrt[6])
```

Maple [A]
time = 0.02, size = 39, normalized size = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{6}\right)\sqrt{6}}{12}+\frac{\ln\left(1+2x^2+x\sqrt{6}\right)\sqrt{6}}{12}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{6}\right)\sqrt{6}}{12}+\frac{\ln\left(1+2x^2+x\sqrt{6}\right)\sqrt{6}}{12}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-2*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/12*ln(1+2*x^2-x*6^(1/2))*6^(1/2)+1/12*ln(1+2*x^2+x*6^(1/2))*6^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 2*x^2 + 1), x)
```

Fricas [A]
time = 0.37, size = 45, normalized size = 0.90

$$\frac{1}{12}\,\sqrt{6}\,\log\left(\frac{4\,x^4+10\,x^2+2\,\sqrt{6}\,(2\,x^3+x)+1}{4\,x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="fricas")
[Out] 1/12*sqrt(6)*log((4*x^4 + 10*x^2 + 2*sqrt(6)*(2*x^3 + x) + 1)/(4*x^4 - 2*x^
2 + 1))
```

Sympy [A]
time = 0.03, size = 46, normalized size = 0.92

$$-\frac{\sqrt{6}\,\log\left(x^2-\frac{\sqrt{6}\,x}{2}+\frac{1}{2}\right)}{12}+\frac{\sqrt{6}\,\log\left(x^2+\frac{\sqrt{6}\,x}{2}+\frac{1}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-2*x**2+1),x)
[Out] -sqrt(6)*log(x**2 - sqrt(6)*x/2 + 1/2)/12 + sqrt(6)*log(x**2 + sqrt(6)*x/2
+ 1/2)/12
```

Giac [A]
time = 4.41, size = 40, normalized size = 0.80

$$\frac{1}{12}\,\sqrt{6}\,\log\left(x^2+\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{12}\,\sqrt{6}\,\log\left(x^2-\sqrt{3}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-2*x^2+1),x, algorithm="giac")
[Out] 1/12*sqrt(6)*log(x^2 + sqrt(3)*(1/4)^(1/4)*x + 1/2) - 1/12*sqrt(6)*log(x^2
- sqrt(3)*(1/4)^(1/4)*x + 1/2)
```

Mupad [B]
time = 0.07, size = 20, normalized size = 0.40

$$\frac{\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\,x}{2\,x^2+1}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 2*x^2 + 1),x)
[Out] (6^(1/2)*atanh((6^(1/2)*x)/(2*x^2 + 1)))/6
```

3.64 $\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx$

Optimal. Leaf size=50

$$-\frac{\log\left(1-\sqrt{7}\,x+2x^2\right)}{2\sqrt{7}}+\frac{\log\left(1+\sqrt{7}\,x+2x^2\right)}{2\sqrt{7}}$$

```
[Out] -1/14*ln(1+2*x^2-x*7^(1/2))*7^(1/2)+1/14*ln(1+2*x^2+x*7^(1/2))*7^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1178, 642}

$$\frac{\log\left(2x^2+\sqrt{7}\,x+1\right)}{2\sqrt{7}}-\frac{\log\left(2x^2-\sqrt{7}\,x+1\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[7]*x + 2*x^2]/Sqrt[7] + Log[1 + Sqrt[7]*x + 2*x^2]/(2*Sqr
t[7])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1-3x^2+4x^4}\,dx &= -\frac{\int \frac{\frac{\sqrt{7}}{2}+2x}{-\frac{1}{2}-\frac{\sqrt{7}\,x}{2}-x^2}\,dx}{2\sqrt{7}}-\frac{\int \frac{\frac{\sqrt{7}}{2}-2x}{-\frac{1}{2}+\frac{\sqrt{7}\,x}{2}-x^2}\,dx}{2\sqrt{7}}\\ &= -\frac{\log\left(1-\sqrt{7}\,x+2x^2\right)}{2\sqrt{7}}+\frac{\log\left(1+\sqrt{7}\,x+2x^2\right)}{2\sqrt{7}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.84

$$\frac{-\log\left(-1+\sqrt{7}\,x-2x^2\right)+\log\left(1+\sqrt{7}\,x+2x^2\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 3*x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[7]*x - 2*x^2] + Log[1 + Sqrt[7]*x + 2*x^2])/(2*Sqrt[7])
```

Maple [A]
time = 0.03, size = 39, normalized size = 0.78

method	result	size
default	$-\frac{\ln\left(1+2x^2-x\sqrt{7}\right)\sqrt{7}}{14}+\frac{\ln\left(1+2x^2+x\sqrt{7}\right)\sqrt{7}}{14}$	39
risch	$-\frac{\ln\left(1+2x^2-x\sqrt{7}\right)\sqrt{7}}{14}+\frac{\ln\left(1+2x^2+x\sqrt{7}\right)\sqrt{7}}{14}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-3*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/14*ln(1+2*x^2-x*7^(1/2))*7^(1/2)+1/14*ln(1+2*x^2+x*7^(1/2))*7^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="maxima")
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 3*x^2 + 1), x)
```

Fricas [A]
time = 0.34, size = 45, normalized size = 0.90

$$\frac{1}{14}\sqrt{7}\log\left(\frac{4\,x^4+11\,x^2+2\,\sqrt{7}\,(2\,x^3+x)+1}{4\,x^4-3\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((4*x^4 + 11*x^2 + 2*sqrt(7)*(2*x^3 + x) + 1)/(4*x^4 - 3*x^
2 + 1))
```

Sympy [A]
time = 0.03, size = 46, normalized size = 0.92

$$-\frac{\sqrt{7}\log\left(x^2-\frac{\sqrt{7}\,x}{2}+\frac{1}{2}\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\frac{\sqrt{7}\,x}{2}+\frac{1}{2}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-3*x**2+1),x)
```

```
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x/2 + 1/2)/14 + sqrt(7)*log(x**2 + sqrt(7)*x/2
+ 1/2)/14
```

Giac [A]
time = 3.28, size = 41, normalized size = 0.82

$$\frac{1}{14}\sqrt{7}\log\left(x^2+\frac{1}{2}\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)-\frac{1}{14}\sqrt{7}\log\left(x^2-\frac{1}{2}\sqrt{14}\left(\frac{1}{4}\right)^{\frac{1}{4}}x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*log(x^2 + 1/2*sqrt(14)*(1/4)^(1/4)*x + 1/2) - 1/14*sqrt(7)*log
(x^2 - 1/2*sqrt(14)*(1/4)^(1/4)*x + 1/2)
```

Mupad [B]
time = 4.39, size = 20, normalized size = 0.40

$$\frac{\sqrt{7}\,\mathrm{atanh}\left(\frac{\sqrt{7}\,x}{2\,x^2+1}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 3*x^2 + 1),x)
```

```
[Out] (7^(1/2)*atanh((7^(1/2)*x)/(2*x^2 + 1)))/7
```

3.65 $\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {28, 21, 212}

$$\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] ArcTanh[Sqrt[2]*x]/Sqrt[2]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1-2x^2}{1-4x^2+4x^4}\,dx = 4\int \frac{1-2x^2}{(-2+4x^2)^2}\,dx$$
$$= \int \frac{1}{1-2x^2}\,dx$$
$$= \frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.
time = 0.00, size = 32, normalized size = 2.29

$$\frac{-\log\left(\sqrt{2}-2x\right)+\log\left(\sqrt{2}+2x\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 4*x^2 + 4*x^4),x]
```

```
[Out] (-Log[Sqrt[2] - 2*x] + Log[Sqrt[2] + 2*x])/(2*Sqrt[2])
```

Maple [A]
time = 0.01, size = 12, normalized size = 0.86

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sqrt{2}\,x\right)\sqrt{2}}{2}$	12
risch	$\frac{\sqrt{2}\,\ln\left(2x+\sqrt{2}\right)}{4}-\frac{\sqrt{2}\,\ln\left(2x-\sqrt{2}\right)}{4}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-4*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctanh(2^(1/2)*x)*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.50, size = 25, normalized size = 1.79

$$-\frac{1}{4}\,\sqrt{2}\,\log\left(\frac{2\,x-\sqrt{2}}{2\,x+\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log((2*x - sqrt(2))/(2*x + sqrt(2)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.
time = 0.32, size = 29, normalized size = 2.07

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2x^2+2\sqrt{2}x+1}{2x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*x^2 + 2*sqrt(2)*x + 1)/(2*x^2 - 1))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.
time = 0.03, size = 32, normalized size = 2.29

$$-\frac{\sqrt{2}\log\left(x-\frac{\sqrt{2}}{2}\right)}{4}+\frac{\sqrt{2}\log\left(x+\frac{\sqrt{2}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-4*x**2+1),x)
```

```
[Out] -sqrt(2)*log(x - sqrt(2)/2)/4 + sqrt(2)*log(x + sqrt(2)/2)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.
time = 3.17, size = 29, normalized size = 2.07

$$\frac{1}{4}\sqrt{2}\log\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{4}\sqrt{2}\log\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(x + 1/2*sqrt(2))) - 1/4*sqrt(2)*log(abs(x - 1/2*sqrt(2)
))
```

Mupad [B]
time = 4.33, size = 11, normalized size = 0.79

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 4*x^2 + 1),x)
[Out] (2^(1/2)*atanh(2^(1/2)*x))/2
```

3.66 $\int \frac{1-2x^2}{1-5x^2+4x^4} \, dx$

Optimal. Leaf size=39

$$-\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(1+x)+\frac{1}{6}\log(1+2x)$$

```
[Out] -1/6*ln(1-2*x)-1/6*ln(1-x)+1/6*ln(1+x)+1/6*ln(1+2*x)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 630, 31}

$$-\frac{1}{6}\log(1-2x)-\frac{1}{6}\log(1-x)+\frac{1}{6}\log(x+1)+\frac{1}{6}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

```
[Out] -1/6*Log[1 - 2*x] - Log[1 - x]/6 + Log[1 + x]/6 + Log[1 + 2*x]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1-2x^2}{1-5x^2+4x^4}\,dx = -\left(\frac{1}{4}\int \frac{1}{-\frac{1}{2}-\frac{x}{2}+x^2}\,dx\right) - \frac{1}{4}\int \frac{1}{-\frac{1}{2}+\frac{x}{2}+x^2}\,dx$$
$$= -\left(\frac{1}{6}\int \frac{1}{-1+x}\,dx\right) - \frac{1}{6}\int \frac{1}{-\frac{1}{2}+x}\,dx + \frac{1}{6}\int \frac{1}{\frac{1}{2}+x}\,dx + \frac{1}{6}\int \frac{1}{1+x}\,dx$$
$$= -\frac{1}{6}\log(1-2x) - \frac{1}{6}\log(1-x) + \frac{1}{6}\log(1+x) + \frac{1}{6}\log(1+2x)$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 0.79

$$-\frac{1}{6}\log\left(1-3x+2x^2\right) + \frac{1}{6}\log\left(1+3x+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 5*x^2 + 4*x^4),x]
```

```
[Out] -1/6*Log[1 - 3*x + 2*x^2] + Log[1 + 3*x + 2*x^2]/6
```

Maple [A]
time = 0.02, size = 30, normalized size = 0.77

method	result	size
risch	$-\frac{\ln(2x^2-3x+1)}{6}+\frac{\ln(2x^2+3x+1)}{6}$	28
default	$\frac{\ln(2x+1)}{6}-\frac{\ln(-1+x)}{6}-\frac{\ln(2x-1)}{6}+\frac{\ln(1+x)}{6}$	30
norman	$\frac{\ln(2x+1)}{6}-\frac{\ln(-1+x)}{6}-\frac{\ln(2x-1)}{6}+\frac{\ln(1+x)}{6}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-5*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln(2*x+1)-1/6*ln(-1+x)-1/6*ln(2*x-1)+1/6*ln(1+x)
```

Maxima [A]
time = 0.31, size = 29, normalized size = 0.74

$$\frac{1}{6}\,\log\left(2\,x+1\right) - \frac{1}{6}\,\log\left(2\,x-1\right) + \frac{1}{6}\,\log\left(x+1\right) - \frac{1}{6}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*log(2*x + 1) - 1/6*log(2*x - 1) + 1/6*log(x + 1) - 1/6*log(x - 1)
```

Fricas [A]
time = 0.33, size = 27, normalized size = 0.69

$$\frac{1}{6}\,\log\left(2\,x^2+3\,x+1\right)-\frac{1}{6}\,\log\left(2\,x^2-3\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*log(2*x^2 + 3*x + 1) - 1/6*log(2*x^2 - 3*x + 1)
```

Sympy [A]
time = 0.03, size = 29, normalized size = 0.74

$$-\frac{\log\left(x^2-\frac{3x}{2}+\frac{1}{2}\right)}{6}+\frac{\log\left(x^2+\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-5*x**2+1),x)
```

```
[Out] -log(x**2 - 3*x/2 + 1/2)/6 + log(x**2 + 3*x/2 + 1/2)/6
```

Giac [A]
time = 5.38, size = 33, normalized size = 0.85

$$\frac{1}{6}\,\log\left(|2\,x+1|\right)-\frac{1}{6}\,\log\left(|2\,x-1|\right)+\frac{1}{6}\,\log\left(|x+1|\right)-\frac{1}{6}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*log(abs(2*x + 1)) - 1/6*log(abs(2*x - 1)) + 1/6*log(abs(x + 1)) - 1/6*l
og(abs(x - 1))
```

Mupad [B]
time = 0.10, size = 15, normalized size = 0.38

$$\frac{\mathrm{atanh}\left(\frac{3\,x}{2\,x^2+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 5*x^2 + 1),x)
```

```
[Out] atanh((3*x)/(2*x^2 + 1))/3
```

3.67 $\int \frac{1-2x^2}{1-6x^2+4x^4} \, dx$

Optimal. Leaf size=48

$$-\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}\,x}{\sqrt{5}}\right)}{\sqrt{10}}+\frac{\tanh^{-1}\left(\frac{1+2\sqrt{2}\,x}{\sqrt{5}}\right)}{\sqrt{10}}$$

```
[Out] -1/10*arctanh(1/5*(1-2*x*2^(1/2))*5^(1/2))*10^(1/2)+1/10*arctanh(1/5*(1+2*x
*2^(1/2))*5^(1/2))*10^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 212}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{2}\,x+1}{\sqrt{5}}\right)}{\sqrt{10}}-\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}\,x}{\sqrt{5}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
```

```
[Out] -(ArcTanh[(1 - 2*Sqrt[2]*x)/Sqrt[5]]/Sqrt[10]) + ArcTanh[(1 + 2*Sqrt[2]*x)/
Sqrt[5]]/Sqrt[10]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1-2x^2}{1-6x^2+4x^4}\,dx &= -\left(\frac{1}{4}\int \frac{1}{-\frac{1}{2}-\frac{x}{\sqrt{2}}+x^2}\,dx\right)-\frac{1}{4}\int \frac{1}{-\frac{1}{2}+\frac{x}{\sqrt{2}}+x^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{5}{2}-x^2}\,dx, x, -\frac{1}{\sqrt{2}}+2x\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{5}{2}-x^2}\,dx, x, \frac{1}{\sqrt{2}}+2x\right)\\ &= -\frac{\tanh^{-1}\left(\frac{1-2\sqrt{2}\,x}{\sqrt{5}}\right)}{\sqrt{10}}+\frac{\tanh^{-1}\left(\frac{1+2\sqrt{2}\,x}{\sqrt{5}}\right)}{\sqrt{10}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.88

$$\frac{-\log\left(-1+\sqrt{10}\,x-2x^2\right)+\log\left(1+\sqrt{10}\,x+2x^2\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)/(1 - 6*x^2 + 4*x^4),x]
[Out] (-Log[-1 + Sqrt[10]*x - 2*x^2] + Log[1 + Sqrt[10]*x + 2*x^2])/(2*Sqrt[10])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(39) = 78.
time = 0.03, size = 82, normalized size = 1.71

method	result	size
risch	$\frac{\sqrt{10}\,\ln\left(\sqrt{10}\,x+2x^2+1\right)}{20}-\frac{\sqrt{10}\,\ln\left(-\sqrt{10}\,x+2x^2+1\right)}{20}$	39
default	$\frac{2\left(\sqrt{5}-1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}-2\sqrt{2}}\right)}{5\left(2\sqrt{10}-2\sqrt{2}\right)}+\frac{2\sqrt{5}\,\left(\sqrt{5}+1\right)\operatorname{arctanh}\left(\frac{8x}{2\sqrt{10}+2\sqrt{2}}\right)}{5\left(2\sqrt{10}+2\sqrt{2}\right)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2+1)/(4*x^4-6*x^2+1),x,method=_RETURNVERBOSE)
[Out] 2/5*(5^(1/2)-1)*5^(1/2)/(2*10^(1/2)-2*2^(1/2))*arctanh(8*x/(2*10^(1/2)-2*2^
(1/2)))+2/5*5^(1/2)*(5^(1/2)+1)/(2*10^(1/2)+2*2^(1/2))*arctanh(8*x/(2*10^(1
/2)+2*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^2 - 1)/(4*x^4 - 6*x^2 + 1), x)
```

Fricas [A]
time = 0.32, size = 45, normalized size = 0.94

$$\frac{1}{20}\sqrt{10}\,\log\left(\frac{4\,x^4+14\,x^2+2\,\sqrt{10}\,(2\,x^3+x)+1}{4\,x^4-6\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(10)*log((4*x^4 + 14*x^2 + 2*sqrt(10)*(2*x^3 + x) + 1)/(4*x^4 - 6*
x^2 + 1))
```

Sympy [A]
time = 0.04, size = 46, normalized size = 0.96

$$-\frac{\sqrt{10}\,\log\left(x^2-\frac{\sqrt{10}\,x}{2}+\frac{1}{2}\right)}{20}+\frac{\sqrt{10}\,\log\left(x^2+\frac{\sqrt{10}\,x}{2}+\frac{1}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2+1)/(4*x**4-6*x**2+1),x)
```

```
[Out] -sqrt(10)*log(x**2 - sqrt(10)*x/2 + 1/2)/20 + sqrt(10)*log(x**2 + sqrt(10)*
x/2 + 1/2)/20
```

Giac [A]
time = 3.37, size = 77, normalized size = 1.60

$$\frac{1}{20}\sqrt{10}\log\left(\left|x+\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)+\frac{1}{20}\sqrt{10}\log\left(\left|x+\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right)-\frac{1}{20}\sqrt{10}\log\left(\left|x-\frac{1}{4}\sqrt{10}+\frac{1}{4}\sqrt{2}\right|\right)-\frac{1}{20}\sqrt{10}\log\left(\left|x-\frac{1}{4}\sqrt{10}-\frac{1}{4}\sqrt{2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2+1)/(4*x^4-6*x^2+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10)*log(abs(x + 1/4*sqrt(10) + 1/4*sqrt(2))) + 1/20*sqrt(10)*log(
abs(x + 1/4*sqrt(10) - 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt(1
0) + 1/4*sqrt(2))) - 1/20*sqrt(10)*log(abs(x - 1/4*sqrt(10) - 1/4*sqrt(2)))
```

Mupad [B]
time = 0.13, size = 20, normalized size = 0.42

$$\frac{\sqrt{10}\,\operatorname{atanh}\left(\frac{\sqrt{10}\,x}{2\,x^2+1}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 - 1)/(4*x^4 - 6*x^2 + 1),x)
[Out] (10^(1/2)*atanh((10^(1/2)*x)/(2*x^2 + 1)))/10
```

3.68 $\int \frac{1+x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=62

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}$$

```
[Out] -arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))/(2+b)^(1/2)+arctan((2*x+(2-b)^(1/2)
)/(2+b)^(1/2))/(2+b)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{\sqrt{b+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -(ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]) + ArcTan[(Sqrt[2 - b
] + 2*x)/Sqrt[2 + b]]/Sqrt[2 + b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+x^2}{1+bx^2+x^4}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{2-b}\,x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{2-b}\,x+x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{-2-b-x^2}\,dx, x, -\sqrt{2-b}+2x\right) - \text{Subst}\left(\int \frac{1}{-2-b-x^2}\,dx, x, \sqrt{2-\right.$$
$$= \frac{\tan^{-1}\left(\frac{-\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{\sqrt{2+b}}$$

Mathematica [A]
time = 0.04, size = 124, normalized size = 2.00

$$\frac{\frac{\left(2-b+\sqrt{-4+b^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b-\sqrt{-4+b^2}}}\right)}{\sqrt{b-\sqrt{-4+b^2}}} + \frac{\left(-2+b+\sqrt{-4+b^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b+\sqrt{-4+b^2}}}\right)}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2}\sqrt{-4+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + b*x^2 + x^4),x]
[Out] (((2 - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sq
rt[b - Sqrt[-4 + b^2]] + ((-2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt
[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(50) = 100.
time = 0.04, size = 124, normalized size = 2.00

method	result
risch	$-\frac{\ln\left(-x^2\sqrt{-2-b}+x(2+b)+\sqrt{-2-b}\right)}{2\sqrt{-2-b}} + \frac{\ln\left(-x^2\sqrt{-2-b}+(-2-b)x+\sqrt{-2-b}\right)}{2\sqrt{-2-b}}$
default	$\frac{\left(-2+\sqrt{(b-2)(2+b)}+b\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\sqrt{2\sqrt{(b-2)(2+b)}+2b}} + \frac{\left(2+\sqrt{(b-2)(2+b)}-b\right)\arctan\left(-\right.}{\sqrt{(b-2)(2+b)}\sqrt{-2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-2+((b-2)*(2+b))^(1/2)+b)/((b-2)*(2+b))^(1/2)/(2*((b-2)*(2+b))^(1/2)+2*b)^
(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))+(2+((b-2)*(2+b))^(1/2)-
b)/((b-2)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((
b-2)*(2+b))^(1/2)+2*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + b*x^2 + 1), x)
```

Fricas [A]
time = 0.33, size = 101, normalized size = 1.63

$$\left[-\frac{\sqrt{-b-2}\,\log\left(\frac{x^4-(b+4)x^2-2\,(x^3-x)\sqrt{-b-2}+1}{x^4+bx^2+1}\right)}{2\,(b+2)}, \frac{\sqrt{b+2}\,\arctan\left(\frac{x^3+(b+1)x}{\sqrt{b+2}}\right)+\sqrt{b+2}\,\arctan\left(\frac{x}{\sqrt{b+2}}\right)}{b+2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b - 2)*log((x^4 - (b + 4)*x^2 - 2*(x^3 - x)*sqrt(-b - 2) + 1)/(
x^4 + b*x^2 + 1))/(b + 2), (sqrt(b + 2)*arctan((x^3 + (b + 1)*x)/sqrt(b + 2
)) + sqrt(b + 2)*arctan(x/sqrt(b + 2)))/(b + 2)]
```

Sympy [A]
time = 0.17, size = 88, normalized size = 1.42

$$-\frac{\sqrt{-\frac{1}{b+2}}\,\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b+2}}-2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2}+\frac{\sqrt{-\frac{1}{b+2}}\,\log\left(x^2+x\left(b\sqrt{-\frac{1}{b+2}}+2\sqrt{-\frac{1}{b+2}}\right)-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+b*x**2+1),x)
```

```
[Out] -sqrt(-1/(b + 2))*log(x**2 + x*(-b*sqrt(-1/(b + 2)) - 2*sqrt(-1/(b + 2))) -
 1)/2 + sqrt(-1/(b + 2))*log(x**2 + x*(b*sqrt(-1/(b + 2)) + 2*sqrt(-1/(b +
2))) - 1)/2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 0.06, size = 73, normalized size = 1.18

$$\frac{\operatorname{atan}\left(\frac{x}{\sqrt{b+2}}\right)+\operatorname{atan}\left((b+2)\left(x\left(\frac{1}{\sqrt{b+2}}+\frac{\frac{4}{b+2}-1}{(b-2)\sqrt{b+2}}\right)+\frac{x^3\left(\frac{2b}{b+2}-1\right)}{(b-2)\sqrt{b+2}}\right)\right)}{\sqrt{b+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(b*x^2 + x^4 + 1),x)
```

```
[Out] (atan(x/(b + 2)^(1/2)) + atan((b + 2)*(x*(1/(b + 2)^(1/2) + (4/(b + 2) - 1)
/((b - 2)*(b + 2)^(1/2))) + (x^3*((2*b)/(b + 2) - 1))/((b - 2)*(b + 2)^(1/2
)))))/(b + 2)^(1/2)
```

3.69 $\int \frac{1+x^2}{1+5x^2+x^4} \, dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{7}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{7}}$$

[Out] 1/7*arctan(x*2^(1/2)/(5+21^(1/2))^(1/2))*7^(1/2)+1/7*arctan(x*(1/2*7^(1/2)+1/2*3^(1/2)))*7^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1177, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{7}}+\frac{\text{ArcTan}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/(1 + 5*x^2 + x^4),x]

[Out] ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[7] + ArcTan[Sqrt[(5 + Sqrt[21])/2]*x]/Sqrt[7]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1+x^2}{1+5x^2+x^4}\,dx = \frac{1}{14}\left(7-\sqrt{21}\right)\int \frac{1}{\frac{5}{2}-\frac{\sqrt{21}}{2}+x^2}\,dx + \frac{1}{14}\left(7+\sqrt{21}\right)\int \frac{1}{\frac{5}{2}+\frac{\sqrt{21}}{2}+x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{7}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{7}}$$

Mathematica [A]
time = 0.08, size = 83, normalized size = 1.69

$$\frac{\left(-3+\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}\,x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}} + \frac{\left(3+\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] ((-3 + Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sqrt[21])]
+ ((3 + Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[42*(5 + Sqrt[21])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(38) = 76$.
time = 0.05, size = 82, normalized size = 1.67

method	result	size
risch	$\frac{\sqrt{7}\,\arctan\left(\frac{x\sqrt{7}}{7}\right)}{7} + \frac{\sqrt{7}\,\arctan\left(\frac{x^3\sqrt{7}}{7}+\frac{6x\sqrt{7}}{7}\right)}{7}$	35
default	$\frac{2\left(-3+\sqrt{21}\right)\sqrt{21}\,\arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)} + \frac{2\left(3+\sqrt{21}\right)\sqrt{21}\,\arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+5*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*(-3+21^(1/2))*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*3
^(1/2)))+2/21*(3+21^(1/2))*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctan(4*x/(2*7^(
1/2)+2*3^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 + 5*x^2 + 1), x)
```

Fricas [A]
time = 0.35, size = 31, normalized size = 0.63

$$\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(x^{3}+6x\right)\right)+\frac{1}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="fricas")
[Out] 1/7*sqrt(7)*arctan(1/7*sqrt(7)*(x^3 + 6*x)) + 1/7*sqrt(7)*arctan(1/7*sqrt(7
)*x)
```

Sympy [A]
time = 0.04, size = 41, normalized size = 0.84

$$\frac{\sqrt{7}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{7}x}{7}\right)+2\operatorname{atan}\left(\frac{\sqrt{7}x^{3}}{7}+\frac{6\sqrt{7}x}{7}\right)\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+5*x**2+1),x)
[Out] sqrt(7)*(2*atan(sqrt(7)*x/7) + 2*atan(sqrt(7)*x**3/7 + 6*sqrt(7)*x/7))/14
```

Giac [A]
time = 3.88, size = 26, normalized size = 0.53

$$\frac{1}{14}\sqrt{7}\left(\pi\operatorname{sgn}(x)+2\arctan\left(\frac{\sqrt{7}\left(x^{2}-1\right)}{7x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")
[Out] 1/14*sqrt(7)*(pi*sgn(x) + 2*arctan(1/7*sqrt(7)*(x^2 - 1)/x))
```

Mupad [B]
time = 0.08, size = 29, normalized size = 0.59

$$\frac{\sqrt{7}\left(\mathrm{atan}\left(\frac{\sqrt{7}\,x^3}{7}+\frac{6\,\sqrt{7}\,x}{7}\right)+\mathrm{atan}\left(\frac{\sqrt{7}\,x}{7}\right)\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(5*x^2 + x^4 + 1),x)
[Out] (7^(1/2)*(atan((6*7^(1/2)*x)/7 + (7^(1/2)*x^3)/7) + atan((7^(1/2)*x)/7)))/7
```

3.70 $\int \frac{1+x^2}{1+4x^2+x^4}\,dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

```
[Out] 1/6*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/6*arctan(x/(1/2*6^(1/2)+1
/2*2^(1/2)))*6^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1177, 209}

$$\frac{\text{ArcTan}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}}+\frac{\text{ArcTan}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[6] + ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[6]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1+x^2}{1+4x^2+x^4}\,dx = \frac{1}{6}\left(3-\sqrt{3}\right)\int \frac{1}{2-\sqrt{3}+x^2}\,dx + \frac{1}{6}\left(3+\sqrt{3}\right)\int \frac{1}{2+\sqrt{3}+x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{6}} + \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{6}}$$

Mathematica [A]
time = 0.04, size = 81, normalized size = 1.88

$$\frac{\left(-1+\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\left(1+\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ((-1 + Sqrt[3])*ArcTan[x/Sqrt[2 - Sqrt[3]]])/(2*Sqrt[3*(2 - Sqrt[3])]) + ((
1 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3*(2 + Sqrt[3])])
```

Maple [A]
time = 0.05, size = 70, normalized size = 1.63

method	result	size
risch	$\frac{\sqrt{6}\,\arctan\left(\frac{x\sqrt{6}}{6}\right)}{6}+\frac{\sqrt{6}\,\arctan\left(\frac{x^3\sqrt{6}}{6}+\frac{5x\sqrt{6}}{6}\right)}{6}$	35
default	$\frac{\left(1+\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\sqrt{6}+3\sqrt{2}}+\frac{\left(\sqrt{3}-1\right)\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+4*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(1+3^(1/2))*3^(1/2)/(6^(1/2)+2^(1/2))*arctan(2*x/(6^(1/2)+2^(1/2)))+1/3
*(3^(1/2)-1)*3^(1/2)/(6^(1/2)-2^(1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 + 4*x^2 + 1), x)
```

Fricas [A]
time = 0.34, size = 31, normalized size = 0.72

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}\left(x^{3}+5x\right)\right)+\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="fricas")
[Out] 1/6*sqrt(6)*arctan(1/6*sqrt(6)*(x^3 + 5*x)) + 1/6*sqrt(6)*arctan(1/6*sqrt(6
)*x)
```

Sympy [A]
time = 0.04, size = 41, normalized size = 0.95

$$\frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{6}\right)+2\operatorname{atan}\left(\frac{\sqrt{6}x^{3}}{6}+\frac{5\sqrt{6}x}{6}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+4*x**2+1),x)
[Out] sqrt(6)*(2*atan(sqrt(6)*x/6) + 2*atan(sqrt(6)*x**3/6 + 5*sqrt(6)*x/6))/12
```

Giac [A]
time = 3.58, size = 26, normalized size = 0.60

$$\frac{1}{12}\sqrt{6}\left(\pi\operatorname{sgn}(x)+2\arctan\left(\frac{\sqrt{6}\left(x^{2}-1\right)}{6x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")
[Out] 1/12*sqrt(6)*(pi*sgn(x) + 2*arctan(1/6*sqrt(6)*(x^2 - 1)/x))
```

Mupad [B]
time = 0.08, size = 29, normalized size = 0.67

$$\frac{\sqrt{6}\left(\operatorname{atan}\left(\frac{\sqrt{6}x^{3}}{6}+\frac{5\sqrt{6}x}{6}\right)+\operatorname{atan}\left(\frac{\sqrt{6}x}{6}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(4*x^2 + x^4 + 1),x)
[Out] (6^(1/2)*(atan((5*6^(1/2)*x)/6 + (6^(1/2)*x^3)/6) + atan((6^(1/2)*x)/6)))/6
```

3.71 $\int \frac{1+x^2}{1+3x^2+x^4} \, dx$

Optimal. Leaf size=49

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt{5}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{\sqrt{5}}$$

```
[Out] 1/5*arctan(x*2^(1/2)/(3+5^(1/2))^(1/2))*5^(1/2)+1/5*arctan(x*(1/2+1/2*5^(1/
2)))*5^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1177, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt{5}}+\frac{\text{ArcTan}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]/Sqrt[5] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]/S
qrt[5]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1+x^2}{1+3x^2+x^4}\,dx = \frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx + \frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt{5}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{\sqrt{5}}$$

Mathematica [**A**]
time = 0.05, size = 83, normalized size = 1.69

$$\frac{\left(-1+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}\,x\right)}{\sqrt{10\left(3-\sqrt{5}\right)}} + \frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ((-1 + Sqrt[5])*ArcTan[Sqrt[2/(3 - Sqrt[5])]*x])/Sqrt[10*(3 - Sqrt[5])] + (
(1 + Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x])/Sqrt[10*(3 + Sqrt[5])]
```

Maple [**A**]
time = 0.04, size = 66, normalized size = 1.35

method	result	size
risch	$\frac{\arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\sqrt{5}\,\arctan\left(\frac{x^3\sqrt{5}}{5}+\frac{4x\sqrt{5}}{5}\right)}{5}$	35
default	$\frac{2\sqrt{5}\,\left(\sqrt{5}+1\right)\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}+\frac{2\left(\sqrt{5}-1\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*5^(1/2)*(5^(1/2)+1)/(2*5^(1/2)+2)*arctan(4*x/(2*5^(1/2)+2))+2/5*(5^(1/2
)-1)*5^(1/2)/(2*5^(1/2)-2)*arctan(4*x/(2*5^(1/2)-2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 + 3*x^2 + 1), x)
```

Fricas [A]
time = 0.39, size = 31, normalized size = 0.63

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\left(x^3+4x\right)\right)+\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="fricas")
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(x^3 + 4*x)) + 1/5*sqrt(5)*arctan(1/5*sqrt(5
)*x)
```

Sympy [A]
time = 0.04, size = 41, normalized size = 0.84

$$\frac{\sqrt{5}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{5}x^3}{5}+\frac{4\sqrt{5}x}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+3*x**2+1),x)
[Out] sqrt(5)*(2*atan(sqrt(5)*x/5) + 2*atan(sqrt(5)*x**3/5 + 4*sqrt(5)*x/5))/10
```

Giac [A]
time = 3.70, size = 26, normalized size = 0.53

$$\frac{1}{10}\sqrt{5}\left(\pi\mathrm{sgn}(x)+2\arctan\left(\frac{\sqrt{5}\left(x^2-1\right)}{5x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")
[Out] 1/10*sqrt(5)*(pi*sgn(x) + 2*arctan(1/5*sqrt(5)*(x^2 - 1)/x))
```

Mupad [B]
time = 4.39, size = 29, normalized size = 0.59

$$\frac{\sqrt{5}\left(\mathrm{atan}\left(\frac{\sqrt{5}x^3}{5}+\frac{4\sqrt{5}x}{5}\right)+\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(3*x^2 + x^4 + 1),x)
[Out] (5^(1/2)*(atan((4*5^(1/2)*x)/5 + (5^(1/2)*x^3)/5) + atan((5^(1/2)*x)/5)))/5
```

3.72 $\int \frac{1+x^2}{1+2x^2+x^4}\,dx$

Optimal. Leaf size=2

$$\tan^{-1}(x)$$

```
[Out] arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {28, 209}

$$\text{ArcTan}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1+2x^2+x^4}\,dx &= \int \frac{1}{1+x^2}\,dx\\ &= \tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 2, normalized size = 1.00

$$\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[x]
```

Maple [A]
time = 0.01, size = 3, normalized size = 1.50

method	result	size
default	$\arctan(x)$	3
risch	$\arctan(x)$	3

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x)
```

Maxima [A]
time = 0.51, size = 2, normalized size = 1.00

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A]
time = 0.33, size = 2, normalized size = 1.00

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A]
time = 0.02, size = 2, normalized size = 1.00

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+2*x**2+1),x)
```

```
[Out] atan(x)
```

Giac [A]
time = 4.26, size = 2, normalized size = 1.00

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] arctan(x)
```

Mupad [B]
time = 4.33, size = 2, normalized size = 1.00

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(2*x^2 + x^4 + 1),x)
```

```
[Out] atan(x)
```

3.73 $\int \frac{1+x^2}{1+x^2+x^4}\,dx$

Optimal. Leaf size=38

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+x^2}{1+x^2+x^4}\,dx = \frac{1}{2}\int \frac{1}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right)$$
$$= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 99, normalized size = 2.61

$$\frac{\left(-i+\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1-i\sqrt{3}\right)}} + \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{3}\right)}}\right)}{\sqrt{6\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x^2 + x^4),x]
```

```
[Out] ((-I + Sqrt[3])*ArcTan[x/Sqrt[(1 - I*Sqrt[3])/2]])/Sqrt[6*(1 - I*Sqrt[3])]
+ ((I + Sqrt[3])*ArcTan[x/Sqrt[(1 + I*Sqrt[3])/2]])/Sqrt[6*(1 + I*Sqrt[3])]
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.89

method	result	size
default	$\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$	34
risch	$\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{x^3\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{3}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))
```

Maxima [A]
time = 0.51, size = 33, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="maxima")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1))
```

Fricas [A]
time = 0.35, size = 31, normalized size = 0.82

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x^3+2x)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*x)
```

Sympy [A]
time = 0.04, size = 41, normalized size = 1.08

$$\frac{\sqrt{3}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{2\sqrt{3}x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+x**2+1),x)
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/6
```

Giac [A]
time = 3.83, size = 26, normalized size = 0.68

$$\frac{1}{6}\sqrt{3}\left(\pi\operatorname{sgn}(x)+2\arctan\left(\frac{\sqrt{3}(x^2-1)}{3x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*(x^2 - 1)/x))
```

Mupad [B]
time = 0.08, size = 29, normalized size = 0.76

$$\frac{\sqrt{3}\,\left(\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\,\sqrt{3}\,x}{3}\right)+\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^2 + x^4 + 1),x)
[Out] (3^(1/2)*(atan((2*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) + atan((3^(1/2)*x)/3)))/3
```

3.74 $\int \frac{1+x^2}{1+x^4}\,dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(-1+x*2^(1/2))*2^(1/2)+1/2*arctan(1+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1176, 631, 210}

$$\frac{\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{\sqrt{2}}-\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x^4),x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/Sqrt[2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rubi steps

$$\int \frac{1+x^2}{1+x^4}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,x\right)}{\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\,x\right)}{\sqrt{2}}$$
$$= -\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.86

$$\frac{-\tan^{-1}\left(1-\sqrt{2}\,x\right)+\tan^{-1}\left(1+\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x^4),x]
[Out] (-ArcTan[1 - Sqrt[2]*x] + ArcTan[1 + Sqrt[2]*x])/Sqrt[2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(27) = 54$.
time = 0.15, size = 104, normalized size = 2.97

method	result
risch	$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^3}{2}+\frac{\sqrt{2}\,x}{2}\right)}{2}$
default	$\frac{\sqrt{2}\,\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{8}+\frac{\sqrt{2}\,\left(\ln\left(\frac{1+x^2-\sqrt{2}\,x}{1+x^2+\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1 \cdots\right.\right.}{8}$
meijerg	$\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*2^(1/2)*(ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))+2*arctan(2^(1/2)*x+1)+
2*arctan(2^(1/2)*x-1))+1/8*2^(1/2)*(ln((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))
+2*arctan(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))
```

Maxima [A]
time = 0.50, size = 39, normalized size = 1.11

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)))
```

Fricas [A]
time = 0.36, size = 29, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^3+x\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + x)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*
x)
```

Sympy [A]
time = 0.03, size = 39, normalized size = 1.11

$$\frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^3}{2}+\frac{\sqrt{2}\,x}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+1),x)
```

```
[Out] sqrt(2)*(2*atan(sqrt(2)*x/2) + 2*atan(sqrt(2)*x**3/2 + sqrt(2)*x/2))/4
```

Giac [A]
time = 4.33, size = 39, normalized size = 1.11

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/2*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)))
```

Mupad [B]
time = 4.37, size = 29, normalized size = 0.83

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\,x^3}{2}+\frac{\sqrt{2}\,x}{2}\right)+\operatorname{atan}\left(\frac{\sqrt{2}\,x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 + 1),x)
[Out] (2^(1/2)*(atan((2^(1/2)*x)/2 + (2^(1/2)*x^3)/2) + atan((2^(1/2)*x)/2)))/2
```

3.75 $\int \frac{1+x^2}{1-x^2+x^4} \, dx$

Optimal. Leaf size=23

$$-\tan^{-1}\left(\sqrt{3}-2x\right)+\tan^{-1}\left(\sqrt{3}+2x\right)$$

```
[Out] arctan(2*x-3^(1/2))+arctan(2*x+3^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1175, 632, 210}

$$\text{ArcTan}\left(2x+\sqrt{3}\right)-\text{ArcTan}\left(\sqrt{3}-2x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x] + ArcTan[Sqrt[3] + 2*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+x^2}{1-x^2+x^4}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}+2x\right) - \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{3}+2x\right)$$
$$= -\tan^{-1}\left(\sqrt{3}-2x\right) + \tan^{-1}\left(\sqrt{3}+2x\right)$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 0.52

$$-\tan^{-1}\left(\frac{x}{-1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - x^2 + x^4),x]
[Out] -ArcTan[x/(-1 + x^2)]
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.87

method	result	size
risch	$\arctan\left(x^3\right) + \arctan\left(x\right)$	8
default	$\arctan\left(2x-\sqrt{3}\right) + \arctan\left(2x+\sqrt{3}\right)$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
[Out] arctan(2*x-3^(1/2))+arctan(2*x+3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 - x^2 + 1), x)
```

Fricas [A]
time = 0.33, size = 7, normalized size = 0.30

$$\arctan\left(x^3\right) + \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(x^3) + arctan(x)
```

Sympy [A]
time = 0.03, size = 7, normalized size = 0.30

$$\operatorname{atan}(x) + \operatorname{atan}(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-x**2+1),x)
```

```
[Out] atan(x) + atan(x**3)
```

Giac [A]
time = 4.48, size = 30, normalized size = 1.30

$$\frac{1}{4}\,\pi \operatorname{sgn}(x) + \frac{1}{2}\,\arctan\left(\frac{x^4 - 3\,x^2 + 1}{2\,(x^3 - x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) + 1/2*arctan(1/2*(x^4 - 3*x^2 + 1)/(x^3 - x))
```

Mupad [B]
time = 4.31, size = 7, normalized size = 0.30

$$\operatorname{atan}(x^3) + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 - x^2 + 1),x)
```

```
[Out] atan(x^3) + atan(x)
```

3.76 $\int \frac{1+x^2}{1-2x^2+x^4} \, dx$

Optimal. Leaf size=11

$$\frac{x}{1-x^2}$$

```
[Out] x/(-x^2+1)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {28, 391}

$$\frac{x}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] x/(1 - x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1-2x^2+x^4} \, dx &= \int \frac{1+x^2}{(-1+x^2)^2} \, dx\\ &= \frac{x}{1-x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 0.91

$$-\frac{x}{-1+x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] -(x/(-1 + x^2))
```

Maple [A]
time = 0.01, size = 16, normalized size = 1.45

method	result	size
gosper	$-\frac{x}{x^2-1}$	11
norman	$-\frac{x}{x^2-1}$	11
risch	$-\frac{x}{x^2-1}$	11
default	$-\frac{1}{2(-1+x)}-\frac{1}{2(1+x)}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-1+x)-1/2/(1+x)
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] -x/(x^2 - 1)
```

Fricas [A]
time = 0.34, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] -x/(x^2 - 1)
```

Sympy [A]
time = 0.02, size = 7, normalized size = 0.64

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-2*x**2+1),x)
```

```
[Out] -x/(x**2 - 1)
```

Giac [A]
time = 3.61, size = 11, normalized size = 1.00

$$-\frac{1}{x-\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/(x - 1/x)
```

Mupad [B]
time = 4.34, size = 10, normalized size = 0.91

$$-\frac{x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 - 2*x^2 + 1),x)
```

```
[Out] -x/(x^2 - 1)
```

3.77 $\int \frac{1+x^2}{1-3x^2+x^4}\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}\log\left(1-\sqrt{5}-2x\right)+\frac{1}{2}\log\left(1+\sqrt{5}-2x\right)-\frac{1}{2}\log\left(1-\sqrt{5}+2x\right)-\frac{1}{2}\log\left(1+\sqrt{5}+2x\right)$$

```
[Out] 1/2*ln(1-2*x-5^(1/2))-1/2*ln(1+2*x-5^(1/2))+1/2*ln(1-2*x+5^(1/2))-1/2*ln(1+
2*x+5^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1175, 630, 31}

$$\frac{1}{2}\log\left(-2x-\sqrt{5}+1\right)+\frac{1}{2}\log\left(-2x+\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x-\sqrt{5}+1\right)-\frac{1}{2}\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] Log[1 - Sqrt[5] - 2*x]/2 + Log[1 + Sqrt[5] - 2*x]/2 - Log[1 - Sqrt[5] + 2*x
]/2 - Log[1 + Sqrt[5] + 2*x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+x^2}{1-3x^2+x^4}\,dx = \frac{1}{2}\int \frac{1}{1-\sqrt{5}\,x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+\sqrt{5}\,x+x^2}\,dx$$
$$= \frac{1}{2}\int \frac{1}{\frac{1}{2}\left(-1-\sqrt{5}\right)+x}\,dx - \frac{1}{2}\int \frac{1}{\frac{1}{2}\left(1-\sqrt{5}\right)+x}\,dx + \frac{1}{2}\int \frac{1}{\frac{1}{2}\left(-1+\sqrt{5}\right)+x}\,dx$$
$$= \frac{1}{2}\log\left(1-\sqrt{5}-2x\right) + \frac{1}{2}\log\left(1+\sqrt{5}-2x\right) - \frac{1}{2}\log\left(1-\sqrt{5}+2x\right) - \frac{1}{2}\log\left(1+\right.$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 0.45

$$-\frac{1}{2}\log\left(1-x-x^2\right) + \frac{1}{2}\log\left(1+x-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 3*x^2 + x^4),x]
[Out] -1/2*Log[1 - x - x^2] + Log[1 + x - x^2]/2
```

Maple [A]
time = 0.02, size = 22, normalized size = 0.34

method	result	size
default	$\frac{\ln(x^2-x-1)}{2} - \frac{\ln(x^2+x-1)}{2}$	22
norman	$\frac{\ln(x^2-x-1)}{2} - \frac{\ln(x^2+x-1)}{2}$	22
risch	$\frac{\ln(x^2-x-1)}{2} - \frac{\ln(x^2+x-1)}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-3*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2-x-1)-1/2*ln(x^2+x-1)
```

Maxima [A]
time = 0.29, size = 21, normalized size = 0.32

$$-\frac{1}{2}\log\left(x^2+x-1\right) + \frac{1}{2}\log\left(x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="maxima")
[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

Fricas [A]
time = 0.38, size = 21, normalized size = 0.32

$$-\frac{1}{2}\log\left(x^2+x-1\right)+\frac{1}{2}\log\left(x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="fricas")
[Out] -1/2*log(x^2 + x - 1) + 1/2*log(x^2 - x - 1)
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.29

$$\frac{\log\left(x^2-x-1\right)}{2}-\frac{\log\left(x^2+x-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-3*x**2+1),x)
[Out] log(x**2 - x - 1)/2 - log(x**2 + x - 1)/2
```

Giac [A]
time = 4.05, size = 43, normalized size = 0.66

$$-\frac{1}{4}\log\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|x+\frac{1}{x-\frac{1}{x}}-\frac{1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")
[Out] -1/4*log(abs(x + 1/(x - 1/x) - 1/x + 2)) + 1/4*log(abs(x + 1/(x - 1/x) - 1/
x - 2))
```

Mupad [B]
time = 0.26, size = 12, normalized size = 0.18

$$-\mathrm{atanh}\left(\frac{x}{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 - 3*x^2 + 1),x)
[Out] -atanh(x/(x^2 - 1))
```

3.78 $\int \frac{1+x^2}{1-4x^2+x^4} \, dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{3}+\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(x*2^(1/2)-3^(1/2))*2^(1/2)-1/2*arctanh(x*2^(1/2)+3^(1/2))*2^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1175, 632, 212}

$$\frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}\,x\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}\,x+\sqrt{3}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] ArcTanh[Sqrt[3] - Sqrt[2]*x]/Sqrt[2] - ArcTanh[Sqrt[3] + Sqrt[2]*x]/Sqrt[2]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\int \frac{1+x^2}{1-4x^2+x^4} dx = \frac{1}{2} \int \frac{1}{1-\sqrt{6}x+x^2} dx + \frac{1}{2} \int \frac{1}{1+\sqrt{6}x+x^2} dx$$
$$= -\text{Subst}\left(\int \frac{1}{2-x^2} dx, x, -\sqrt{6}+2x\right) - \text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \sqrt{6}+2x\right)$$
$$= \frac{\tanh^{-1}\left(\sqrt{3}-\sqrt{2}x\right)}{\sqrt{2}} - \frac{\tanh^{-1}\left(\sqrt{3}+\sqrt{2}x\right)}{\sqrt{2}}$$

Mathematica [**A**]
time = 0.01, size = 40, normalized size = 0.93

$$\frac{\log\left(1+\sqrt{2}x-x^2\right)-\log\left(-1+\sqrt{2}x+x^2\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 4*x^2 + x^4),x]
[Out] (Log[1 + Sqrt[2]*x - x^2] - Log[-1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(33) = 66$.
time = 0.04, size = 70, normalized size = 1.63

method	result	size
risch	$\frac{\sqrt{2}\,\ln\left(x^2-\sqrt{2}\,x-1\right)}{4}-\frac{\sqrt{2}\,\ln\left(x^2+\sqrt{2}\,x-1\right)}{4}$	35
default	$-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\left(\sqrt{6}-\sqrt{2}\right)}-\frac{\left(\sqrt{3}+3\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-4*x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/3*(-3+3^(1/2))*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-2^(1/2)))-
1/3*(3^(1/2)+3)*3^(1/2)/(6^(1/2)+2^(1/2))*arctanh(2*x/(6^(1/2)+2^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 - 4*x^2 + 1), x)
```

Fricas [A]
time = 0.35, size = 36, normalized size = 0.84

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4-2\sqrt{2}\,(x^3-x)+1}{x^4-4x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*log((x^4 - 2*sqrt(2)*(x^3 - x) + 1)/(x^4 - 4*x^2 + 1))
```

Sympy [A]
time = 0.03, size = 39, normalized size = 0.91

$$\frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x-1\right)}{4}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}\,x-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-4*x**2+1),x)
[Out] sqrt(2)*log(x**2 - sqrt(2)*x - 1)/4 - sqrt(2)*log(x**2 + sqrt(2)*x - 1)/4
```

Giac [A]
time = 3.08, size = 39, normalized size = 0.91

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}-\frac{2}{x}\right|}{\left|2x+2\sqrt{2}-\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")
[Out] 1/4*sqrt(2)*log(abs(2*x - 2*sqrt(2) - 2/x)/abs(2*x + 2*sqrt(2) - 2/x))
```

Mupad [B]
time = 4.40, size = 18, normalized size = 0.42

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{x^2-1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 - 4*x^2 + 1),x)
[Out] -(2^(1/2)*atanh((2^(1/2)*x)/(x^2 - 1)))/2
```

3.79 $\int \frac{1+x^2}{1-5x^2+x^4} \, dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}+2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/3*arctanh(1/3*(-2*x+7^(1/2))*3^(1/2))*3^(1/2)-1/3*arctanh(1/3*(2*x+7^(1/2
))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1175, 632, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{2x+\sqrt{7}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] ArcTanh[(Sqrt[7] - 2*x)/Sqrt[3]]/Sqrt[3] - ArcTanh[(Sqrt[7] + 2*x)/Sqrt[3]]
/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{1-5x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1}{1-\sqrt{7}\,x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1+\sqrt{7}\,x+x^2}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{3-x^2}\,dx,x,-\sqrt{7}+2x\right)-\text{Subst}\left(\int \frac{1}{3-x^2}\,dx,x,\sqrt{7}+2x\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{7}-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}+2x}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.87

$$\frac{\log\left(1+\sqrt{3}\,x-x^2\right)-\log\left(-1+\sqrt{3}\,x+x^2\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] (Log[1 + Sqrt[3]*x - x^2] - Log[-1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(37) = 74.
time = 0.04, size = 82, normalized size = 1.78

method	result	size
risch	$\frac{\sqrt{3}\,\ln\left(x^2-x\sqrt{3}-1\right)}{6}-\frac{\sqrt{3}\,\ln\left(x^2+x\sqrt{3}-1\right)}{6}$	35
default	$-\frac{2\sqrt{21}\,\left(7+\sqrt{21}\right)\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}-\frac{2\left(-7+\sqrt{21}\right)\sqrt{21}\,\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4-5*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*21^(1/2)*(7+21^(1/2))/(2*7^(1/2)+2*3^(1/2))*arctanh(4*x/(2*7^(1/2)+2*
3^(1/2)))-2/21*(-7+21^(1/2))*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctanh(4*x/(2*
7^(1/2)-2*3^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + 1)/(x^4 - 5*x^2 + 1), x)
```

Fricas [A]
time = 0.34, size = 39, normalized size = 0.85

$$\frac{1}{6}\sqrt{3}\log\left(\frac{x^4+x^2-2\sqrt{3}\,(x^3-x)+1}{x^4-5\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*log((x^4 + x^2 - 2*sqrt(3)*(x^3 - x) + 1)/(x^4 - 5*x^2 + 1))
```

Sympy [A]
time = 0.03, size = 39, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}\,x-1\right)}{6}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}\,x-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4-5*x**2+1),x)
[Out] sqrt(3)*log(x**2 - sqrt(3)*x - 1)/6 - sqrt(3)*log(x**2 + sqrt(3)*x - 1)/6
```

Giac [A]
time = 3.76, size = 39, normalized size = 0.85

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2\,x-2\sqrt{3}-\frac{2}{x}\right|}{\left|2\,x+2\sqrt{3}-\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) - 2/x)/abs(2*x + 2*sqrt(3) - 2/x))
```

Mupad [B]
time = 4.47, size = 18, normalized size = 0.39

$$-\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{x^2-1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^4 - 5*x^2 + 1),x)
[Out] -(3^(1/2)*atanh((3^(1/2)*x)/(x^2 - 1)))/3
```

3.80 $\int \frac{1-x^2}{1+bx^2+x^4} \, dx$

Optimal. Leaf size=62

$$-\frac{\log\left(1-\sqrt{2-b}\,x+x^2\right)}{2\sqrt{2-b}}+\frac{\log\left(1+\sqrt{2-b}\,x+x^2\right)}{2\sqrt{2-b}}$$

```
[Out] -1/2*ln(1+x^2-x*(2-b)^(1/2))/(2-b)^(1/2)+1/2*ln(1+x^2+x*(2-b)^(1/2))/(2-b)^
(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1178, 642}

$$\frac{\log\left(\sqrt{2-b}\,x+x^2+1\right)}{2\sqrt{2-b}}-\frac{\log\left(-\sqrt{2-b}\,x+x^2+1\right)}{2\sqrt{2-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[2 - b]*x + x^2]/Sqrt[2 - b] + Log[1 + Sqrt[2 - b]*x + x^2
]/(2*Sqrt[2 - b])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^2}{1+bx^2+x^4}\,dx = -\frac{\int \frac{\sqrt{2-b}+2x}{-1-\sqrt{2-b}\,x-x^2}\,dx}{2\sqrt{2-b}} - \frac{\int \frac{\sqrt{2-b}-2x}{-1+\sqrt{2-b}\,x-x^2}\,dx}{2\sqrt{2-b}}$$
$$= -\frac{\log\left(1-\sqrt{2-b}\,x+x^2\right)}{2\sqrt{2-b}} + \frac{\log\left(1+\sqrt{2-b}\,x+x^2\right)}{2\sqrt{2-b}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 125 vs. 2(62) = 124.
time = 0.04, size = 125, normalized size = 2.02

$$\frac{\frac{\left(2+b-\sqrt{-4+b^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b-\sqrt{-4+b^2}}}\right)}{\sqrt{b-\sqrt{-4+b^2}}} - \frac{\left(2+b+\sqrt{-4+b^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b+\sqrt{-4+b^2}}}\right)}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2}\sqrt{-4+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2 + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^2]]])/Sq
rt[b - Sqrt[-4 + b^2]] - ((2 + b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[
b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sqrt[-4 + b^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(50) = 100.
time = 0.03, size = 128, normalized size = 2.06

method	result
risch	$-\frac{\ln\left(-x^2\sqrt{2-b}+(2-b)x-\sqrt{2-b}\right)}{2\sqrt{2-b}} + \frac{\ln\left(-x^2\sqrt{2-b}+x(b-2)-\sqrt{2-b}\right)}{2\sqrt{2-b}}$
default	$\frac{\left(-2-\sqrt{(b-2)(2+b)}-b\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\sqrt{2\sqrt{(b-2)(2+b)}+2b}} + \frac{\left(2-\sqrt{(b-2)(2+b)}+b\right)\arctan\left(-\right.}{\sqrt{(b-2)(2+b)}\sqrt{-2\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-2-((b-2)*(2+b))^(1/2)-b)/((b-2)*(2+b))^(1/2)/(2*((b-2)*(2+b))^(1/2)+2*b)^
(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))+(2-((b-2)*(2+b))^(1/2)+
b)/((b-2)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*arctan(2*x/(-2*((
b-2)*(2+b))^(1/2)+2*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + b*x^2 + 1), x)
```

Fricas [A]
time = 0.35, size = 100, normalized size = 1.61

$$\left[-\frac{\sqrt{-b+2}\,\log\left(\frac{x^4-(b-4)x^2+2\,(x^3+x)\sqrt{-b+2}+1}{x^4+bx^2+1}\right)}{2\,(b-2)},\ \frac{\sqrt{b-2}\,\arctan\left(\frac{x^3+(b-1)x}{\sqrt{b-2}}\right)-\sqrt{b-2}\,\arctan\left(\frac{x}{\sqrt{b-2}}\right)}{b-2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b + 2)*log((x^4 - (b - 4)*x^2 + 2*(x^3 + x)*sqrt(-b + 2) + 1)/(
x^4 + b*x^2 + 1))/(b - 2), (sqrt(b - 2)*arctan((x^3 + (b - 1)*x)/sqrt(b - 2
)) - sqrt(b - 2)*arctan(x/sqrt(b - 2)))/(b - 2)]
```

Sympy [A]
time = 0.17, size = 87, normalized size = 1.40

$$\frac{\sqrt{-\frac{1}{b-2}}\,\log\left(x^2+x\left(-b\sqrt{-\frac{1}{b-2}}+2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2}-\frac{\sqrt{-\frac{1}{b-2}}\,\log\left(x^2+x\left(b\sqrt{-\frac{1}{b-2}}-2\sqrt{-\frac{1}{b-2}}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+b*x**2+1),x)
```

```
[Out] sqrt(-1/(b - 2))*log(x**2 + x*(-b*sqrt(-1/(b - 2)) + 2*sqrt(-1/(b - 2))) +
1)/2 - sqrt(-1/(b - 2))*log(x**2 + x*(b*sqrt(-1/(b - 2)) - 2*sqrt(-1/(b - 2
))) + 1)/2
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, need to choose a branch for
the root of a polynomial with parameters. This might be wrong.The choice wa
s done
```

Mupad [B]
time = 4.34, size = 76, normalized size = 1.23

$$-\frac{\operatorname{atan}\left(\frac{x}{\sqrt{b-2}}\right)-\operatorname{atan}\left((b-2)\,\left(x\,\left(\frac{1}{\sqrt{b-2}}+\frac{\frac{4}{b-2}+1}{\sqrt{b-2}\,(b+2)}\right)+\frac{x^3\,\left(\frac{2\,b}{b-2}-1\right)}{\sqrt{b-2}\,(b+2)}\right)\right)}{\sqrt{b-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(b*x^2 + x^4 + 1),x)
```

```
[Out] -(atan(x/(b - 2)^(1/2)) - atan((b - 2)*(x*(1/(b - 2)^(1/2) + (4/(b - 2) + 1
)/((b - 2)^(1/2)*(b + 2))) + (x^3*((2*b)/(b - 2) - 1))/((b - 2)^(1/2)*(b +
2)))))/(b - 2)^(1/2)
```

3.81 $\int \frac{1-x^2}{1+5x^2+x^4} \, dx$

Optimal. Leaf size=50

$$-\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctan(x*2^(1/2)/(5+21^(1/2))^(1/2))*3^(1/2)+1/3*arctan(x*(1/2*7^(1/2)
+1/2*3^(1/2)))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1177, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 5*x^2 + x^4),x]
```

```
[Out] -(ArcTan[Sqrt[2/(5 + Sqrt[21])]*x]/Sqrt[3]) + ArcTan[Sqrt[(5 + Sqrt[21])/2]
*x]/Sqrt[3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^2}{1+5x^2+x^4}\,dx = \frac{1}{6}\left(-3+\sqrt{21}\right)\int \frac{1}{\frac{5}{2}-\frac{\sqrt{21}}{2}+x^2}\,dx - \frac{1}{6}\left(3+\sqrt{21}\right)\int \frac{1}{\frac{5}{2}+\frac{\sqrt{21}}{2}+x^2}\,dx$$
$$= -\frac{\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}\,x\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.05, size = 87, normalized size = 1.74

$$\frac{\left(7-\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5-\sqrt{21}}}\,x\right)}{\sqrt{42\left(5-\sqrt{21}\right)}} + \frac{\left(-7-\sqrt{21}\right)\tan^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}\,x\right)}{\sqrt{42\left(5+\sqrt{21}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 5*x^2 + x^4),x]
[Out] ((7 - Sqrt[21])*ArcTan[Sqrt[2/(5 - Sqrt[21])]*x])/Sqrt[42*(5 - Sqrt[21])] +
 ((-7 - Sqrt[21])*ArcTan[Sqrt[2/(5 + Sqrt[21])]*x])/Sqrt[42*(5 + Sqrt[21])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(38) = 76.
time = 0.03, size = 82, normalized size = 1.64

method	result	size
risch	$-\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\sqrt{3}\,\arctan\left(\frac{x^3\sqrt{3}}{3}+\frac{4x\sqrt{3}}{3}\right)}{3}$	35
default	$-\frac{2\left(-7+\sqrt{21}\right)\sqrt{21}\,\arctan\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}-\frac{2\sqrt{21}\,\left(7+\sqrt{21}\right)\arctan\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+5*x^2+1),x,method=_RETURNVERBOSE)
[Out] -2/21*(-7+21^(1/2))*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctan(4*x/(2*7^(1/2)-2*
3^(1/2)))-2/21*21^(1/2)*(7+21^(1/2))/(2*7^(1/2)+2*3^(1/2))*arctan(4*x/(2*7^
(1/2)+2*3^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 5*x^2 + 1), x)
```

Fricas [A]
time = 0.40, size = 31, normalized size = 0.62

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+4x\right)\right)-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 4*x)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*x)
```

Sympy [A]
time = 0.04, size = 42, normalized size = 0.84

$$-\frac{\sqrt{3}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)-2\operatorname{atan}\left(\frac{\sqrt{3}x^3}{3}+\frac{4\sqrt{3}x}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+5*x**2+1),x)
```

```
[Out] -sqrt(3)*(2*atan(sqrt(3)*x/3) - 2*atan(sqrt(3)*x**3/3 + 4*sqrt(3)*x/3))/6
```

Giac [A]
time = 3.89, size = 26, normalized size = 0.52

$$\frac{1}{6}\sqrt{3}\left(\pi\operatorname{sgn}(x)-2\arctan\left(\frac{\sqrt{3}\left(x^2+1\right)}{3x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+5*x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(pi*sgn(x) - 2*arctan(1/3*sqrt(3)*(x^2 + 1)/x))
```

Mupad [B]
time = 0.08, size = 31, normalized size = 0.62

$$\frac{\sqrt{3}\,\left(\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{4\,\sqrt{3}\,x}{3}\right)-\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(5*x^2 + x^4 + 1),x)
[Out] (3^(1/2)*(atan((4*3^(1/2)*x)/3 + (3^(1/2)*x^3)/3) - atan((3^(1/2)*x)/3)))/3
```

3.82 $\int \frac{1-x^2}{1+4x^2+x^4} dx$

Optimal. Leaf size=44

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x/(1/2*6^(1/2)-1/2*2^(1/2)))*2^(1/2)-1/2*arctan(x/(1/2*6^(1/2)+1
/2*2^(1/2)))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1177, 209}

$$\frac{\text{ArcTan}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 4*x^2 + x^4),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/Sqrt[2] - ArcTan[x/Sqrt[2 + Sqrt[3]]]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^2}{1+4x^2+x^4}\,dx = \frac{1}{2}\left(-1-\sqrt{3}\right)\int \frac{1}{2+\sqrt{3}+x^2}\,dx + \frac{1}{2}\left(-1+\sqrt{3}\right)\int \frac{1}{2-\sqrt{3}+x^2}\,dx$$
$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{\sqrt{2}}$$

Mathematica [**A**]
time = 0.03, size = 82, normalized size = 1.86

$$\frac{-\left(\left(-3+\sqrt{3}\right)\sqrt{2+\sqrt{3}}\,\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)\right)-\sqrt{2-\sqrt{3}}\left(3+\sqrt{3}\right)\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)/(1 + 4*x^2 + x^4),x]

[Out] (-((-3 + Sqrt[3])*Sqrt[2 + Sqrt[3]]*ArcTan[x/Sqrt[2 - Sqrt[3]]]) - Sqrt[2 - Sqrt[3]]*(3 + Sqrt[3])*ArcTan[x/Sqrt[2 + Sqrt[3]]])/(2*Sqrt[3])

Maple [**A**]
time = 0.03, size = 70, normalized size = 1.59

method	result	size
risch	$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{2}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x^{3}}{2}+\frac{3\sqrt{2}\,x}{2}\right)}{2}$	35
default	$-\frac{\left(\sqrt{3}+3\right)\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\left(\sqrt{6}+\sqrt{2}\right)}-\frac{\left(-3+\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\left(\sqrt{6}-\sqrt{2}\right)}$	70

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^4+4*x^2+1),x,method=_RETURNVERBOSE)

[Out] -1/3*(3^(1/2)+3)*3^(1/2)/(6^(1/2)+2^(1/2))*arctan(2*x/(6^(1/2)+2^(1/2)))-1/3*(-3+3^(1/2))*3^(1/2)/(6^(1/2)-2^(1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(x^4 + 4*x^2 + 1), x)
```

Fricas [A]
time = 0.33, size = 31, normalized size = 0.70

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^{3}+3x\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^3 + 3*x)) - 1/2*sqrt(2)*arctan(1/2*sqrt(2
)*x)
```

Sympy [A]
time = 0.04, size = 42, normalized size = 0.95

$$-\frac{\sqrt{2}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)-2\operatorname{atan}\left(\frac{\sqrt{2}x^{3}}{2}+\frac{3\sqrt{2}x}{2}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+4*x**2+1),x)
[Out] -sqrt(2)*(2*atan(sqrt(2)*x/2) - 2*atan(sqrt(2)*x**3/2 + 3*sqrt(2)*x/2))/4
```

Giac [A]
time = 3.25, size = 26, normalized size = 0.59

$$\frac{1}{4}\sqrt{2}\left(\pi\operatorname{sgn}(x)-2\arctan\left(\frac{\sqrt{2}\left(x^{2}+1\right)}{2x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+4*x^2+1),x, algorithm="giac")
[Out] 1/4*sqrt(2)*(pi*sgn(x) - 2*arctan(1/2*sqrt(2)*(x^2 + 1)/x))
```

Mupad [B]
time = 0.08, size = 31, normalized size = 0.70

$$\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}x^{3}}{2}+\frac{3\sqrt{2}x}{2}\right)-\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(4*x^2 + x^4 + 1),x)
[Out] (2^(1/2)*(atan((3*2^(1/2)*x)/2 + (2^(1/2)*x^3)/2) - atan((2^(1/2)*x)/2)))/2
```

3.83 $\int \frac{1-x^2}{1+3x^2+x^4}\,dx$

Optimal. Leaf size=39

$$-\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)+\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)$$

```
[Out] -arctan(x*2^(1/2)/(3+5^(1/2))^(1/2))+arctan(x*(1/2+1/2*5^(1/2)))
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1177, 209}

$$\text{ArcTan}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)-\text{ArcTan}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[2/(3 + Sqrt[5])]*x] + ArcTan[Sqrt[(3 + Sqrt[5])/2]*x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[c*d^2 - a*e^2, 0] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^2}{1+3x^2+x^4}\,dx=\frac{1}{2}\left(-1-\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx+\frac{1}{2}\left(-1+\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx$$

$$=-\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)+\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 0.26

$$\tan^{-1}\left(\frac{x}{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] ArcTan[x/(1 + x^2)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 0.03, size = 66, normalized size = 1.69

method	result	size
risch	$-\arctan(x)+\arctan(x^3+2x)$	14
default	$-\frac{2\left(5+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}-\frac{2\left(-5+\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(5+5^(1/2))*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x/(2*5^(1/2)+2))-2/5*(-5+5^
(1/2))*5^(1/2)/(2*5^(1/2)-2)*arctan(4*x/(2*5^(1/2)-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 + 3*x^2 + 1), x)
```

Fricas [A]
time = 0.37, size = 13, normalized size = 0.33

$$\arctan\left(x^3+2\,x\right)-\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(x^3 + 2*x) - arctan(x)
```

Sympy [A]
time = 0.03, size = 10, normalized size = 0.26

$$-\operatorname{atan}(x) + \operatorname{atan}\left(x^3 + 2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+3*x**2+1),x)
[Out] -atan(x) + atan(x**3 + 2*x)
```

Giac [A]
time = 3.38, size = 26, normalized size = 0.67

$$\frac{1}{4}\pi\operatorname{sgn}(x) - \frac{1}{2}\arctan\left(\frac{x^4 + x^2 + 1}{2\left(x^3 + x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+3*x^2+1),x, algorithm="giac")
[Out] 1/4*pi*sgn(x) - 1/2*arctan(1/2*(x^4 + x^2 + 1)/(x^3 + x))
```

Mupad [B]
time = 4.31, size = 13, normalized size = 0.33

$$\operatorname{atan}\left(x^3 + 2\,x\right) - \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(3*x^2 + x^4 + 1),x)
[Out] atan(2*x + x^3) - atan(x)
```

3.84 $\int \frac{1-x^2}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=9

$$\frac{x}{1+x^2}$$

```
[Out] x/(x^2+1)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {28, 391}

$$\frac{x}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{1+2x^2+x^4} \, dx &= \int \frac{1-x^2}{\left(1+x^2\right)^2} \, dx \\
&= \frac{x}{1+x^2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$\frac{x}{1+x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + 2*x^2 + x^4),x]
```

```
[Out] x/(1 + x^2)
```

Maple [A]
time = 0.01, size = 10, normalized size = 1.11

method	result	size
gosper	$\frac{x}{x^2+1}$	10
default	$\frac{x}{x^2+1}$	10
norman	$\frac{x}{x^2+1}$	10
risch	$\frac{x}{x^2+1}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] x/(x^2+1)
```

Maxima [A]
time = 0.27, size = 9, normalized size = 1.00

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] x/(x^2 + 1)
```

Fricas [A]
time = 0.36, size = 9, normalized size = 1.00

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] x/(x^2 + 1)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.56

$$\frac{x}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+2*x**2+1),x)
```

```
[Out] x/(x**2 + 1)
```

Giac [A]
time = 4.59, size = 7, normalized size = 0.78

$$\frac{1}{x+\frac{1}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/(x + 1/x)
```

Mupad [B]
time = 0.03, size = 9, normalized size = 1.00

$$\frac{x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(2*x^2 + x^4 + 1),x)
```

```
[Out] x/(x^2 + 1)
```

3.85 $\int \frac{1-x^2}{1+x^2+x^4} \, dx$

Optimal. Leaf size=25

$$-\frac{1}{2}\log\left(1-x+x^2\right)+\frac{1}{2}\log\left(1+x+x^2\right)$$

```
[Out] -1/2*ln(x^2-x+1)+1/2*ln(x^2+x+1)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1178, 642}

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1+x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1+2x}{-1-x-x^2}\,dx\right)-\frac{1}{2}\int \frac{1-2x}{-1+x-x^2}\,dx\\ &= -\frac{1}{2}\log\left(1-x+x^2\right)+\frac{1}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$-\frac{1}{2}\log\left(1-x+x^2\right)+\frac{1}{2}\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - x + x^2] + Log[1 + x + x^2]/2
```

Maple [A]
time = 0.02, size = 22, normalized size = 0.88

method	result	size
default	$-\frac{\ln\left(x^{2}-x+1\right)}{2}+\frac{\ln\left(x^{2}+x+1\right)}{2}$	22
norman	$-\frac{\ln\left(x^{2}-x+1\right)}{2}+\frac{\ln\left(x^{2}+x+1\right)}{2}$	22
risch	$-\frac{\ln\left(x^{2}-x+1\right)}{2}+\frac{\ln\left(x^{2}+x+1\right)}{2}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(x^2-x+1)+1/2*ln(x^2+x+1)
```

Maxima [A]
time = 0.27, size = 21, normalized size = 0.84

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

Fricas [A]
time = 0.32, size = 21, normalized size = 0.84

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{1}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 + x + 1) - 1/2*log(x^2 - x + 1)
```

Sympy [A]
time = 0.03, size = 19, normalized size = 0.76

$$-\frac{\log\left(x^2 - x + 1\right)}{2} + \frac{\log\left(x^2 + x + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+x**2+1),x)
[Out] -log(x**2 - x + 1)/2 + log(x**2 + x + 1)/2
```

Giac [A]
time = 3.35, size = 35, normalized size = 1.40

$$\frac{1}{4}\log\left(\left|x + \frac{1}{x + \frac{1}{x}} + \frac{1}{x} + 2\right|\right) - \frac{1}{4}\log\left(\left|x + \frac{1}{x + \frac{1}{x}} + \frac{1}{x} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/4*log(abs(x + 1/(x + 1/x) + 1/x + 2)) - 1/4*log(abs(x + 1/(x + 1/x) + 1/x
 - 2))
```

Mupad [B]
time = 0.06, size = 10, normalized size = 0.40

$$\operatorname{atanh}\left(\frac{x}{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^2 + x^4 + 1),x)
[Out] atanh(x/(x^2 + 1))
```

3.86 $\int \frac{1-x^2}{1+x^4} \, dx$

Optimal. Leaf size=46

$$-\frac{\log \left(1-\sqrt{2}\, x+x^2\right)}{2 \sqrt{2}}+\frac{\log \left(1+\sqrt{2}\, x+x^2\right)}{2 \sqrt{2}}$$

```
[Out] -1/4*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/4*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1179, 642}

$$\frac{\log \left(x^2+\sqrt{2}\, x+1\right)}{2 \sqrt{2}}-\frac{\log \left(x^2-\sqrt{2}\, x+1\right)}{2 \sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 + x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[2]*x + x^2]/Sqrt[2] + Log[1 + Sqrt[2]*x + x^2]/(2*Sqrt[2]
)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{1+x^4} \, dx &= -\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\, x-x^2} \, dx}{2 \sqrt{2}}-\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}\, x-x^2} \, dx}{2 \sqrt{2}}\\
&= -\frac{\log \left(1-\sqrt{2}\, x+x^2\right)}{2 \sqrt{2}}+\frac{\log \left(1+\sqrt{2}\, x+x^2\right)}{2 \sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.87

$$\frac{-\log\left(-1+\sqrt{2}\,x-x^2\right)+\log\left(1+\sqrt{2}\,x+x^2\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 + x^4),x]
[Out] (-Log[-1 + Sqrt[2]*x - x^2] + Log[1 + Sqrt[2]*x + x^2])/(2*Sqrt[2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(34) = 68.
time = 0.15, size = 104, normalized size = 2.26

method	result
risch	$-\frac{\ln\left(1+x^2-\sqrt{2}\,x\right)\sqrt{2}}{4}+\frac{\ln\left(1+x^2+\sqrt{2}\,x\right)\sqrt{2}}{4}$
default	$\frac{\sqrt{2}\,\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{8}-\frac{\sqrt{2}\,\left(\ln\left(\frac{1+x^2-\sqrt{2}\,x}{1+x^2+\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)\right.}{8}$
meijerg	$-\frac{x^3\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}-\frac{x^3\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^4)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^4)^{\frac{1}{4}}}\right)}{4(x^4)^{\frac{3}{4}}}+\frac{x^3\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^4)^{\frac{1}{4}}+\sqrt{x^4}\right)}{8(x^4)^{\frac{3}{4}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*2^(1/2)*(ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))+2*arctan(2^(1/2)*x+1)+
2*arctan(2^(1/2)*x-1))-1/8*2^(1/2)*(ln((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))
+2*arctan(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))
```

Maxima [A]
time = 0.50, size = 34, normalized size = 0.74

$$\frac{1}{4}\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{4}\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="maxima")
[Out] 1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A]
time = 0.32, size = 34, normalized size = 0.74

$$\frac{1}{4}\sqrt{2}\,\log\left(\frac{x^4+4\,x^2+2\,\sqrt{2}\,(x^3+x)+1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 4*x^2 + 2*sqrt(2)*(x^3 + x) + 1)/(x^4 + 1))
```

Sympy [**A**]
time = 0.03, size = 39, normalized size = 0.85

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{4}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/4 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/4
```

Giac [**A**]
time = 3.28, size = 34, normalized size = 0.74

$$\frac{1}{4}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{1}{4}\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Mupad [**B**]
time = 0.06, size = 18, normalized size = 0.39

$$\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,x}{x^2+1}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 + 1),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*x)/(x^2 + 1)))/2
```

3.87 $\int \frac{1-x^2}{1-x^2+x^4}\,dx$

Optimal. Leaf size=46

$$-\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{2\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{2\sqrt{3}}$$

```
[Out] -1/6*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1178, 642}

$$\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - x^2 + x^4),x]
```

```
[Out] -1/2*Log[1 - Sqrt[3]*x + x^2]/Sqrt[3] + Log[1 + Sqrt[3]*x + x^2]/(2*Sqrt[3]
)
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1-x^2+x^4}\,dx &= -\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}\,x-x^2}\,dx}{2\sqrt{3}}-\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}\,x-x^2}\,dx}{2\sqrt{3}}\\ &= -\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{2\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.87

$$\frac{-\log\left(-1+\sqrt{3}\,x-x^2\right)+\log\left(1+\sqrt{3}\,x+x^2\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(2*Sqrt[3])
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.76

method	result	size
default	$-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	35
risch	$-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{6}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/6*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Fricas [A]
time = 0.38, size = 39, normalized size = 0.85

$$\frac{1}{6}\sqrt{3}\,\log\left(\frac{x^4+5\,x^2+2\,\sqrt{3}\,(x^3+x)+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2 + 1))
```

Sympy [A]
time = 0.03, size = 39, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)}{6}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-x**2+1),x)
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/6 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/6
```

Giac [A]
time = 4.62, size = 39, normalized size = 0.85

$$-\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2\,x-2\sqrt{3}+\frac{2}{x}\right|}{\left|2\,x+2\sqrt{3}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-x^2+1),x, algorithm="giac")
[Out] -1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3) + 2/x)/abs(2*x + 2*sqrt(3) + 2/x))
```

Mupad [B]
time = 4.31, size = 18, normalized size = 0.39

$$\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{x^2+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 - x^2 + 1),x)
[Out] (3^(1/2)*atanh((3^(1/2)*x)/(x^2 + 1)))/3
```

3.88 $\int \frac{1-x^2}{1-2x^2+x^4} \, dx$

Optimal. Leaf size=2

$$\tanh^{-1}(x)$$

```
[Out] arctanh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {28, 21, 213}

$$\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] ArcTanh[x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^2}{1-2x^2+x^4} \, dx &= \int \frac{1-x^2}{(-1+x^2)^2} \, dx \\
&= -\int \frac{1}{-1+x^2} \, dx \\
&= \tanh^{-1}(x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(2) = 4$.
time = 0.00, size = 19, normalized size = 9.50

$$-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 2*x^2 + x^4),x]
[Out] -1/2*Log[1 - x] + Log[1 + x]/2
```

Maple [A]
time = 0.01, size = 3, normalized size = 1.50

method	result	size
default	$\operatorname{arctanh}(x)$	3
norman	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
risch	$-\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-2*x^2+1),x,method=_RETURNVERBOSE)
[Out] arctanh(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.
time = 0.28, size = 13, normalized size = 6.50

$$\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="maxima")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(2) = 4$.
time = 0.34, size = 13, normalized size = 6.50

$$\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="fricas")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(2) = 4$.
time = 0.03, size = 12, normalized size = 6.00

$$-\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-2*x**2+1),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(2) = 4$.
time = 3.84, size = 15, normalized size = 7.50

$$\frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

Mupad [B]
time = 4.30, size = 2, normalized size = 1.00

$$\operatorname{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 - 2*x^2 + 1),x)
```

```
[Out] atanh(x)
```

3.89 $\int \frac{1-x^2}{1-3x^2+x^4}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\tanh^{-1}\left(\frac{1+2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] -1/5*arctanh(1/5*(1-2*x)*5^(1/2))*5^(1/2)+1/5*arctanh(1/5*(1+2*x)*5^(1/2))*
5^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1175, 632, 212}

$$\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)}{\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 3*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(1 - 2*x)/Sqrt[5]]/Sqrt[5]) + ArcTanh[(1 + 2*x)/Sqrt[5]]/Sqrt[5]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x^2}{1-3x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-x+x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+x+x^2}\,dx\\
&= \text{Subst}\left(\int \frac{1}{5-x^2}\,dx,x,-1+2x\right)+\text{Subst}\left(\int \frac{1}{5-x^2}\,dx,x,1+2x\right)\\
&= \frac{\tanh^{-1}\left(\frac{-1+2x}{\sqrt{5}}\right)}{\sqrt{5}}+\frac{\tanh^{-1}\left(\frac{1+2x}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 1.05

$$\frac{-\log\left(-1+\sqrt{5}\,x-x^2\right)+\log\left(1+\sqrt{5}\,x+x^2\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 3*x^2 + x^4),x]
[Out] (-Log[-1 + Sqrt[5]*x - x^2] + Log[1 + Sqrt[5]*x + x^2])/(2*Sqrt[5])
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.89

method	result	size
default	$\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{5}+\frac{\mathrm{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)\sqrt{5}}{5}$	34
risch	$\frac{\sqrt{5}\,\ln\left(x^2+x\sqrt{5}+1\right)}{10}-\frac{\sqrt{5}\,\ln\left(x^2-x\sqrt{5}+1\right)}{10}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-3*x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))+1/5*arctanh(1/5*(2*x+1)*5^(1/2))*5
^(1/2)
```

Maxima [A]
time = 0.53, size = 55, normalized size = 1.45

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+1}{2\,x+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}-1}{2\,x+\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/10*sqrt(5)*l
og((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1))
```

Fricas [A]
time = 0.40, size = 39, normalized size = 1.03

$$\frac{1}{10}\sqrt{5}\log\left(\frac{x^4+7x^2+2\sqrt{5}(x^3+x)+1}{x^4-3x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*log((x^4 + 7*x^2 + 2*sqrt(5)*(x^3 + x) + 1)/(x^4 - 3*x^2 + 1))
```

Sympy [A]
time = 0.03, size = 39, normalized size = 1.03

$$-\frac{\sqrt{5}\log\left(x^2-\sqrt{5}x+1\right)}{10}+\frac{\sqrt{5}\log\left(x^2+\sqrt{5}x+1\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-3*x**2+1),x)
```

```
[Out] -sqrt(5)*log(x**2 - sqrt(5)*x + 1)/10 + sqrt(5)*log(x**2 + sqrt(5)*x + 1)/1
0
```

Giac [A]
time = 3.81, size = 39, normalized size = 1.03

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x-2\sqrt{5}+\frac{2}{x}\right|}{\left|2x+2\sqrt{5}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*log(abs(2*x - 2*sqrt(5) + 2/x)/abs(2*x + 2*sqrt(5) + 2/x))
```

Mupad [B]
time = 0.11, size = 18, normalized size = 0.47

$$\frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,x}{x^2+1}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 - 3*x^2 + 1),x)
[Out] (5^(1/2)*atanh((5^(1/2)*x)/(x^2 + 1)))/5
```

3.90 $\int \frac{1-x^2}{1-4x^2+x^4} \, dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{1-\sqrt{2}\,x}{\sqrt{3}}\right)}{\sqrt{6}}+\frac{\tanh^{-1}\left(\frac{1+\sqrt{2}\,x}{\sqrt{3}}\right)}{\sqrt{6}}$$

```
[Out] -1/6*arctanh(1/3*(1-x*2^(1/2))*3^(1/2))*6^(1/2)+1/6*arctanh(1/3*(1+x*2^(1/2
))*3^(1/2))*6^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1175, 632, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x+1}{\sqrt{3}}\right)}{\sqrt{6}}-\frac{\tanh^{-1}\left(\frac{1-\sqrt{2}\,x}{\sqrt{3}}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(1 - Sqrt[2]*x)/Sqrt[3]]/Sqrt[6]) + ArcTanh[(1 + Sqrt[2]*x)/Sqrt[
3]]/Sqrt[6]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1-4x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{2}\,x+x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+\sqrt{2}\,x+x^2}\,dx\\ &= \text{Subst}\left(\int \frac{1}{6-x^2}\,dx,x,-\sqrt{2}+2x\right)+\text{Subst}\left(\int \frac{1}{6-x^2}\,dx,x,\sqrt{2}+2x\right)\\ &= \frac{\tanh^{-1}\left(\frac{-1+\sqrt{2}\,x}{\sqrt{3}}\right)}{\sqrt{6}}+\frac{\tanh^{-1}\left(\frac{1+\sqrt{2}\,x}{\sqrt{3}}\right)}{\sqrt{6}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.85

$$\frac{-\log\left(-1+\sqrt{6}\,x-x^2\right)+\log\left(1+\sqrt{6}\,x+x^2\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 4*x^2 + x^4),x]
```

```
[Out] (-Log[-1 + Sqrt[6]*x - x^2] + Log[1 + Sqrt[6]*x + x^2])/(2*Sqrt[6])
```

Maple [A]
time = 0.03, size = 70, normalized size = 1.49

method	result	size
risch	$\frac{\sqrt{6}\,\ln\left(x^2+x\sqrt{6}+1\right)}{12}-\frac{\sqrt{6}\,\ln\left(x^2-x\sqrt{6}+1\right)}{12}$	35
default	$\frac{\left(\sqrt{3}-1\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)}{3\sqrt{6}-3\sqrt{2}}+\frac{\left(1+\sqrt{3}\right)\sqrt{3}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)}{3\sqrt{6}+3\sqrt{2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-4*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(3^(1/2)-1)*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-2^(1/2)))+1/
3*(1+3^(1/2))*3^(1/2)/(6^(1/2)+2^(1/2))*arctanh(2*x/(6^(1/2)+2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(x^4 - 4*x^2 + 1), x)
```

Fricas [A]
time = 0.33, size = 39, normalized size = 0.83

$$\frac{1}{12}\sqrt{6}\log\left(\frac{x^4+8x^2+2\sqrt{6}(x^3+x)+1}{x^4-4x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="fricas")
[Out] 1/12*sqrt(6)*log((x^4 + 8*x^2 + 2*sqrt(6)*(x^3 + x) + 1)/(x^4 - 4*x^2 + 1))
```

Sympy [A]
time = 0.03, size = 39, normalized size = 0.83

$$-\frac{\sqrt{6}\log\left(x^2-\sqrt{6}x+1\right)}{12}+\frac{\sqrt{6}\log\left(x^2+\sqrt{6}x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-4*x**2+1),x)
[Out] -sqrt(6)*log(x**2 - sqrt(6)*x + 1)/12 + sqrt(6)*log(x**2 + sqrt(6)*x + 1)/1
2
```

Giac [A]
time = 3.39, size = 39, normalized size = 0.83

$$-\frac{1}{12}\sqrt{6}\log\left(\frac{\left|2x-2\sqrt{6}+\frac{2}{x}\right|}{\left|2x+2\sqrt{6}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-4*x^2+1),x, algorithm="giac")
[Out] -1/12*sqrt(6)*log(abs(2*x - 2*sqrt(6) + 2/x)/abs(2*x + 2*sqrt(6) + 2/x))
```

Mupad [B]
time = 4.32, size = 18, normalized size = 0.38

$$\frac{\sqrt{6}\operatorname{atanh}\left(\frac{\sqrt{6}x}{x^2+1}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 - 4*x^2 + 1),x)
[Out] (6^(1/2)*atanh((6^(1/2)*x)/(x^2 + 1)))/6
```

3.91 $\int \frac{1-x^2}{1-5x^2+x^4}\,dx$

Optimal. Leaf size=46

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3}+2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] -1/7*arctanh(1/7*(-2*x+3^(1/2))*7^(1/2))*7^(1/2)+1/7*arctanh(1/7*(2*x+3^(1/
2))*7^(1/2))*7^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1175, 632, 212}

$$\frac{\tanh^{-1}\left(\frac{2x+\sqrt{3}}{\sqrt{7}}\right)}{\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/(1 - 5*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[3] - 2*x)/Sqrt[7]]/Sqrt[7]) + ArcTanh[(Sqrt[3] + 2*x)/Sqrt[
7]]/Sqrt[7]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{1-5x^2+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1}{-1-\sqrt{3}\,x+x^2}\,dx\right)-\frac{1}{2}\int \frac{1}{-1+\sqrt{3}\,x+x^2}\,dx\\ &= \text{Subst}\left(\int \frac{1}{7-x^2}\,dx,x,-\sqrt{3}+2x\right)+\text{Subst}\left(\int \frac{1}{7-x^2}\,dx,x,\sqrt{3}+2x\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}-2x}{\sqrt{7}}\right)}{\sqrt{7}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3}+2x}{\sqrt{7}}\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 40, normalized size = 0.87

$$\frac{-\log\left(-1+\sqrt{7}\,x-x^2\right)+\log\left(1+\sqrt{7}\,x+x^2\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/(1 - 5*x^2 + x^4),x]
[Out] (-Log[-1 + Sqrt[7]*x - x^2] + Log[1 + Sqrt[7]*x + x^2])/(2*Sqrt[7])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(37) = 74.
time = 0.02, size = 82, normalized size = 1.78

method	result	size
risch	$\frac{\sqrt{7}\,\ln\left(x^2+x\sqrt{7}+1\right)}{14}-\frac{\sqrt{7}\,\ln\left(x^2-x\sqrt{7}+1\right)}{14}$	35
default	$\frac{2\left(3+\sqrt{21}\right)\sqrt{21}\,\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}+2\sqrt{3}}\right)}{21\left(2\sqrt{7}+2\sqrt{3}\right)}+\frac{2\left(-3+\sqrt{21}\right)\sqrt{21}\,\operatorname{arctanh}\left(\frac{4x}{2\sqrt{7}-2\sqrt{3}}\right)}{21\left(2\sqrt{7}-2\sqrt{3}\right)}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^4-5*x^2+1),x,method=_RETURNVERBOSE)
[Out] 2/21*(3+21^(1/2))*21^(1/2)/(2*7^(1/2)+2*3^(1/2))*arctanh(4*x/(2*7^(1/2)+2*3
^(1/2)))+2/21*(-3+21^(1/2))*21^(1/2)/(2*7^(1/2)-2*3^(1/2))*arctanh(4*x/(2*7
^(1/2)-2*3^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="maxima")
[Out] -integrate((x^2 - 1)/(x^4 - 5*x^2 + 1), x)
```

Fricas [A]
time = 0.38, size = 39, normalized size = 0.85

$$\frac{1}{14}\sqrt{7}\log\left(\frac{x^4+9x^2+2\sqrt{7}\,(x^3+x)+1}{x^4-5x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="fricas")
[Out] 1/14*sqrt(7)*log((x^4 + 9*x^2 + 2*sqrt(7)*(x^3 + x) + 1)/(x^4 - 5*x^2 + 1))
```

Sympy [A]
time = 0.03, size = 39, normalized size = 0.85

$$-\frac{\sqrt{7}\log\left(x^2-\sqrt{7}\,x+1\right)}{14}+\frac{\sqrt{7}\log\left(x^2+\sqrt{7}\,x+1\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**4-5*x**2+1),x)
[Out] -sqrt(7)*log(x**2 - sqrt(7)*x + 1)/14 + sqrt(7)*log(x**2 + sqrt(7)*x + 1)/1
4
```

Giac [A]
time = 3.11, size = 39, normalized size = 0.85

$$-\frac{1}{14}\sqrt{7}\log\left(\frac{\left|2x-2\sqrt{7}+\frac{2}{x}\right|}{\left|2x+2\sqrt{7}+\frac{2}{x}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^4-5*x^2+1),x, algorithm="giac")
[Out] -1/14*sqrt(7)*log(abs(2*x - 2*sqrt(7) + 2/x)/abs(2*x + 2*sqrt(7) + 2/x))
```

Mupad [B]
time = 4.39, size = 18, normalized size = 0.39

$$\frac{\sqrt{7}\,\mathrm{atanh}\left(\frac{\sqrt{7}\,x}{x^2+1}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/(x^4 - 5*x^2 + 1),x)
[Out] (7^(1/2)*atanh((7^(1/2)*x)/(x^2 + 1)))/7
```

3.92 $\int \frac{-1-3x^2}{1+2x^2+9x^4}\,dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(1/2*(1-3*x)*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+3*x)*2^(1/2))*2^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 3*x^2)/(1 + 2*x^2 + 9*x^4),x]
```

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2
])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\int -\frac{1+3x^2}{1+2x^2+9x^4}\,dx &= -\left(\frac{1}{6}\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx\right)-\frac{1}{6}\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,-\frac{2}{3}+2x\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,\frac{2}{3}+2x\right)\\
&= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 99, normalized size = 2.30

$$
-\frac{\left(-i+\sqrt{2}\right)\tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(i+\sqrt{2}\right)\tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 3*x^2)/(1 + 2*x^2 + 9*x^4),x]
```

```
[Out] -1/2*((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/Sqrt[2*(1 - (2*
I)*Sqrt[2])] - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*Sqr
t[2*(1 + (2*I)*Sqrt[2])])
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.79

method	result	size
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)}{4}$	34
risch	$-\frac{\sqrt{2}\,\arctan\left(\frac{3\sqrt{2}\,x}{4}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{9\sqrt{2}\,x^3}{4}+\frac{5\sqrt{2}\,x}{4}\right)}{4}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2-1)/(9*x^4+2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(
1/2))
```

Maxima [A]
time = 0.53, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

Fricas [A]
time = 0.32, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(9x^3+5x\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{3}{4}\sqrt{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sqr
t(2)*x)
```

Sympy [A]
time = 0.04, size = 46, normalized size = 1.07

$$-\frac{\sqrt{2}\cdot\left(2\operatorname{atan}\left(\frac{3\sqrt{2}x}{4}\right)+2\operatorname{atan}\left(\frac{9\sqrt{2}x^3}{4}+\frac{5\sqrt{2}x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2-1)/(9*x**4+2*x**2+1),x)
```

```
[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4))
/8
```

Giac [A]
time = 5.18, size = 33, normalized size = 0.77

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2-1)/(9*x^4+2*x^2+1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

Mupad [B]
time = 4.38, size = 29, normalized size = 0.67

$$-\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{9\sqrt{2}\,x^3}{4}+\frac{5\sqrt{2}\,x}{4}\right)+\operatorname{atan}\left(\frac{3\sqrt{2}\,x}{4}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 + 1)/(2*x^2 + 9*x^4 + 1),x)
```

```
[Out] -(2^(1/2)*(atan((5*2^(1/2)*x)/4 + (9*2^(1/2)*x^3)/4) + atan((3*2^(1/2)*x)/4
)))/4
```

3.93 $\int \frac{1+3x^2}{-1-2x^2-9x^4} \, dx$

Optimal. Leaf size=43

$$\frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(1/2*(1-3*x)*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+3*x)*2^(1/2))*2^(
1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{3x+1}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

```
[Out] ArcTan[(1 - 3*x)/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + 3*x)/Sqrt[2]]/(2*Sqrt[2
])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+3x^2}{-1-2x^2-9x^4}\,dx &= -\left(\frac{1}{6}\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx\right)-\frac{1}{6}\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,-\frac{2}{3}+2x\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx,x,\frac{2}{3}+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{1-3x}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+3x}{\sqrt{2}}\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 99, normalized size = 2.30

$$
-\frac{\left(-i+\sqrt{2}\right)\tan^{-1}\left(\frac{3x}{\sqrt{1-2i\sqrt{2}}}\right)}{2\sqrt{2\left(1-2i\sqrt{2}\right)}}-\frac{\left(i+\sqrt{2}\right)\tan^{-1}\left(\frac{3x}{\sqrt{1+2i\sqrt{2}}}\right)}{2\sqrt{2\left(1+2i\sqrt{2}\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x^2)/(-1 - 2*x^2 - 9*x^4),x]
```

```
[Out] -1/2*((-I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 - (2*I)*Sqrt[2]]])/Sqrt[2*(1 - (2*
I)*Sqrt[2])] - ((I + Sqrt[2])*ArcTan[(3*x)/Sqrt[1 + (2*I)*Sqrt[2]]])/(2*Sqr
t[2*(1 + (2*I)*Sqrt[2])])
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.79

method	result	size
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{(6x-2)\sqrt{2}}{4}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{(6x+2)\sqrt{2}}{4}\right)}{4}$	34
risch	$-\frac{\sqrt{2}\,\arctan\left(\frac{3\sqrt{2}\,x}{4}\right)}{4}-\frac{\sqrt{2}\,\arctan\left(\frac{9\sqrt{2}\,x^3}{4}+\frac{5\sqrt{2}\,x}{4}\right)}{4}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+1)/(-9*x^4-2*x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(6*x-2)*2^(1/2))-1/4*2^(1/2)*arctan(1/4*(6*x+2)*2^(
1/2))
```

Maxima [A]
time = 0.51, size = 33, normalized size = 0.77

$$-\frac{1}{4}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,(3\,x+1)\right)-\frac{1}{4}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,(3\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

Fricas [A]
time = 0.35, size = 33, normalized size = 0.77

$$-\frac{1}{4}\,\sqrt{2}\,\arctan\left(\frac{1}{4}\,\sqrt{2}\,(9\,x^3+5\,x)\right)-\frac{1}{4}\,\sqrt{2}\,\arctan\left(\frac{3}{4}\,\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(9*x^3 + 5*x)) - 1/4*sqrt(2)*arctan(3/4*sqr
t(2)*x)
```

Sympy [A]
time = 0.04, size = 46, normalized size = 1.07

$$-\frac{\sqrt{2}\cdot\left(2\,\mathrm{atan}\left(\frac{3\sqrt{2}\,x}{4}\right)+2\,\mathrm{atan}\left(\frac{9\sqrt{2}\,x^3}{4}+\frac{5\sqrt{2}\,x}{4}\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+1)/(-9*x**4-2*x**2-1),x)
```

```
[Out] -sqrt(2)*(2*atan(3*sqrt(2)*x/4) + 2*atan(9*sqrt(2)*x**3/4 + 5*sqrt(2)*x/4))
/8
```

Giac [A]
time = 5.40, size = 33, normalized size = 0.77

$$-\frac{1}{4}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,(3\,x+1)\right)-\frac{1}{4}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,(3\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+1)/(-9*x^4-2*x^2-1),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)
*(3*x - 1))
```

Mupad [B]
time = 0.00, size = 29, normalized size = 0.67

$$-\frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{9\sqrt{2}\,x^3}{4}+\frac{5\sqrt{2}\,x}{4}\right)+\operatorname{atan}\left(\frac{3\sqrt{2}\,x}{4}\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x^2 + 1)/(2*x^2 + 9*x^4 + 1),x)
```

```
[Out] -(2^(1/2)*(atan((5*2^(1/2)*x)/4 + (9*2^(1/2)*x^3)/4) + atan((3*2^(1/2)*x)/4
)))/4
```

3.94 $\int \frac{3+2x^2}{1-2x^2+x^4} \, dx$

Optimal. Leaf size=21

$$\frac{5x}{2(1-x^2)} + \frac{1}{2}\tanh^{-1}(x)$$

[Out] 5/2*x/(-x^2+1)+1/2*arctanh(x)

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {28, 393, 213}

$$\frac{5x}{2(1-x^2)} + \frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]

[Out] (5*x)/(2*(1 - x^2)) + ArcTanh[x]/2

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\int \frac{3+2x^2}{1-2x^2+x^4}\,dx = \int \frac{3+2x^2}{(-1+x^2)^2}\,dx$$
$$= \frac{5x}{2(1-x^2)} - \frac{1}{2}\int \frac{1}{-1+x^2}\,dx$$
$$= \frac{5x}{2(1-x^2)} + \frac{1}{2}\tanh^{-1}(x)$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{10x}{-1+x^2} - \log(1-x) + \log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)/(1 - 2*x^2 + x^4),x]
```

```
[Out] ((-10*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [A]
time = 0.03, size = 28, normalized size = 1.33

method	result	size
norman	$-\frac{5x}{2(x^2-1)} - \frac{\ln(-1+x)}{4} + \frac{\ln(1+x)}{4}$	24
risch	$-\frac{5x}{2(x^2-1)} - \frac{\ln(-1+x)}{4} + \frac{\ln(1+x)}{4}$	24
default	$-\frac{5}{4(-1+x)} - \frac{\ln(-1+x)}{4} - \frac{5}{4(1+x)} + \frac{\ln(1+x)}{4}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+3)/(x^4-2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -5/4/(-1+x)-1/4*ln(-1+x)-5/4/(1+x)+1/4*ln(1+x)
```

Maxima [A]
time = 0.28, size = 23, normalized size = 1.10

$$-\frac{5\,x}{2\,(x^2-1)} + \frac{1}{4}\,\log\left(x+1\right) - \frac{1}{4}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] -5/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.
time = 0.35, size = 34, normalized size = 1.62

$$\frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-10\,x}{4\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 10*x)/(x^2 - 1)
```

Sympy [A]
time = 0.03, size = 22, normalized size = 1.05

$$-\frac{5x}{2x^2-2}-\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+3)/(x**4-2*x**2+1),x)
```

```
[Out] -5*x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

Giac [A]
time = 4.50, size = 25, normalized size = 1.19

$$-\frac{5\,x}{2\,(x^2-1)}+\frac{1}{4}\log(|x+1|)-\frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+3)/(x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] -5/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.81

$$\frac{\operatorname{atanh}(x)}{2}-\frac{5\,x}{2\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 3)/(x^4 - 2*x^2 + 1),x)
```

```
[Out] atanh(x)/2 - (5*x)/(2*(x^2 - 1))
```

3.95 $\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx$

Optimal. Leaf size=28

$$\frac{5}{2}\tanh^{-1}(x)-\frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}\,x\right)$$

```
[Out] 5/2*arctanh(x)-7/10*arctanh(1/5*x*15^(1/2))*15^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1180, 213}

$$\frac{5}{2}\tanh^{-1}(x)-\frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (5*ArcTanh[x])/2 - (7*Sqrt[3/5]*ArcTanh[Sqrt[3/5]*x])/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{5-8x^2+3x^4}\,dx &= -\left(\frac{15}{2}\int \frac{1}{-3+3x^2}\,dx\right)+\frac{21}{2}\int \frac{1}{-5+3x^2}\,dx\\ &=\frac{5}{2}\tanh^{-1}(x)-\frac{7}{2}\sqrt{\frac{3}{5}}\tanh^{-1}\left(\sqrt{\frac{3}{5}}\,x\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 1.89

$$\frac{1}{20}\left(7\sqrt{15}\log\left(\sqrt{15}-3x\right)-25\log(1-x)+25\log(1+x)-7\sqrt{15}\log\left(\sqrt{15}+3x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (7*Sqrt[15]*Log[Sqrt[15] - 3*x] - 25*Log[1 - x] + 25*Log[1 + x] - 7*Sqrt[15
]*Log[Sqrt[15] + 3*x])/20
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.93

method	result	size
default	$-\frac{7\,\mathrm{arctanh}\left(\frac{x\sqrt{15}}{5}\right)\sqrt{15}}{10}-\frac{5\ln(-1+x)}{4}+\frac{5\ln(1+x)}{4}$	26
risch	$\frac{7\sqrt{15}\,\ln\left(3x-\sqrt{15}\right)}{20}-\frac{7\sqrt{15}\,\ln\left(3x+\sqrt{15}\right)}{20}-\frac{5\ln(-1+x)}{4}+\frac{5\ln(1+x)}{4}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(3*x^4-8*x^2+5),x,method=_RETURNVERBOSE)
```

```
[Out] -7/10*arctanh(1/5*x*15^(1/2))*15^(1/2)-5/4*ln(-1+x)+5/4*ln(1+x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.
time = 0.56, size = 38, normalized size = 1.36

$$\frac{7}{20}\sqrt{15}\log\left(\frac{3x-\sqrt{15}}{3x+\sqrt{15}}\right)+\frac{5}{4}\log\left(x+1\right)-\frac{5}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="maxima")
```

```
[Out] 7/20*sqrt(15)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 5/4*log(x + 1) - 5/4
*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(17) = 34$.
time = 0.34, size = 49, normalized size = 1.75

$$\frac{7}{20}\sqrt{5}\sqrt{3}\log\left(-\frac{2\sqrt{5}\sqrt{3}x-3x^2-5}{3x^2-5}\right)+\frac{5}{4}\log\left(x+1\right)-\frac{5}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="fricas")
```

```
[Out] 7/20*sqrt(5)*sqrt(3)*log(-(2*sqrt(5)*sqrt(3)*x - 3*x^2 - 5)/(3*x^2 - 5)) +
5/4*log(x + 1) - 5/4*log(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.
time = 0.31, size = 53, normalized size = 1.89

$$-\frac{5\log(x-1)}{4}+\frac{5\log(x+1)}{4}+\frac{7\sqrt{15}\log\left(x-\frac{\sqrt{15}}{3}\right)}{20}-\frac{7\sqrt{15}\log\left(x+\frac{\sqrt{15}}{3}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(3*x**4-8*x**2+5),x)
```

```
[Out] -5*log(x - 1)/4 + 5*log(x + 1)/4 + 7*sqrt(15)*log(x - sqrt(15)/3)/20 - 7*sq
rt(15)*log(x + sqrt(15)/3)/20
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.
time = 6.42, size = 44, normalized size = 1.57

$$\frac{7}{20}\sqrt{15}\log\left(\frac{\left|6x-2\sqrt{15}\right|}{\left|6x+2\sqrt{15}\right|}\right)+\frac{5}{4}\log\left(|x+1|\right)-\frac{5}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(3*x^4-8*x^2+5),x, algorithm="giac")
```

```
[Out] 7/20*sqrt(15)*log(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15))) + 5/4*log(ab
s(x + 1)) - 5/4*log(abs(x - 1))
```

Mupad [B]
time = 4.39, size = 17, normalized size = 0.61

$$\frac{5\,\mathrm{atanh}(x)}{2}-\frac{7\sqrt{15}\,\mathrm{atanh}\left(\frac{\sqrt{15}\,x}{5}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(3*x^4 - 8*x^2 + 5),x)
```

```
[Out] (5*atanh(x))/2 - (7*15^(1/2)*atanh((15^(1/2)*x)/5))/10
```

3.96 $\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}(d+e)\tanh^{-1}(x)-\frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}\,x\right)}{2\sqrt{15}}$$

```
[Out] 1/2*(d+e)*arctanh(x)-1/30*(3*d+5*e)*arctanh(1/5*x*15^(1/2))*15^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1180, 213}

$$\frac{1}{2}(d+e)\tanh^{-1}(x)-\frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}\,x\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] ((d + e)*ArcTanh[x])/2 - ((3*d + 5*e)*ArcTanh[Sqrt[3/5]*x])/(2*Sqrt[15])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{d+ex^2}{5-8x^2+3x^4}\,dx = -\left(\frac{1}{2}(3(d+e))\int \frac{1}{-3+3x^2}\,dx\right) + \frac{1}{2}(3d+5e)\int \frac{1}{-5+3x^2}\,dx$$
$$= \frac{1}{2}(d+e)\tanh^{-1}(x) - \frac{(3d+5e)\tanh^{-1}\left(\sqrt{\frac{3}{5}}\,x\right)}{2\sqrt{15}}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 2.00

$$\frac{1}{60}\left(\sqrt{15}\,(3d+5e)\log\left(\sqrt{15}-3x\right) - 15(d+e)\log(1-x) + 15(d+e)\log(1+x) - \sqrt{15}\,(3d+5e)\log\left(\sqrt{15}+3x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(5 - 8*x^2 + 3*x^4),x]
```

```
[Out] (Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] - 3*x] - 15*(d + e)*Log[1 - x] + 15*(d +
 e)*Log[1 + x] - Sqrt[15]*(3*d + 5*e)*Log[Sqrt[15] + 3*x])/60
```

Maple [A]
time = 0.03, size = 45, normalized size = 1.25

method	result
default	$-\frac{\left(\frac{3d}{2}+\frac{5e}{2}\right)\operatorname{arctanh}\left(\frac{x\sqrt{15}}{5}\right)\sqrt{15}}{15}+\left(-\frac{d}{4}-\frac{e}{4}\right)\ln\left(-1+x\right)+\left(\frac{d}{4}+\frac{e}{4}\right)\ln\left(1+x\right)$
risch	$-\frac{\ln(1-x)d}{4}-\frac{\ln(1-x)e}{4}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(15\_Z^2-9d^2-30de-25e^2\right)}\_R\ln\left(\left(12\_R^3+(-9d^2-24de-17e^2)\_R\right)x+3d\_R^2-3c\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(3*x^4-8*x^2+5),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(3/2*d+5/2*e)*arctanh(1/5*x*15^(1/2))*15^(1/2)+(-1/4*d-1/4*e)*ln(-1+x
)+(1/4*d+1/4*e)*ln(1+x)
```

Maxima [A]
time = 0.49, size = 54, normalized size = 1.50

$$\frac{1}{60}\sqrt{15}\,(3\,d+5\,e)\log\left(\frac{3\,x-\sqrt{15}}{3\,x+\sqrt{15}}\right)+\frac{1}{4}\,(d+e)\log\left(x+1\right)-\frac{1}{4}\,(d+e)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log((3*x - sqrt(15))/(3*x + sqrt(15))) + 1/4*(d +
 e)*log(x + 1) - 1/4*(d + e)*log(x - 1)
```

Fricas [A]
time = 0.36, size = 58, normalized size = 1.61

$$\frac{1}{60}\sqrt{15}\,(3\,d+5\,e)\log\left(\frac{3\,x^2-2\,\sqrt{15}\,x+5}{3\,x^2-5}\right)+\frac{1}{4}\,(d+e)\log\left(x+1\right)-\frac{1}{4}\,(d+e)\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="fricas")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log((3*x^2 - 2*sqrt(15)*x + 5)/(3*x^2 - 5)) + 1/4
*(d + e)*log(x + 1) - 1/4*(d + e)*log(x - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(31) = 62$.
time = 0.80, size = 474, normalized size = 13.17

$$\frac{(d+e)\log\left(x+\frac{-51d^3(d+e)-180d^2e(d+e)-225de^2(d+e)+60d(d+e)^3-100e^3(d+e)+75e(d+e)^3}{9d^4+24d^3e-40de^3-25e^4}\right)}{4}-\frac{(d+e)\log\left(x+\frac{51d^3(d+e)+180d^2e(d+e)+225de^2(d+e)-60d(d+e)^3+100e^3(d+e)-75e(d+e)^3}{9d^4+24d^3e-40de^3-25e^4}\right)}{4}+\frac{\sqrt{15}\cdot(3d+5e)\log\left(x+\frac{-\frac{17\sqrt{15}d^3\cdot(3d+5e)}{5}-12\sqrt{15}d^2e(3d+5e)-15\sqrt{15}de^2\cdot(3d+5e)+\frac{4\sqrt{15}d(3d+5e)^3}{15}-\frac{20\sqrt{15}e^3\cdot(3d+5e)}{3}+\frac{\sqrt{15}e(3d+5e)^3}{3}}{9d^4+24d^3e-40de^3-25e^4}\right)}{60}-\frac{\sqrt{15}\cdot(3d+5e)\log\left(x+\frac{\frac{17\sqrt{15}d^3\cdot(3d+5e)}{5}+12\sqrt{15}d^2e(3d+5e)+15\sqrt{15}de^2\cdot(3d+5e)-\frac{4\sqrt{15}d(3d+5e)^3}{15}+\frac{20\sqrt{15}e^3\cdot(3d+5e)}{3}-\frac{\sqrt{15}e(3d+5e)^3}{3}}{9d^4+24d^3e-40de^3-25e^4}\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(3*x**4-8*x**2+5),x)
```

```
[Out] (d + e)*log(x + (-51*d**3*(d + e) - 180*d**2*e*(d + e) - 225*d*e**2*(d + e)
 + 60*d*(d + e)**3 - 100*e**3*(d + e) + 75*e*(d + e)**3)/(9*d**4 + 24*d**3*
e - 40*d*e**3 - 25*e**4))/4 - (d + e)*log(x + (51*d**3*(d + e) + 180*d**2*e
*(d + e) + 225*d*e**2*(d + e) - 60*d*(d + e)**3 + 100*e**3*(d + e) - 75*e*(
d + e)**3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))/4 + sqrt(15)*(3*d +
5*e)*log(x + (-17*sqrt(15)*d**3*(3*d + 5*e)/5 - 12*sqrt(15)*d**2*e*(3*d + 5
*e) - 15*sqrt(15)*d*e**2*(3*d + 5*e) + 4*sqrt(15)*d*(3*d + 5*e)**3/15 - 20*
sqrt(15)*e**3*(3*d + 5*e)/3 + sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**
3*e - 40*d*e**3 - 25*e**4))/60 - sqrt(15)*(3*d + 5*e)*log(x + (17*sqrt(15)*
d**3*(3*d + 5*e)/5 + 12*sqrt(15)*d**2*e*(3*d + 5*e) + 15*sqrt(15)*d*e**2*(3
*d + 5*e) - 4*sqrt(15)*d*(3*d + 5*e)**3/15 + 20*sqrt(15)*e**3*(3*d + 5*e)/3
 - sqrt(15)*e*(3*d + 5*e)**3/3)/(9*d**4 + 24*d**3*e - 40*d*e**3 - 25*e**4))
/60
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.
time = 5.12, size = 60, normalized size = 1.67

$$\frac{1}{60}\sqrt{15}\,(3\,d+5\,e)\log\left(\frac{\left|6\,x-2\,\sqrt{15}\right|}{\left|6\,x+2\,\sqrt{15}\right|}\right)+\frac{1}{4}\,(d+e)\log\left(|x+1|\right)-\frac{1}{4}\,(d+e)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(3*x^4-8*x^2+5),x, algorithm="giac")
```

```
[Out] 1/60*sqrt(15)*(3*d + 5*e)*log(abs(6*x - 2*sqrt(15))/abs(6*x + 2*sqrt(15)))
+ 1/4*(d + e)*log(abs(x + 1)) - 1/4*(d + e)*log(abs(x - 1))
```

Mupad [B]
time = 4.39, size = 290, normalized size = 8.06

$$\frac{\sqrt{15}\,\mathrm{atanh}\left(\frac{54\sqrt{15}\,d^3\,x}{25\left(-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3\right)}-\frac{6\sqrt{15}\,e^3\,x}{-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3}-\frac{18\sqrt{15}\,d\,e^2\,x}{5\left(-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3\right)}+\frac{18\sqrt{15}\,d^2\,e\,x}{5\left(-\frac{54\,d^3}{5}-18\,d^2\,e+18\,d\,e^2+30\,e^3\right)}\right)(3\,d+5\,e)}{30}-\mathrm{atanh}\left(\frac{18\,d^3\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3}-\frac{30\,e^3\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3}-\frac{30\,d\,e^2\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3}+\frac{18\,d^2\,e\,x}{-18\,d^3-18\,d^2\,e+30\,d\,e^2+30\,e^3}\right)\left(\frac{d}{2}+\frac{e}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(3*x^4 - 8*x^2 + 5),x)
```

```
[Out] (15^(1/2)*atanh((54*15^(1/2)*d^3*x)/(25*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 +
 30*e^3)) - (6*15^(1/2)*e^3*x)/(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)
- (18*15^(1/2)*d*e^2*x)/(5*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)) + (
18*15^(1/2)*d^2*e*x)/(5*(18*d*e^2 - 18*d^2*e - (54*d^3)/5 + 30*e^3)))*(3*d
+ 5*e))/30 - atanh((18*d^3*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3) - (30
*e^3*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 + 30*e^3) - (30*d*e^2*x)/(30*d*e^2 -
18*d^2*e - 18*d^3 + 30*e^3) + (18*d^2*e*x)/(30*d*e^2 - 18*d^2*e - 18*d^3 +
30*e^3))*(d/2 + e/2)
```

3.97 $\int \frac{3+x^2}{1+3x^2+x^4}\,dx$

Optimal. Leaf size=74

$$-\frac{1}{10}\sqrt{180-80\sqrt{5}}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{10}}$$

```
[Out] 1/20*arctan(x*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)-1/10*arctan(x*2
^(1/2)/(3+5^(1/2))^(1/2))*(10-4*5^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$,
Rules used = {1180, 209}

$$\frac{\left(3+\sqrt{5}\right)^{3/2}\text{ArcTan}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{10}}-\frac{1}{10}\sqrt{180-80\sqrt{5}}\,\text{ArcTan}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] -1/10*(Sqrt[180 - 80*Sqrt[5]]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]) + ((3 + Sqrt
[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{3+x^2}{1+3x^2+x^4}\,dx = \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx + \frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx$$

$$= -\frac{1}{5}\sqrt{45-20\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right) + \frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{10}}$$

Mathematica [A]
time = 0.03, size = 73, normalized size = 0.99

$$\frac{-\left(3-\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right)+\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2)/(1 + 3*x^2 + x^4),x]
```

```
[Out] (-((3 - Sqrt[5])^(3/2)*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x]) + (3 + Sqrt[5])^(3/
2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x])/(2*Sqrt[10])
```

Maple [A]
time = 0.04, size = 66, normalized size = 0.89

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+3\_Z^2+1\right)}\frac{\left(\_R^2+3\right)\ln\left(x-\_R\right)}{2\_R^3+3\_R}\right)}{2}$	40
default	$\frac{2\left(\sqrt{5}-3\right)\sqrt{5}\,\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}+\frac{2\sqrt{5}\left(3+\sqrt{5}\right)\arctan\left(\frac{4x}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^4+3*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(5^(1/2)-3)*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x/(2*5^(1/2)+2))+2/5*5^(1/2)
*(3+5^(1/2))/(2*5^(1/2)-2)*arctan(4*x/(2*5^(1/2)-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 3)/(x^4 + 3*x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(44) = 88$.
time = 0.45, size = 124, normalized size = 1.68

$$\frac{2}{5}\sqrt{5}\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{4x^2+2\sqrt{5}+6}\left(\sqrt{5}+3\right)\sqrt{-4\sqrt{5}+9}-\frac{1}{2}\left(\sqrt{5}x+3x\right)\sqrt{-4\sqrt{5}+9}\right)+\frac{2}{5}\sqrt{5}\sqrt{4\sqrt{5}+9}\arctan\left(-\frac{1}{4}\left(2\sqrt{5}x-\sqrt{4x^2-2\sqrt{5}+6}\left(\sqrt{5}-3\right)-6x\right)\sqrt{4\sqrt{5}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(-4*sqrt(5) + 9)*arctan(1/4*sqrt(4*x^2 + 2*sqrt(5) + 6)*(sq
rt(5) + 3)*sqrt(-4*sqrt(5) + 9) - 1/2*(sqrt(5)*x + 3*x)*sqrt(-4*sqrt(5) + 9
)) + 2/5*sqrt(5)*sqrt(4*sqrt(5) + 9)*arctan(-1/4*(2*sqrt(5)*x - sqrt(4*x^2
- 2*sqrt(5) + 6)*(sqrt(5) - 3) - 6*x)*sqrt(4*sqrt(5) + 9))
```

Sympy [A]
time = 0.08, size = 46, normalized size = 0.62

$$2\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right)\operatorname{atan}\left(\frac{2x}{-1+\sqrt{5}}\right)-2\cdot\left(\frac{1}{2}-\frac{\sqrt{5}}{5}\right)\operatorname{atan}\left(\frac{2x}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)/(x**4+3*x**2+1),x)
```

```
[Out] 2*(sqrt(5)/5 + 1/2)*atan(2*x/(-1 + sqrt(5))) - 2*(1/2 - sqrt(5)/5)*atan(2*x
/(1 + sqrt(5)))
```

Giac [A]
time = 7.92, size = 41, normalized size = 0.55

$$\frac{1}{5}\left(2\sqrt{5}-5\right)\arctan\left(\frac{2x}{\sqrt{5}+1}\right)+\frac{1}{5}\left(2\sqrt{5}+5\right)\arctan\left(\frac{2x}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^4+3*x^2+1),x, algorithm="giac")
```

```
[Out] 1/5*(2*sqrt(5) - 5)*arctan(2*x/(sqrt(5) + 1)) + 1/5*(2*sqrt(5) + 5)*arctan(
2*x/(sqrt(5) - 1))
```

Mupad [B]
time = 0.11, size = 117, normalized size = 1.58

$$2\,\mathrm{atanh}\left(\frac{80x\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}-56}-\frac{48\sqrt{5}\,x\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}-56}\right)\sqrt{\frac{\sqrt{5}}{5}-\frac{9}{20}}-2\,\mathrm{atanh}\left(\frac{80x\sqrt{-\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}+56}+\frac{48\sqrt{5}\,x\sqrt{-\frac{\sqrt{5}}{5}-\frac{9}{20}}}{24\sqrt{5}+56}\right)\sqrt{-\frac{\sqrt{5}}{5}-\frac{9}{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3)/(3*x^2 + x^4 + 1),x)
```

```
[Out] 2*atanh((80*x*(5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) - 56) - (48*5^(1/2)*x*(
5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) - 56))*(5^(1/2)/5 - 9/20)^(1/2) - 2*at
anh((80*x*(- 5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) + 56) + (48*5^(1/2)*x*(-
5^(1/2)/5 - 9/20)^(1/2))/(24*5^(1/2) + 56))*(- 5^(1/2)/5 - 9/20)^(1/2)
```

3.98 $\int \frac{a+bx^2}{1+x^2+x^4}\,dx$

Optimal. Leaf size=83

$$-\frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}(a-b)\log\left(1-x+x^2\right)+\frac{1}{4}(a-b)\log\left(1+x+x^2\right)$$

```
[Out] -1/4*(a-b)*ln(x^2-x+1)+1/4*(a-b)*ln(x^2+x+1)-1/6*(a+b)*arctan(1/3*(1-2*x)*3
^(1/2))*3^(1/2)+1/6*(a+b)*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1183, 648, 632, 210, 642}

$$-\frac{(a+b)\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] -1/2*((a + b)*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3] + ((a + b)*ArcTan[(1 + 2*x
)/Sqrt[3]])/(2*Sqrt[3]) - ((a - b)*Log[1 - x + x^2])/4 + ((a - b)*Log[1 + x
 + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{1+x^2+x^4}\,dx &= \frac{1}{2}\int \frac{a-(a-b)x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{a+(a-b)x}{1+x+x^2}\,dx\\ &= \frac{1}{4}(a-b)\int \frac{1+2x}{1+x+x^2}\,dx+\frac{1}{4}(-a+b)\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}(a+b)\int \frac{1}{1-x+x^2}\,dx\\ &= -\frac{1}{4}(a-b)\log\left(1-x+x^2\right)+\frac{1}{4}(a-b)\log\left(1+x+x^2\right)+\frac{1}{2}(-a-b)\mathrm{Subst}\left(\int \frac{1}{-3-x}\right.\\ &= -\frac{(a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{(a+b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}(a-b)\log\left(1-x+x^2\right)+\frac{1}{4}(a-\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 97, normalized size = 1.17

$$\frac{\left(2ia+\left(-i+\sqrt{3}\right)b\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{\sqrt{6+6i\sqrt{3}}}+\frac{\left(-2ia+\left(i+\sqrt{3}\right)b\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{\sqrt{6-6i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4),x]
```

```
[Out] (((2*I)*a + (-I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/Sqrt[6 + (6*I)*
Sqrt[3]] + (((-2*I)*a + (I + Sqrt[3])*b)*ArcTan[((I + Sqrt[3])*x)/2])/Sqrt[
6 - (6*I)*Sqrt[3]]
```

Maple [A]
time = 0.06, size = 78, normalized size = 0.94

method	result
default	$\frac{(a-b)\ln(x^2+x+1)}{4}+\frac{\left(\frac{a}{2}+\frac{b}{2}\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{(-a+b)\ln(x^2-x+1)}{4}+\frac{\left(\frac{a}{2}+\frac{b}{2}\right)\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{3}$
risch	$-\frac{a\ln(4a^2x^2-4ab\,x^2+4b^2x^2-4a^2x+4abx-4b^2x+4a^2-4ab+4b^2)}{4}+\frac{b\ln(4a^2x^2-4ab\,x^2+4b^2x^2-4a^2x+4abx-4b^2x+4a^2-4ab+4b^2)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(a-b)*ln(x^2+x+1)+1/3*(1/2*a+1/2*b)*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)
+1/4*(-a+b)*ln(x^2-x+1)+1/3*(1/2*a+1/2*b)*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2
))
```

Maxima [A]
time = 0.49, size = 69, normalized size = 0.83

$$\frac{1}{6}\sqrt{3}\,(a+b)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{1}{6}\sqrt{3}\,(a+b)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

Fricas [A]
time = 0.36, size = 69, normalized size = 0.83

$$\frac{1}{6}\sqrt{3}\,(a+b)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{1}{6}\sqrt{3}\,(a+b)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{4}(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}(a-b)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.67, size = 740, normalized size = 8.92

$$\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)\log\left(x+\frac{2a^3\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)+6a^2b\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)-12ab^2\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)+24a\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)^3+2b^3\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)-48b\left(-\frac{a}{4}+\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)^3}{a^4-a^3b+ab^3-b^4}\right)+\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)\log\left(x+\frac{2a^3\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)+6a^2b\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)-12ab^2\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)+24a\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)^3+2b^3\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)-48b\left(-\frac{a}{4}+\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)^3}{a^4-a^3b+ab^3-b^4}\right)+\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)\log\left(x+\frac{2a^3\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)+6a^2b\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)-12ab^2\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)+24a\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)^3+2b^3\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)-48b\left(\frac{a}{4}-\frac{b}{4}-\frac{\sqrt{3}\,i(a+b)}{12}\right)^3}{a^4-a^3b+ab^3-b^4}\right)+\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)\log\left(x+\frac{2a^3\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)+6a^2b\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)-12ab^2\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)+24a\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)^3+2b^3\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)-48b\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,i(a+b)}{12}\right)^3}{a^4-a^3b+ab^3-b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+1),x)
```

```
[Out] (-a/4 + b/4 - sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(-a/4 + b/4 - sqrt(3)*I
*(a + b)/12) + 6*a**2*b*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12) - 12*a*b**2*(-a
/4 + b/4 - sqrt(3)*I*(a + b)/12) + 24*a*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12)
**3 + 2*b**3*(-a/4 + b/4 - sqrt(3)*I*(a + b)/12) - 48*b*(-a/4 + b/4 - sqrt(
3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4)) + (-a/4 + b/4 + sqrt(
3)*I*(a + b)/12)*log(x + (2*a**3*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) + 6*a*
*2*b*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12) - 12*a*b**2*(-a/4 + b/4 + sqrt(3)*
I*(a + b)/12) + 24*a*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(-a/4
+ b/4 + sqrt(3)*I*(a + b)/12) - 48*b*(-a/4 + b/4 + sqrt(3)*I*(a + b)/12)**3
)/(a**4 - a**3*b + a*b**3 - b**4)) + (a/4 - b/4 - sqrt(3)*I*(a + b)/12)*log
(x + (2*a**3*(a/4 - b/4 - sqrt(3)*I*(a + b)/12) + 6*a**2*b*(a/4 - b/4 - sqr
t(3)*I*(a + b)/12) - 12*a*b**2*(a/4 - b/4 - sqrt(3)*I*(a + b)/12) + 24*a*(a
/4 - b/4 - sqrt(3)*I*(a + b)/12)**3 + 2*b**3*(a/4 - b/4 - sqrt(3)*I*(a + b)
/12) - 48*b*(a/4 - b/4 - sqrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3
- b**4)) + (a/4 - b/4 + sqrt(3)*I*(a + b)/12)*log(x + (2*a**3*(a/4 - b/4 +
sqrt(3)*I*(a + b)/12) + 6*a**2*b*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) - 12*a*
b**2*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) + 24*a*(a/4 - b/4 + sqrt(3)*I*(a +
b)/12)**3 + 2*b**3*(a/4 - b/4 + sqrt(3)*I*(a + b)/12) - 48*b*(a/4 - b/4 + s
qrt(3)*I*(a + b)/12)**3)/(a**4 - a**3*b + a*b**3 - b**4))
```

Giac [A]
time = 6.42, size = 69, normalized size = 0.83

$$\frac{1}{6}\,\sqrt{3}\,(a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\,\sqrt{3}\,(a+b)\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{4}\,(a-b)\log\left(x^2+x+1\right)-\frac{1}{4}\,(a-b)\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*(a + b)*arc
tan(1/3*sqrt(3)*(2*x - 1)) + 1/4*(a - b)*log(x^2 + x + 1) - 1/4*(a - b)*log
(x^2 - x + 1)
```

Mupad [B]
time = 4.50, size = 827, normalized size = 9.96

$$-\mathrm{atan}\left(\frac{\left(x\,\left(-4\,a^2+4\,a\,b+2\,b^2\right)+\left(12\,a+24\,x\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\,1i+\left(x\,\left(-4\,a^2+4\,a\,b+2\,b^2\right)-\left(12\,a-24\,x\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\,1i}{\left(x\,\left(-4\,a^2+4\,a\,b+2\,b^2\right)+\left(12\,a+24\,x\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)-\left(x\,\left(-4\,a^2+4\,a\,b+2\,b^2\right)-\left(12\,a-24\,x\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{b}{4}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)-2\,a\,b^2+2\,a^2\,b+2\,b^3}\right)\,\left(\frac{\sqrt{3}\,a}{6}+\frac{\sqrt{3}\,b}{6}+\frac{a\,1i}{2}-\frac{b\,1i}{2}\right)-\mathrm{atan}\left(\frac{\left(x\,\left(-4\,a^2+4\,a\,b+2\,b^2\right)+\left(12\,a+24\,x\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\,1i+\left(x\,\left(-4\,a^2+4\,a\,b+2\,b^2\right)-\left(12\,a-24\,x\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\,1i}{\left(x\,\left(-4\,a^2+4\,a\,b+2\,b^2\right)+\left(12\,a+24\,x\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)-\left(x\,\left(-4\,a^2+4\,a\,b+2\,b^2\right)-\left(12\,a-24\,x\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)\right)\,\left(\frac{a}{4}-\frac{b}{4}+\frac{\sqrt{3}\,a\,1i}{12}+\frac{\sqrt{3}\,b\,1i}{12}\right)-2\,a\,b^2+2\,a^2\,b+2\,b^3}\right)\,\left(\frac{\sqrt{3}\,a}{6}+\frac{\sqrt{3}\,b}{6}-\frac{a\,1i}{2}+\frac{b\,1i}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2 + x^4 + 1),x)
```

```
[Out] - atan(((x*(4*a*b - 4*a^2 + 2*b^2) + (12*a + 24*x*(b/4 - a/4 + (3^(1/2)*a*1
i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)
/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i + (x*(4*a*b -
4*a^2 + 2*b^2) - (12*a - 24*x*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1
```

```
i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (
3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i)/((x*(4*a*b - 4*a^2 + 2*b^2) + (12
*a + 24*x*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 +
 (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (
3^(1/2)*b*1i)/12) - (x*(4*a*b - 4*a^2 + 2*b^2) - (12*a - 24*x*(b/4 - a/4 +
(3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3
^(1/2)*b*1i)/12))*(b/4 - a/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12) - 2*a
*b^2 + 2*a^2*b + 2*b^3))*((a*1i)/2 - (b*1i)/2 + (3^(1/2)*a)/6 + (3^(1/2)*b)
/6) - atan(((x*(4*a*b - 4*a^2 + 2*b^2) + (12*a + 24*x*(a/4 - b/4 + (3^(1/2)
*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b
*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i + (x*(4*a*
b - 4*a^2 + 2*b^2) - (12*a - 24*x*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)
*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4
 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12)*1i)/((x*(4*a*b - 4*a^2 + 2*b^2) +
 (12*a + 24*x*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b
/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12
 + (3^(1/2)*b*1i)/12) - (x*(4*a*b - 4*a^2 + 2*b^2) - (12*a - 24*x*(a/4 - b/
4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12
+ (3^(1/2)*b*1i)/12))*(a/4 - b/4 + (3^(1/2)*a*1i)/12 + (3^(1/2)*b*1i)/12) -
 2*a*b^2 + 2*a^2*b + 2*b^3))*((b*1i)/2 - (a*1i)/2 + (3^(1/2)*a)/6 + (3^(1/2
)*b)/6)
```

3.99 $\int \frac{a+bx^2}{\left(1+x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=119

$$\frac{x(a+b-(a-2b)x^2)}{6\left(1+x^2+x^4\right)}-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}}-\frac{1}{8}(2a-b)\log\left(1-x+x^2\right)+\frac{1}{8}(2a-$$

```
[Out] 1/6*x*(a+b-(a-2*b)*x^2)/(x^4+x^2+1)-1/8*(2*a-b)*ln(x^2-x+1)+1/8*(2*a-b)*ln(
x^2+x+1)-1/36*(4*a+b)*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/36*(4*a+b)*arct
an(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1192, 1183, 648, 632, 210, 642}

$$-\frac{(4a+b)\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{12\sqrt{3}}-\frac{1}{8}(2a-b)\log\left(x^2-x+1\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)+\frac{x\left(-\left(x^2(a-2b)\right)+a+b\right)}{6\left(x^4+x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(a + b - (a - 2*b)*x^2))/(6*(1 + x^2 + x^4)) - ((4*a + b)*ArcTan[(1 - 2*
x)/Sqrt[3]])/(12*Sqrt[3]) + ((4*a + b)*ArcTan[(1 + 2*x)/Sqrt[3]])/(12*Sqrt[
3]) - ((2*a - b)*Log[1 - x + x^2])/8 + ((2*a - b)*Log[1 + x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{(1+x^2+x^4)^2}\,dx &= \frac{x(a+b-(a-2b)x^2)}{6(1+x^2+x^4)}+\frac{1}{6}\int \frac{5a-b+(-a+2b)x^2}{1+x^2+x^4}\,dx\\ &= \frac{x(a+b-(a-2b)x^2)}{6(1+x^2+x^4)}+\frac{1}{12}\int \frac{5a-b-(6a-3b)x}{1-x+x^2}\,dx+\frac{1}{12}\int \frac{5a-b+(6a-3b)x}{1+x+x^2}\,d\\ &= \frac{x(a+b-(a-2b)x^2)}{6(1+x^2+x^4)}+\frac{1}{8}(2a-b)\int \frac{1+2x}{1+x+x^2}\,dx+\frac{1}{8}(-2a+b)\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= \frac{x(a+b-(a-2b)x^2)}{6(1+x^2+x^4)}-\frac{1}{8}(2a-b)\log\left(1-x+x^2\right)+\frac{1}{8}(2a-b)\log\left(1+x+x^2\right)+\\ &= \frac{x(a+b-(a-2b)x^2)}{6(1+x^2+x^4)}-\frac{(4a+b)\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{12\sqrt{3}}+\frac{(4a+b)\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{12\sqrt{3}}-\frac{1}{8}(2a-\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 147, normalized size = 1.24

$$\frac{x(a+b-ax^2+2bx^2)}{6(1+x^2+x^4)}-\frac{\left(\left(-11i+\sqrt{3}\right)a-2\left(-2i+\sqrt{3}\right)b\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x\right)}{6\sqrt{6+6i\sqrt{3}}}-\frac{\left(\left(11i+\sqrt{3}\right)a-2\left(2i+\sqrt{3}\right)b\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x\right)}{6\sqrt{6-6i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(1 + x^2 + x^4)^2,x]
```

```
[Out] (x*(a + b - a*x^2 + 2*b*x^2))/(6*(1 + x^2 + x^4)) - (((-11*I + Sqrt[3])*a -
 2*(-2*I + Sqrt[3])*b)*ArcTan[((-I + Sqrt[3])*x)/2])/(6*Sqrt[6 + (6*I)*Sqrt
[3]]) - (((11*I + Sqrt[3])*a - 2*(2*I + Sqrt[3])*b)*ArcTan[((I + Sqrt[3])*x
)/2])/(6*Sqrt[6 - (6*I)*Sqrt[3]])
```

Maple [A]
time = 0.07, size = 136, normalized size = 1.14

method	result
default	$\frac{\left(-\frac{a}{3}+\frac{2b}{3}\right)x-\frac{2a}{3}+\frac{b}{3}}{4x^2+4x+4}+\frac{(6a-3b)\ln\left(x^2+x+1\right)}{24}+\frac{\left(2a+\frac{b}{2}\right)\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{18}-\frac{\left(\frac{a}{3}-\frac{2b}{3}\right)x-\frac{2a}{3}+\frac{b}{3}}{4(x^2-x+1)}-\frac{(6a-3b)\ln\left(x^2-\right.}{24}$
risch	$\frac{\sqrt{3}\,b\arctan\left(\frac{62\sqrt{3}\,a^2x}{3\left(31a^2-25ab+7b^2\right)}-\frac{50\sqrt{3}\,axb}{3\left(31a^2-25ab+7b^2\right)}+\frac{14\sqrt{3}\,b^2x}{3\left(31a^2-25ab+7b^2\right)}-\frac{31\sqrt{3}\,a^2}{3\left(31a^2-25ab+7b^2\right)}+\frac{25\sqrt{3}\,ab}{3\left(31a^2-25ab+7b^2\right)}-\frac{7\sqrt{3}\,b^2}{3\left(31a^2-25ab+\right.}\right.}{36}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((-1/3*a+2/3*b)*x-2/3*a+1/3*b)/(x^2+x+1)+1/24*(6*a-3*b)*ln(x^2+x+1)+1/1
8*(2*a+1/2*b)*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/4*((1/3*a-2/3*b)*x-2/3*
a+1/3*b)/(x^2-x+1)-1/24*(6*a-3*b)*ln(x^2-x+1)-1/18*(-2*a-1/2*b)*3^(1/2)*arc
tan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 105, normalized size = 0.88

$$\frac{1}{36}\sqrt{3}\,(4\,a+b)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{36}\sqrt{3}\,(4\,a+b)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{8}\,(2\,a-b)\log\left(x^2+x+1\right)-\frac{1}{8}\,(2\,a-b)\log\left(x^2-x+1\right)-\frac{(a-2\,b)x^3-(a+b)x}{6\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(4*a +
b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^2 + x + 1) - 1/8*(2*
a - b)*log(x^2 - x + 1) - 1/6*((a - 2*b)*x^3 - (a + b)*x)/(x^4 + x^2 + 1)
```

Fricas [A]
time = 0.36, size = 185, normalized size = 1.55

$$-\frac{12\,(a-2\,b)x^3-2\sqrt{3}\,((4\,a+b)x^4+(4\,a+b)x^2+4\,a+b)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)-2\sqrt{3}\,((4\,a+b)x^4+(4\,a+b)x^2+4\,a+b)\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-12\,(a+b)x-9\,((2\,a-b)x^4+(2\,a-b)x^2+2\,a-b)\log\left(x^2+x+1\right)+9\,((2\,a-b)x^4+(2\,a-b)x^2+2\,a-b)\log\left(x^2-x+1\right)}{72\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/72*(12*(a - 2*b)*x^3 - 2*sqrt(3)*((4*a + b)*x^4 + (4*a + b)*x^2 + 4*a +
b)*arctan(1/3*sqrt(3)*(2*x + 1)) - 2*sqrt(3)*((4*a + b)*x^4 + (4*a + b)*x^2
 + 4*a + b)*arctan(1/3*sqrt(3)*(2*x - 1)) - 12*(a + b)*x - 9*((2*a - b)*x^4
 + (2*a - b)*x^2 + 2*a - b)*log(x^2 + x + 1) + 9*((2*a - b)*x^4 + (2*a - b)
*x^2 + 2*a - b)*log(x^2 - x + 1))/(x^4 + x^2 + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.99, size = 874, normalized size = 7.34

$$\frac{x^3(-a+2b)+x(a+b)}{6x^4+6x^2+6}+\left(-\frac{a}{4}+\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)\log\left(x+\frac{76a^3\left(-\frac{a}{4}+\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)+948a^2b\left(-\frac{a}{4}+\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)-816ab^2\left(-\frac{a}{4}+\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)+12096a\left(-\frac{a}{4}+\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)^3+148b^3\left(-\frac{a}{4}+\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)-8640b\left(-\frac{a}{4}+\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)^3}{248a^4-262a^3b+75a^2b^2+11ab^3-7b^4}\right)+\left(-\frac{a}{4}+\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)\log\left(x+\frac{76a^3\left(-\frac{a}{4}+\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)+948a^2b\left(-\frac{a}{4}+\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)-816ab^2\left(-\frac{a}{4}+\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)+12096a\left(-\frac{a}{4}+\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)^3+148b^3\left(-\frac{a}{4}+\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)-8640b\left(-\frac{a}{4}+\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)^3}{248a^4-262a^3b+75a^2b^2+11ab^3-7b^4}\right)+\left(\frac{a}{4}-\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)\log\left(x+\frac{76a^3\left(\frac{a}{4}-\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)+948a^2b\left(\frac{a}{4}-\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)-816ab^2\left(\frac{a}{4}-\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)+12096a\left(\frac{a}{4}-\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)^3+148b^3\left(\frac{a}{4}-\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)-8640b\left(\frac{a}{4}-\frac{b}{8}-\frac{\sqrt{3}\,i(4a+b)}{72}\right)^3}{248a^4-262a^3b+75a^2b^2+11ab^3-7b^4}\right)+\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)\log\left(x+\frac{76a^3\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)+948a^2b\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)-816ab^2\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)+12096a\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)^3+148b^3\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)-8640b\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,i(4a+b)}{72}\right)^3}{248a^4-262a^3b+75a^2b^2+11ab^3-7b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+1)**2,x)
```

```
[Out] (x**3*(-a + 2*b) + x*(a + b))/(6*x**4 + 6*x**2 + 6) + (-a/4 + b/8 - sqrt(3)
*I*(4*a + b)/72)*log(x + (76*a**3*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) + 9
48*a**2*b*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(-a/4 + b/8 -
sqrt(3)*I*(4*a + b)/72) + 12096*a*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72)**3
+ 148*b**3*(-a/4 + b/8 - sqrt(3)*I*(4*a + b)/72) - 8640*b*(-a/4 + b/8 - sqr
t(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a*b**3
- 7*b**4)) + (-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*(-a/4 +
 b/8 + sqrt(3)*I*(4*a + b)/72) + 948*a**2*b*(-a/4 + b/8 + sqrt(3)*I*(4*a +
b)/72) - 816*a*b**2*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72) + 12096*a*(-a/4 +
 b/8 + sqrt(3)*I*(4*a + b)/72)**3 + 148*b**3*(-a/4 + b/8 + sqrt(3)*I*(4*a +
 b)/72) - 8640*b*(-a/4 + b/8 + sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*
a**3*b + 75*a**2*b**2 + 11*a*b**3 - 7*b**4)) + (a/4 - b/8 - sqrt(3)*I*(4*a
+ b)/72)*log(x + (76*a**3*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) + 948*a**2*b
*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 816*a*b**2*(a/4 - b/8 - sqrt(3)*I*(
4*a + b)/72) + 12096*a*(a/4 - b/8 - sqrt(3)*I*(4*a + b)/72)**3 + 148*b**3*(
a/4 - b/8 - sqrt(3)*I*(4*a + b)/72) - 8640*b*(a/4 - b/8 - sqrt(3)*I*(4*a +
b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2 + 11*a*b**3 - 7*b**4)) + (
a/4 - b/8 + sqrt(3)*I*(4*a + b)/72)*log(x + (76*a**3*(a/4 - b/8 + sqrt(3)*I
*(4*a + b)/72) + 948*a**2*b*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) - 816*a*b*
*2*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) + 12096*a*(a/4 - b/8 + sqrt(3)*I*(4
*a + b)/72)**3 + 148*b**3*(a/4 - b/8 + sqrt(3)*I*(4*a + b)/72) - 8640*b*(a/
4 - b/8 + sqrt(3)*I*(4*a + b)/72)**3)/(248*a**4 - 262*a**3*b + 75*a**2*b**2
 + 11*a*b**3 - 7*b**4))
```

Giac [A]
time = 7.11, size = 109, normalized size = 0.92

$$\frac{1}{36}\sqrt{3}\,(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)+\frac{1}{36}\sqrt{3}\,(4a+b)\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{8}(2a-b)\log\left(x^2+x+1\right)-\frac{1}{8}(2a-b)\log\left(x^2-x+1\right)-\frac{ax^3-2bx^3-ax-bx}{6\,(x^4+x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(4*a + b)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/36*sqrt(3)*(4*a +
b)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*(2*a - b)*log(x^2 + x + 1) - 1/8*(2*
a - b)*log(x^2 - x + 1) - 1/6*(a*x^3 - 2*b*x^3 - a*x - b*x)/(x^4 + x^2 + 1)
```

Mupad [B]
time = 4.49, size = 897, normalized size = 7.54

$$\mathrm{atan}\left(\frac{\left(\left(2b-10a+24x\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)\right)\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)-x\left(\frac{59a^2}{18}-\frac{19ab}{9}+\frac{b^2}{9}\right)\right)\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)1i+\left(\left(10a-2b+24x\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)\right)\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)-x\left(\frac{59a^2}{18}-\frac{19ab}{9}+\frac{b^2}{9}\right)\right)\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)1i}{\frac{19ab^2}{36}-\frac{29a^2b}{36}+\frac{31a^3}{108}-\frac{7b^3}{54}+\left(\left(2b-10a+24x\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)\right)\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)-x\left(\frac{59a^2}{18}-\frac{19ab}{9}+\frac{b^2}{9}\right)\right)\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)-\left(\left(10a-2b+24x\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)\right)\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)-x\left(\frac{59a^2}{18}-\frac{19ab}{9}+\frac{b^2}{9}\right)\right)\left(\frac{b}{8}-\frac{a}{4}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)}\right)\left(\frac{\sqrt{3}\,a}{9}+\frac{\sqrt{3}\,b}{36}+\frac{a\,1i}{2}-\frac{b\,1i}{4}\right)+\mathrm{atan}\left(\frac{\left(\left(2b-10a+24x\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)\right)\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)-x\left(\frac{59a^2}{18}-\frac{19ab}{9}+\frac{b^2}{9}\right)\right)\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)1i+\left(\left(10a-2b+24x\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)\right)\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)-x\left(\frac{59a^2}{18}-\frac{19ab}{9}+\frac{b^2}{9}\right)\right)\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)1i}{\frac{19ab^2}{36}-\frac{29a^2b}{36}+\frac{31a^3}{108}-\frac{7b^3}{54}+\left(\left(2b-10a+24x\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)\right)\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)-x\left(\frac{59a^2}{18}-\frac{19ab}{9}+\frac{b^2}{9}\right)\right)\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)-\left(\left(10a-2b+24x\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)\right)\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)-x\left(\frac{59a^2}{18}-\frac{19ab}{9}+\frac{b^2}{9}\right)\right)\left(\frac{a}{4}-\frac{b}{8}+\frac{\sqrt{3}\,a\,1i}{18}+\frac{\sqrt{3}\,b\,1i}{72}\right)}\right)\left(\frac{\sqrt{3}\,a}{9}+\frac{\sqrt{3}\,b}{36}-\frac{a\,1i}{2}+\frac{b\,1i}{4}\right)-\frac{x^3\left(\frac{a}{6}-\frac{b}{3}\right)-x\left(\frac{a}{6}+\frac{b}{6}\right)}{x^4+x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2 + x^4 + 1)^2,x)
```

```
[Out] atan((((2*b - 10*a + 24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/7
2))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 -
(19*a*b)/9 + b^2/9))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)*1i
 + ((10*a - 2*b + 24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))
*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19
*a*b)/9 + b^2/9))*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)*1i)/(
(19*a*b^2)/36 - (29*a^2*b)/36 + (31*a^3)/108 - (7*b^3)/54 + ((2*b - 10*a +
24*x*(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(b/8 - a/4 + (3^(
1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*
(b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - ((10*a - 2*b + 24*x*(
b/8 - a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(b/8 - a/4 + (3^(1/2)*a
*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*(b/8 -
 a/4 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)))*((a*1i)/2 - (b*1i)/4 + (3^(
1/2)*a)/9 + (3^(1/2)*b)/36) + atan((((2*b - 10*a + 24*x*(a/4 - b/8 + (3^(1/
2)*a*1i)/18 + (3^(1/2)*b*1i)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)
*b*1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*
1i)/18 + (3^(1/2)*b*1i)/72)*1i + ((10*a - 2*b + 24*x*(a/4 - b/8 + (3^(1/2)*
a*1i)/18 + (3^(1/2)*b*1i)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*
1i)/72) - x*((59*a^2)/18 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*1i)
/18 + (3^(1/2)*b*1i)/72)*1i)/((19*a*b^2)/36 - (29*a^2*b)/36 + (31*a^3)/108
- (7*b^3)/54 + ((2*b - 10*a + 24*x*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2
)*b*1i)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a
^2)/18 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1
i)/72) - ((10*a - 2*b + 24*x*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i
)/72))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72) - x*((59*a^2)/18
 - (19*a*b)/9 + b^2/9))*(a/4 - b/8 + (3^(1/2)*a*1i)/18 + (3^(1/2)*b*1i)/72)
))*((b*1i)/4 - (a*1i)/2 + (3^(1/2)*a)/9 + (3^(1/2)*b)/36) - (x^3*(a/6 - b/3
) - x*(a/6 + b/6))/(x^2 + x^4 + 1)
```

3.100 $\int \frac{a+bx^2}{2+x^2+x^4} \, dx$

Optimal. Leaf size=234

$$-\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(a+\sqrt{2}\,b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(a+\sqrt{2}\,b\right)\tan$$

```
[Out] -1/28*arctan((-2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(a+b*2^(1/2))
*(-14+28*2^(1/2))^(1/2)+1/28*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2)
)^(1/2))*(a+b*2^(1/2))*(-14+28*2^(1/2))^(1/2)-1/4*ln(x^2+2^(1/2)-x*(-1+2*2^
(1/2))^(1/2))*(a-b*2^(1/2))/(-2+4*2^(1/2))^(1/2)+1/4*ln(x^2+2^(1/2)+x*(-1+2
*2^(1/2))^(1/2))*(a-b*2^(1/2))/(-2+4*2^(1/2))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1183, 648, 632, 210, 642}

$$-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{2}\,b\right)\text{ArcTan}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(a+\sqrt{2}\,b\right)\text{ArcTan}\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)-\frac{\left(a-\sqrt{2}\,b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(a-\sqrt{2}\,b\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}\,x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(2 + x^2 + x^4),x]
```

```
[Out] -1/2*(Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]
] - 2*x)/Sqrt[1 + 2*Sqrt[2]]]) + (Sqrt[(-1 + 2*Sqrt[2])/14]*(a + Sqrt[2]*b)
*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqrt[2]]])/2 - ((a - Sqrt[2
]*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt[2*(-1 + 2*Sqrt[2]
)]) + ((a - Sqrt[2]*b)*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(4*Sqrt
[2*(-1 + 2*Sqrt[2])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{a+bx^2}{2+x^2+x^4}\,dx = \frac{\int \frac{\sqrt{-1+2\sqrt{2}}\,a-\left(a-\sqrt{2}\,b\right)x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\int \frac{\sqrt{-1+2\sqrt{2}}\,a+\left(a-\sqrt{2}\,b\right)x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

$$=\frac{1}{8}\left(\sqrt{2}\,a+2b\right)\int\frac{1}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx+\frac{1}{8}\left(\sqrt{2}\,a+2b\right)\int\frac{1}{\sqrt{2}+\sqrt{-1+\cdots}}$$

$$=-\frac{\left(a-\sqrt{2}\,b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\left(a-\sqrt{2}\,b\right)\log\left(\sqrt{2}+\sqrt{-1+\cdots}\right.}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

$$=-\frac{\left(a+\sqrt{2}\,b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\left(a+\sqrt{2}\,b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 111, normalized size = 0.47

$$\frac{\left(-2ia+\left(i+\sqrt{7}\right)b\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{14-14i\sqrt{7}}}+\frac{\left(2ia+\left(-i+\sqrt{7}\right)b\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{14+14i\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(2 + x^2 + x^4),x]
```

```
[Out] (((-2*I)*a + (I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[14 -
(14*I)*Sqrt[7]] + (((2*I)*a + (-I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 + I*Sqrt[7
])/2]])/Sqrt[14 + (14*I)*Sqrt[7]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(162) = 324.
time = 0.09, size = 371, normalized size = 1.59

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2+2\right)}\frac{\left(\_R^2 b+a\right)\ln\left(x-\_R\right)}{2\_R^3+\_R}\right)}{2}$
default	$\frac{\left(-\sqrt{2}\,\sqrt{-1+2\sqrt{2}}\,a+4\sqrt{2}\,\sqrt{-1+2\sqrt{2}}\,b-4\sqrt{-1+2\sqrt{2}}\,a+2\sqrt{-1+2\sqrt{2}}\,b\right)\ln\left(x^2+\sqrt{2}-x\sqrt{}\right.}{56}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/56*(-2^(1/2)*(-1+2*2^(1/2))^(1/2)*a+4*2^(1/2)*(-1+2*2^(1/2))^(1/2)*b-4*(-
1+2*2^(1/2))^(1/2)*a+2*(-1+2*2^(1/2))^(1/2)*b)*ln(x^2+2^(1/2)-x*(-1+2*2^(1/
2))^(1/2))+1/14*(7*2^(1/2)*a+1/2*(-2^(1/2)*(-1+2*2^(1/2))^(1/2)*a+4*2^(1/2)
*(-1+2*2^(1/2))^(1/2)*b-4*(-1+2*2^(1/2))^(1/2)*a+2*(-1+2*2^(1/2))^(1/2)*b)*
(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))
/(1+2*2^(1/2))^(1/2))+1/56*(2^(1/2)*(-1+2*2^(1/2))^(1/2)*a-4*2^(1/2)*(-1+2*
2^(1/2))^(1/2)*b+4*(-1+2*2^(1/2))^(1/2)*a-2*(-1+2*2^(1/2))^(1/2)*b)*ln(x^2+
2^(1/2)+x*(-1+2*2^(1/2))^(1/2))+1/14*(7*2^(1/2)*a-1/2*(2^(1/2)*(-1+2*2^(1/2
))^(1/2)*a-4*2^(1/2)*(-1+2*2^(1/2))^(1/2)*b+4*(-1+2*2^(1/2))^(1/2)*a-2*(-1+
2*2^(1/2))^(1/2)*b)*(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2)*arctan((2*x+(
-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(x^4 + x^2 + 2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4122 vs. $2(167) = 334$.
time = 0.58, size = 4122, normalized size = 17.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="fricas")
```

```
[Out] 1/112*(28*sqrt(2)*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*
b^4)^(1/4)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a
^2*b^2 + 4*b^4)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sq
rt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^
2))/(a^4 - 4*a^2*b^2 + 4*b^4))*arctan(1/56*(7*sqrt(2)*sqrt(1/7)*(8*a^4 - 16
*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(3/4)*sqrt(2*(a^8 - 2*a^7*b + a^6*
b^2 + 4*a^5*b^3 - 12*a^4*b^4 + 8*a^3*b^5 + 4*a^2*b^6 - 16*a*b^7 + 16*b^8)*x
^2 + sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(s
qrt(7)*sqrt(2)*(a^4*b - 4*a^2*b^3 + 4*b^5)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 -
 4*a*b^3 + 4*b^4)*x - sqrt(7)*(a^7 - a^6*b - 2*a^5*b^2 + 4*a^4*b^3 - 4*a^3*
b^4 - 4*a^2*b^5 + 8*a*b^6)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3
 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2
- 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)) + 2*sqrt(2)*(a^6 - a^5*b - 2*a
^4*b^2 + 4*a^3*b^3 - 4*a^2*b^4 - 4*a*b^5 + 8*b^6)*sqrt(a^4 - 2*a^3*b + 5*a^
2*b^2 - 4*a*b^3 + 4*b^4))*(sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3
 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*a - 2*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)
*(a^2*b - a*b^2 + 2*b^3))*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 1
6*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*
a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)) - 16*sqrt(7)*sqrt(2)*(a^8 - 3*a^7*
b + 7*a^6*b^2 - 7*a^5*b^3 + 14*a^3*b^5 - 28*a^2*b^6 + 24*a*b^7 - 16*b^8)*sq
rt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^
4) + 14*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(3/4)
*(sqrt(2)*(a^5 - a^4*b + 2*a^2*b^3 - 4*a*b^4)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^
2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*x - 2*(a^6*b - 2*a^5*b^2
 + 3*a^4*b^3 - 6*a^2*b^5 + 8*a*b^6 - 8*b^7)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*x
)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4
```

```
 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a
^2*b^2 + 4*b^4)) + 8*sqrt(7)*(a^10 - 4*a^9*b + 12*a^8*b^2 - 20*a^7*b^3 + 21
*a^6*b^4 - 42*a^4*b^6 + 80*a^3*b^7 - 96*a^2*b^8 + 64*a*b^9 - 32*b^10)*sqrt(
a^4 - 4*a^2*b^2 + 4*b^4))/(a^12 - 4*a^11*b + 10*a^10*b^2 - 12*a^9*b^3 - 3*a
^8*b^4 + 40*a^7*b^5 - 84*a^6*b^6 + 80*a^5*b^7 - 12*a^4*b^8 - 96*a^3*b^9 + 1
60*a^2*b^10 - 128*a*b^11 + 64*b^12)) + 28*sqrt(2)*sqrt(1/7)*(8*a^4 - 16*a^3
*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 -
 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*sqrt((4*a^4 - 8*a^3*b + 20*
a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*
b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4))*arctan(1/56*
(7*sqrt(2)*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(3
/4)*sqrt(2*(a^8 - 2*a^7*b + a^6*b^2 + 4*a^5*b^3 - 12*a^4*b^4 + 8*a^3*b^5 +
4*a^2*b^6 - 16*a*b^7 + 16*b^8)*x^2 - sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b
^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sqrt(2)*(a^4*b - 4*a^2*b^3 + 4*b^5)*
sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*x - sqrt(7)*(a^7 - a^6*b
- 2*a^5*b^2 + 4*a^4*b^3 - 4*a^3*b^4 - 4*a^2*b^5 + 8*a*b^6)*x)*sqrt((4*a^4 -
 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*
a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)
) + 2*sqrt(2)*(a^6 - a^5*b - 2*a^4*b^2 + 4*a^3*b^3 - 4*a^2*b^4 - 4*a*b^5 +
8*b^6)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*(sqrt(2)*sqrt(a^4
 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*a -
 2*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*(a^2*b - a*b^2 + 2*b^3))*sqrt((4*a^4 - 8*a
^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*
b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)) +
16*sqrt(7)*sqrt(2)*(a^8 - 3*a^7*b + 7*a^6*b^2 - 7*a^5*b^3 + 14*a^3*b^5 - 28
*a^2*b^6 + 24*a*b^7 - 16*b^8)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*
b^4)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4) + 14*sqrt(1/7)*(8*a^4 - 16*a^3*b + 40*a^
2*b^2 - 32*a*b^3 + 32*b^4)^(3/4)*(sqrt(2)*(a^5 - a^4*b + 2*a^2*b^3 - 4*a*b^
4)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*sqrt(a^4 - 4*a^2*b^2 +
 4*b^4)*x - 2*(a^6*b - 2*a^5*b^2 + 3*a^4*b^3 - 6*a^2*b^5 + 8*a*b^6 - 8*b^7)
*sqrt(a^4 - 4*a^2*b^2 + 4*b^4)*x)*sqrt((4*a^4 - 8*a^3*b + 20*a^2*b^2 - 16*a
*b^3 + 16*b^4 - sqrt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(
a^2 - 8*a*b + 2*b^2))/(a^4 - 4*a^2*b^2 + 4*b^4)) - 8*sqrt(7)*(a^10 - 4*a^9*
b + 12*a^8*b^2 - 20*a^7*b^3 + 21*a^6*b^4 - 42*a^4*b^6 + 80*a^3*b^7 - 96*a^2
*b^8 + 64*a*b^9 - 32*b^10)*sqrt(a^4 - 4*a^2*b^2 + 4*b^4))/(a^12 - 4*a^11*b
+ 10*a^10*b^2 - 12*a^9*b^3 - 3*a^8*b^4 + 40*a^7*b^5 - 84*a^6*b^6 + 80*a^5*b
^7 - 12*a^4*b^8 - 96*a^3*b^9 + 160*a^2*b^10 - 128*a*b^11 + 64*b^12)) - sqrt
(1/7)*(8*a^4 - 16*a^3*b + 40*a^2*b^2 - 32*a*b^3 + 32*b^4)^(1/4)*(sqrt(7)*sq
rt(2)*sqrt(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4)*(a^2 - 8*a*b + 2*b^
2) + 4*sqrt(7)*(a^4 - 2*a^3*b + 5*a^2*b^2 - 4*a*b^3 + 4*b^4))*sqrt((4*a^4 -
 8*a^3*b + 20*a^2*b^2 - 16*a*b^3 + 16*b^4 - sqr...
```

Sympy [A]

time = 0.70, size = 122, normalized size = 0.52

$$\mathrm{RootSum}\left(1568t^4 + t^2(-28a^2 + 224ab - 56b^2) + a^4 - 2a^3b + 5a^2b^2 - 4ab^3 + 4b^4, \left(t \mapsto t\log\left(x + \frac{112t^3a - 448t^3b + 6ta^3 + 12ta^2b - 48tab^2 + 8tb^3}{a^4 - a^3b + 2ab^3 - 4b^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+2),x)
```

```
[Out] RootSum(1568*_t**4 + _t**2*(-28*a**2 + 224*a*b - 56*b**2) + a**4 - 2*a**3*b
 + 5*a**2*b**2 - 4*a*b**3 + 4*b**4, Lambda(_t, _t*log(x + (112*_t**3*a - 44
8*_t**3*b + 6*_t*a**3 + 12*_t*a**2*b - 48*_t*a*b**2 + 8*_t*b**3)/(a**4 - a*
*3*b + 2*a*b**3 - 4*b**4))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(167) = 334$.
time = 6.97, size = 604, normalized size = 2.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2),x, algorithm="giac")
```

```
[Out] -1/896*sqrt(7)*(sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*sqr
t(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 3*2^(3/4)*b*(sqrt(2) + 4
)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) - 8*s
qrt(7)*2^(1/4)*a*sqrt(2*sqrt(2) + 8) + 8*2^(1/4)*a*sqrt(-2*sqrt(2) + 8))*ar
ctan(2*2^(3/4)*sqrt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2
) + 4)) - 1/896*sqrt(7)*(sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4
) + 3*sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 3*2^(3/4)*b*(sq
rt(2) + 4)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) +
 8) - 8*sqrt(7)*2^(1/4)*a*sqrt(2*sqrt(2) + 8) + 8*2^(1/4)*a*sqrt(-2*sqrt(2)
 + 8))*arctan(2*2^(3/4)*sqrt(1/2)*(x - 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sq
rt(sqrt(2) + 4)) - 1/1792*sqrt(7)*(3*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-
2*sqrt(2) + 8) + sqrt(7)*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) + 2^(
3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*
(sqrt(2) - 4) - 8*sqrt(7)*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) - 8*2^(1/4)*a*sqrt
(2*sqrt(2) + 8))*log(x^2 + 2*2^(1/4)*x*sqrt(-1/8*sqrt(2) + 1/2) + sqrt(2))
+ 1/1792*sqrt(7)*(3*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) +
sqrt(7)*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) + 2^(3/4)*b*sqrt(2*sqr
t(2) + 8)*(sqrt(2) + 4) + 3*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 8
*sqrt(7)*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) - 8*2^(1/4)*a*sqrt(2*sqrt(2) + 8))*
log(x^2 - 2*2^(1/4)*x*sqrt(-1/8*sqrt(2) + 1/2) + sqrt(2))
```

Mupad [B]
time = 4.49, size = 771, normalized size = 3.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2 + x^4 + 2),x)
```

```
[Out] - atan((a^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^2/
112 + b^2/56)^(1/2)*7i)/((7^(1/2)*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4*b
^3 - 7^(1/2)*a*b^2*1i) - (b^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)
*b^2*1i)/56 + a^2/112 + b^2/56)^(1/2)*14i)/((7^(1/2)*a^3*1i)/2 - a*b^2 - 2*
a^2*b + a^3/2 + 4*b^3 - 7^(1/2)*a*b^2*1i) + (7^(1/2)*a^2*x*((7^(1/2)*a^2*1i
)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^2/112 + b^2/56)^(1/2))/((7^(1/2)
*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4*b^3 - 7^(1/2)*a*b^2*1i) - (2*7^(1/
2)*b^2*x*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^2/112 +
 b^2/56)^(1/2))/((7^(1/2)*a^3*1i)/2 - a*b^2 - 2*a^2*b + a^3/2 + 4*b^3 - 7^(
1/2)*a*b^2*1i))*((7^(1/2)*a^2*1i)/112 - (a*b)/14 - (7^(1/2)*b^2*1i)/56 + a^
2/112 + b^2/56)^(1/2)*2i - 2*atanh((7*a^2*x*((7^(1/2)*b^2*1i)/56 - (7^(1/2)
*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2))/((7^(1/2)*a^3*1i)/2 + a*
b^2 + 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i) - (14*b^2*x*((7^(1/2)*b^2
*1i)/56 - (7^(1/2)*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2))/((7^(1
/2)*a^3*1i)/2 + a*b^2 + 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i) + (7^(1
/2)*a^2*x*((7^(1/2)*b^2*1i)/56 - (7^(1/2)*a^2*1i)/112 - (a*b)/14 + a^2/112
+ b^2/56)^(1/2)*1i)/((7^(1/2)*a^3*1i)/2 + a*b^2 + 2*a^2*b - a^3/2 - 4*b^3 -
 7^(1/2)*a*b^2*1i) - (7^(1/2)*b^2*x*((7^(1/2)*b^2*1i)/56 - (7^(1/2)*a^2*1i)
/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2)*2i)/((7^(1/2)*a^3*1i)/2 + a*b^2 +
 2*a^2*b - a^3/2 - 4*b^3 - 7^(1/2)*a*b^2*1i))*((7^(1/2)*b^2*1i)/56 - (7^(1/
2)*a^2*1i)/112 - (a*b)/14 + a^2/112 + b^2/56)^(1/2)
```

3.101 $\int \frac{a+bx^2}{\left(2+x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=316

$$\frac{x\left(3a+2b-(a-4b)x^2\right)}{28\left(2+x^2+x^4\right)}-\frac{1}{56}\sqrt{\frac{1}{14}\left(-1+2\sqrt{2}\right)}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}}{\sqrt{1+2\sqrt{2}}}\right.$$

```
[Out] 1/28*x*(3*a+2*b-(a-4*b)*x^2)/(x^4+x^2+2)-1/784*arctan((-2*x+(-1+2*2^(1/2))^
(1/2))/(1+2*2^(1/2))^(1/2))*(-b*(2-4*2^(1/2))+a*(11-2^(1/2)))*(-14+28*2^(1/
2))^(1/2)+1/784*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-b*
(2-4*2^(1/2))+a*(11-2^(1/2)))*(-14+28*2^(1/2))^(1/2)-1/112*ln(x^2+2^(1/2)-x
*(-1+2*2^(1/2))^(1/2))*(11*a-2*b+(a-4*b)*2^(1/2))/(-2+4*2^(1/2))^(1/2)+1/11
2*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(a*(11+2^(1/2))-2*b-4*b*2^(1/2))/(
-2+4*2^(1/2))^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1192, 1183, 648, 632, 210, 642}

$$-\frac{1}{56}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\text{ArcTan}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{56}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\text{ArcTan}\left(\frac{2x+\sqrt{2\sqrt{2}-1}}{\sqrt{1+2\sqrt{2}}}\right)-\frac{\left(\sqrt{2}(a-4b)+11a-2b\right)\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\left(\left(11+\sqrt{2}\right)a-2\left(2\sqrt{2}b+b\right)\right)\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{112\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{x\left(-\left(x^2(a-4b)\right)+3a+2b\right)}{28\left(x^4+x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```

```
[Out] (x*(3*a + 2*b - (a - 4*b)*x^2))/(28*(2 + x^2 + x^4)) - (Sqrt[(-1 + 2*Sqrt[2
])/14]*((11 - Sqrt[2])*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]]
- 2*x)/Sqrt[1 + 2*Sqrt[2]]])/56 + (Sqrt[(-1 + 2*Sqrt[2])/14]*((11 - Sqrt[2]
)*a - (2 - 4*Sqrt[2])*b)*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*x)/Sqrt[1 + 2*Sqr
t[2]]])/56 - ((11*a + Sqrt[2]*(a - 4*b) - 2*b)*Log[Sqrt[2] - Sqrt[-1 + 2*Sq
rt[2]]*x + x^2])/(112*Sqrt[2*(-1 + 2*Sqrt[2])]) + (((11 + Sqrt[2])*a - 2*(b
 + 2*Sqrt[2]*b))*Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2])/(112*Sqrt[2*(
-1 + 2*Sqrt[2])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{a+bx^2}{(2+x^2+x^4)^2}\,dx = \frac{x(3a+2b-(a-4b)x^2)}{28\,(2+x^2+x^4)} + \frac{1}{28}\int \frac{11a-2b+(-a+4b)x^2}{2+x^2+x^4}\,dx$$

$$= \frac{x(3a+2b-(a-4b)x^2)}{28\,(2+x^2+x^4)} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}\,(11a-2b)-\left(11a-2b-\sqrt{2}\,(-a+4b)\right)x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,dx}{56\sqrt{2\left(-1+2\sqrt{2}\right)}} + \cdots$$

$$= \frac{x(3a+2b-(a-4b)x^2)}{28\,(2+x^2+x^4)} - \frac{\left(11a+\sqrt{2}\,(a-4b)-2b\right)\int \frac{-\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x+x^2}\,d\cdots}{112\sqrt{2\left(-1+2\sqrt{2}\right)}}\cdots$$

$$= \frac{x(3a+2b-(a-4b)x^2)}{28\,(2+x^2+x^4)} - \frac{\left(11a+\sqrt{2}\,(a-4b)-2b\right)\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}\,x\cdots\right.}{112\sqrt{2\left(-1+2\sqrt{2}\right)}}\cdots$$

$$= \frac{x(3a+2b-(a-4b)x^2)}{28\,(2+x^2+x^4)} - \frac{\left(\left(11-\sqrt{2}\right)a-\left(2-4\sqrt{2}\right)b\right)\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}\cdots}}{\sqrt{1+2\sqrt{2}\cdots}}\right.}{56\sqrt{2\left(1+2\sqrt{2}\right)}}\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.12, size = 165, normalized size = 0.52

$$\frac{-ax(-3+x^2)+2b(x+2x^3)}{28\,(2+x^2+x^4)} - \frac{\left(\left(23i+\sqrt{7}\right)a-4\left(2i+\sqrt{7}\right)b\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{28\sqrt{14-14i\sqrt{7}}} - \frac{\left(\left(-23i+\sqrt{7}\right)a-4\left(-2i+\sqrt{7}\right)b\right)\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{28\sqrt{14+14i\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(2 + x^2 + x^4)^2,x]
```

```
[Out] (-(a*x*(-3 + x^2)) + 2*b*(x + 2*x^3))/(28*(2 + x^2 + x^4)) - (((23*I + Sqrt
[7])*a - 4*(2*I + Sqrt[7])*b)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/(28*Sqrt[1
4 - (14*I)*Sqrt[7]]) - (((-23*I + Sqrt[7])*a - 4*(-2*I + Sqrt[7])*b)*ArcTan
[x/Sqrt[(1 + I*Sqrt[7])/2]])/(28*Sqrt[14 + (14*I)*Sqrt[7]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 604 vs. 2(235) = 470.
time = 0.21, size = 605, normalized size = 1.91

method	result
risch	$\frac{\left(\frac{b}{7}-\frac{a}{28}\right)x^3+\left(\frac{b}{14}+\frac{3a}{28}\right)x}{x^4+x^2+2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2+2\right)}\frac{\left((-a+4b)\_R^2-2b+11a\right)\ln\left(x-\_R\right)}{2\_R^3+\_R}\right)}{56}$
default	$-\frac{-\frac{\left(-14a-28\sqrt{2}\,a+112b\sqrt{2}+56b\right)x}{1+2\sqrt{2}}+\frac{\sqrt{-1+2\sqrt{2}}\,\left(-70a-42\sqrt{2}\,a+56b\sqrt{2}+28b\right)}{1+2\sqrt{2}}}{784\left(x^2+\sqrt{2}-x\sqrt{-1+2\sqrt{2}}\right)}-\frac{\left(107\sqrt{2}\,\sqrt{-1+2\sqrt{2}}\,a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(x^4+x^2+2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/784*(-(-14*a-28*2^(1/2)*a+112*b*2^(1/2)+56*b)/(1+2*2^(1/2))*x+1/(1+2*2^(
1/2))*(-1+2*2^(1/2))^(1/2)*(-70*a-42*2^(1/2)*a+56*b*2^(1/2)+28*b))/(x^2+2^(
1/2)-x*(-1+2*2^(1/2))^(1/2))-1/784/(1+2*2^(1/2))*(1/2*(107*2^(1/2)*(-1+2*2^
(1/2))^(1/2)*a-50*2^(1/2)*(-1+2*2^(1/2))^(1/2)*b+106*(-1+2*2^(1/2))^(1/2)*a
-88*(-1+2*2^(1/2))^(1/2)*b)*ln(x^2+2^(1/2)-x*(-1+2*2^(1/2))^(1/2))+2*(-77*2
^(1/2)*a+14*b*2^(1/2)-308*a+56*b+1/2*(107*2^(1/2)*(-1+2*2^(1/2))^(1/2)*a-50
*2^(1/2)*(-1+2*2^(1/2))^(1/2)*b+106*(-1+2*2^(1/2))^(1/2)*a-88*(-1+2*2^(1/2)
)^(1/2)*b)*(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1
/2))^(1/2))/(1+2*2^(1/2))^(1/2)))+1/784*((-14*a-28*2^(1/2)*a+112*b*2^(1/2)+
56*b)/(1+2*2^(1/2))*x+1/(1+2*2^(1/2))*(-1+2*2^(1/2))^(1/2)*(-70*a-42*2^(1/2
)*a+56*b*2^(1/2)+28*b))/(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))+1/784/(1+2*2^(
1/2))*(1/2*(107*2^(1/2)*(-1+2*2^(1/2))^(1/2)*a-50*2^(1/2)*(-1+2*2^(1/2))^(1
/2)*b+106*(-1+2*2^(1/2))^(1/2)*a-88*(-1+2*2^(1/2))^(1/2)*b)*ln(x^2+2^(1/2)+
x*(-1+2*2^(1/2))^(1/2))+2*(308*a+77*2^(1/2)*a-56*b-14*b*2^(1/2)-1/2*(107*2^
(1/2)*(-1+2*2^(1/2))^(1/2)*a-50*2^(1/2)*(-1+2*2^(1/2))^(1/2)*b+106*(-1+2*2^
(1/2))^(1/2)*a-88*(-1+2*2^(1/2))^(1/2)*b)*(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2
))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -1/28*((a - 4*b)*x^3 - (3*a + 2*b)*x)/(x^4 + x^2 + 2) + 1/28*integrate(-((a
 - 4*b)*x^2 - 11*a + 2*b)/(x^4 + x^2 + 2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5168 vs. $2(235) = 470$.

time = 0.76, size = 5168, normalized size = 16.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/21952*(196*2^(3/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 233
2*a*b^3 + 484*b^4)^(3/4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3
+ 16*b^4)*(x^4 + x^2 + 2)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 1
8656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 -
 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b
- 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*arctan(-1/98*(7*2^(3/4)*sqrt(2/7)*(4489
*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(3/4)*(sqrt(2)*(12
529*a^5 - 15137*a^4*b + 5836*a^3*b^2 + 728*a^2*b^3 - 1216*a*b^4 + 176*b^5)*
sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*sqrt(289*
a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*x + 2*(76313*a^7 - 44394
2*a^6*b + 663081*a^5*b^2 - 467548*a^4*b^3 + 131584*a^3*b^4 + 29280*a^2*b^5
- 31504*a*b^6 + 7744*b^7)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3
 + 16*b^4)*x)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 +
 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3
+ 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^
2 + 32*a*b^3 + 16*b^4)) + 2^(3/4)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2
332*a*b^3 + 484*b^4)^(3/4)*sqrt(14*(1297321*a^8 - 2662982*a^7*b + 2090965*a
^6*b^2 - 461012*a^5*b^3 - 389252*a^4*b^4 + 284608*a^3*b^5 - 40592*a^2*b^6 -
 21824*a*b^7 + 7744*b^8)*x^2 + 2^(1/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5
757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt(7)*sqrt(2)*(289*a^5 - 1292*
a^4*b + 424*a^3*b^2 + 512*a^2*b^3 - 112*a*b^4 - 64*b^5)*sqrt(4489*a^4 - 710
2*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*x + sqrt(7)*(212993*a^7 - 30
7445*a^6*b + 109644*a^5*b^2 + 49580*a^4*b^3 - 46928*a^3*b^4 + 4944*a^2*b^5
+ 4160*a*b^6 - 704*b^7)*x)*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 -
18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2
- 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b
 - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 14*sqrt(2)*(19363*a^6 - 24429*a^5*b
+ 5526*a^4*b^2 + 5512*a^3*b^3 - 3264*a^2*b^4 - 144*a*b^5 + 352*b^6)*sqrt(44
89*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4))*(sqrt(2)*sqrt(4
489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*sqrt(289*a^4 -
136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(11*a - 2*b) + 2*sqrt(289*a^4
- 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(67*a^3 - 321*a^2*b + 234*a*
b^2 - 88*b^3))*sqrt((35912*a^4 - 56816*a^3*b + 46056*a^2*b^2 - 18656*a*b^3
+ 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3
 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b
^2 + 32*a*b^3 + 16*b^4)) + 28*sqrt(7)*sqrt(2)*(5112971*a^8 - 13336819*a^7*b
```

```
 + 16286963*a^6*b^2 - 11087881*a^5*b^3 + 3832430*a^4*b^4 + 31472*a^3*b^5 -
641872*a^2*b^6 + 265232*a*b^7 - 42592*b^8)*sqrt(4489*a^4 - 7102*a^3*b + 575
7*a^2*b^2 - 2332*a*b^3 + 484*b^4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 +
32*a*b^3 + 16*b^4) - 14*sqrt(7)*(342569057*a^10 - 1164554336*a^9*b + 191056
3290*a^8*b^2 - 1899507084*a^7*b^3 + 1202743689*a^6*b^4 - 444943548*a^5*b^5
+ 39640020*a^4*b^6 + 52482144*a^3*b^7 - 31032144*a^2*b^8 + 8092480*a*b^9 -
937024*b^10)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4))/(
5823673969*a^12 - 21167699940*a^11*b + 35767517046*a^10*b^2 - 35275656244*a
^9*b^3 + 20402417889*a^8*b^4 - 4776986736*a^7*b^5 - 2357320224*a^6*b^6 + 25
13410560*a^5*b^7 - 896035104*a^4*b^8 + 51772160*a^3*b^9 + 75829248*a^2*b^10
 - 28621824*a*b^11 + 3748096*b^12)) + 196*2^(3/4)*sqrt(2/7)*(4489*a^4 - 710
2*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(3/4)*sqrt(289*a^4 - 136*a^3
*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*(x^4 + x^2 + 2)*sqrt((35912*a^4 - 568
16*a^3*b + 46056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 -
 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100
*b^2))/(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4))*arctan(-1/9
8*(7*2^(3/4)*sqrt(2/7)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 +
 484*b^4)^(3/4)*(sqrt(2)*(12529*a^5 - 15137*a^4*b + 5836*a^3*b^2 + 728*a^2*
b^3 - 1216*a*b^4 + 176*b^5)*sqrt(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 233
2*a*b^3 + 484*b^4)*sqrt(289*a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b
^4)*x + 2*(76313*a^7 - 443942*a^6*b + 663081*a^5*b^2 - 467548*a^4*b^3 + 131
584*a^3*b^4 + 29280*a^2*b^5 - 31504*a*b^6 + 7744*b^7)*sqrt(289*a^4 - 136*a^
3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)*x)*sqrt((35912*a^4 - 56816*a^3*b + 4
6056*a^2*b^2 - 18656*a*b^3 + 3872*b^4 - sqrt(2)*sqrt(4489*a^4 - 7102*a^3*b
+ 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)*(211*a^2 - 428*a*b + 100*b^2))/(289*
a^4 - 136*a^3*b - 120*a^2*b^2 + 32*a*b^3 + 16*b^4)) + 2^(3/4)*(4489*a^4 - 7
102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(3/4)*sqrt(14*(1297321*a^8
 - 2662982*a^7*b + 2090965*a^6*b^2 - 461012*a^5*b^3 - 389252*a^4*b^4 + 2846
08*a^3*b^5 - 40592*a^2*b^6 - 21824*a*b^7 + 7744*b^8)*x^2 - 2^(1/4)*sqrt(2/7
)*(4489*a^4 - 7102*a^3*b + 5757*a^2*b^2 - 2332*a*b^3 + 484*b^4)^(1/4)*(sqrt
(7)*sqrt(2)*(289*a^5 - 1292*a^4*b + 424*a^3*b^2...
```

Sympy [A]
time = 0.95, size = 165, normalized size = 0.52

$$\frac{x^3(-a+4b)+x(3a+2b)}{28x^4+28x^2+56}+\operatorname{RootSum}\left(240945152t^4+t^2(-1157968a^2+2348864ab-548800b^2)+4489a^4-7102a^3b+5757a^2b^2-2332ab^3+484b^4,\left(t\mapsto t\log\left(x+\frac{2634240t^3a-3161088t^3b+11996ta^3+12792ta^2b-21936tab^2+4384tb^3}{1139a^4-1169a^3b+318a^2b^2+124ab^3-88b^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(x**4+x**2+2)**2,x)
```

```
[Out] (x**3*(-a + 4*b) + x*(3*a + 2*b))/(28*x**4 + 28*x**2 + 56) + RootSum(240945
152*_t**4 + _t**2*(-1157968*a**2 + 2348864*a*b - 548800*b**2) + 4489*a**4 -
 7102*a**3*b + 5757*a**2*b**2 - 2332*a*b**3 + 484*b**4, Lambda(_t, _t*log(x
 + (2634240*_t**3*a - 3161088*_t**3*b + 11996*_t*a**3 + 12792*_t*a**2*b - 2
1936*_t*a*b**2 + 4384*_t*b**3)/(1139*a**4 - 1169*a**3*b + 318*a**2*b**2 + 1
24*a*b**3 - 88*b**4))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1112 vs. $2(235) = 470$.
time = 7.94, size = 1112, normalized size = 3.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(x^4+x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/25088*sqrt(7)*(sqrt(7)*2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) - 4*sq
rt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*sqrt(7)*2^(3/4)*a*sqr
t(2*sqrt(2) + 8)*(sqrt(2) - 4) - 12*sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(
sqrt(2) - 4) - 3*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) + 12*2^(3/4)*
b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*a*(sqrt(2) - 4)*sqrt(-2*sqrt
(2) + 8) + 4*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) + 88*sqrt(7)*2^(1
/4)*a*sqrt(2*sqrt(2) + 8) - 16*sqrt(7)*2^(1/4)*b*sqrt(2*sqrt(2) + 8) - 88*2
^(1/4)*a*sqrt(-2*sqrt(2) + 8) + 16*2^(1/4)*b*sqrt(-2*sqrt(2) + 8))*arctan(2
*2^(3/4)*sqrt(1/2)*(x + 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2) + 4)
) + 1/25088*sqrt(7)*(sqrt(7)*2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) -
4*sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*sqrt(7)*2^(3/4)*a
*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 12*sqrt(7)*2^(3/4)*b*sqrt(2*sqrt(2) +
8)*(sqrt(2) - 4) - 3*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) + 12*2^(3
/4)*b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) - 2^(3/4)*a*(sqrt(2) - 4)*sqrt(-2*
sqrt(2) + 8) + 4*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) + 88*sqrt(7)*
2^(1/4)*a*sqrt(2*sqrt(2) + 8) - 16*sqrt(7)*2^(1/4)*b*sqrt(2*sqrt(2) + 8) -
88*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) + 16*2^(1/4)*b*sqrt(-2*sqrt(2) + 8))*arct
an(2*2^(3/4)*sqrt(1/2)*(x - 2^(1/4)*sqrt(-1/8*sqrt(2) + 1/2))/sqrt(sqrt(2)
+ 4)) + 1/50176*sqrt(7)*(3*sqrt(7)*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-2*sqrt(2)
+ 8) - 12*sqrt(7)*2^(3/4)*b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) + sqrt(7)*2^
(3/4)*a*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8) - 4*sqrt(7)*2^(3/4)*b*(sqrt(2) -
 4)*sqrt(-2*sqrt(2) + 8) + 2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) - 4*
2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) + 3*2^(3/4)*a*sqrt(2*sqrt(2) +
8)*(sqrt(2) - 4) - 12*2^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) + 88*sqrt
(7)*2^(1/4)*a*sqrt(-2*sqrt(2) + 8) - 16*sqrt(7)*2^(1/4)*b*sqrt(-2*sqrt(2) +
 8) + 88*2^(1/4)*a*sqrt(2*sqrt(2) + 8) - 16*2^(1/4)*b*sqrt(2*sqrt(2) + 8))*
log(x^2 + 2*2^(1/4)*x*sqrt(-1/8*sqrt(2) + 1/2) + sqrt(2)) - 1/50176*sqrt(7)
*(3*sqrt(7)*2^(3/4)*a*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) - 12*sqrt(7)*2^(3/
4)*b*(sqrt(2) + 4)*sqrt(-2*sqrt(2) + 8) + sqrt(7)*2^(3/4)*a*(sqrt(2) - 4)*s
qrt(-2*sqrt(2) + 8) - 4*sqrt(7)*2^(3/4)*b*(sqrt(2) - 4)*sqrt(-2*sqrt(2) + 8
) + 2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) + 4) - 4*2^(3/4)*b*sqrt(2*sqrt(2
) + 8)*(sqrt(2) + 4) + 3*2^(3/4)*a*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) - 12*2
^(3/4)*b*sqrt(2*sqrt(2) + 8)*(sqrt(2) - 4) + 88*sqrt(7)*2^(1/4)*a*sqrt(-2*s
qrt(2) + 8) - 16*sqrt(7)*2^(1/4)*b*sqrt(-2*sqrt(2) + 8) + 88*2^(1/4)*a*sqrt
(2*sqrt(2) + 8) - 16*2^(1/4)*b*sqrt(2*sqrt(2) + 8))*log(x^2 - 2*2^(1/4)*x*s
```

```
qrt(-1/8*sqrt(2) + 1/2) + sqrt(2)) - 1/28*(a*x^3 - 4*b*x^3 - 3*a*x - 2*b*x)
/(x^4 + x^2 + 2)
```

Mupad [B]
time = 4.50, size = 1491, normalized size = 4.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^2 + x^4 + 2)^2,x)
```

```
[Out] atan((b^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3
136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/(
4*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a
^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1
/2)*a^2*b*39i)/3136)) - (a^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 -
 (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*1i
)/3136)^(1/2)*17i)/(16*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3
*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a
*b^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)) + (a*b*x*((7^(1/2)*a^2*17i)/1254
4 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21
952 - (7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/(4*((7^(1/2)*a^3*187i)/6272 + (7^(1/
2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^
3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)) - (17*7^(1/2)
*a^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 +
 (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2))/(112*((7^
(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a^2*b)/
3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1/2)*a^
2*b*39i)/3136)) + (7^(1/2)*b^2*x*((7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952
 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b^2)/21952 - (7^(1/2)*a*b*
1i)/3136)^(1/2))/(28*((7^(1/2)*a^3*187i)/6272 + (7^(1/2)*b^3*1i)/784 + (3*a
*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 - (7^(1/2)*a*b
^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)) + (7^(1/2)*a*b*x*((7^(1/2)*a^2*17i
)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/87808 + (25*b
^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2))/(28*((7^(1/2)*a^3*187i)/6272 + (7
^(1/2)*b^3*1i)/784 + (3*a*b^2)/1568 - (183*a^2*b)/3136 + (255*a^3)/6272 + (
9*b^3)/784 - (7^(1/2)*a*b^2*9i)/1568 - (7^(1/2)*a^2*b*39i)/3136)))*((7^(1/2
)*a^2*17i)/12544 - (107*a*b)/21952 - (7^(1/2)*b^2*1i)/3136 + (211*a^2)/8780
8 + (25*b^2)/21952 - (7^(1/2)*a*b*1i)/3136)^(1/2)*2i - atan((a^2*x*((7^(1/2
)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/8780
8 + (25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2)*17i)/(16*((3*a*b^2)/1568
- (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*
a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/313
6)) - (b^2*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/2
1952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2)*1i)/
```

```
(4*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*
a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(
1/2)*a^2*b*39i)/3136)) - (a*b*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/
12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)*a*b*1
i)/3136)^(1/2)*1i)/(4*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3
*187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*
b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/3136)) - (17*7^(1/2)*a^2*x*((7^(1/2)*b^2
*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + (
25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2))/(112*((3*a*b^2)/1568 - (7^(1/
2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/627
2 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/3136)) + (7
^(1/2)*b^2*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2*17i)/12544 - (107*a*b)/2
1952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)*a*b*1i)/3136)^(1/2))/(28
*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)*a^3*187i)/6272 - (183*a^
2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2)*a*b^2*9i)/1568 + (7^(1/
2)*a^2*b*39i)/3136)) + (7^(1/2)*a*b*x*((7^(1/2)*b^2*1i)/3136 - (7^(1/2)*a^2
*17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 + (7^(1/2)
*a*b*1i)/3136)^(1/2))/(28*((3*a*b^2)/1568 - (7^(1/2)*b^3*1i)/784 - (7^(1/2)
*a^3*187i)/6272 - (183*a^2*b)/3136 + (255*a^3)/6272 + (9*b^3)/784 + (7^(1/2
)*a*b^2*9i)/1568 + (7^(1/2)*a^2*b*39i)/3136)))*((7^(1/2)*b^2*1i)/3136 - (7^
(1/2)*a^2*17i)/12544 - (107*a*b)/21952 + (211*a^2)/87808 + (25*b^2)/21952 +
 (7^(1/2)*a*b*1i)/3136)^(1/2)*2i - (x^3*(a/28 - b/7) - x*((3*a)/28 + b/14))
/(x^2 + x^4 + 2)
```

3.102 $\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}\,x^2+x^4}\,dx$

Optimal. Leaf size=160

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}+2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2+\sqrt{2}}\,x+x^2\right)+\frac{1}{4}\sqrt{1$$

[Out] -1/8*ln(1+x^2-x*(2+2^(1/2))^(1/2))*(4+2*2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2+2^(1/2))^(1/2))*(4+2*2^(1/2))^(1/2)-1/2*arctan((-2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)+1/2*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)

Rubi [A]
time = 0.10, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$,
Rules used = {1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2+\sqrt{2}}}-\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2+\sqrt{2}}\,x+1\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2+\sqrt{2}}\,x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/Sqrt[2 + Sqrt[2]]
+ ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2 + Sqrt[2]])
 - (Sqrt[1 + 1/Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/4 + (Sqrt[1 + 1
/Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}\,x^2+x^4}\,dx = \frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2+\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2+\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2+\sqrt{2}}}$$

$$= \frac{1}{4}\sqrt{3-2\sqrt{2}}\int \frac{1}{1-\sqrt{2+\sqrt{2}}\,x+x^2}\,dx+\frac{1}{4}\sqrt{3-2\sqrt{2}}\int \frac{1}{1+\sqrt{2+\sqrt{2}}\,x$$

$$= -\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2+\sqrt{2}}\,x+x^2\right)+\frac{1}{4}\sqrt{1+\frac{1}{\sqrt{2}}}\log\left(1+\sqrt{2+\sqrt{2}}$$

$$= -\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(2-\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.03, size = 53, normalized size = 0.33

$$\frac{\sqrt{-1-i}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1-i}}\right)+\sqrt{-1+i}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{-1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] - x^2)/(1 - Sqrt[2]*x^2 + x^4),x]
```

```
[Out] (Sqrt[-1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[-1 - I]] + Sqrt[-1 + I]*ArcTan[(2^(1/
4)*x)/Sqrt[-1 + I]])/2^(3/4)
```

Maple [A]
time = 0.11, size = 145, normalized size = 0.91

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-\_Z^2\,\mathrm{RootOf}\left(\_Z^2-2,index=1\right)+1\right)}\frac{\left(\_R^2-\sqrt{2}\right)\ln\left(x-\_R\right)}{-2\_R^3+\_R\sqrt{2}}\right)}{2}$
default	$\frac{\sqrt{2}\left(-\frac{\sqrt{2+\sqrt{2}}\,\ln\left(1+x^2-x\sqrt{2+\sqrt{2}}\right)}{2}+\frac{2\left(-\frac{\sqrt{2}}{2}+1\right)\arctan\left(\frac{2x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}\right)}{4}+\frac{\sqrt{2}\left(\frac{\sqrt{2+\sqrt{2}}}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)*(-1/2*(2+2^(1/2))^(1/2)*ln(1+x^2-x*(2+2^(1/2))^(1/2))+2*(-1/2*2
^(1/2)+1)/(2-2^(1/2))^(1/2)*arctan((2*x-(2+2^(1/2))^(1/2))/(2-2^(1/2))^(1/2
)))+1/4*2^(1/2)*(1/2*(2+2^(1/2))^(1/2)*ln(1+x^2+x*(2+2^(1/2))^(1/2))+2*(-1/
2*2^(1/2)+1)/(2-2^(1/2))^(1/2)*arctan((2*x+(2+2^(1/2))^(1/2))/(2-2^(1/2))^(
1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - sqrt(2))/(x^4 - sqrt(2)*x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(119) = 238$.
time = 0.50, size = 251, normalized size = 1.57

$$\frac{1}{8}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}\log\left(4x^2+2\left(\sqrt{2}x+2x\right)\sqrt{-2\sqrt{2}+4}+4\right)-\frac{1}{8}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}\log\left(4x^2-2\left(\sqrt{2}x+2x\right)\sqrt{-2\sqrt{2}+4}+4\right)-\frac{1}{2}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{1}{2}\sqrt{4x^2+2\left(\sqrt{2}x+2x\right)\sqrt{-2\sqrt{2}+4}+4}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}-\left(\sqrt{2}x+x\right)\sqrt{-2\sqrt{2}+4}-\sqrt{2}-1\right)-\frac{1}{2}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{1}{2}\sqrt{4x^2-2\left(\sqrt{2}x+2x\right)\sqrt{-2\sqrt{2}+4}+4}\left(\sqrt{2}+1\right)\sqrt{-2\sqrt{2}+4}-\left(\sqrt{2}x+x\right)\sqrt{-2\sqrt{2}+4}+\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log(4*x^2 + 2*(sqrt(2)*x + 2*x)*sqrt
(-2*sqrt(2) + 4) + 4) - 1/8*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log(4*x^2 -
2*(sqrt(2)*x + 2*x)*sqrt(-2*sqrt(2) + 4) + 4) - 1/2*sqrt(-2*sqrt(2) + 4)*ar
ctan(1/2*sqrt(4*x^2 + 2*(sqrt(2)*x + 2*x)*sqrt(-2*sqrt(2) + 4) + 4)*(sqrt(2
) + 1)*sqrt(-2*sqrt(2) + 4) - (sqrt(2)*x + x)*sqrt(-2*sqrt(2) + 4) - sqrt(2
) - 1) - 1/2*sqrt(-2*sqrt(2) + 4)*arctan(1/2*sqrt(4*x^2 - 2*(sqrt(2)*x + 2*
x)*sqrt(-2*sqrt(2) + 4) + 4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) - (sqrt(2)*
x + x)*sqrt(-2*sqrt(2) + 4) + sqrt(2) + 1)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2**(1/2))/(1+x**4-x**2*2**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(128*_t**4 - 16*sqrt(2)*_t**2 + 1) c
ontains an element of the set of generators.
```

Giac [A]
time = 6.58, size = 122, normalized size = 0.76

$$\frac{1}{4}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{-2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{\sqrt{2}+2}}{\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\log\left(x^2+x\sqrt{\sqrt{2}+2}+1\right)-\frac{1}{8}\sqrt{2\sqrt{2}+4}\log\left(x^2-x\sqrt{\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(1+x^4-x^2*2^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-2*sqrt(2) + 4)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2
)) + 1/4*sqrt(-2*sqrt(2) + 4)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2
) + 2)) + 1/8*sqrt(2*sqrt(2) + 4)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/8*
sqrt(2*sqrt(2) + 4)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1)
```

Mupad [B]
time = 4.96, size = 121, normalized size = 0.76

$$-\mathrm{atan}\left(x\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i}-\frac{\sqrt{2}\,\sqrt{8}\,x\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}}{2}\right)\sqrt{\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i}-\mathrm{atan}\left(x\sqrt{\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i}+\frac{\sqrt{2}\,\sqrt{8}\,x\sqrt{\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}}{2}\right)\sqrt{\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1\mathrm{i}}{32}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(1/2) - x^2)/(x^4 - 2^(1/2)*x^2 + 1),x)
```

```
[Out] - atan(x*(2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*2i - (2^(1/2)*8^(1/2)*x*(2^(1
/2)/16 - (8^(1/2)*1i)/32)^(1/2))/2)*(2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*2i
 - atan(x*(2^(1/2)/16 + (8^(1/2)*1i)/32)^(1/2)*2i + (2^(1/2)*8^(1/2)*x*(2^(
1/2)/16 + (8^(1/2)*1i)/32)^(1/2))/2)*(2^(1/2)/16 + (8^(1/2)*1i)/32)^(1/2)*2
i
```

3.103 $\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}\,x^2+x^4}\,dx$

Optimal. Leaf size=172

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}+2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right)+\frac{1}{4}\sqrt{}$$

```
[Out] -1/8*ln(1+x^2-x*(2-2^(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)+1/8*ln(1+x^2+x*(2-2^
(1/2))^(1/2))*(4-2*2^(1/2))^(1/2)-1/2*arctan((-2*x+(2-2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2))/(2-2^(1/2))^(1/2)+1/2*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2))/(2-2^(1/2))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2-\sqrt{2}}}-\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2-\sqrt{2-\sqrt{2}}\,x+1\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\log\left(x^2+\sqrt{2-\sqrt{2}}\,x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/Sqrt[2 - Sqrt[2]]
+ ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2 - Sqrt[2]])
 - (Sqrt[1 - 1/Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/4 + (Sqrt[1 - 1
/Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}\,x^2+x^4}\,dx &= \frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2-\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{2\sqrt{2-\sqrt{2}}}\\
&= \frac{\left(1-\sqrt{2}\right)\int \frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}}+\frac{\left(-1+\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}\,x+x^2}\,dx}{4\sqrt{2-\sqrt{2}}}+\frac{1}{4}\sqrt{3}\\
&= -\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\,\log\left(1-\sqrt{2-\sqrt{2}}\,x+x^2\right)+\frac{1}{4}\sqrt{1-\frac{1}{\sqrt{2}}}\,\log\left(1+\sqrt{2-\sqrt{2}}\right.\\
&= -\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(2+\sqrt{2}\right)}\,\tan^{-1}\left(\frac{\sqrt{2-}}{\sqrt{}}\right.
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.02, size = 53, normalized size = 0.31

$$\frac{\sqrt{1-i}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1-i}}\right)+\sqrt{1+i}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\,x}{\sqrt{1+i}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] + x^2)/(1 + Sqrt[2]*x^2 + x^4),x]
```

```
[Out] (Sqrt[1 - I]*ArcTan[(2^(1/4)*x)/Sqrt[1 - I]] + Sqrt[1 + I]*ArcTan[(2^(1/4)*
x)/Sqrt[1 + I]])/2^(3/4)
```

Maple [A]
time = 0.10, size = 149, normalized size = 0.87

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\_Z^4+\_Z^2\,\mathrm{RootOf}\left(\_Z^2-2,index=1\right)\right)}\frac{\left(\_R^2+\sqrt{2}\right)\ln\left(x-\_R\right)}{2\_R^3+\_R\sqrt{2}}\right)}{2}$
default	$\frac{\sqrt{2}\left(-\frac{\sqrt{2-\sqrt{2}}\,\ln\left(1+x^2-x\sqrt{2-\sqrt{2}}\right)}{2}+\frac{2\left(1+\frac{\sqrt{2}}{2}\right)\arctan\left(\frac{2x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{\sqrt{2+\sqrt{2}}}\right)}{4}+\frac{\sqrt{2}\left(\frac{\sqrt{2-\sqrt{2}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*2^(1/2)*(-1/2*(2-2^(1/2))^(1/2)*ln(1+x^2-x*(2-2^(1/2))^(1/2))+2*(1+1/2*
2^(1/2))/(2+2^(1/2))^(1/2)*arctan((2*x-(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
))+1/4*2^(1/2)*(1/2*(2-2^(1/2))^(1/2)*ln(1+x^2+x*(2-2^(1/2))^(1/2))+2*(1+1/
2*2^(1/2))/(2+2^(1/2))^(1/2)*arctan((2*x+(2-2^(1/2))^(1/2))/(2+2^(1/2))^(1/
2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + sqrt(2)*x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(127) = 254$.

time = 0.39, size = 255, normalized size = 1.48

$$-\frac{1}{8}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-1\right)\log\left(4x^2+2\left(\sqrt{2}x-2x\right)\sqrt{2\sqrt{2}+4}+4\right)+\frac{1}{8}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-1\right)\log\left(4x^2-2\left(\sqrt{2}x-2x\right)\sqrt{2\sqrt{2}+4}+4\right)-\frac{1}{2}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{1}{2}\sqrt{4x^2+2\left(\sqrt{2}x-2x\right)\sqrt{2\sqrt{2}+4}+4}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-1\right)-\left(\sqrt{2}x-x\right)\sqrt{2\sqrt{2}+4}+\sqrt{2}-1\right)-\frac{1}{2}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{1}{2}\sqrt{4x^2-2\left(\sqrt{2}x-2x\right)\sqrt{2\sqrt{2}+4}+4}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-1\right)-\left(\sqrt{2}x-x\right)\sqrt{2\sqrt{2}+4}-\sqrt{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(4*x^2 + 2*(sqrt(2)*x - 2*x)*sqrt
(2*sqrt(2) + 4) + 4) + 1/8*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(4*x^2 - 2*
(sqrt(2)*x - 2*x)*sqrt(2*sqrt(2) + 4) + 4) - 1/2*sqrt(2*sqrt(2) + 4)*arctan
(1/2*sqrt(4*x^2 + 2*(sqrt(2)*x - 2*x)*sqrt(2*sqrt(2) + 4) + 4)*sqrt(2*sqrt(
2) + 4)*(sqrt(2) - 1) - (sqrt(2)*x - x)*sqrt(2*sqrt(2) + 4) + sqrt(2) - 1)
- 1/2*sqrt(2*sqrt(2) + 4)*arctan(1/2*sqrt(4*x^2 - 2*(sqrt(2)*x - 2*x)*sqrt(
2*sqrt(2) + 4) + 4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1) - (sqrt(2)*x - x)*sqr
t(2*sqrt(2) + 4) - sqrt(2) + 1)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2**(1/2))/(1+x**4+x**2*2**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(128*_t**4 + 16*sqrt(2)*_t**2 + 1) c
ontains an element of the set of generators.
```

Giac [A]

time = 6.66, size = 126, normalized size = 0.73

$$\frac{1}{4}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{4}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-2\sqrt{2}+4}\log\left(x^2+x\sqrt{-\sqrt{2}+2}+1\right)-\frac{1}{8}\sqrt{-2\sqrt{2}+4}\log\left(x^2-x\sqrt{-\sqrt{2}+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(1+x^4+x^2*2^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2*sqrt(2) + 4)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
) + 1/4*sqrt(2*sqrt(2) + 4)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2)
+ 2)) + 1/8*sqrt(-2*sqrt(2) + 4)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/8*
sqrt(-2*sqrt(2) + 4)*log(x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

Mupad [B]

time = 4.95, size = 121, normalized size = 0.70

$$\mathrm{atan}\left(x\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1i}{32}}\,2i+\frac{\sqrt{2}\,\sqrt{8}\,x\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1i}{32}}}{2}\right)\sqrt{-\frac{\sqrt{2}}{16}-\frac{\sqrt{8}\,1i}{32}}\,2i+\mathrm{atan}\left(x\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1i}{32}}\,2i-\frac{\sqrt{2}\,\sqrt{8}\,x\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1i}{32}}}{2}\right)\sqrt{-\frac{\sqrt{2}}{16}+\frac{\sqrt{8}\,1i}{32}}\,2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(1/2) + x^2)/(2^(1/2)*x^2 + x^4 + 1),x)
```

```
[Out] atan(x*(- 2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2)*2i + (2^(1/2)*8^(1/2)*x*(- 2^
(1/2)/16 - (8^(1/2)*1i)/32)^(1/2))/2)*(- 2^(1/2)/16 - (8^(1/2)*1i)/32)^(1/2
)*2i + atan(x*((8^(1/2)*1i)/32 - 2^(1/2)/16)^(1/2)*2i - (2^(1/2)*8^(1/2)*x*
((8^(1/2)*1i)/32 - 2^(1/2)/16)^(1/2))/2)*((8^(1/2)*1i)/32 - 2^(1/2)/16)^(1/
2)*2i
```

3.104 $\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=160

$$\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}}-\frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}}-\frac{\left(1+\sqrt{2}\right)\log\left(1-\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}}+$$

```
[Out] -1/4*ln(1+x^2-x*(2-b)^(1/2))*(1+2^(1/2))/(2-b)^(1/2)+1/4*ln(1+x^2+x*(2-b)^(
1/2))*(1+2^(1/2))/(2-b)^(1/2)+1/2*arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1
-2^(1/2))/(2+b)^(1/2)-1/2*arctan((2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1-2^(1/2))
/(2+b)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1183, 648, 632, 210, 642}

$$\frac{\left(1-\sqrt{2}\right)\text{ArcTan}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}-\frac{\left(1-\sqrt{2}\right)\text{ArcTan}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}-\frac{\left(1+\sqrt{2}\right)\log\left(-\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}+\frac{\left(1+\sqrt{2}\right)\log\left(\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] ((1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - (
(1 - Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) - ((
1 + Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) + ((1 + Sqrt[2])
*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{2}-x^2}{1+bx^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{2-b}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2-b}\,x+x^2}\,dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b}+\left(1+\sqrt{2}\right)x}{1+\sqrt{2-b}\,x+x^2}\,dx}{2\sqrt{2-b}}$$

$$= \frac{1}{4}\left(-1+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2-b}\,x+x^2}\,dx + \frac{1}{4}\left(-1+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2-b}\,x+x^2}\,dx -$$

$$= -\frac{\left(1+\sqrt{2}\right)\log\left(1-\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}} + \frac{\left(1+\sqrt{2}\right)\log\left(1+\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}} + \frac{1}{2}\Big($$

$$= \frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{\left(1-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} - \frac{\left(1+\sqrt{2}\right)\,\mathrm{l}}{}$$

Mathematica [A]
time = 0.05, size = 137, normalized size = 0.86

$$\frac{\frac{\left(2\sqrt{2}+b-\sqrt{-4+b^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b-\sqrt{-4+b^2}}}\right)}{\sqrt{b-\sqrt{-4+b^2}}} - \frac{\left(2\sqrt{2}+b+\sqrt{-4+b^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b+\sqrt{-4+b^2}}}\right)}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2}\,\sqrt{-4+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] - x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2*Sqrt[2] + b - Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^
2]]])/Sqrt[b - Sqrt[-4 + b^2]] - ((2*Sqrt[2] + b + Sqrt[-4 + b^2])*ArcTan[(
Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*Sq
rt[-4 + b^2])
```

Maple [A]
time = 0.07, size = 136, normalized size = 0.85

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2 b+1\right)}\frac{\left(-\_R^2+\sqrt{2}\right)\ln\left(x-\_R\right)}{2\_R^3+\_R b}\right)}{2}$
default	$\frac{\left(-\sqrt{(b-2)(2+b)}-b-2\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\,\sqrt{2\sqrt{(b-2)(2+b)}+2b}}+\frac{\left(-\sqrt{(b-2)(2+b)}+b+2\sqrt{2}\right.}{\sqrt{(b-2)(2+b)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2^(1/2))/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-((b-2)*(2+b))^(1/2)-b-2*2^(1/2))/((b-2)*(2+b))^(1/2)/(2*((b-2)*(2+b))^(1/
2)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))+(-((b-2)*(2+b))
^(1/2)+b+2*2^(1/2))/((b-2)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*
arctan(2*x/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - sqrt(2))/(x^4 + b*x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 517 vs. 2(123) = 246.
time = 0.38, size = 517, normalized size = 3.23

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3b+4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(2(2b^2-9)x+\sqrt{\frac{1}{2}}\left(2b^3-3\sqrt{2}(b^2-4)-8b-\frac{2b^4-14b^2-\sqrt{2}(b^3-4b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3b+4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3b+4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(2(2b^2-9)x-\sqrt{\frac{1}{2}}\left(2b^3-3\sqrt{2}(b^2-4)-8b-\frac{2b^4-14b^2-\sqrt{2}(b^3-4b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3b+4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3b+4\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\left(2(2b^2-9)x+\sqrt{\frac{1}{2}}\left(2b^3-3\sqrt{2}(b^2-4)-8b+\frac{2b^4-14b^2-\sqrt{2}(b^3-4b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3b+4\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3b+4\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\left(2(2b^2-9)x-\sqrt{\frac{1}{2}}\left(2b^3-3\sqrt{2}(b^2-4)-8b+\frac{2b^4-14b^2-\sqrt{2}(b^3-4b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3b+4\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*
b^2 - 9)*x + sqrt(1/2)*(2*b^3 - 3*sqrt(2)*(b^2 - 4) - 8*b - (2*b^4 - 14*b^2
 - sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b + 4*sqrt(2) + sqrt(b
^2 - 4))/(b^2 - 4))) + 1/2*sqrt(1/2)*sqrt(-(3*b + 4*sqrt(2) + sqrt(b^2 - 4)
)/(b^2 - 4))*log(2*(2*b^2 - 9)*x - sqrt(1/2)*(2*b^3 - 3*sqrt(2)*(b^2 - 4) -
 8*b - (2*b^4 - 14*b^2 - sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*
b + 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) - 1/2*sqrt(1/2)*sqrt(-(3*b + 4*s
qrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*x + sqrt(1/2)*(2*b^3 -
 3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*b^2 - sqrt(2)*(b^3 - 4*b) + 24)/sq
rt(b^2 - 4))*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))) + 1/2*sqrt
(1/2)*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*
x - sqrt(1/2)*(2*b^3 - 3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*b^2 - sqrt(2
)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b + 4*sqrt(2) - sqrt(b^2 - 4))/
(b^2 - 4)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(128) = 256$.
time = 1.41, size = 1469, normalized size = 9.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] -RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 + 16*sqrt(2)*b**
2 - 48*b - 64*sqrt(2)) + 2*b**2 + 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**3*
(64*b**12/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*
b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729)
 + 672*sqrt(2)*b**11/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b
**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*
b**2 + 729) + 5760*b**10/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(
2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3
402*b**2 + 729) + 12064*sqrt(2)*b**9/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8
+ 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt
(2)*b**3 - 3402*b**2 + 729) + 17744*b**8/(8*b**10 + 88*sqrt(2)*b**9 + 828*b
**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*
sqrt(2)*b**3 - 3402*b**2 + 729) - 27480*sqrt(2)*b**7/(8*b**10 + 88*sqrt(2)*
b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*
b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 154608*b**6/(8*b**10 + 88*sqr
t(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 +
2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 141376*sqrt(2)*b**5/(8*b
**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 5310*sq
rt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 69072*b**4/
(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 6470*b**6 + 531
```

```
0*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + 61704*s
qrt(2)*b**3/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 + 647
0*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2 + 72
9) + 78192*b**2/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)*b**7 +
 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 3402*b**2
+ 729) - 2592*sqrt(2)*b/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2
)*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 34
02*b**2 + 729) - 15552/(8*b**10 + 88*sqrt(2)*b**9 + 828*b**8 + 2144*sqrt(2)
*b**7 + 6470*b**6 + 5310*sqrt(2)*b**5 + 2781*b**4 - 2322*sqrt(2)*b**3 - 340
2*b**2 + 729)) + _t*(16*b**7/(4*b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqr
t(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) + 116*sqrt(2)*b**6/(4*b**6 + 28*s
qrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) +
 668*b**5/(4*b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**
2 - 27*sqrt(2)*b - 81) + 942*sqrt(2)*b**4/(4*b**6 + 28*sqrt(2)*b**5 + 152*b
**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) + 1226*b**3/(4*b**6
+ 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b -
 81) + 144*sqrt(2)*b**2/(4*b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*
b**3 + 189*b**2 - 27*sqrt(2)*b - 81) - 378*b/(4*b**6 + 28*sqrt(2)*b**5 + 15
2*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2)*b - 81) - 108*sqrt(2)/(4*
b**6 + 28*sqrt(2)*b**5 + 152*b**4 + 192*sqrt(2)*b**3 + 189*b**2 - 27*sqrt(2
)*b - 81)) + x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(123) = 246$.
time = 5.60, size = 1501, normalized size = 9.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b + 2)*b^4 + sqrt(2)*sqrt(b - 2)*b^4 - sqrt(2)*sqrt(b^2 -
 4)*sqrt(b + 2)*b^3 - sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b^3 - sqrt(2)*sqrt(
b + 2)*sqrt(b - 2)*b^3 - 3*sqrt(2)*b^4 + 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2
)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b^3 - sqrt(2)*sqrt(b + 2)*b^3 - s
qrt(2)*sqrt(b - 2)*b^3 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + sqrt(2)*sq
rt(b^2 - 4)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b^2 + 3*sqrt(
2)*b^3 - 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2)*b - sqrt(2)*sqrt(b
^2 - 4)*b^2 - 10*sqrt(2)*sqrt(b + 2)*b^2 - 2*sqrt(b^2 - 4)*sqrt(b + 2)*b^2
- 6*sqrt(2)*sqrt(b - 2)*b^2 - 2*sqrt(b^2 - 4)*sqrt(b - 2)*b^2 - 2*sqrt(b +
2)*sqrt(b - 2)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b + 4*sqrt(2)*sqrt
(b^2 - 4)*sqrt(b - 2)*b + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b + 24*sqrt(2)*
b^2 + 2*sqrt(b^2 - 4)*b^2 - 12*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2
) - 4*sqrt(2)*sqrt(b^2 - 4)*b + 6*sqrt(2)*sqrt(b + 2)*b + 4*sqrt(b^2 - 4)*s
qrt(b + 2)*b + 2*sqrt(2)*sqrt(b - 2)*b + 4*sqrt(b^2 - 4)*sqrt(b - 2)*b + 4*
```

```
sqrt(b + 2)*sqrt(b - 2)*b + 6*b^2 + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2) + 4
*sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2) + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2) - 6*
sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2) - 12*sqrt(2)*b - 4*sqrt(b^2 - 4)*b -
2*sqrt(b + 2)*b - 2*sqrt(b - 2)*b - 4*sqrt(2)*sqrt(b^2 - 4) + 20*sqrt(2)*sq
rt(b + 2) + 8*sqrt(b^2 - 4)*sqrt(b + 2) + 4*sqrt(2)*sqrt(b - 2) + 8*sqrt(b^
2 - 4)*sqrt(b - 2) + 8*sqrt(b + 2)*sqrt(b - 2) - 48*sqrt(2) - 8*sqrt(b^2 -
4) + 4*sqrt(b + 2) - 4*sqrt(b - 2) - 24)*arctan(x/sqrt(1/2*b + 1/2*sqrt(b^2
 - 4)))/(b^4 - 2*b^3 - 7*b^2 + 8*b + 12) + 1/4*(sqrt(2)*sqrt(b + 2)*b^4 - s
qrt(2)*sqrt(b - 2)*b^4 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b^3 - sqrt(2)*sq
rt(b^2 - 4)*sqrt(b - 2)*b^3 - sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b^3 + 3*sqrt(
2)*b^4 - 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt
(b^2 - 4)*b^3 - sqrt(2)*sqrt(b + 2)*b^3 + sqrt(2)*sqrt(b - 2)*b^3 - sqrt(2)
*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b^2 + sq
rt(2)*sqrt(b + 2)*sqrt(b - 2)*b^2 - 3*sqrt(2)*b^3 + 3*sqrt(2)*sqrt(b^2 - 4)
*sqrt(b + 2)*sqrt(b - 2)*b - sqrt(2)*sqrt(b^2 - 4)*b^2 - 10*sqrt(2)*sqrt(b
+ 2)*b^2 + 2*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + 6*sqrt(2)*sqrt(b - 2)*b^2 - 2*
sqrt(b^2 - 4)*sqrt(b - 2)*b^2 - 2*sqrt(b + 2)*sqrt(b - 2)*b^2 - 4*sqrt(2)*s
qrt(b^2 - 4)*sqrt(b + 2)*b + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b + 4*sqrt
(2)*sqrt(b + 2)*sqrt(b - 2)*b - 24*sqrt(2)*b^2 + 2*sqrt(b^2 - 4)*b^2 + 12*s
qrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2) - 4*sqrt(2)*sqrt(b^2 - 4)*b +
6*sqrt(2)*sqrt(b + 2)*b - 4*sqrt(b^2 - 4)*sqrt(b + 2)*b - 2*sqrt(2)*sqrt(b
- 2)*b + 4*sqrt(b^2 - 4)*sqrt(b - 2)*b + 4*sqrt(b + 2)*sqrt(b - 2)*b - 6*b^
2 - 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2) + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b -
2) + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2) + 6*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b
 - 2) + 12*sqrt(2)*b - 4*sqrt(b^2 - 4)*b - 2*sqrt(b + 2)*b + 2*sqrt(b - 2)*
b - 4*sqrt(2)*sqrt(b^2 - 4) + 20*sqrt(2)*sqrt(b + 2) - 8*sqrt(b^2 - 4)*sqrt
(b + 2) - 4*sqrt(2)*sqrt(b - 2) + 8*sqrt(b^2 - 4)*sqrt(b - 2) + 8*sqrt(b +
2)*sqrt(b - 2) + 48*sqrt(2) - 8*sqrt(b^2 - 4) + 4*sqrt(b + 2) + 4*sqrt(b -
2) + 24)*arctan(x/sqrt(1/2*b - 1/2*sqrt(b^2 - 4)))/(b^4 - 2*b^3 - 7*b^2 + 8
*b + 12)
```

Mupad [B]
time = 1.07, size = 1227, normalized size = 7.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(1/2) - x^2)/(b*x^2 + x^4 + 1),x)
```

```
[Out] atan((x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b
^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*(-(4*2^(1/2)*b^2 -
16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64
*b^2 + 128))^(3/2)*256i + b^2*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^
3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i -
b^4*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6
```

```
 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*(-(4*2^(1/2)*b^2 - 1
6*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(3/2)*128i - b^5*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3
 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3/2)*16i +
2^(1/2)*b*x*(-(4*2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4
 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - 2^(1/2)*b^3*x*(-(4*
2^(1/2)*b^2 - 16*2^(1/2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(2^(1/2)*b^3 - 4*2^(1/2)*b + 2^(1/2)*(
48*b^2 - 12*b^4 + b^6 - 64)^(1/2) + 2*b^2 - 8))*(-(4*2^(1/2)*b^2 - 16*2^(1/
2) - 12*b + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 1
28))^(1/2)*2i - atan((x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b
^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*((12
*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(3/2)*256i + b^2*x*((12*b + 16*2^(1/2) - 4*2^(1/
2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128)
)^(1/2)*8i - b^4*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 -
12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*((12*b +
 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(
8*b^4 - 64*b^2 + 128))^(3/2)*128i - b^5*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b
^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3
/2)*16i + 2^(1/2)*b*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2
 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - 2^(1/2)*b^
3*x*((12*b + 16*2^(1/2) - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 -
64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(4*2^(1/2)*b - 2^(1/2)*b^3 + 2
^(1/2)*(48*b^2 - 12*b^4 + b^6 - 64)^(1/2) - 2*b^2 + 8))*((12*b + 16*2^(1/2)
 - 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(1/2)*2i
```

3.105 $\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx$

Optimal. Leaf size=160

$$-\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}}+\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}}+\frac{\left(1-\sqrt{2}\right)\log\left(1-\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}}$$

```
[Out] 1/4*ln(1+x^2-x*(2-b)^(1/2))*(1-2^(1/2))/(2-b)^(1/2)-1/4*ln(1+x^2+x*(2-b)^(1
/2))*(1-2^(1/2))/(2-b)^(1/2)-1/2*arctan((-2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1+
2^(1/2))/(2+b)^(1/2)+1/2*arctan((2*x+(2-b)^(1/2))/(2+b)^(1/2))*(1+2^(1/2))/
(2+b)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1183, 648, 632, 210, 642}

$$-\frac{\left(1+\sqrt{2}\right)\text{ArcTan}\left(\frac{\sqrt{2-b}-2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}+\frac{\left(1+\sqrt{2}\right)\text{ArcTan}\left(\frac{\sqrt{2-b}+2x}{\sqrt{b+2}}\right)}{2\sqrt{b+2}}+\frac{\left(1-\sqrt{2}\right)\log\left(-\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}-\frac{\left(1-\sqrt{2}\right)\log\left(\sqrt{2-b}\,x+x^2+1\right)}{4\sqrt{2-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] -1/2*((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] - 2*x)/Sqrt[2 + b]])/Sqrt[2 + b] +
((1 + Sqrt[2])*ArcTan[(Sqrt[2 - b] + 2*x)/Sqrt[2 + b]])/(2*Sqrt[2 + b]) + (
(1 - Sqrt[2])*Log[1 - Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b]) - ((1 - Sqrt[2]
)*Log[1 + Sqrt[2 - b]*x + x^2])/(4*Sqrt[2 - b])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt{2}+x^2}{1+bx^2+x^4}\,dx = \frac{\int \frac{\sqrt{2}\sqrt{2-b}-\left(-1+\sqrt{2}\right)x}{1-\sqrt{2-b}\,x+x^2}\,dx}{2\sqrt{2-b}} + \frac{\int \frac{\sqrt{2}\sqrt{2-b}+\left(-1+\sqrt{2}\right)x}{1+\sqrt{2-b}\,x+x^2}\,dx}{2\sqrt{2-b}}$$

$$= \frac{1}{4}\left(1+\sqrt{2}\right)\int \frac{1}{1-\sqrt{2-b}\,x+x^2}\,dx + \frac{1}{4}\left(1+\sqrt{2}\right)\int \frac{1}{1+\sqrt{2-b}\,x+x^2}\,dx + \frac{\left(1-\right.}{}$$

$$= \frac{\left(1-\sqrt{2}\right)\log\left(1-\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}} - \frac{\left(1-\sqrt{2}\right)\log\left(1+\sqrt{2-b}\,x+x^2\right)}{4\sqrt{2-b}} + \frac{1}{2}\left(-1\right.$$

$$= -\frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}-2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} + \frac{\left(1+\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{2-b}+2x}{\sqrt{2+b}}\right)}{2\sqrt{2+b}} + \frac{\left(1-\sqrt{2}\right)\mathrm{l}}{}$$

Mathematica [A]
time = 0.03, size = 136, normalized size = 0.85

$$\frac{\frac{\left(2\sqrt{2}-b+\sqrt{-4+b^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b-\sqrt{-4+b^2}}}\right)}{\sqrt{b-\sqrt{-4+b^2}}} + \frac{\left(-2\sqrt{2}+b+\sqrt{-4+b^2}\right)\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b+\sqrt{-4+b^2}}}\right)}{\sqrt{b+\sqrt{-4+b^2}}}}{\sqrt{2}\sqrt{-4+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] + x^2)/(1 + b*x^2 + x^4),x]
```

```
[Out] (((2*Sqrt[2] - b + Sqrt[-4 + b^2])*ArcTan[(Sqrt[2]*x)/Sqrt[b - Sqrt[-4 + b^
2]]])/Sqrt[b - Sqrt[-4 + b^2]] + ((-2*Sqrt[2] + b + Sqrt[-4 + b^2])*ArcTan[
(Sqrt[2]*x)/Sqrt[b + Sqrt[-4 + b^2]]])/Sqrt[b + Sqrt[-4 + b^2]])/(Sqrt[2]*S
qrt[-4 + b^2])
```

Maple [A]
time = 0.06, size = 132, normalized size = 0.82

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+\_Z^2 b+1\right)}\frac{\left(\_R^2+\sqrt{2}\right)\ln\left(x-\_R\right)}{2\_R^3+\_R b}\right)}{2}$
default	$\frac{\left(\sqrt{(b-2)(2+b)}+b-2\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{(b-2)(2+b)}+2b}}\right)}{\sqrt{(b-2)(2+b)}\sqrt{2\sqrt{(b-2)(2+b)}+2b}}+\frac{\left(\sqrt{(b-2)(2+b)}-b+2\sqrt{2}\right)}{\sqrt{(b-2)(2+b)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2^(1/2))/(x^4+b*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] (((b-2)*(2+b))^(1/2)+b-2*2^(1/2))/((b-2)*(2+b))^(1/2)/(2*((b-2)*(2+b))^(1/2
)+2*b)^(1/2)*arctan(2*x/(2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))+(((b-2)*(2+b))^(
1/2)-b+2*2^(1/2))/((b-2)*(2+b))^(1/2)/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2)*ar
ctan(2*x/(-2*((b-2)*(2+b))^(1/2)+2*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + sqrt(2))/(x^4 + b*x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(123) = 246.
time = 0.38, size = 513, normalized size = 3.21

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3b-4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(2(2b^2-9)x+\sqrt{\frac{1}{2}}\left(2b^3+3\sqrt{2}(b^2-4)-8b-\frac{2b^4-14b^2+\sqrt{2}(b^3-4b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3b-4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3b-4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\log\left(2(2b^2-9)x-\sqrt{\frac{1}{2}}\left(2b^3+3\sqrt{2}(b^2-4)-8b-\frac{2b^4-14b^2+\sqrt{2}(b^3-4b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3b-4\sqrt{2}+\sqrt{b^2-4}}{b^2-4}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3b-4\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\left(2(2b^2-9)x+\sqrt{\frac{1}{2}}\left(2b^3+3\sqrt{2}(b^2-4)-8b+\frac{2b^4-14b^2+\sqrt{2}(b^3-4b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3b-4\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{3b-4\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\log\left(2(2b^2-9)x-\sqrt{\frac{1}{2}}\left(2b^3+3\sqrt{2}(b^2-4)-8b+\frac{2b^4-14b^2+\sqrt{2}(b^3-4b)+24}{\sqrt{b^2-4}}\right)\sqrt{-\frac{3b-4\sqrt{2}-\sqrt{b^2-4}}{b^2-4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b
^2 - 9)*x + sqrt(1/2)*(2*b^3 + 3*sqrt(2)*(b^2 - 4) - 8*b - (2*b^4 - 14*b^2
+ sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^
2 - 4))/(b^2 - 4))) - 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sqrt(2) + sqrt(b^2 - 4))
/(b^2 - 4))*log(2*(2*b^2 - 9)*x - sqrt(1/2)*(2*b^3 + 3*sqrt(2)*(b^2 - 4) -
8*b - (2*b^4 - 14*b^2 + sqrt(2)*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b
 - 4*sqrt(2) + sqrt(b^2 - 4))/(b^2 - 4))) + 1/2*sqrt(1/2)*sqrt(-(3*b - 4*sq
rt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*x + sqrt(1/2)*(2*b^3 +
3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*b^2 + sqrt(2)*(b^3 - 4*b) + 24)/sqr
t(b^2 - 4))*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))) - 1/2*sqrt(
1/2)*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(b^2 - 4))*log(2*(2*b^2 - 9)*x
 - sqrt(1/2)*(2*b^3 + 3*sqrt(2)*(b^2 - 4) - 8*b + (2*b^4 - 14*b^2 + sqrt(2)
*(b^3 - 4*b) + 24)/sqrt(b^2 - 4))*sqrt(-(3*b - 4*sqrt(2) - sqrt(b^2 - 4))/(
b^2 - 4)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(128) = 256$.
time = 1.44, size = 1467, normalized size = 9.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2**(1/2))/(x**4+b*x**2+1),x)
```

```
[Out] RootSum(_t**4*(16*b**4 - 128*b**2 + 256) + _t**2*(12*b**3 - 16*sqrt(2)*b**2
 - 48*b + 64*sqrt(2)) + 2*b**2 - 6*sqrt(2)*b + 9, Lambda(_t, _t*log(_t**3*(
64*b**12/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b
**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729)
- 672*sqrt(2)*b**11/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b*
*7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b
**2 + 729) + 5760*b**10/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2
)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 34
02*b**2 + 729) - 12064*sqrt(2)*b**9/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 -
 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(
2)*b**3 - 3402*b**2 + 729) + 17744*b**8/(8*b**10 - 88*sqrt(2)*b**9 + 828*b*
*8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*s
qrt(2)*b**3 - 3402*b**2 + 729) + 27480*sqrt(2)*b**7/(8*b**10 - 88*sqrt(2)*b
**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b
**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 154608*b**6/(8*b**10 - 88*sqrt
(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2
781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) + 141376*sqrt(2)*b**5/(8*b*
*10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310*sqr
t(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 69072*b**4/(
8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470*b**6 - 5310
```

```
*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729) - 61704*sq
rt(2)*b**3/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 + 6470
*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 + 729
) + 78192*b**2/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*b**7 +
6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402*b**2 +
 729) + 2592*sqrt(2)*b/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)
*b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 340
2*b**2 + 729) - 15552/(8*b**10 - 88*sqrt(2)*b**9 + 828*b**8 - 2144*sqrt(2)*
b**7 + 6470*b**6 - 5310*sqrt(2)*b**5 + 2781*b**4 + 2322*sqrt(2)*b**3 - 3402
*b**2 + 729)) + _t*(16*b**7/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt
(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) - 116*sqrt(2)*b**6/(4*b**6 - 28*sq
rt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) +
668*b**5/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2
 + 27*sqrt(2)*b - 81) - 942*sqrt(2)*b**4/(4*b**6 - 28*sqrt(2)*b**5 + 152*b*
*4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) + 1226*b**3/(4*b**6 -
 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b -
81) - 144*sqrt(2)*b**2/(4*b**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b
**3 + 189*b**2 + 27*sqrt(2)*b - 81) - 378*b/(4*b**6 - 28*sqrt(2)*b**5 + 152
*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)*b - 81) + 108*sqrt(2)/(4*b
**6 - 28*sqrt(2)*b**5 + 152*b**4 - 192*sqrt(2)*b**3 + 189*b**2 + 27*sqrt(2)
*b - 81)) + x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(123) = 246$.
time = 5.18, size = 1501, normalized size = 9.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2^(1/2))/(x^4+b*x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b + 2)*b^4 + sqrt(2)*sqrt(b - 2)*b^4 - sqrt(2)*sqrt(b^2 -
 4)*sqrt(b + 2)*b^3 - sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b^3 - sqrt(2)*sqrt(
b + 2)*sqrt(b - 2)*b^3 - 3*sqrt(2)*b^4 + 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2
)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*b^3 - sqrt(2)*sqrt(b + 2)*b^3 - s
qrt(2)*sqrt(b - 2)*b^3 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + sqrt(2)*sq
rt(b^2 - 4)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b^2 + 3*sqrt(
2)*b^3 - 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2)*b - sqrt(2)*sqrt(b
^2 - 4)*b^2 - 10*sqrt(2)*sqrt(b + 2)*b^2 + 2*sqrt(b^2 - 4)*sqrt(b + 2)*b^2
- 6*sqrt(2)*sqrt(b - 2)*b^2 + 2*sqrt(b^2 - 4)*sqrt(b - 2)*b^2 + 2*sqrt(b +
2)*sqrt(b - 2)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b + 4*sqrt(2)*sqrt
(b^2 - 4)*sqrt(b - 2)*b + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b + 24*sqrt(2)*
b^2 - 2*sqrt(b^2 - 4)*b^2 - 12*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2
) - 4*sqrt(2)*sqrt(b^2 - 4)*b + 6*sqrt(2)*sqrt(b + 2)*b - 4*sqrt(b^2 - 4)*s
qrt(b + 2)*b + 2*sqrt(2)*sqrt(b - 2)*b - 4*sqrt(b^2 - 4)*sqrt(b - 2)*b - 4*
```

```
sqrt(b + 2)*sqrt(b - 2)*b - 6*b^2 + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2) + 4
*sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2) + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2) + 6*
sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2) - 12*sqrt(2)*b + 4*sqrt(b^2 - 4)*b +
2*sqrt(b + 2)*b + 2*sqrt(b - 2)*b - 4*sqrt(2)*sqrt(b^2 - 4) + 20*sqrt(2)*sq
rt(b + 2) - 8*sqrt(b^2 - 4)*sqrt(b + 2) + 4*sqrt(2)*sqrt(b - 2) - 8*sqrt(b^
2 - 4)*sqrt(b - 2) - 8*sqrt(b + 2)*sqrt(b - 2) - 48*sqrt(2) + 8*sqrt(b^2 -
4) - 4*sqrt(b + 2) + 4*sqrt(b - 2) + 24)*arctan(x/sqrt(1/2*b + 1/2*sqrt(b^2
 - 4)))/(b^4 - 2*b^3 - 7*b^2 + 8*b + 12) + 1/4*(sqrt(2)*sqrt(b + 2)*b^4 - s
qrt(2)*sqrt(b - 2)*b^4 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*b^3 - sqrt(2)*sq
rt(b^2 - 4)*sqrt(b - 2)*b^3 - sqrt(2)*sqrt(b + 2)*sqrt(b - 2)*b^3 + 3*sqrt(
2)*b^4 - 3*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2)*b^2 + sqrt(2)*sqrt
(b^2 - 4)*b^3 - sqrt(2)*sqrt(b + 2)*b^3 + sqrt(2)*sqrt(b - 2)*b^3 - sqrt(2)
*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b^2 + sq
rt(2)*sqrt(b + 2)*sqrt(b - 2)*b^2 - 3*sqrt(2)*b^3 + 3*sqrt(2)*sqrt(b^2 - 4)
*sqrt(b + 2)*sqrt(b - 2)*b - sqrt(2)*sqrt(b^2 - 4)*b^2 - 10*sqrt(2)*sqrt(b
+ 2)*b^2 - 2*sqrt(b^2 - 4)*sqrt(b + 2)*b^2 + 6*sqrt(2)*sqrt(b - 2)*b^2 + 2*
sqrt(b^2 - 4)*sqrt(b - 2)*b^2 + 2*sqrt(b + 2)*sqrt(b - 2)*b^2 - 4*sqrt(2)*s
qrt(b^2 - 4)*sqrt(b + 2)*b + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b - 2)*b + 4*sqrt
(2)*sqrt(b + 2)*sqrt(b - 2)*b - 24*sqrt(2)*b^2 - 2*sqrt(b^2 - 4)*b^2 + 12*s
qrt(2)*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b - 2) - 4*sqrt(2)*sqrt(b^2 - 4)*b +
6*sqrt(2)*sqrt(b + 2)*b + 4*sqrt(b^2 - 4)*sqrt(b + 2)*b - 2*sqrt(2)*sqrt(b
- 2)*b - 4*sqrt(b^2 - 4)*sqrt(b - 2)*b - 4*sqrt(b + 2)*sqrt(b - 2)*b + 6*b^
2 - 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b + 2) + 4*sqrt(2)*sqrt(b^2 - 4)*sqrt(b -
2) + 4*sqrt(2)*sqrt(b + 2)*sqrt(b - 2) - 6*sqrt(b^2 - 4)*sqrt(b + 2)*sqrt(b
 - 2) + 12*sqrt(2)*b + 4*sqrt(b^2 - 4)*b + 2*sqrt(b + 2)*b - 2*sqrt(b - 2)*
b - 4*sqrt(2)*sqrt(b^2 - 4) + 20*sqrt(2)*sqrt(b + 2) + 8*sqrt(b^2 - 4)*sqrt
(b + 2) - 4*sqrt(2)*sqrt(b - 2) - 8*sqrt(b^2 - 4)*sqrt(b - 2) - 8*sqrt(b +
2)*sqrt(b - 2) + 48*sqrt(2) + 8*sqrt(b^2 - 4) - 4*sqrt(b + 2) - 4*sqrt(b -
2) - 24)*arctan(x/sqrt(1/2*b - 1/2*sqrt(b^2 - 4)))/(b^4 - 2*b^3 - 7*b^2 + 8
*b + 12)
```

Mupad [B]
time = 5.25, size = 1227, normalized size = 7.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(1/2) + x^2)/(b*x^2 + x^4 + 1),x)
```

```
[Out] atan((x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b
^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*(-(16*2^(1/2) - 12*
b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64
*b^2 + 128))^(3/2)*256i + b^2*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^
3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i -
b^4*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6
```

```
 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*(-(16*2^(1/2) - 12*b
 - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(3/2)*128i - b^5*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3
 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3/2)*16i -
2^(1/2)*b*x*(-(16*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4
 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i + 2^(1/2)*b^3*x*(-(16
*2^(1/2) - 12*b - 4*2^(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(2^(1/2)*b^3 - 4*2^(1/2)*b + 2^(1/2)*(
48*b^2 - 12*b^4 + b^6 - 64)^(1/2) - 2*b^2 + 8))*(-(16*2^(1/2) - 12*b - 4*2^
(1/2)*b^2 + 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 1
28))^(1/2)*2i - atan((x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b
^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i - b*x*((12
*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2
))/(8*b^4 - 64*b^2 + 128))^(3/2)*256i + b^2*x*((12*b - 16*2^(1/2) + 4*2^(1/
2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128)
)^(1/2)*8i - b^4*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 -
12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*4i + b^3*x*((12*b -
 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(
8*b^4 - 64*b^2 + 128))^(3/2)*128i - b^5*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b
^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(3
/2)*16i - 2^(1/2)*b*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2
 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*32i + 2^(1/2)*b^
3*x*((12*b - 16*2^(1/2) + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 -
64)^(1/2))/(8*b^4 - 64*b^2 + 128))^(1/2)*8i)/(4*2^(1/2)*b - 2^(1/2)*b^3 + 2
^(1/2)*(48*b^2 - 12*b^4 + b^6 - 64)^(1/2) + 2*b^2 - 8))*((12*b - 16*2^(1/2)
 + 4*2^(1/2)*b^2 - 3*b^3 + (48*b^2 - 12*b^4 + b^6 - 64)^(1/2))/(8*b^4 - 64*
b^2 + 128))^(1/2)*2i
```

3.106 $\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx$

Optimal. Leaf size=114

$$-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{3}\log\left(a-\sqrt{3}\sqrt{a}\,x+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(a+\sqrt{3}\sqrt{a}\,x+x^2\right)}{4\sqrt{a}}$$

```
[Out] 1/2*arctan(-3^(1/2)+2*x/a^(1/2))/a^(1/2)+1/2*arctan(3^(1/2)+2*x/a^(1/2))/a^
(1/2)-1/4*ln(a+x^2-x*3^(1/2)*a^(1/2))*3^(1/2)/a^(1/2)+1/4*ln(a+x^2+x*3^(1/2
)*a^(1/2))*3^(1/2)/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1183, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\text{ArcTan}\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2\sqrt{a}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] -1/2*ArcTan[Sqrt[3] - (2*x)/Sqrt[a]]/Sqrt[a] + ArcTan[Sqrt[3] + (2*x)/Sqrt[
a]]/(2*Sqrt[a]) - (Sqrt[3]*Log[a - Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a]) +
(Sqrt[3]*Log[a + Sqrt[3]*Sqrt[a]*x + x^2])/(4*Sqrt[a])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2a-x^2}{a^2-ax^2+x^4}\,dx = \frac{\int \frac{2\sqrt{3}\,a^{3/2}-3ax}{a-\sqrt{3}\,\sqrt{a}\,x+x^2}\,dx}{2\sqrt{3}\,a^{3/2}} + \frac{\int \frac{2\sqrt{3}\,a^{3/2}+3ax}{a+\sqrt{3}\,\sqrt{a}\,x+x^2}\,dx}{2\sqrt{3}\,a^{3/2}}$$

$$= \frac{1}{4}\int \frac{1}{a-\sqrt{3}\,\sqrt{a}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{a+\sqrt{3}\,\sqrt{a}\,x+x^2}\,dx - \frac{\sqrt{3}\int \frac{-\sqrt{3}\,\sqrt{a}+2x}{a-\sqrt{3}\,\sqrt{a}\,x+x^2}\,d}{4\sqrt{a}}$$

$$= -\frac{\sqrt{3}\,\log\left(a-\sqrt{3}\,\sqrt{a}\,x+x^2\right)}{4\sqrt{a}} + \frac{\sqrt{3}\,\log\left(a+\sqrt{3}\,\sqrt{a}\,x+x^2\right)}{4\sqrt{a}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\right.}{2}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{3}\,\log\left(a-\sqrt{3}\,\sqrt{a}\,x+x^2\right)}{4\sqrt{a}} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 115, normalized size = 1.01

$$\frac{\sqrt[4]{-1}\left(-\sqrt{i+\sqrt{3}}\,\left(3i+\sqrt{3}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\,\sqrt{a}}\right)+\sqrt{-i+\sqrt{3}}\,\left(-3i+\sqrt{3}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\,\sqrt{a}}\right)\right)}{2\sqrt{6}\,\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*a - x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[-
I + Sqrt[3]]*Sqrt[a])]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1 +
 I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])]))/(2*Sqrt[6]*Sqrt[a])
```

Maple [A]
time = 0.04, size = 90, normalized size = 0.79

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-a\_Z^{2}+a^{2}\right)}\frac{\left(-\_R^{2}+2a\right)\ln\left(x-\_R\right)}{2\_R^{3}-\_Ra}\right)}{2}$	48
default	$\frac{-\frac{\sqrt{3}\,\ln\left(x\sqrt{3}\,\sqrt{a}-x^{2}-a\right)}{2}-\arctan\left(\frac{\sqrt{3}\,\sqrt{a}-2x}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\frac{\sqrt{3}\,\ln\left(a+x^{2}+x\sqrt{3}\,\sqrt{a}\right)}{2}+\arctan\left(\frac{2x+\sqrt{3}\,\sqrt{a}}{\sqrt{a}}\right)}{2\sqrt{a}}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*a)/(x^4-a*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^(1/2)*(-1/2*3^(1/2)*ln(x*3^(1/2)*a^(1/2)-x^2-a)-arctan((3^(1/2)*a^(1/
2)-2*x)/a^(1/2)))+1/2/a^(1/2)*(1/2*3^(1/2)*ln(a+x^2+x*3^(1/2)*a^(1/2))+arct
an((2*x+3^(1/2)*a^(1/2))/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*a)/(x^4 - a*x^2 + a^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(80) = 160.
time = 0.39, size = 512, normalized size = 4.49

$$\frac{1}{24}\left(\sqrt{3}a\sqrt{\frac{1}{a^{2}}}+2\sqrt{3}\right)\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{1}{4}}\log\left(a^{2}\sqrt{\frac{1}{a^{2}}}+x^{2}+\frac{1}{6}\left(\sqrt{3}a^{2}\sqrt{\frac{1}{a^{2}}}x+2\sqrt{3}ax\right)\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{1}{4}}\right)-\frac{1}{24}\left(\sqrt{3}a\sqrt{\frac{1}{a^{2}}}+2\sqrt{3}\right)\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{1}{4}}\log\left(a^{2}\sqrt{\frac{1}{a^{2}}}+x^{2}-\frac{1}{6}\left(\sqrt{3}a^{2}\sqrt{\frac{1}{a^{2}}}x+2\sqrt{3}ax\right)\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{1}{4}}\right)-\frac{1}{2}\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{1}{4}}\arctan\left(\frac{1}{18}\left(\sqrt{6}a^{2}\sqrt{\frac{1}{a^{2}}}+2\sqrt{6}a\right)\sqrt{6a^{2}\sqrt{\frac{1}{a^{2}}}+6x^{2}+\left(\sqrt{3}a^{2}\sqrt{\frac{1}{a^{2}}}x+2\sqrt{3}ax\right)\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{1}{4}}}\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{3}{4}}-\frac{1}{3}\left(a^{2}\sqrt{\frac{1}{a^{2}}}x+2ax\right)\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{3}{4}}-\frac{1}{3}\sqrt{3}a\sqrt{\frac{1}{a^{2}}}-\frac{2}{3}\sqrt{3}\right)-\frac{1}{2}\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{1}{4}}\arctan\left(\frac{1}{18}\left(\sqrt{6}a^{2}\sqrt{\frac{1}{a^{2}}}+2\sqrt{6}a\right)\sqrt{6a^{2}\sqrt{\frac{1}{a^{2}}}+6x^{2}-\left(\sqrt{3}a^{2}\sqrt{\frac{1}{a^{2}}}x+2\sqrt{3}ax\right)\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{1}{4}}}\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{3}{4}}-\frac{1}{3}\left(a^{2}\sqrt{\frac{1}{a^{2}}}x+2ax\right)\sqrt{-4a\sqrt{\frac{1}{a^{2}}}+8}\,\frac{1}{a^{2}}^{\frac{3}{4}}+\frac{1}{3}\sqrt{3}a\sqrt{\frac{1}{a^{2}}}+\frac{2}{3}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(3)*a*sqrt(a^(-2)) + 2*sqrt(3))*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-
2))^(1/4)*log(a^2*sqrt(a^(-2)) + x^2 + 1/6*(sqrt(3)*a^2*sqrt(a^(-2))*x + 2*
sqrt(3)*a*x)*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4)) - 1/24*(sqrt(3)*a*
sqrt(a^(-2)) + 2*sqrt(3))*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4)*log(a^
2*sqrt(a^(-2)) + x^2 - 1/6*(sqrt(3)*a^2*sqrt(a^(-2))*x + 2*sqrt(3)*a*x)*sqr
t(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4)) - 1/2*sqrt(-4*a*sqrt(a^(-2)) + 8)*
(a^(-2))^(1/4)*arctan(1/18*(sqrt(6)*a^2*sqrt(a^(-2)) + 2*sqrt(6)*a)*sqrt(6*
```

```
a^2*sqrt(a^(-2)) + 6*x^2 + (sqrt(3)*a^2*sqrt(a^(-2))*x + 2*sqrt(3)*a*x)*sqr
t(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4))*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2
))^(3/4) - 1/3*(a^2*sqrt(a^(-2))*x + 2*a*x)*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^
(-2))^(3/4) - 1/3*sqrt(3)*a*sqrt(a^(-2)) - 2/3*sqrt(3)) - 1/2*sqrt(-4*a*sqr
t(a^(-2)) + 8)*(a^(-2))^(1/4)*arctan(1/18*(sqrt(6)*a^2*sqrt(a^(-2)) + 2*sqr
t(6)*a)*sqrt(6*a^2*sqrt(a^(-2)) + 6*x^2 - (sqrt(3)*a^2*sqrt(a^(-2))*x + 2*s
qrt(3)*a*x)*sqrt(-4*a*sqrt(a^(-2)) + 8)*(a^(-2))^(1/4))*sqrt(-4*a*sqrt(a^(-
2)) + 8)*(a^(-2))^(3/4) - 1/3*(a^2*sqrt(a^(-2))*x + 2*a*x)*sqrt(-4*a*sqrt(a
^(-2)) + 8)*(a^(-2))^(3/4) + 1/3*sqrt(3)*a*sqrt(a^(-2)) + 2/3*sqrt(3))
```

Sympy [**A**]
time = 0.10, size = 27, normalized size = 0.24

$$-\operatorname{RootSum}\left(16t^4a^2 - 4t^2a + 1, \left(t \mapsto t\log\left(-2ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*a)/(x**4-a*x**2+a**2),x)
```

```
[Out] -RootSum(16*_t**4*a**2 - 4*_t**2*a + 1, Lambda(_t, _t*log(-2*_t*a + x)))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a)/(x^4-a*x^2+a^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-16,[2,0]%%%}+%%%{-4,[0,1]%%%},0,%%%{64,[4,0]
%%%}+%%%{8,[2,2]%%%}+%%%{16,[2,1]%%%}+%%%{6,[0,2]%%%},0,%%%{-64,[4,2]%%%}+%
%%{-128,[4,1]%%%
```

Mupad [**B**]
time = 4.48, size = 133, normalized size = 1.17

$$-\frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{\frac{1}{8\,a}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8\,a}}\,1{}\mathrm{i}+\sqrt{3}\,x\,\sqrt{\frac{1}{8\,a}+\frac{\sqrt{3}\,1{}\mathrm{i}}{8\,a}}\right)\,\sqrt{\frac{1+\sqrt{3}\,1{}\mathrm{i}}{a}}\,1{}\mathrm{i}}{4}-\frac{\sqrt{8}\,\mathrm{atan}\left(x\,\sqrt{\frac{1}{8\,a}-\frac{\sqrt{3}\,1{}\mathrm{i}}{8\,a}}\,1{}\mathrm{i}-\sqrt{3}\,x\,\sqrt{\frac{1}{8\,a}-\frac{\sqrt{3}\,1{}\mathrm{i}}{8\,a}}\right)\,\sqrt{-\frac{-1+\sqrt{3}\,1{}\mathrm{i}}{a}}\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a - x^2)/(a^2 - a*x^2 + x^4),x)
```

```
[Out] - (8^(1/2)*atan(x*((3^(1/2)*1i)/(8*a) + 1/(8*a))^(1/2)*1i + 3^(1/2)*x*((3^(
1/2)*1i)/(8*a) + 1/(8*a))^(1/2))*((3^(1/2)*1i + 1)/a)^(1/2)*1i)/4 - (8^(1/2
)*atan(x*(1/(8*a) - (3^(1/2)*1i)/(8*a))^(1/2)*1i - 3^(1/2)*x*(1/(8*a) - (3^
(1/2)*1i)/(8*a))^(1/2))*(-(3^(1/2)*1i - 1)/a)^(1/2)*1i)/4
```

3.107 $\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}\,x^2+x^4}\,dx$

Optimal. Leaf size=122

$$-\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}-\frac{\sqrt{3}\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}\,x+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{a}+\sqrt{3}\sqrt[4]{a}\right.}{4\sqrt[4]{a}}$$

```
[Out] 1/2*arctan(2*x/a^(1/4)-3^(1/2))/a^(1/4)+1/2*arctan(2*x/a^(1/4)+3^(1/2))/a^(
1/4)-1/4*ln(x^2-a^(1/4)*x*3^(1/2)+a^(1/2))*3^(1/2)/a^(1/4)+1/4*ln(x^2+a^(1/
4)*x*3^(1/2)+a^(1/2))*3^(1/2)/a^(1/4)
```

Rubi [A]
time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1183, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}}+\frac{\text{ArcTan}\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2\sqrt[4]{a}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[4]{a}\,x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}+\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[4]{a}\,x+\sqrt{a}+x^2\right)}{4\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] -1/2*ArcTan[Sqrt[3] - (2*x)/a^(1/4)]/a^(1/4) + ArcTan[Sqrt[3] + (2*x)/a^(1/
4)]/(2*a^(1/4)) - (Sqrt[3]*Log[Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/
4)) + (Sqrt[3]*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*a^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2\sqrt{a}-x^2}{a-\sqrt{a}\,x^2+x^4}\,dx = \frac{\int \frac{2\sqrt{3}\,a^{3/4}-3\sqrt{a}\,x}{\sqrt{a}-\sqrt{3}\,\sqrt[4]{a}\,x+x^2}\,dx}{2\sqrt{3}\,a^{3/4}} + \frac{\int \frac{2\sqrt{3}\,a^{3/4}+3\sqrt{a}\,x}{\sqrt{a}+\sqrt{3}\,\sqrt[4]{a}\,x+x^2}\,dx}{2\sqrt{3}\,a^{3/4}}$$

$$= \frac{1}{4}\int \frac{1}{\sqrt{a}-\sqrt{3}\,\sqrt[4]{a}\,x+x^2}\,dx + \frac{1}{4}\int \frac{1}{\sqrt{a}+\sqrt{3}\,\sqrt[4]{a}\,x+x^2}\,dx - \frac{\sqrt{3}\int \frac{-\sqrt{3}}{\sqrt{a}-\sqrt{3}}}{4\sqrt[4]{a}}$$

$$= -\frac{\sqrt{3}\,\log\left(\sqrt{a}-\sqrt{3}\,\sqrt[4]{a}\,x+x^2\right)}{4\sqrt[4]{a}} + \frac{\sqrt{3}\,\log\left(\sqrt{a}+\sqrt{3}\,\sqrt[4]{a}\,x+x^2\right)}{4\sqrt[4]{a}} + \frac{\text{Subst}(}{}$$

$$= -\frac{\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} + \frac{\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}} - \frac{\sqrt{3}\,\log\left(\sqrt{a}-\sqrt{3}\,\sqrt[4]{a}\,x+x\right.}{4\sqrt[4]{a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 115, normalized size = 0.94

$$\frac{\sqrt[4]{-1}\left(-\sqrt{i+\sqrt{3}}\left(3i+\sqrt{3}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\,\sqrt[4]{a}}\right)+\sqrt{-i+\sqrt{3}}\left(-3i+\sqrt{3}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\,\sqrt[4]{a}}\right)\right)}{2\sqrt{6}\,\sqrt[4]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[a] - x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(-(Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[-
I + Sqrt[3]]*a^(1/4))]) + Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTanh[((1 +
 I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))]))/(2*Sqrt[6]*a^(1/4))
```

Maple [A]
time = 0.06, size = 94, normalized size = 0.77

method	result	size
default	$\frac{-\frac{\sqrt{3}\,\ln\left(a^{\frac{1}{4}} x \sqrt{3}-x^{2}-\sqrt{a}\right)}{2}-\arctan\left(\frac{a^{\frac{1}{4}} \sqrt{3}-2 x}{a^{\frac{1}{4}}}\right)}{2 a^{\frac{1}{4}}}+\frac{\frac{\sqrt{3}\,\ln\left(x^{2}+a^{\frac{1}{4}} x \sqrt{3}+\sqrt{a}\right)}{2}+\arctan\left(\frac{2 x+a^{\frac{1}{4}} \sqrt{3}}{a^{\frac{1}{4}}}\right)}{2 a^{\frac{1}{4}}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^(1/4)*(-1/2*3^(1/2)*ln(a^(1/4)*x*3^(1/2)-x^2-a^(1/2))-arctan((a^(1/4)
*3^(1/2)-2*x)/a^(1/4)))+1/2/a^(1/4)*(1/2*3^(1/2)*ln(x^2+a^(1/4)*x*3^(1/2)+a
^(1/2))+arctan((2*x+a^(1/4)*3^(1/2))/a^(1/4)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*sqrt(a))/(x^4 - sqrt(a)*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(84) = 168.
time = 0.34, size = 251, normalized size = 2.06

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}\log\left(-\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a}}+\sqrt{a}}{a}}+x\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}\log\left(\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{-\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}\log\left(-\sqrt{\frac{1}{2}}\sqrt{a}\sqrt{-\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a}}-\sqrt{a}}{a}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a)*log(sqrt(1/2)*sqrt(a
)*sqrt((sqrt(3)*a*sqrt(-1/a) + sqrt(a))/a) + x) - 1/2*sqrt(1/2)*sqrt((sqrt(
3)*a*sqrt(-1/a) + sqrt(a))/a)*log(-sqrt(1/2)*sqrt(a)*sqrt((sqrt(3)*a*sqrt(-
1/a) + sqrt(a))/a) + x) + 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(
a))/a)*log(sqrt(1/2)*sqrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + x)
 - 1/2*sqrt(1/2)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a)*log(-sqrt(1/2)*s
qrt(a)*sqrt(-(sqrt(3)*a*sqrt(-1/a) - sqrt(a))/a) + x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*a**(1/2))/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(64*_t**4*a - 16*_t**2*sqrt(a) + 1)
contains an element of the set of generators.
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*a^(1/2))/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 5.06, size = 159, normalized size = 1.30

$$2\,\mathrm{atanh}\left(x\,\sqrt{\frac{1}{8\,\sqrt{a}}-\frac{\sqrt{-27\,a^3}}{24\,a^2}}-\frac{9\,a^{3/2}\,x\,\sqrt{\frac{1}{8\,\sqrt{a}}-\frac{\sqrt{-27\,a^3}}{24\,a^2}}}{\sqrt{-27\,a^3}}\right)\,\sqrt{\frac{1}{8\,\sqrt{a}}-\frac{\sqrt{-27\,a^3}}{24\,a^2}}+2\,\mathrm{atanh}\left(x\,\sqrt{\frac{\sqrt{-27\,a^3}}{24\,a^2}+\frac{1}{8\,\sqrt{a}}}+\frac{9\,a^{3/2}\,x\,\sqrt{\frac{\sqrt{-27\,a^3}}{24\,a^2}+\frac{1}{8\,\sqrt{a}}}}{\sqrt{-27\,a^3}}\right)\,\sqrt{\frac{\sqrt{-27\,a^3}}{24\,a^2}+\frac{1}{8\,\sqrt{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a^(1/2) - x^2)/(a + x^4 - a^(1/2)*x^2),x)
```

```
[Out] 2*atanh(x*(1/(8*a^(1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2) - (9*a^(3/2)*x*(
1/(8*a^(1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2))/(-27*a^3)^(1/2))*(1/(8*a^(
1/2)) - (-27*a^3)^(1/2)/(24*a^2))^(1/2) + 2*atanh(x*((-27*a^3)^(1/2)/(24*a^
2) + 1/(8*a^(1/2)))^(1/2) + (9*a^(3/2)*x*((-27*a^3)^(1/2)/(24*a^2) + 1/(8*a
^(1/2)))^(1/2))/(-27*a^3)^(1/2))*((-27*a^3)^(1/2)/(24*a^2) + 1/(8*a^(1/2)))
^(1/2)
```

3.108 $\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx$

Optimal. Leaf size=124

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\,x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\,x+x^2\right)}{4\sqrt[3]{b}}$$

```
[Out] -1/4*ln(b^(2/3)-b^(1/3)*x+x^2)/b^(1/3)+1/4*ln(b^(2/3)+b^(1/3)*x+x^2)/b^(1/3
)-1/2*arctan(1/3*(b^(1/3)-2*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)+1/2*arctan(
1/3*(b^(1/3)+2*x)/b^(1/3)*3^(1/2))*3^(1/2)/b^(1/3)
```

Rubi [A]
time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1183, 648, 631, 210, 642}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}}-\frac{\log\left(b^{2/3}-\sqrt[3]{b}\,x+x^2\right)}{4\sqrt[3]{b}}+\frac{\log\left(b^{2/3}+\sqrt[3]{b}\,x+x^2\right)}{4\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

```
[Out] -1/2*(Sqrt[3]*ArcTan[(b^(1/3) - 2*x)/(Sqrt[3]*b^(1/3))])/b^(1/3) + (Sqrt[3]
*ArcTan[(b^(1/3) + 2*x)/(Sqrt[3]*b^(1/3))])/(2*b^(1/3)) - Log[b^(2/3) - b^(
1/3)*x + x^2]/(4*b^(1/3)) + Log[b^(2/3) + b^(1/3)*x + x^2]/(4*b^(1/3))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{2b^{2/3}+x^2}{b^{4/3}+b^{2/3}x^2+x^4}\,dx = \frac{\int \frac{2b-b^{2/3}x}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx}{2b} + \frac{\int \frac{2b+b^{2/3}x}{b^{2/3}+\sqrt[3]{b}\,x+x^2}\,dx}{2b}$$

$$= \frac{3}{4}\int \frac{1}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx + \frac{3}{4}\int \frac{1}{b^{2/3}+\sqrt[3]{b}\,x+x^2}\,dx - \frac{\int \frac{-\sqrt[3]{b}+2x}{b^{2/3}-\sqrt[3]{b}\,x+x^2}\,dx}{4\sqrt[3]{b}} + \frac{\int$$

$$= -\frac{\log\left(b^{2/3}-\sqrt[3]{b}\,x+x^2\right)}{4\sqrt[3]{b}} + \frac{\log\left(b^{2/3}+\sqrt[3]{b}\,x+x^2\right)}{4\sqrt[3]{b}} + \frac{3\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{2\sqrt[3]{b}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{b}+2x}{\sqrt{3}\sqrt[3]{b}}\right)}{2\sqrt[3]{b}} - \frac{\log\left(b^{2/3}-\sqrt[3]{b}\,x+x^2\right)}{4\sqrt[3]{b}} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 115, normalized size = 0.93

$$\frac{\sqrt[4]{-1}\left(\sqrt{-i+\sqrt{3}}\left(-3i+\sqrt{3}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt[3]{b}}\right)-\sqrt{i+\sqrt{3}}\left(3i+\sqrt{3}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\sqrt[3]{b}}\right)\right)}{2\sqrt{6}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*b^(2/3) + x^2)/(b^(4/3) + b^(2/3)*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*(Sqrt[-I + Sqrt[3]]*(-3*I + Sqrt[3])*ArcTan[((1 + I)*x)/(Sqrt[I
 + Sqrt[3]]*b^(1/3))] - Sqrt[I + Sqrt[3]]*(3*I + Sqrt[3])*ArcTanh[((1 + I)*
x)/(Sqrt[-I + Sqrt[3]]*b^(1/3))]))/(2*Sqrt[6]*b^(1/3))
```

Maple [A]
time = 0.06, size = 87, normalized size = 0.70

method	result	size
default	$\frac{\frac{\ln\left(b^{\frac{2}{3}}+b^{\frac{1}{3}}x+x^{2}\right)}{2}+\arctan\left(\frac{\left(b^{\frac{1}{3}}+2x\right)\sqrt{3}}{3b^{\frac{1}{3}}}\right)\sqrt{3}}{2b^{\frac{1}{3}}}+\frac{-\frac{\ln\left(b^{\frac{2}{3}}-b^{\frac{1}{3}}x+x^{2}\right)}{2}+\sqrt{3}\,\arctan\left(\frac{\left(-b^{\frac{1}{3}}+2x\right)\sqrt{3}}{3b^{\frac{1}{3}}}\right)}{2b^{\frac{1}{3}}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b^(1/3)*(1/2*ln(b^(2/3)+b^(1/3)*x+x^2)+arctan(1/3*(b^(1/3)+2*x)/b^(1/3)
*3^(1/2))*3^(1/2))+1/2/b^(1/3)*(-1/2*ln(b^(2/3)-b^(1/3)*x+x^2)+3^(1/2)*arct
an(1/3*(-b^(1/3)+2*x)*3^(1/2)/b^(1/3)))
```

Maxima [A]
time = 0.51, size = 88, normalized size = 0.71

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{2\,b^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{2\,b^{\frac{1}{3}}}+\frac{\log\left(x^{2}+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}}-\frac{\log\left(x^{2}-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/b^(1/3))/b^(1/3) + 1/2*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - b^(1/3))/b^(1/3))/b^(1/3) + 1/4*log(x^2 + b^(1
/3)*x + b^(2/3))/b^(1/3) - 1/4*log(x^2 - b^(1/3)*x + b^(2/3))/b^(1/3)
```

Fricas [A]
time = 0.37, size = 264, normalized size = 2.13

$$\left[\frac{\sqrt{3}\,b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\,\log\left(\frac{2\,x^{3}+\sqrt{3}\,\left(2\,b^{\frac{2}{3}}x^{2}+bx-b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3\,b^{\frac{2}{3}}x-b}{x^{3}+b}\right)+\sqrt{3}\,b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\,\log\left(\frac{2\,x^{3}+\sqrt{3}\,\left(2\,b^{\frac{2}{3}}x^{2}-bx-b^{\frac{4}{3}}\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}-3\,b^{\frac{2}{3}}x+b}{x^{3}-b}\right)+b^{\frac{2}{3}}\log\left(x^{2}+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)-b^{\frac{2}{3}}\log\left(x^{2}-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b},\ \frac{2\,\sqrt{3}\,b^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\,x+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)-2\,\sqrt{3}\,b^{\frac{2}{3}}\arctan\left(-\frac{\sqrt{3}\,\left(2\,x-b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)+b^{\frac{2}{3}}\log\left(x^{2}+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)-b^{\frac{2}{3}}\log\left(x^{2}-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(3)*b*sqrt(-1/b^(2/3))*log((2*x^3 + sqrt(3)*(2*b^(2/3)*x^2 + b*x
 - b^(4/3))*sqrt(-1/b^(2/3)) - 3*b^(2/3)*x - b)/(x^3 + b)) + sqrt(3)*b*sqrt(
-1/b^(2/3))*log((2*x^3 + sqrt(3)*(2*b^(2/3)*x^2 - b*x - b^(4/3))*sqrt(-1/b^
(2/3)) - 3*b^(2/3)*x + b)/(x^3 - b)) + b^(2/3)*log(x^2 + b^(1/3)*x + b^(2/3
)) - b^(2/3)*log(x^2 - b^(1/3)*x + b^(2/3)))/b, 1/4*(2*sqrt(3)*b^(2/3)*arct
an(1/3*sqrt(3)*(2*x + b^(1/3))/b^(1/3)) - 2*sqrt(3)*b^(2/3)*arctan(-1/3*sqr
```

```
t(3)*(2*x - b^(1/3))/b^(1/3)) + b^(2/3)*log(x^2 + b^(1/3)*x + b^(2/3)) - b^
(2/3)*log(x^2 - b^(1/3)*x + b^(2/3)))/b]
```

Sympy [C] Result contains complex when optimal does not.
time = 0.12, size = 143, normalized size = 1.15

$$\frac{\left(-\frac{1}{4}-\frac{\sqrt{3}\,i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}-\frac{\sqrt{3}\,i}{4}\right)+x\right)+\left(-\frac{1}{4}+\frac{\sqrt{3}\,i}{4}\right)\log\left(2\sqrt[3]{b}\left(-\frac{1}{4}+\frac{\sqrt{3}\,i}{4}\right)+x\right)+\left(\frac{1}{4}-\frac{\sqrt{3}\,i}{4}\right)\log\left(2\sqrt[3]{b}\left(\frac{1}{4}-\frac{\sqrt{3}\,i}{4}\right)+x\right)+\left(\frac{1}{4}+\frac{\sqrt{3}\,i}{4}\right)\log\left(2\sqrt[3]{b}\left(\frac{1}{4}+\frac{\sqrt{3}\,i}{4}\right)+x\right)}{\sqrt[3]{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b**(2/3)+x**2)/(b**(4/3)+b**(2/3)*x**2+x**4),x)
```

```
[Out] ((-1/4 - sqrt(3)*I/4)*log(2*b**(1/3)*(-1/4 - sqrt(3)*I/4) + x) + (-1/4 + sq
rt(3)*I/4)*log(2*b**(1/3)*(-1/4 + sqrt(3)*I/4) + x) + (1/4 - sqrt(3)*I/4)*l
og(2*b**(1/3)*(1/4 - sqrt(3)*I/4) + x) + (1/4 + sqrt(3)*I/4)*log(2*b**(1/3)
*(1/4 + sqrt(3)*I/4) + x))/b**(1/3)
```

Giac [A]
time = 4.24, size = 92, normalized size = 0.74

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-b^{\frac{1}{3}}\right)}{3|b|^{\frac{1}{3}}}\right)}{2|b|^{\frac{1}{3}}}+\frac{\log\left(x^2+b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}-\frac{\log\left(x^2-b^{\frac{1}{3}}x+b^{\frac{2}{3}}\right)}{4b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*b^(2/3)+x^2)/(b^(4/3)+b^(2/3)*x^2+x^4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3) +
 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - b^(1/3))/abs(b)^(1/3))/abs(b)^(1/3)
+ 1/4*log(x^2 + b^(1/3)*x + b^(2/3))/b^(1/3) - 1/4*log(x^2 - b^(1/3)*x + b^
(2/3))/b^(1/3)
```

Mupad [B]
time = 0.24, size = 133, normalized size = 1.07

$$\frac{\sqrt{8}\,\mathrm{atan}\left(x\sqrt{-\frac{1}{8b^{2/3}}-\frac{\sqrt{3}\,1\mathrm{i}}{8b^{2/3}}}\,1\mathrm{i}+\sqrt{3}\,x\sqrt{-\frac{1}{8b^{2/3}}-\frac{\sqrt{3}\,1\mathrm{i}}{8b^{2/3}}}\right)\sqrt{-\frac{1+\sqrt{3}\,1\mathrm{i}}{b^{2/3}}}\,1\mathrm{i}}{4}+\frac{\sqrt{8}\,\mathrm{atan}\left(x\sqrt{-\frac{1}{8b^{2/3}}+\frac{\sqrt{3}\,1\mathrm{i}}{8b^{2/3}}}\,1\mathrm{i}-\sqrt{3}\,x\sqrt{-\frac{1}{8b^{2/3}}+\frac{\sqrt{3}\,1\mathrm{i}}{8b^{2/3}}}\right)\sqrt{\frac{-1+\sqrt{3}\,1\mathrm{i}}{b^{2/3}}}\,1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*b^(2/3) + x^2)/(b^(4/3) + x^4 + b^(2/3)*x^2),x)
```

```
[Out] (8^(1/2)*atan(x*(- (3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)))^(1/2)*1i + 3^(
1/2)*x*(- (3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)))^(1/2))*(-(3^(1/2)*1i +
1)/b^(2/3))^(1/2)*1i)/4 + (8^(1/2)*atan(x*((3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*
b^(2/3)))^(1/2)*1i - 3^(1/2)*x*((3^(1/2)*1i)/(8*b^(2/3)) - 1/(8*b^(2/3)))^(
1/2))*((3^(1/2)*1i - 1)/b^(2/3))^(1/2)*1i)/4
```

3.109 $\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx$

Optimal. Leaf size=136

$$-\frac{(A+aB)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(A+aB)\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{(A-aB)\log\left(a-\sqrt{3}\sqrt{a}\,x+x^2\right)}{4\sqrt{3}\,a^{3/2}}+\cdots$$

```
[Out] 1/2*(B*a+A)*arctan(-3^(1/2)+2*x/a^(1/2))/a^(3/2)+1/2*(B*a+A)*arctan(3^(1/2)
+2*x/a^(1/2))/a^(3/2)-1/12*(-B*a+A)*ln(a+x^2-x*3^(1/2)*a^(1/2))/a^(3/2)*3^(
1/2)+1/12*(-B*a+A)*ln(a+x^2+x*3^(1/2)*a^(1/2))/a^(3/2)*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {1183, 648, 631, 210, 642}

$$-\frac{(aB+A)\text{ArcTan}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(aB+A)\text{ArcTan}\left(\frac{2x}{\sqrt{a}}+\sqrt{3}\right)}{2a^{3/2}}-\frac{(A-aB)\log\left(-\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{3}\,a^{3/2}}+\frac{(A-aB)\log\left(\sqrt{3}\sqrt{a}\,x+a+x^2\right)}{4\sqrt{3}\,a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] -1/2*((A + a*B)*ArcTan[Sqrt[3] - (2*x)/Sqrt[a]])/a^(3/2) + ((A + a*B)*ArcTa
n[Sqrt[3] + (2*x)/Sqrt[a]])/(2*a^(3/2)) - ((A - a*B)*Log[a - Sqrt[3]*Sqrt[a
]*x + x^2])/(4*Sqrt[3]*a^(3/2)) + ((A - a*B)*Log[a + Sqrt[3]*Sqrt[a]*x + x^
2])/(4*Sqrt[3]*a^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^2}{a^2-ax^2+x^4}\,dx = \frac{\int \frac{\sqrt{3}\sqrt{a}A-(A-aB)x}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}}+\frac{\int \frac{\sqrt{3}\sqrt{a}A+(A-aB)x}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{2\sqrt{3}a^{3/2}}$$

$$= -\frac{(A-aB)\int \frac{-\sqrt{3}\sqrt{a}+2x}{a-\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{3}a^{3/2}}+\frac{(A-aB)\int \frac{\sqrt{3}\sqrt{a}+2x}{a+\sqrt{3}\sqrt{a}x+x^2}\,dx}{4\sqrt{3}a^{3/2}}+\frac{(A+aB)\int \cdots}{4\cdots}$$

$$= -\frac{(A-aB)\log\left(a-\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}}+\frac{(A-aB)\log\left(a+\sqrt{3}\sqrt{a}x+x^2\right)}{4\sqrt{3}a^{3/2}}+\frac{(A-\cdots}{\cdots}$$

$$= -\frac{(A+aB)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{(A+aB)\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{(A-aB)\log\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 130, normalized size = 0.96

$$\frac{\sqrt[4]{-1}\left(\frac{\left(-2iA+\left(-i+\sqrt{3}\right)aB\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\sqrt{a}}\right)}{\sqrt{-i+\sqrt{3}}}-\frac{\left(2iA+\left(i+\sqrt{3}\right)aB\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\sqrt{a}}\right)}{\sqrt{i+\sqrt{3}}}\right)}{\sqrt{6}\,a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a^2 - a*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*a*B)*ArcTan[((1 + I)*x)/(Sqrt[-I +
 Sqrt[3]]*Sqrt[a])])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (I + Sqrt[3])*a*B)*Ar
cTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*Sqrt[a])])/Sqrt[I + Sqrt[3]]))/(Sqrt[6
]*a^(3/2))
```

Maple [A]
time = 0.04, size = 186, normalized size = 1.37

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^2+a^2\right)}\frac{\left(B\_R^2+A\right)\ln\left(x-\_R\right)}{2\_R^3-\_Ra}\right)}{2}$
default	$\frac{-\frac{\left(-B\sqrt{3}\,a^2+A\sqrt{3}\,a\right)\ln\left(x\sqrt{3}\,\sqrt{a}-x^2-a\right)}{2}+\frac{2\left(-3Aa^{\frac{3}{2}}+\frac{\left(-B\sqrt{3}\,a^2+A\sqrt{3}\,a\right)\sqrt{3}\,\sqrt{a}}{2}\right)\arctan\left(\frac{\sqrt{3}\,\sqrt{a}-2x}{\sqrt{a}}\right)}{\sqrt{a}}}{6a^{\frac{5}{2}}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(x^4-a*x^2+a^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^(5/2)*(-1/2*(-B*3^(1/2)*a^2+A*3^(1/2)*a)*ln(x*3^(1/2)*a^(1/2)-x^2-a)+
2*(-3*A*a^(3/2)+1/2*(-B*3^(1/2)*a^2+A*3^(1/2)*a)*3^(1/2)*a^(1/2))/a^(1/2)*a
rctan((3^(1/2)*a^(1/2)-2*x)/a^(1/2)))+1/6/a^(5/2)*(1/2*(-B*3^(1/2)*a^2+A*3^
(1/2)*a)*ln(a+x^2+x*3^(1/2)*a^(1/2))+2*(3*A*a^(3/2)-1/2*(-B*3^(1/2)*a^2+A*3
^(1/2)*a)*3^(1/2)*a^(1/2))/a^(1/2)*arctan((2*x+3^(1/2)*a^(1/2))/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - a*x^2 + a^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5405 vs. $2(104) = 208$.
time = 2.01, size = 5405, normalized size = 39.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="fricas")

[Out] 1/4*(4*(1/9)^(1/4)*a^6*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^
3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3
 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*
((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4)*sqrt(
(B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6)*arctan((18*sqrt(1/3)*(1/9)^(3/4)*(sqrt
(1/3)*A*a^10*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)
/a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) - sqrt(1/3)*(B^3*a^10 + A*B
^2*a^9 + A^2*B*a^8)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt((B^8*a^
8 + 2*A*B^7*a^7 + A^2*B^6*a^6 - 2*A^3*B^5*a^5 - 4*A^4*B^4*a^4 - 2*A^5*B^3*a
^3 + A^6*B^2*a^2 + 2*A^7*B*a + A^8)*x^2 + 3*sqrt(1/3)*(1/9)^(1/4)*((B^5*a^1
0 - 2*A^2*B^3*a^8 + A^4*B*a^6)*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^
2 + 2*A^3*B*a + A^4)/a^6) - (A*B^6*a^8 + A^2*B^5*a^7 - A^3*B^4*a^6 - 2*A^4*
B^3*a^5 - A^5*B^2*a^4 + A^6*B*a^3 + A^7*a^2)*x)*sqrt((2*B^4*a^4 + 4*A*B^3*a
^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sq
rt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4
 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*
a + A^4)/a^6)^(1/4) + (B^6*a^10 + A*B^5*a^9 - A^2*B^4*a^8 - 2*A^3*B^3*a^7 -
 A^4*B^2*a^6 + A^5*B*a^5 + A^6*a^4)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2
*a^2 + 2*A^3*B*a + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2
 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*
B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 +
 A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4
) - 18*sqrt(1/3)*(1/9)^(3/4)*(sqrt(1/3)*(A*B^4*a^14 + A^2*B^3*a^13 - A^4*B*
a^11 - A^5*a^10)*x*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a
+ A^4)/a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) - sqrt(1/3)*(B^7*a^14
 + 2*A*B^6*a^13 + 2*A^2*B^5*a^12 - 2*A^4*B^3*a^10 - 2*A^5*B^2*a^9 - A^6*B*a
^8)*x*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*
a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*s
qrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(B^4*a^
4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B
*a + A^4)/a^6)^(3/4) + 2*sqrt(1/3)*(B^8*a^14 + 3*A*B^7*a^13 + 5*A^2*B^6*a^1
2 + 4*A^3*B^5*a^11 - 4*A^5*B^3*a^9 - 5*A^6*B^2*a^8 - 3*A^7*B*a^7 - A^8*a^6)
*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt((
B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6) + sqrt(1/3)*(B^10*a^13 + 4*A*B^9*a^12 +
 9*A^2*B^8*a^11 + 12*A^3*B^7*a^10 + 9*A^4*B^6*a^9 - 9*A^6*B^4*a^7 - 12*A^7*
B^3*a^6 - 9*A^8*B^2*a^5 - 4*A^9*B*a^4 - A^10*a^3)*sqrt((B^4*a^4 - 2*A^2*B^2
*a^2 + A^4)/a^6))/(B^12*a^12 + 4*A*B^11*a^11 + 8*A^2*B^10*a^10 + 8*A^3*B^9*
a^9 - 12*A^5*B^7*a^7 - 18*A^6*B^6*a^6 - 12*A^7*B^5*a^5 + 8*A^9*B^3*a^3 + 8*
A^10*B^2*a^2 + 4*A^11*B*a + A^12)) + 4*(1/9)^(1/4)*a^6*sqrt((2*B^4*a^4 + 4*
A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4 + A^2*
a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))/(
B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2
*A^3*B*a + A^4)/a^6)^(3/4)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/a^6)*arctan
```

```
((18*sqrt(1/3)*(1/9)^(3/4)*(sqrt(1/3)*A*a^10*sqrt((B^4*a^4 + 2*A*B^3*a^3 +
3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/
a^6) - sqrt(1/3)*(B^3*a^10 + A*B^2*a^9 + A^2*B*a^8)*sqrt((B^4*a^4 - 2*A^2*B
^2*a^2 + A^4)/a^6))*sqrt((B^8*a^8 + 2*A*B^7*a^7 + A^2*B^6*a^6 - 2*A^3*B^5*a
^5 - 4*A^4*B^4*a^4 - 2*A^5*B^3*a^3 + A^6*B^2*a^2 + 2*A^7*B*a + A^8)*x^2 - 3
*sqrt(1/3)*(1/9)^(1/4)*((B^5*a^10 - 2*A^2*B^3*a^8 + A^4*B*a^6)*x*sqrt((B^4*
a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6) - (A*B^6*a^8 + A^
2*B^5*a^7 - A^3*B^4*a^6 - 2*A^4*B^3*a^5 - A^5*B^2*a^4 + A^6*B*a^3 + A^7*a^2
)*x)*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B
^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 +
 2*A^3*B*a + A^4)/a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^
3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)^(1/4) + (B^6*a^10 + A*B^5*a^9
 - A^2*B^4*a^8 - 2*A^3*B^3*a^7 - A^4*B^2*a^6 + A^5*B*a^5 + A^6*a^4)*sqrt((B
^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6))*sqrt((2*B^4*a
^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (B^2*a^5 + 4*A*B*a^4
 + A^2*a^3)*sqrt((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/
a^6))/(B^4*a^4 - 2*A^2*B^2*a^2 + A^4))*((B^4*a^4 + 2*A*B^3*a^3 + 3*A^2*B^2*
a^2 + 2*A^3*B*a + A^4)/a^6)^(3/4) - 18*sqrt(1/3)*(1/9)^(3/4)*(sqrt(1/3)*(A*
B^4*a^14 + A^2*B^3*a^13 - A^4*B*a^11 - A^5*a^10)*x*sqrt((B^4*a^4 + 2*A*B^3*
a^3 + 3*A^2*B^2*a^2 + 2*A^3*B*a + A^4)/a^6)*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 +
 A^4)/a^6) - sqrt(1/3)*(B^7*a^14 + 2*A*B^6*a^13 + 2*A^2*B^5*a^12 - 2*A^4*B^
3*a^10 - 2*A^5*B^2*a^9 - A^6*B*a^8)*x*sqrt((B^4*a^4 - 2*A^2*B^2*a^2 + A^4)/
a^6))*sqrt((2*B^4*a^4 + 4*A*B^3*a^3 + 6*A^2*B^2*a^2 + 4*A^3*B*a + 2*A^4 + (
B^2*a^5 + 4*A*B*a^4 + A^2*a^3)*sqrt((B^4*a^4 + ...
```

Sympy [A]
time = 1.03, size = 172, normalized size = 1.26

$$\operatorname{RootSum}\left(144t^4a^6 + t^2\cdot\left(12A^2a^3 + 48ABa^4 + 12B^2a^5\right) + A^4 + 2A^3Ba + 3A^2B^2a^2 + 2AB^3a^3 + B^4a^4, \left(t \mapsto t\log\left(x + \frac{24t^3Aa^5 + 48t^3Ba^6 - 2tA^3a^2 + 6tA^2Ba^3 + 12tAB^2a^4 + 2tB^3a^5}{-A^4 - A^3Ba + AB^3a^3 + B^4a^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(x**4-a*x**2+a**2),x)
```

```
[Out] RootSum(144*_t**4*a**6 + _t**2*(12*A**2*a**3 + 48*A*B*a**4 + 12*B**2*a**5)
+ A**4 + 2*A**3*B*a + 3*A**2*B**2*a**2 + 2*A*B**3*a**3 + B**4*a**4, Lambda(
_t, _t*log(x + (24*_t**3*A*a**5 + 48*_t**3*B*a**6 - 2*_t*A**3*a**2 + 6*_t*A
**2*B*a**3 + 12*_t*A*B**2*a**4 + 2*_t*B**3*a**5)/(-A**4 - A**3*B*a + A*B**3
*a**3 + B**4*a**4))))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(x^4-a*x^2+a^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-16,[2,0]%%%}+%%%{-4,[0,1]%%%},0,%%%{64,[4,0]
%%%}+%%%{8,[2,2]%%%}+%%%{16,[2,1]%%%}+%%%{6,[0,2]%%%},0,%%%{-64,[4,2]%%%}+%
%%{-128,[4,1]%%%
```

Mupad [B]
time = 4.59, size = 1007, normalized size = 7.40

$$\mathrm{atan}\left(\frac{A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1{}\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1{}\mathrm{i}}{24\,a}}\,6{}\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1{}\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1{}\mathrm{i}}+\frac{2\,\sqrt{3}\,A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1{}\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1{}\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1{}\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1{}\mathrm{i}}-\frac{B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1{}\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1{}\mathrm{i}}{24\,a}}\,6{}\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1{}\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1{}\mathrm{i}}-\frac{2\,\sqrt{3}\,B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}-\frac{\sqrt{3}\,A^2\,1{}\mathrm{i}}{24\,a^3}+\frac{\sqrt{3}\,B^2\,1{}\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2+\frac{\sqrt{3}\,A^3\,1{}\mathrm{i}}{a}-A\,B^2\,a-\sqrt{3}\,A\,B^2\,a\,1{}\mathrm{i}}\right)\,\sqrt{-\frac{A^2+B^2\,a^2+4\,A\,B\,a+\sqrt{3}\,A^2\,1{}\mathrm{i}-\sqrt{3}\,B^2\,a^2\,1{}\mathrm{i}}{24\,a^3}}\,2{}\mathrm{i}+\mathrm{atan}\left(\frac{A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1{}\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1{}\mathrm{i}}{24\,a}}\,6{}\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1{}\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1{}\mathrm{i}}-\frac{2\,\sqrt{3}\,A^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1{}\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1{}\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1{}\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1{}\mathrm{i}}-\frac{B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1{}\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1{}\mathrm{i}}{24\,a}}\,6{}\mathrm{i}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1{}\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1{}\mathrm{i}}+\frac{2\,\sqrt{3}\,B^2\,a^2\,x\,\sqrt{-\frac{A^2}{24\,a^3}-\frac{B^2}{24\,a}-\frac{A\,B}{6\,a^2}+\frac{\sqrt{3}\,A^2\,1{}\mathrm{i}}{24\,a^3}-\frac{\sqrt{3}\,B^2\,1{}\mathrm{i}}{24\,a}}}{2\,A^2\,B+\frac{A^3}{a}-2\,B^3\,a^2-\frac{\sqrt{3}\,A^3\,1{}\mathrm{i}}{a}-A\,B^2\,a+\sqrt{3}\,A\,B^2\,a\,1{}\mathrm{i}}\right)\,\sqrt{-\frac{A^2+B^2\,a^2+4\,A\,B\,a-\sqrt{3}\,A^2\,1{}\mathrm{i}+\sqrt{3}\,B^2\,a^2\,1{}\mathrm{i}}{24\,a^3}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a^2 - a*x^2 + x^4),x)
```

```
[Out] atan((A^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^
3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 +
 (3^(1/2)*A^3*1i)/a - A*B^2*a - 3^(1/2)*A*B^2*a*1i) + (2*3^(1/2)*A^2*x*((3^
(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24*a^3
) - (A*B)/(6*a^2))^(1/2))/(2*A^2*B + A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*1i)/a
 - A*B^2*a - 3^(1/2)*A*B^2*a*1i) - (B^2*a^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^
2/(24*a) - (3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2)*
6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*1i)/a - A*B^2*a - 3^(1/2)*A
*B^2*a*1i) - (2*3^(1/2)*B^2*a^2*x*((3^(1/2)*B^2*1i)/(24*a) - B^2/(24*a) - (
3^(1/2)*A^2*1i)/(24*a^3) - A^2/(24*a^3) - (A*B)/(6*a^2))^(1/2))/(2*A^2*B +
A^3/a - 2*B^3*a^2 + (3^(1/2)*A^3*1i)/a - A*B^2*a - 3^(1/2)*A*B^2*a*1i))*(-(
3^(1/2)*A^2*1i + A^2 + B^2*a^2 - 3^(1/2)*B^2*a^2*1i + 4*A*B*a)/(24*a^3))^(1
/2)*2i + atan((A^2*x*((3^(1/2)*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3)
 - (3^(1/2)*B^2*1i)/(24*a) - (A*B)/(6*a^2))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*
B^3*a^2 - (3^(1/2)*A^3*1i)/a - A*B^2*a + 3^(1/2)*A*B^2*a*1i) - (2*3^(1/2)*A
^2*x*((3^(1/2)*A^2*1i)/(24*a^3) - B^2/(24*a) - A^2/(24*a^3) - (3^(1/2)*B^2*
1i)/(24*a) - (A*B)/(6*a^2))^(1/2))/(2*A^2*B + A^3/a - 2*B^3*a^2 - (3^(1/2)*
A^3*1i)/a - A*B^2*a + 3^(1/2)*A*B^2*a*1i) - (B^2*a^2*x*((3^(1/2)*A^2*1i)/(2
4*a^3) - B^2/(24*a) - A^2/(24*a^3) - (3^(1/2)*B^2*1i)/(24*a) - (A*B)/(6*a^2
))^(1/2)*6i)/(2*A^2*B + A^3/a - 2*B^3*a^2 - (3^(1/2)*A^3*1i)/a - A*B^2*a +
3^(1/2)*A*B^2*a*1i) + (2*3^(1/2)*B^2*a^2*x*((3^(1/2)*A^2*1i)/(24*a^3) - B^2
/(24*a) - A^2/(24*a^3) - (3^(1/2)*B^2*1i)/(24*a) - (A*B)/(6*a^2))^(1/2))/(2
*A^2*B + A^3/a - 2*B^3*a^2 - (3^(1/2)*A^3*1i)/a - A*B^2*a + 3^(1/2)*A*B^2*a
*1i))*(-(A^2 - 3^(1/2)*A^2*1i + B^2*a^2 + 3^(1/2)*B^2*a^2*1i + 4*A*B*a)/(24
*a^3))^(1/2)*2i
```

3.110 $\int \frac{A+Bx^2}{a-\sqrt{a}\,x^2+x^4}\,dx$

Optimal. Leaf size=160

$$-\frac{\left(A+\sqrt{a}\,B\right)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{\left(A+\sqrt{a}\,B\right)\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{\left(A-\sqrt{a}\,B\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}\right.}{4\sqrt{3}\,a^{3/4}}$$

```
[Out] -1/12*ln(x^2-a^(1/4)*x*3^(1/2)+a^(1/2))*(A-B*a^(1/2))/a^(3/4)*3^(1/2)+1/12*
ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))*(A-B*a^(1/2))/a^(3/4)*3^(1/2)+1/2*arctan(
2*x/a^(1/4)-3^(1/2))*(A+B*a^(1/2))/a^(3/4)+1/2*arctan(2*x/a^(1/4)+3^(1/2))*
(A+B*a^(1/2))/a^(3/4)
```

Rubi [A]
time = 0.08, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1183, 648, 631, 210, 642}

$$-\frac{\left(\sqrt{a}\,B+A\right)\text{ArcTan}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{\left(\sqrt{a}\,B+A\right)\text{ArcTan}\left(\frac{2x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}}-\frac{\left(A-\sqrt{a}\,B\right)\log\left(-\sqrt{3}\sqrt[4]{a}\,x+\sqrt{a}+x^2\right)}{4\sqrt{3}\,a^{3/4}}+\frac{\left(A-\sqrt{a}\,B\right)\log\left(\sqrt{3}\sqrt[4]{a}\,x+\sqrt{a}+x^2\right)}{4\sqrt{3}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] -1/2*((A + Sqrt[a]*B)*ArcTan[Sqrt[3] - (2*x)/a^(1/4)])/a^(3/4) + ((A + Sqrt
[a]*B)*ArcTan[Sqrt[3] + (2*x)/a^(1/4)])/(2*a^(3/4)) - ((A - Sqrt[a]*B)*Log[
Sqrt[a] - Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3]*a^(3/4)) + ((A - Sqrt[a]*B)*
Log[Sqrt[a] + Sqrt[3]*a^(1/4)*x + x^2])/(4*Sqrt[3]*a^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^2}{a-\sqrt{a}\,x^2+x^4}\,dx=\frac{\int \frac{\sqrt{3}\,\sqrt[4]{a}\,A-\left(A-\sqrt{a}\,B\right)x}{\sqrt{a}-\sqrt{3}\,\sqrt[4]{a}\,x+x^2}\,dx}{2\sqrt{3}\,a^{3/4}}+\frac{\int \frac{\sqrt{3}\,\sqrt[4]{a}\,A+\left(A-\sqrt{a}\,B\right)x}{\sqrt{a}+\sqrt{3}\,\sqrt[4]{a}\,x+x^2}\,dx}{2\sqrt{3}\,a^{3/4}}$$

$$=\frac{1}{4}\left(\frac{A}{\sqrt{a}}+B\right)\int \frac{1}{\sqrt{a}-\sqrt{3}\,\sqrt[4]{a}\,x+x^2}\,dx+\frac{1}{4}\left(\frac{A}{\sqrt{a}}+B\right)\int \frac{1}{\sqrt{a}+\sqrt{3}\,\sqrt[4]{a}\,x$$

$$=-\frac{\left(A-\sqrt{a}\,B\right)\log\left(\sqrt{a}-\sqrt{3}\,\sqrt[4]{a}\,x+x^2\right)}{4\sqrt{3}\,a^{3/4}}+\frac{\left(A-\sqrt{a}\,B\right)\log\left(\sqrt{a}+\sqrt{3}\,\sqrt[4]{a}\,x\right.}{4\sqrt{3}\,a^{3/4}}$$

$$=-\frac{\left(A+\sqrt{a}\,B\right)\tan^{-1}\left(\sqrt{3}-\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}+\frac{\left(A+\sqrt{a}\,B\right)\tan^{-1}\left(\sqrt{3}+\frac{2x}{\sqrt[4]{a}}\right)}{2a^{3/4}}-\frac{(A}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 138, normalized size = 0.86

$$\frac{\sqrt[4]{-1}\left(\frac{\left(-2iA+\left(-i+\sqrt{3}\right)\sqrt{a}\,B\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-i+\sqrt{3}}\,\sqrt[4]{a}}\right)}{\sqrt{-i+\sqrt{3}}}-\frac{\left(2iA+\left(i+\sqrt{3}\right)\sqrt{a}\,B\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{i+\sqrt{3}}\,\sqrt[4]{a}}\right)}{\sqrt{i+\sqrt{3}}}\right)}{\sqrt{6}\,a^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a]*x^2 + x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-2*I)*A + (-I + Sqrt[3])*Sqrt[a]*B)*ArcTan[((1 + I)*x)/(Sqr
t[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((2*I)*A + (I + Sqrt[3])*S
qrt[a]*B)*ArcTanh[((1 + I)*x)/(Sqrt[I + Sqrt[3]]*a^(1/4))])/Sqrt[I + Sqrt[3
]]))/(Sqrt[6]*a^(3/4))
```

Maple [A]
time = 0.06, size = 190, normalized size = 1.19

method	result
default	$\frac{-\frac{\left(A\sqrt{3}\,\sqrt{a}-B\sqrt{3}\,a\right)\ln\left(a^{\frac{1}{4}}x\sqrt{3}-x^{2}-\sqrt{a}\right)}{2}+\frac{2\left(-3Aa^{\frac{3}{4}}+\frac{\left(A\sqrt{3}\,\sqrt{a}-B\sqrt{3}\,a\right)a^{\frac{1}{4}}\sqrt{3}}{2}\right)\arctan\left(\frac{a^{\frac{1}{4}}\sqrt{3}-2x}{a^{\frac{1}{4}}}\right)}{a^{\frac{1}{4}}}}{6a^{\frac{5}{4}}}+ -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^(5/4)*(-1/2*(A*3^(1/2)*a^(1/2)-B*3^(1/2)*a)*ln(a^(1/4)*x*3^(1/2)-x^2-
a^(1/2))+2*(-3*A*a^(3/4)+1/2*(A*3^(1/2)*a^(1/2)-B*3^(1/2)*a)*a^(1/4)*3^(1/2
))/a^(1/4)*arctan((a^(1/4)*3^(1/2)-2*x)/a^(1/4)))+1/6/a^(5/4)*(1/2*(A*3^(1/
2)*a^(1/2)-B*3^(1/2)*a)*ln(x^2+a^(1/4)*x*3^(1/2)+a^(1/2))+2*(3*A*a^(3/4)-1/
2*(A*3^(1/2)*a^(1/2)-B*3^(1/2)*a)*a^(1/4)*3^(1/2))/a^(1/4)*arctan((2*x+a^(1
/4)*3^(1/2))/a^(1/4)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(x^4 - sqrt(a)*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. 2(116) = 232.
time = 0.45, size = 1141, normalized size = 7.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a
 + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^3 - A^6)*x + 3*sqrt
(1/6)*(A*B^4*a^3 - A^5*a - sqrt(1/3)*(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2
 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - sqrt(1/3)*(A*B^2*a^3
- A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*
a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2
)*sqrt(a))/a^2)) - 1/2*sqrt(1/6)*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^
4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^
3 - A^6)*x - 3*sqrt(1/6)*(A*B^4*a^3 - A^5*a - sqrt(1/3)*(2*B^3*a^4 + A^2*B*
a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a - sq
rt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqr
t(a))*sqrt(-(4*A*B*a + 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/
a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) + 1/2*sqrt(1/6)*sqrt(-(4*A*B*a - 3*sqrt
(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))
/a^2)*log(2*(B^6*a^3 - A^6)*x + 3*sqrt(1/6)*(A*B^4*a^3 - A^5*a + sqrt(1/3)*
(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) - (A^2*B^3
*a^2 - A^4*B*a + sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(B^4*a^2 - 2*A^2*B^2
*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 -
2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2)) - 1/2*sqrt(1/6)*sqrt
(-(4*A*B*a - 3*sqrt(1/3)*a^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^
2*a + A^2)*sqrt(a))/a^2)*log(2*(B^6*a^3 - A^6)*x - 3*sqrt(1/6)*(A*B^4*a^3 -
 A^5*a + sqrt(1/3)*(2*B^3*a^4 + A^2*B*a^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A
^4)/a^3) - (A^2*B^3*a^2 - A^4*B*a + sqrt(1/3)*(A*B^2*a^3 - A^3*a^2)*sqrt(-(
B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3))*sqrt(a))*sqrt(-(4*A*B*a - 3*sqrt(1/3)*a^
2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a + A^4)/a^3) + (B^2*a + A^2)*sqrt(a))/a^2))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+x**4-x**2*a**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(64*_t**4*a - 16*_t**2*B**2*sqrt(a)
+ B**4) contains an element of the set of generators.
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+x^4-x^2*a^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 4.99, size = 1155, normalized size = 7.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + x^4 - a^(1/2)*x^2),x)
```

```
[Out] - 2*atanh((6*A^2*x*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^
2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(2*A^2
*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^2)
- (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (6*B^2*a*x*((B^2*(-27*a^3)^(1/2))/(72*a^
2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) -
 (A*B)/(6*a))^(1/2))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^
3*(-27*a^3)^(1/2))/(3*a^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a)) - (2*A^2*x*(-27
*a^3)^(1/2)*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*
a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(3*a^(3/2)*(2
*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a
^2) - (A*B^2*(-27*a^3)^(1/2))/(3*a))) + (2*B^2*x*(-27*a^3)^(1/2)*((B^2*(-27
*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*(-27*a^3)^(1/2))/(72*a^3) -
 A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2))/(3*a^(1/2)*(2*A^2*B - 2*B^3*a + A^3
/a^(1/2) - A*B^2*a^(1/2) + (A^3*(-27*a^3)^(1/2))/(3*a^2) - (A*B^2*(-27*a^3)
^(1/2))/(3*a))))*((B^2*(-27*a^3)^(1/2))/(72*a^2) - B^2/(24*a^(1/2)) - (A^2*
(-27*a^3)^(1/2))/(72*a^3) - A^2/(24*a^(3/2)) - (A*B)/(6*a))^(1/2) - 2*atanh
((6*A^2*x*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3
/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2))/(2*A^2*B - 2*B^
3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*B^2*
(-27*a^3)^(1/2))/(3*a)) - (6*B^2*a*x*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/
(24*a^(1/2)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6
*a))^(1/2))/(2*A^2*B - 2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^
3)^(1/2))/(3*a^2) + (A*B^2*(-27*a^3)^(1/2))/(3*a)) + (2*A^2*x*(-27*a^3)^(1/
2)*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2)) -
(B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2))/(3*a^(3/2)*(2*A^2*B -
2*B^3*a + A^3/a^(1/2) - A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*
B^2*(-27*a^3)^(1/2))/(3*a))) - (2*B^2*x*(-27*a^3)^(1/2)*((A^2*(-27*a^3)^(1/
2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2)) - (B^2*(-27*a^3)^(1/2))/
(72*a^2) - (A*B)/(6*a))^(1/2))/(3*a^(1/2)*(2*A^2*B - 2*B^3*a + A^3/a^(1/2)
- A*B^2*a^(1/2) - (A^3*(-27*a^3)^(1/2))/(3*a^2) + (A*B^2*(-27*a^3)^(1/2))/(
3*a))))*((A^2*(-27*a^3)^(1/2))/(72*a^3) - B^2/(24*a^(1/2)) - A^2/(24*a^(3/2
)) - (B^2*(-27*a^3)^(1/2))/(72*a^2) - (A*B)/(6*a))^(1/2)
```

3.111 $\int \frac{A+Bx^2}{a-\sqrt{ac}\,x^2+cx^4}\,dx$

Optimal. Leaf size=414

$$-\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}-2\sqrt{c}\,x}{\sqrt{2\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\,\sqrt{c}\,\sqrt{2\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}+\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}+2\sqrt{c}\,x}{\sqrt{2\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\,\sqrt{c}\,\sqrt{2\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}$$

```
[Out] -1/2*arctan((-2*x*c^(1/2)+(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))/(2*a^(1/2)
*c^(1/2)-(a*c)^(1/2))^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(1/2)/c^(1/2)/(2*a^(1/
2)*c^(1/2)-(a*c)^(1/2))^(1/2)+1/2*arctan((2*x*c^(1/2)+(2*a^(1/2)*c^(1/2)+(a
*c)^(1/2))^(1/2))/(2*a^(1/2)*c^(1/2)-(a*c)^(1/2))^(1/2))*(B*a^(1/2)+A*c^(1/
2))/a^(1/2)/c^(1/2)/(2*a^(1/2)*c^(1/2)-(a*c)^(1/2))^(1/2)-1/4*ln(a^(1/2)+x^
2*c^(1/2)-x*(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))*(A-B*a^(1/2)/c^(1/2))/a^
(1/2)/(2*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2)+1/4*ln(a^(1/2)+x^2*c^(1/2)+x*(2
*a^(1/2)*c^(1/2)+(a*c)^(1/2))^(1/2))*(A-B*a^(1/2)/c^(1/2))/a^(1/2)/(2*a^(1/
2)*c^(1/2)+(a*c)^(1/2))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1183, 648, 632, 210, 642}

$$-\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\text{ArcTan}\left(\frac{\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}-2\sqrt{c}\,x}{\sqrt{2\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\,\sqrt{c}\,\sqrt{2\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}+\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\text{ArcTan}\left(\frac{\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}+2\sqrt{c}\,x}{\sqrt{2\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{a}\,\sqrt{c}\,\sqrt{2\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}-\frac{\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\log\left(-x\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{a}\,\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}}+\frac{\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\log\left(x\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{a}\,\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```

```
[Out] -1/2*((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]] -
 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/(Sqrt[a]*Sqrt[c]*Sqrt[2
*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[(Sqrt[2*Sq
rt[a]*Sqrt[c] + Sqrt[a*c]] + 2*Sqrt[c]*x)/Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c
]]])/(2*Sqrt[a]*Sqrt[c]*Sqrt[2*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]) - ((A - (Sqrt[
a]*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c
]*x^2])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]) + ((A - (Sqrt[a]*B)
/Sqrt[c])*Log[Sqrt[a] + Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]]*x + Sqrt[c]*x^2
])/(4*Sqrt[a]*Sqrt[2*Sqrt[a]*Sqrt[c] + Sqrt[a*c]])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^2}{a-\sqrt{ac}\,x^2+cx^4}\,dx = \frac{\int \frac{\frac{A\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}-\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}\,x}{\sqrt{c}}+x^2}dx}{2\sqrt{a}\,\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}} + \frac{\int \frac{\frac{A\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}}{\sqrt{c}}+\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}\,x}{\sqrt{c}}+x^2}dx}{2\sqrt{a}\,\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}}$$

$$= \frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}\,x}{\sqrt{c}}+x^2}dx}{4c} + \frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{\ldots}}{}}}{4\ldots}$$

$$= -\frac{\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{a}\,\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}} + \frac{\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\ldots}{\ldots}$$

$$= -\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\tan^{-1}\left(\frac{\sqrt{2\sqrt{a}\,\sqrt{c}+\sqrt{ac}}-2\sqrt{c}\,x}{\sqrt{2\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}\right)}{2\sqrt{c}\,\sqrt{2\sqrt{a}\,\sqrt{c}-\sqrt{ac}}} + \frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\tan^{-1}\left(\ldots\right.}{2\sqrt{c}\,\sqrt{\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 247, normalized size = 0.60

$$\frac{\frac{\left(\sqrt{3}\,\sqrt{a}\,B\sqrt{c}-i\left(2Ac+B\sqrt{ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{-i\sqrt{3}\,\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}\right)}{\sqrt{-i\sqrt{3}\,\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}+\frac{\left(\sqrt{3}\,\sqrt{a}\,B\sqrt{c}+i\left(2Ac+B\sqrt{ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{i\sqrt{3}\,\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}\right)}{\sqrt{i\sqrt{3}\,\sqrt{a}\,\sqrt{c}-\sqrt{ac}}}}{\sqrt{6}\,\sqrt{a}\,c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a*c]*x^2 + c*x^4),x]
```

```
[Out] (((Sqrt[3]*Sqrt[a]*B*Sqrt[c] - I*(2*A*c + B*Sqrt[a*c]))*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[(-I)*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt[a*c]]])/Sqrt[(-I)*Sqrt[3]*
Sqrt[a]*Sqrt[c] - Sqrt[a*c]] + ((Sqrt[3]*Sqrt[a]*B*Sqrt[c] + I*(2*A*c + B*S
qrt[a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[I*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt
[a*c]]])/Sqrt[I*Sqrt[3]*Sqrt[a]*Sqrt[c] - Sqrt[a*c]])/(Sqrt[6]*Sqrt[a]*c)
```

Maple [A]
time = 0.08, size = 347, normalized size = 0.84

method	result
default	$\frac{\frac{\left(-B\sqrt{3}\,(ac)^{\frac{3}{4}}a^{\frac{3}{2}}+A\sqrt{3}\,(ac)^{\frac{3}{4}}\sqrt{c}\,a\right)\ln\left(x^{2}\sqrt{c}+x\sqrt{3}\,(ac)^{\frac{1}{4}}+\sqrt{a}\right)}{2\sqrt{c}}+\frac{2\left(3a^{2}cA-\frac{\left(-B\sqrt{3}\,(ac)^{\frac{3}{4}}a^{\frac{3}{2}}+A\sqrt{3}\,(ac)^{\frac{3}{4}}\sqrt{c}\,a\right)\sqrt{}}{2\sqrt{c}}\right.}{\sqrt{4\sqrt{a}\,\sqrt{c}}}}{6a^{\frac{5}{2}}c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^(5/2)/c*(1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/4)*c^
(1/2)*a)/c^(1/2)*ln(x^2*c^(1/2)+x*3^(1/2)*(a*c)^(1/4)+a^(1/2))+2*(3*a^2*c*A
-1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/4)*c^(1/2)*a)*3^(1/
2)*(a*c)^(1/4)/c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((2*x
*c^(1/2)+3^(1/2)*(a*c)^(1/4))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)))+1/6
/a^(5/2)/c*(-1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/4)*c^(1
/2)*a)/c^(1/2)*ln(x*3^(1/2)*(a*c)^(1/4)-x^2*c^(1/2)-a^(1/2))+2*(-3*a^2*c*A+
1/2*(-B*3^(1/2)*(a*c)^(3/4)*a^(3/2)+A*3^(1/2)*(a*c)^(3/4)*c^(1/2)*a)*3^(1/2
)*(a*c)^(1/4)/c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)*arctan((3^(1
/2)*(a*c)^(1/4)-2*x*c^(1/2))/(4*a^(1/2)*c^(1/2)-3*(a*c)^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 - sqrt(a*c)*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1457 vs. 2(289) = 578.
time = 0.64, size = 1457, normalized size = 3.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c +
A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log
```

```
(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3
)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3
*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*
sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c
^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) + 1/2*sqrt(1/6)*s
qrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^
3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A
^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - sqrt(1/3)*(2*B^3*a^4*c^2
 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A
^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt(-(3*sqrt(1/
3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c
 + (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))) - 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)
*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c -
 (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqr
t(1/6)*(A*B^4*a^3*c - A^5*a*c^3 + sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)
*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^
4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*
B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a*c))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(
B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)
*sqrt(a*c))/(a^2*c^2))) + 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*s
qrt(a*c))/(a^2*c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*
c - A^5*a*c^3 + sqrt(1/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 -
2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)) - (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1
/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^3*c^3)))*sqrt(a*c))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a*c))/(a^2*c
^2)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+c*x**4-x**2*(a*c)**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(64*_t**4*a*c**3 - 16*_t**2*B**2*c*s
qrt(a*c) + B**4) contains an element of the set of generators.
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*(a*c)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 5.22, size = 2500, normalized size = 6.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + c*x^4 - x^2*(a*c)^(1/2)),x)
```

```
[Out] - atan(((((12*A*a)/c^2 - (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(B
^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) -
A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*
(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c
*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c
^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)
 + (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3
*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(
3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/
(72*a^3*c^3))^(1/2)*1i - (((12*A*a)/c^2 + (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*
(a*c)^(1/2))*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2
*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/
2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(B^2*a*(-27*a^3*c
^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/
2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(7
2*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*
(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2
) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^
2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)*1i)/((((12*A*a)/c^2 - (2*x*(4*c*(a*c)^
(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3
*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^
2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-
(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2)
- A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*
c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a
*c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) -
 B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)
^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (((12*A*a)/c^2 + (2
*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(B^2*a*(-27*a^3*c^3)^(1/2) -
A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*
```

```
a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^
(1/2))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^3*c^3)^(1/2) - B^2*
a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2
) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*
c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A^2*c*(-27*a^
3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A
^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (2*(B
^3*a + A^2*B*c + A*B^2*(a*c)^(1/2)))/c^4))*(-(B^2*a*(-27*a^3*c^3)^(1/2) - A
^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a
^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(
1/2)*2i - atan(((((12*A*a)/c^2 - (2*x*(4*c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/
2))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^
(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^
2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2)
 - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A
*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3
))^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)))/c^4)*(-(A^2*c*
(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c
*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)
^(1/2))/(72*a^3*c^3))^(1/2)*1i - (((12*A*a)/c^2 + (2*x*(4*c*(a*c)^(3/2) - 1
6*a*c^2*(a*c)^(1/2))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/
2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(
a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))/c^4)*(-(A^2*c*(-
27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(
a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(
1/2))/(72*a^3*c^3))^(1/2) - (2*x*(2*A^2*c^2 - B^2*a*c + 2*A*B*c*(a*c)^(1/2)
))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a
*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) +
4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2)*1i)/((((12*A*a)/c^2 - (2*x*(4*
c*(a*c)^(3/2) - 16*a*c^2*(a*c)^(1/2))*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*
(-27*a^3*c^3)^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^
2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2))
/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)^(1/2) - B^2*a*(a*c
)^(3/2) - A^2*c*(a*c)^(3/2) + 12*A*B*a^2*c^2 + 4*A^2*a*c^2*(a*c)^(1/2) + 4*
B^2*a^2*c*(a*c)^(1/2))/(72*a^3*c^3))^(1/2) + (2*x*(2*A^2*c^2 - B^2*a*c + 2*
A*B*c*(a*c)^(1/2)))/c^4)*(-(A^2*c*(-27*a^3*c^3)^(1/2) - B^2*a*(-27*a^3*c^3)
^(1/2) - B^2*a*(a*c)^(3/2) - A^2*c*(a*c)^(3/2) ...
```

3.112 $\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}\,x^2+cx^4}\,dx$

Optimal. Leaf size=234

$$-\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}-\frac{\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\log\left(\sqrt{\right.}}{4\sqrt{}}$$

```
[Out] -1/12*ln(-a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c^(1/2))*(A-B*a^(1/2)/c^(1/
2))/a^(3/4)/c^(1/4)*3^(1/2)+1/12*ln(a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c
^(1/2))*(A-B*a^(1/2)/c^(1/2))/a^(3/4)/c^(1/4)*3^(1/2)+1/2*arctan(2*c^(1/4)*
x/a^(1/4)-3^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(3/4)/c^(3/4)+1/2*arctan(2*c^(1/
4)*x/a^(1/4)+3^(1/2))*(B*a^(1/2)+A*c^(1/2))/a^(3/4)/c^(3/4)
```

Rubi [A]
time = 0.11, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1183, 648, 631, 210, 642}

$$-\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\text{ArcTan}\left(\sqrt{3}-\frac{2\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\text{ArcTan}\left(\frac{2\sqrt[4]{c}\,x}{\sqrt[4]{a}}+\sqrt{3}\right)}{2a^{3/4}c^{3/4}}-\frac{\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\log\left(-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{3}\,a^{3/4}\sqrt[4]{c}}+\frac{\left(A-\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\log\left(\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{3}\,a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```

```
[Out] -1/2*((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] - (2*c^(1/4)*x)/a^(1/4)])/(a^(
3/4)*c^(3/4)) + ((Sqrt[a]*B + A*Sqrt[c])*ArcTan[Sqrt[3] + (2*c^(1/4)*x)/a^(
1/4)])/(2*a^(3/4)*c^(3/4)) - ((A - (Sqrt[a]*B)/Sqrt[c])*Log[Sqrt[a] - Sqrt[
3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[3]*a^(3/4)*c^(1/4)) + ((A - (S
qrt[a]*B)/Sqrt[c])*Log[Sqrt[a] + Sqrt[3]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/
(4*Sqrt[3]*a^(3/4)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^2}{a-\sqrt{a}\sqrt{c}\,x^2+cx^4}\,dx = \frac{\int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}}-\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{2\sqrt{3}\,a^{3/4}\sqrt[4]{c}}+\frac{\int \frac{\frac{\sqrt{3}\sqrt[4]{a}A}{\sqrt[4]{c}}+\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)x}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{2\sqrt{3}\,a^{3/4}\sqrt[4]{c}}$$

$$=\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c}+\frac{\left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{3}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}}{4c}$$

$$=\frac{\left(\sqrt{a}\,B-A\sqrt{c}\right)\log\left(\sqrt{a}-\sqrt{3}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{3}\,a^{3/4}c^{3/4}}+\frac{\left(A-\frac{\sqrt{a}B}{\sqrt{c}}\right)\log\left(\right.}{4}$$

$$=-\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2a^{3/4}c^{3/4}}+\frac{\left(\sqrt{a}\,B+A\sqrt{c}\right)\tan^{-1}\left(\sqrt{3}+\right.}{2a^{3/4}c^{3/4}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.11, size = 163, normalized size = 0.70

$$\frac{\sqrt[4]{-1}\left(\frac{\left(\left(-i+\sqrt{3}\right)\sqrt{a}\,B-2iA\sqrt{c}\right)\tan^{-1}\left(\frac{(1+i)\sqrt[4]{c}\,x}{\sqrt{-i+\sqrt{3}}\,\sqrt[4]{a}}\right)}{\sqrt{-i+\sqrt{3}}}-\frac{\left(\left(i+\sqrt{3}\right)\sqrt{a}\,B+2iA\sqrt{c}\right)\tanh^{-1}\left(\frac{(1+i)\sqrt[4]{c}\,x}{\sqrt{i+\sqrt{3}}\,\sqrt[4]{a}}\right)}{\sqrt{i+\sqrt{3}}}\right)}{\sqrt{6}\,a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a - Sqrt[a]*Sqrt[c]*x^2 + c*x^4),x]
```

```
[Out] ((-1)^(1/4)*((((-I + Sqrt[3])*Sqrt[a]*B - (2*I)*A*Sqrt[c])*ArcTan[((1 + I)*
c^(1/4)*x)/(Sqrt[-I + Sqrt[3]]*a^(1/4))])/Sqrt[-I + Sqrt[3]] - (((I + Sqrt[
3])*Sqrt[a]*B + (2*I)*A*Sqrt[c])*ArcTanh[((1 + I)*c^(1/4)*x)/(Sqrt[I + Sqrt
[3]]*a^(1/4))])/Sqrt[I + Sqrt[3]]))/(Sqrt[6]*a^(3/4)*c^(3/4))
```

Maple [A]
time = 0.18, size = 279, normalized size = 1.19

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4-\_Z^2\,\mathrm{RootOf}\left(\_Z^2-a,index=1\right)\mathrm{RootOf}\left(\_Z^2-c,index=1\right)+a\right)}\frac{\left(-B\_R^2-A\right)\ln\left(x-\_R\right)}{-2c\_R^3+\_R\sqrt{a}\,\sqrt{c}}\right)}{2}$
default	$\frac{-\frac{\left(A\sqrt{3}\,\sqrt{a}\,c-B\sqrt{3}\,\sqrt{c}\,a\right)\ln\left(a^{\frac{1}{4}}c^{\frac{1}{4}}x\sqrt{3}-x^2\sqrt{c}-\sqrt{a}\right)}{2\sqrt{c}}+\frac{2\left(-3A\,c^{\frac{3}{4}}a^{\frac{3}{4}}+\frac{\left(A\sqrt{3}\,\sqrt{a}\,c-B\sqrt{3}\,\sqrt{c}\,a\right)a^{\frac{1}{4}}\sqrt{3}}{2c^{\frac{1}{4}}}\right)\mathrm{arc}}{\sqrt{\sqrt{a}\,\sqrt{c}}}}{6c^{\frac{3}{4}}a^{\frac{5}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/c^(3/4)/a^(5/4)*(-1/2*(A*3^(1/2)*a^(1/2)*c-B*3^(1/2)*c^(1/2)*a)/c^(1/2)
*ln(a^(1/4)*c^(1/4)*x*3^(1/2)-x^2*c^(1/2)-a^(1/2))+2*(-3*A*c^(3/4)*a^(3/4)+
1/2*(A*3^(1/2)*a^(1/2)*c-B*3^(1/2)*c^(1/2)*a)*a^(1/4)/c^(1/4)*3^(1/2))/(a^(
1/2)*c^(1/2))^(1/2)*arctan((a^(1/4)*c^(1/4)*3^(1/2)-2*x*c^(1/2))/(a^(1/2)*c
^(1/2))^(1/2)))+1/6/c^(3/4)/a^(5/4)*(1/2*(A*3^(1/2)*a^(1/2)*c-B*3^(1/2)*c^(
1/2)*a)/c^(1/2)*ln(a^(1/4)*c^(1/4)*x*3^(1/2)+a^(1/2)+x^2*c^(1/2))+2*(3*A*c^
(3/4)*a^(3/4)-1/2*(A*3^(1/2)*a^(1/2)*c-B*3^(1/2)*c^(1/2)*a)*a^(1/4)/c^(1/4)
*3^(1/2))/(a^(1/2)*c^(1/2))^(1/2)*arctan((a^(1/4)*c^(1/4)*3^(1/2)+2*x*c^(1/
2))/(a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 - sqrt(a)*sqrt(c)*x^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(160) = 320$.
time = 0.99, size = 1469, normalized size = 6.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c +
A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2
))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 - (A
^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt(-
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) - sqrt(1/3)
*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/
(a^3*c^3)))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)))
 + 1/2*sqrt(1/6)*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c
^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
(A^2*B^3*a^2*c - A^4*B*a*c^2 - sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqrt
(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) - sqrt(1/
3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2
)/(a^3*c^3)))*sqrt(-(3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^3*c^3)) + 4*A*B*a*c + (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
)) - 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
 + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*
c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x + 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
 (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqr
t(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) + sqrt(1
/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)/(a^3*c^3)))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
)) + 1/2*sqrt(1/6)*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c
 + A^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*
c^2))*log(-2*(B^6*a^3 - A^6*c^3)*x - 3*sqrt(1/6)*(A*B^4*a^3*c - A^5*a*c^3 -
 (A^2*B^3*a^2*c - A^4*B*a*c^2 + sqrt(1/3)*(A*B^2*a^3*c^2 - A^3*a^2*c^3)*sqr
t(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^3*c^3)))*sqrt(a)*sqrt(c) + sqrt(1
/3)*(2*B^3*a^4*c^2 + A^2*B*a^3*c^3)*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)/(a^3*c^3)))*sqrt((3*sqrt(1/3)*a^2*c^2*sqrt(-(B^4*a^2 - 2*A^2*B^2*a*c + A
```

```
^4*c^2)/(a^3*c^3)) - 4*A*B*a*c - (B^2*a + A^2*c)*sqrt(a)*sqrt(c))/(a^2*c^2)
))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: PolynomialError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(a+c*x**4-x**2*a**(1/2)*c**(1/2)),x)
```

```
[Out] Exception raised: PolynomialError >> 1/(64*_t**4*a*c**5 - 16*_t**2*B**2*sqr
t(a)*c**(7/2) + B**4*c**2) contains an element of the set of generators.
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(a+c*x^4-x^2*a^(1/2)*c^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B]
time = 5.29, size = 1575, normalized size = 6.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + c*x^4 - a^(1/2)*c^(1/2)*x^2),x)
```

```
[Out] - 2*atanh((6*A^2*x*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2
)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(
24*a^(3/2)*c^(1/2)))^(1/2))/((2*A^2*B)/c - (2*B^3*a)/c^2 + A^3/(a^(1/2)*c^(
1/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^2) - (A*B^2*a^(1/2))/c^(3/2) - (
A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^3)) - (6*B^2*a*x*((B^2*(-27*a^3*c^3)^(1/2
))/(72*a^2*c^3) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*
c^3)^(1/2))/(72*a^3*c^2) - A^2/(24*a^(3/2)*c^(1/2)))^(1/2))/(2*A^2*B - (2*B
^3*a)/c + (A^3*c^(1/2))/a^(1/2) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c) - (A*
B^2*a^(1/2))/c^(1/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^2)) - (2*A^2*x*(-
27*a^3*c^3)^(1/2)*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2)
*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(2
4*a^(3/2)*c^(1/2)))^(1/2))/(3*a^(3/2)*c^(7/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^
```

```
4 + A^3/(a^(1/2)*c^(5/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*
a^(1/2))/c^(7/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))) + (2*B^2*x*(-27*
a^3*c^3)^(1/2)*((B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2)*c^
(3/2)) - (A*B)/(6*a*c) - (A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(24*a
^(3/2)*c^(1/2)))^(1/2))/(3*a^(1/2)*c^(9/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^4 +
 A^3/(a^(1/2)*c^(5/2)) + (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*a^(
1/2))/c^(7/2) - (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))))*((B^2*(-27*a^3*c^3
)^(1/2))/(72*a^2*c^3) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - (A^2*(-2
7*a^3*c^3)^(1/2))/(72*a^3*c^2) - A^2/(24*a^(3/2)*c^(1/2)))^(1/2) - 2*atanh(
(6*A^2*x*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2))
 - (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72
*a^2*c^3))^(1/2))/((2*A^2*B)/c - (2*B^3*a)/c^2 + A^3/(a^(1/2)*c^(1/2)) - (A
^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^2) - (A*B^2*a^(1/2))/c^(3/2) + (A*B^2*(-27
*a^3*c^3)^(1/2))/(3*a*c^3)) - (6*B^2*a*x*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3
*c^2) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2))
 - (B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3))^(1/2))/(2*A^2*B - (2*B^3*a)/c +
(A^3*c^(1/2))/a^(1/2) - (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c) - (A*B^2*a^(1/2
))/c^(1/2) + (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^2)) + (2*A^2*x*(-27*a^3*c^3
)^(1/2)*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2))
- (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72*
a^2*c^3))^(1/2))/(3*a^(3/2)*c^(7/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^4 + A^3/(a
^(1/2)*c^(5/2)) - (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*a^(1/2))/c
^(7/2) + (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))) - (2*B^2*x*(-27*a^3*c^3)^(
1/2)*((A^2*(-27*a^3*c^3)^(1/2))/(72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2)) - (
A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72*a^2
*c^3))^(1/2))/(3*a^(1/2)*c^(9/2)*((2*A^2*B)/c^3 - (2*B^3*a)/c^4 + A^3/(a^(1
/2)*c^(5/2)) - (A^3*(-27*a^3*c^3)^(1/2))/(3*a^2*c^4) - (A*B^2*a^(1/2))/c^(7
/2) + (A*B^2*(-27*a^3*c^3)^(1/2))/(3*a*c^5))))*((A^2*(-27*a^3*c^3)^(1/2))/(
72*a^3*c^2) - B^2/(24*a^(1/2)*c^(3/2)) - (A*B)/(6*a*c) - A^2/(24*a^(3/2)*c^
(1/2)) - (B^2*(-27*a^3*c^3)^(1/2))/(72*a^2*c^3))^(1/2)
```

3.113 $\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx$

Optimal. Leaf size=96

$$-\sqrt{\frac{1}{2}\left(-1+\sqrt{13}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)+\sqrt{7+2\sqrt{13}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\right.\right.$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(1+13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))*(
-2+2*13^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(1+13^(1/2))^(1/2),1/6*I*3^(1/2)+1
/6*I*39^(1/2))*(7+2*13^(1/2))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1194, 538, 435, 430}

$$\sqrt{7+2\sqrt{13}}\,F\left(\text{ArcSin}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{13}-1\right)}\,E\left(\text{ArcSin}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)|\frac{1}{6}\left(-7-\sqrt{13}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
```

```
[Out] -(Sqrt[(-1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(1 + Sqrt[13])]*x], (-7 -
 Sqrt[13])/6]) + Sqrt[7 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[13]
)]*x], (-7 - Sqrt[13])/6]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{3-x^2}{\sqrt{3+x^2-x^4}}\,dx = 2\int \frac{3-x^2}{\sqrt{1+\sqrt{13}-2x^2}\,\sqrt{-1+\sqrt{13}+2x^2}}\,dx$$

$$= \left(5+\sqrt{13}\right)\int \frac{1}{\sqrt{1+\sqrt{13}-2x^2}\,\sqrt{-1+\sqrt{13}+2x^2}}\,dx - \int \frac{\sqrt{-1+\sqrt{13}+2}}{\sqrt{1+\sqrt{13}-2x}}$$

$$= -\sqrt{\frac{1}{2}\left(-1+\sqrt{13}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)\middle|\frac{1}{6}\left(-7-\sqrt{13}\right)\right) + \sqrt{7+2\sqrt{13}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.13, size = 103, normalized size = 1.07

$$-\frac{i\left(\left(1+\sqrt{13}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)\middle|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)-\left(-5+\sqrt{13}\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)\middle|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)\right)}{\sqrt{2\left(1+\sqrt{13}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 + x^2 - x^4],x]
```

```
[Out] ((-I)*((1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]*x], (-7 +
 Sqrt[13])/6] - (-5 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[13])]
*x], (-7 + Sqrt[13])/6]))/Sqrt[2*(1 + Sqrt[13])]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(74) = 148.
time = 0.08, size = 200, normalized size = 2.08

method	result

default	$\frac{36\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-6+6\sqrt{13}}}{6},\frac{i\sqrt{3}}{6}+\frac{i\sqrt{39}}{6}\right)-\mathrm{E}\right.}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x^4+x^2+3}\,\left(1+\sqrt{13}\right)}$
elliptic	$\frac{36\sqrt{1-\left(-\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-6+6\sqrt{13}}}{6},\frac{i\sqrt{3}}{6}+\frac{i\sqrt{39}}{6}\right)-\mathrm{E}\right.}{\sqrt{-6+6\sqrt{13}}\,\sqrt{-x^4+x^2+3}\,\left(1+\sqrt{13}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4+x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 36/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*13^
(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)/(1+13^(1/2))*(EllipticF(1/6*x*(-6+6*13
^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))-EllipticE(1/6*x*(-6+6*13^(1/2))
^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2)))+18/(-6+6*13^(1/2))^(1/2)*(1-(-1/6+1/6
*13^(1/2))*x^2)^(1/2)*(1-(-1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4+x^2+3)^(1/2)*
EllipticF(1/6*x*(-6+6*13^(1/2))^(1/2),1/6*I*3^(1/2)+1/6*I*39^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

Fricas [A]
time = 0.10, size = 16, normalized size = 0.17

$$\frac{\sqrt{-x^4+x^2+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-x^4 + x^2 + 3)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2}{\sqrt{-x^4+x^2+3}}\,dx-\int\left(-\frac{3}{\sqrt{-x^4+x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4+x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + x**2 + 3), x) - Integral(-3/sqrt(-x**4 + x**2 +
 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + x^2 + 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2-3}{\sqrt{-x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(x^2 - x^4 + 3)^(1/2),x)
```

```
[Out] -int((x^2 - 3)/(x^2 - x^4 + 3)^(1/2), x)
```

3.114 $\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}} \, dx$

Optimal. Leaf size=25

$$-E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)+4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

```
[Out] -EllipticE(1/3*x*3^(1/2),I*3^(1/2))+4*EllipticF(1/3*x*3^(1/2),I*3^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1194, 21, 434, 435, 430}

$$4F\left(\text{ArcSin}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-E\left(\text{ArcSin}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
```

```
[Out] -EllipticE[ArcSin[x/Sqrt[3]], -3] + 4*EllipticF[ArcSin[x/Sqrt[3]], -3]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3-x^2}{\sqrt{3+2x^2-x^4}}\,dx &= 2\int \frac{3-x^2}{\sqrt{6-2x^2}\,\sqrt{2+2x^2}}\,dx\\ &= \int \frac{\sqrt{6-2x^2}}{\sqrt{2+2x^2}}\,dx\\ &= 8\int \frac{1}{\sqrt{6-2x^2}\,\sqrt{2+2x^2}}\,dx-\int \frac{\sqrt{2+2x^2}}{\sqrt{6-2x^2}}\,dx\\ &= -E\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)+4F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 19, normalized size = 0.76

$$-i\sqrt{3}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 + 2*x^2 - x^4],x]
```

```
[Out] (-I)*Sqrt[3]*EllipticE[I*ArcSinh[x], -1/3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(31) = 62.
time = 0.03, size = 113, normalized size = 4.52

method	result
default	$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)\right)}{3\sqrt{-x^4+2x^2+3}}+\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{x^2}}{\sqrt{-x^4}}$

elliptic	$\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{3}}{3},i\sqrt{3}\right)\right)}{3\sqrt{-x^4+2x^2+3}}+\frac{\sqrt{3}\,\sqrt{-3x^2+9}\,\sqrt{x^2+}}{\sqrt{-x^4+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)*(EllipticF(
1/3*x*3^(1/2),I*3^(1/2))-EllipticE(1/3*x*3^(1/2),I*3^(1/2)))+3^(1/2)*(-3*x^
2+9)^(1/2)*(x^2+1)^(1/2)/(-x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x*3^(1/2),I*3^(
1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

Fricas [A]
time = 0.07, size = 18, normalized size = 0.72

$$\frac{\sqrt{-x^4+2\,x^2+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-x^4 + 2*x^2 + 3)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2}{\sqrt{-x^4+2x^2+3}}\,dx-\int\left(-\frac{3}{\sqrt{-x^4+2x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4+2*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 2*x*
*2 + 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+2*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 2*x^2 + 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^2-3}{\sqrt{-x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(2*x^2 - x^4 + 3)^(1/2),x)
[Out] int(-(x^2 - 3)/(2*x^2 - x^4 + 3)^(1/2), x)
```

3.115 $\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx$

Optimal. Leaf size=96

$$-\sqrt{\frac{1}{2}\left(-3+\sqrt{21}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)+\sqrt{9+2\sqrt{21}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\right.\right.$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(3+21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))*(-
6+2*21^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(3+21^(1/2))^(1/2),1/2*I*3^(1/2)+1/
2*I*7^(1/2))*(9+2*21^(1/2))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1194, 538, 435, 430}

$$\sqrt{9+2\sqrt{21}}\,F\left(\text{ArcSin}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)-\sqrt{\frac{1}{2}\left(\sqrt{21}-3\right)}\,E\left(\text{ArcSin}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
```

```
[Out] -(Sqrt[(-3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(3 + Sqrt[21])]*x], (-5 -
 Sqrt[21])/2]) + Sqrt[9 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(3 + Sqrt[21]
)]*x], (-5 - Sqrt[21])/2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{3-x^2}{\sqrt{3+3x^2-x^4}}\,dx = 2\int \frac{3-x^2}{\sqrt{3+\sqrt{21}-2x^2}\,\sqrt{-3+\sqrt{21}+2x^2}}\,dx$$

$$= \left(3+\sqrt{21}\right)\int \frac{1}{\sqrt{3+\sqrt{21}-2x^2}\,\sqrt{-3+\sqrt{21}+2x^2}}\,dx - \int \frac{\sqrt{-3+\sqrt{21}+}}{\sqrt{3+\sqrt{21}-2}}$$

$$= -\sqrt{\frac{1}{2}\left(-3+\sqrt{21}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5-\sqrt{21}\right)\right)+\frac{1}{2}\sqrt{36+8}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 103, normalized size = 1.07

$$-\frac{i\left(\left(3+\sqrt{21}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)-\left(-3+\sqrt{21}\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)\right)}{\sqrt{2\left(3+\sqrt{21}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 + 3*x^2 - x^4],x]
[Out] ((-I)*((3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21])]*x], (-5 +
 Sqrt[21])/2] - (-3 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[21])]
*x], (-5 + Sqrt[21])/2]))/Sqrt[2*(3 + Sqrt[21])]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(74) = 148.
time = 0.08, size = 204, normalized size = 2.12

method	result

default	$\frac{36\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-18+6\sqrt{21}}}{6},\frac{i\sqrt{3}}{2}+\frac{i\sqrt{7}}{2}\right)-\mathrm{E}\right.}{\sqrt{-18+6\sqrt{21}}\,\sqrt{-x^4+3x^2+3}\,\left(3+\sqrt{21}\right)}$
elliptic	$\frac{36\sqrt{1-\left(-\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-18+6\sqrt{21}}}{6},\frac{i\sqrt{3}}{2}+\frac{i\sqrt{7}}{2}\right)-\mathrm{E}\right.}{\sqrt{-18+6\sqrt{21}}\,\sqrt{-x^4+3x^2+3}\,\left(3+\sqrt{21}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 36/(-18+6*21^(1/2))^(1/2)*(1-(-1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*21
^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^(1/2)/(3+21^(1/2))*(EllipticF(1/6*x*(-18+
6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))-EllipticE(1/6*x*(-18+6*21^(1
/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2)))+18/(-18+6*21^(1/2))^(1/2)*(1-(-1/2
+1/6*21^(1/2))*x^2)^(1/2)*(1-(-1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4+3*x^2+3)^
(1/2)*EllipticF(1/6*x*(-18+6*21^(1/2))^(1/2),1/2*I*3^(1/2)+1/2*I*7^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

Fricas [**A**]
time = 0.07, size = 18, normalized size = 0.19

$$\frac{\sqrt{-x^4+3\,x^2+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-x^4 + 3*x^2 + 3)/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2}{\sqrt{-x^4+3x^2+3}}\,dx-\int\left(-\frac{3}{\sqrt{-x^4+3x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4+3*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 + 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 + 3*x*
*2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4+3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 + 3*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2-3}{\sqrt{-x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(3*x^2 - x^4 + 3)^(1/2),x)
```

```
[Out] int(-(x^2 - 3)/(3*x^2 - x^4 + 3)^(1/2), x)
```

3.116 $\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx$

Optimal. Leaf size=92

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)+\sqrt{5+2\sqrt{13}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\right.\right.$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(-1+13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))*
(2+2*13^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(-1+13^(1/2))^(1/2),1/6*I*39^(1/2)
-1/6*I*3^(1/2))*(5+2*13^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1194, 538, 435, 430}

$$\sqrt{5+2\sqrt{13}}\,F\left(\text{ArcSin}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)-\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}\,E\left(\text{ArcSin}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```

```
[Out] -(Sqrt[(1 + Sqrt[13])/2]*EllipticE[ArcSin[Sqrt[2/(-1 + Sqrt[13])]*x], (-7 +
 Sqrt[13])/6]) + Sqrt[5 + 2*Sqrt[13]]*EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[13
])]*x], (-7 + Sqrt[13])/6]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{3-x^2}{\sqrt{3-x^2-x^4}}\,dx = 2\int \frac{3-x^2}{\sqrt{-1+\sqrt{13}-2x^2}\,\sqrt{1+\sqrt{13}+2x^2}}\,dx$$

$$= \left(7+\sqrt{13}\right)\int \frac{1}{\sqrt{-1+\sqrt{13}-2x^2}\,\sqrt{1+\sqrt{13}+2x^2}}\,dx - \int \frac{\sqrt{1+\sqrt{13}+2x}}{\sqrt{-1+\sqrt{13}-2}}$$

$$= -\sqrt{\frac{1}{2}\left(1+\sqrt{13}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{13}}}\,x\right)|\frac{1}{6}\left(-7+\sqrt{13}\right)\right) + \sqrt{5+2\sqrt{13}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.08, size = 107, normalized size = 1.16

$$-\frac{i\left(\left(-1+\sqrt{13}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)|-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)-\left(-7+\sqrt{13}\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{13}}}\,x\right)|-\frac{7}{6}-\frac{\sqrt{13}}{6}\right)\right)}{\sqrt{2\left(-1+\sqrt{13}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 - x^2 - x^4],x]
```

```
[Out] ((-I)*((-1 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]*x], -7/6
- Sqrt[13]/6] - (-7 + Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[13])]*
x], -7/6 - Sqrt[13]/6]))/Sqrt[2*(-1 + Sqrt[13])]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(74) = 148$.
time = 0.10, size = 204, normalized size = 2.22

method	result

default	$\frac{36\sqrt{1-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{6+6\sqrt{13}}}{6},\frac{i\sqrt{39}}{6}-\frac{i\sqrt{3}}{6}\right)-\text{EllipticE}\right.}{\sqrt{6+6\sqrt{13}}\,\sqrt{-x^4-x^2+3}\,\left(-1+\sqrt{13}\right)}$
elliptic	$\frac{36\sqrt{1-\left(\frac{1}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{6+6\sqrt{13}}}{6},\frac{i\sqrt{39}}{6}-\frac{i\sqrt{3}}{6}\right)-\text{EllipticE}\right.}{\sqrt{6+6\sqrt{13}}\,\sqrt{-x^4-x^2+3}\,\left(-1+\sqrt{13}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4-x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 36/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(1/6-1/6*13^(1/
2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)/(-1+13^(1/2))*(EllipticF(1/6*x*(6+6*13^(1
/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))-EllipticE(1/6*x*(6+6*13^(1/2))^(1/
2),1/6*I*39^(1/2)-1/6*I*3^(1/2)))+18/(6+6*13^(1/2))^(1/2)*(1-(1/6+1/6*13^(1
/2))*x^2)^(1/2)*(1-(1/6-1/6*13^(1/2))*x^2)^(1/2)/(-x^4-x^2+3)^(1/2)*Ellipti
cF(1/6*x*(6+6*13^(1/2))^(1/2),1/6*I*39^(1/2)-1/6*I*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

Fricas [A]
time = 0.09, size = 18, normalized size = 0.20

$$\frac{\sqrt{-x^4-x^2+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-x^4 - x^2 + 3)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2}{\sqrt{-x^4-x^2+3}}\,dx-\int\left(-\frac{3}{\sqrt{-x^4-x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4-x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - x**2 + 3), x) - Integral(-3/sqrt(-x**4 - x**2 +
 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2-3}{\sqrt{-x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(3 - x^4 - x^2)^(1/2),x)
```

```
[Out] int(-(x^2 - 3)/(3 - x^4 - x^2)^(1/2), x)
```

3.117 $\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}} \, dx$

Optimal. Leaf size=27

$$-\sqrt{3}\, E\left(\sin^{-1}(x)|-\frac{1}{3}\right)+2\sqrt{3}\, F\left(\sin^{-1}(x)|-\frac{1}{3}\right)$$

```
[Out] -EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+2*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1194, 538, 435, 430}

$$2\sqrt{3}\, F\left(\text{ArcSin}(x)\left|-\frac{1}{3}\right.\right)-\sqrt{3}\, E\left(\text{ArcSin}(x)\left|-\frac{1}{3}\right.\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] -(Sqrt[3]*EllipticE[ArcSin[x], -1/3]) + 2*Sqrt[3]*EllipticF[ArcSin[x], -1/3
]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c])))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3-x^2}{\sqrt{3-2x^2-x^4}}\,dx &= 2\int \frac{3-x^2}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx\\ &= 12\int \frac{1}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx-\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx\\ &= -\sqrt{3}\,E\left(\sin^{-1}(x)|-\frac{1}{3}\right)+2\sqrt{3}\,F\left(\sin^{-1}(x)|-\frac{1}{3}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 35, normalized size = 1.30

$$-i\left(E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)+2F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x^2)/Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (-I)*(EllipticE[I*ArcSinh[x/Sqrt[3]], -3] + 2*EllipticF[I*ArcSinh[x/Sqrt[3]
], -3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(27) = 54.
time = 0.04, size = 95, normalized size = 3.52

method	result
default	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{\sqrt{-x^4-2x^2+3}}$
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\mathrm{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{\sqrt{-x^4-2x^2+3}}+\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{\sqrt{-x^4-2x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1
/2))-EllipticE(x,1/3*I*3^(1/2)))+(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2
+3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

Fricas [A]
time = 0.08, size = 18, normalized size = 0.67

$$\frac{\sqrt{-x^4 - 2\,x^2 + 3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-x^4 - 2*x^2 + 3)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{-x^4 - 2x^2 + 3}}\,dx - \int \left(-\frac{3}{\sqrt{-x^4 - 2x^2 + 3}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - 2*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 2*x*
*2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-2*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 2*x^2 + 3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int -\frac{x^2-3}{\sqrt{-x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(3 - x^4 - 2*x^2)^(1/2),x)
[Out] int(-(x^2 - 3)/(3 - x^4 - 2*x^2)^(1/2), x)
```

3.118 $\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx$

Optimal. Leaf size=92

$$-\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)+\sqrt{3+2\sqrt{21}}\,F\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\right.\right.$$

```
[Out] -1/2*EllipticE(x*2^(1/2)/(-3+21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))*(
6+2*21^(1/2))^(1/2)+EllipticF(x*2^(1/2)/(-3+21^(1/2))^(1/2),1/2*I*7^(1/2)-1
/2*I*3^(1/2))*(3+2*21^(1/2))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1194, 538, 435, 430}

$$\sqrt{3+2\sqrt{21}}\,F\left(\text{ArcSin}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)-\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}\,E\left(\text{ArcSin}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

```
[Out] -(Sqrt[(3 + Sqrt[21])/2]*EllipticE[ArcSin[Sqrt[2/(-3 + Sqrt[21])]*x], (-5 +
 Sqrt[21])/2]) + Sqrt[3 + 2*Sqrt[21]]*EllipticF[ArcSin[Sqrt[2/(-3 + Sqrt[21
])]*x], (-5 + Sqrt[21])/2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{3-x^2}{\sqrt{3-3x^2-x^4}}\,dx = 2\int \frac{3-x^2}{\sqrt{-3+\sqrt{21}-2x^2}\,\sqrt{3+\sqrt{21}+2x^2}}\,dx$$

$$= \left(9+\sqrt{21}\right)\int \frac{1}{\sqrt{-3+\sqrt{21}-2x^2}\,\sqrt{3+\sqrt{21}+2x^2}}\,dx - \int \frac{\sqrt{3+\sqrt{21}+2}}{\sqrt{-3+\sqrt{21}-}}$$

$$= -\sqrt{\frac{1}{2}\left(3+\sqrt{21}\right)}\,E\left(\sin^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{21}}}\,x\right)|\frac{1}{2}\left(-5+\sqrt{21}\right)\right) + \sqrt{3+2\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 107, normalized size = 1.16

$$-\frac{i\left(\left(-3+\sqrt{21}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)|-\frac{5}{2}-\frac{\sqrt{21}}{2}\right)-\left(-9+\sqrt{21}\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{21}}}\,x\right)|-\frac{5}{2}-\frac{\sqrt{21}}{2}\right)\right)}{\sqrt{2\left(-3+\sqrt{21}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(3 - x^2)/Sqrt[3 - 3*x^2 - x^4],x]
```

```
[Out] ((-I)*((-3 + Sqrt[21])*EllipticE[I*ArcSinh[Sqrt[2/(3 + Sqrt[21])]*x], -5/2
- Sqrt[21]/2] - (-9 + Sqrt[21])*EllipticF[I*ArcSinh[Sqrt[2/(3 + Sqrt[21])]*
x], -5/2 - Sqrt[21]/2]))/Sqrt[2*(-3 + Sqrt[21])]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(74) = 148.
time = 0.08, size = 204, normalized size = 2.22

method	result

default	$\frac{36\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\frac{i\sqrt{7}}{2}-\frac{i\sqrt{3}}{2}\right)-\mathrm{EllipticE}\right.}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}\,\left(-3+\sqrt{21}\right)}$
elliptic	$\frac{36\sqrt{1-\left(\frac{1}{2}+\frac{\sqrt{21}}{6}\right)x^2}\,\sqrt{1-\left(\frac{1}{2}-\frac{\sqrt{21}}{6}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{18+6\sqrt{21}}}{6},\frac{i\sqrt{7}}{2}-\frac{i\sqrt{3}}{2}\right)-\mathrm{EllipticE}\right.}{\sqrt{18+6\sqrt{21}}\,\sqrt{-x^4-3x^2+3}\,\left(-3+\sqrt{21}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 36/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*21^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*21^(1
/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)/(-3+21^(1/2))*(EllipticF(1/6*x*(18+6*2
1^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))-EllipticE(1/6*x*(18+6*21^(1/2))
^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2)))+18/(18+6*21^(1/2))^(1/2)*(1-(1/2+1/6*2
1^(1/2))*x^2)^(1/2)*(1-(1/2-1/6*21^(1/2))*x^2)^(1/2)/(-x^4-3*x^2+3)^(1/2)*E
llipticF(1/6*x*(18+6*21^(1/2))^(1/2),1/2*I*7^(1/2)-1/2*I*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

Fricas [A]
time = 0.10, size = 18, normalized size = 0.20

$$\frac{\sqrt{-x^4-3\,x^2+3}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-x^4 - 3*x^2 + 3)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{\sqrt{-x^4-3x^2+3}}\,dx-\int \left(-\frac{3}{\sqrt{-x^4-3x^2+3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)/(-x**4-3*x**2+3)**(1/2),x)
```

```
[Out] -Integral(x**2/sqrt(-x**4 - 3*x**2 + 3), x) - Integral(-3/sqrt(-x**4 - 3*x*
*2 + 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)/(-x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 3)/sqrt(-x^4 - 3*x^2 + 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{x^2-3}{\sqrt{-x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 3)/(3 - x^4 - 3*x^2)^(1/2),x)
```

```
[Out] int(-(x^2 - 3)/(3 - x^4 - 3*x^2)^(1/2), x)
```

3.119 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=296

$$\frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2}-\frac{2\sqrt[4]{a}\,\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right.\right.}{\sqrt{a+bx^2+cx^4}}$$

```
[Out] 2*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/(a^(1/2)+x^2*c^(1/2))-2*a^(1/4)*c^(1/4)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/2)-(-4*a*c+
b^2)^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)
/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1211, 1117, 1209}

$$\frac{\left(-\sqrt{b^2-4ac}+2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\,\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[a] + Sqrt[c]*x^2) - (2*a^(1/4)*
c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[a]*Sqrt[c] - Sqrt[b^2 - 4*a*c])
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{a+bx^2+cx^4}}\,dx = -\left(\left(2\sqrt{a}\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx\right)+\left(b+2\sqrt{a}\sqrt{c}-\sqrt{b^2-4ac}\right)$$

$$= \frac{2\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}\,x^2}-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E(}{\sqrt{a+bx^2+c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.18, size = 187, normalized size = 0.63

$$-\frac{2i\sqrt{2}\,a\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
+ Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^2 + c*x^4]
```

Maple [A]
time = 0.08, size = 515, normalized size = 1.74

method	result
default	$\frac{b\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
elliptic	$-\frac{\left(-b-2c\,x^2+\sqrt{-4ac+b^2}\right)\sqrt{-\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)}\left(\frac{\left(4ac-b^2\right)\sqrt{2}\,\sqrt{4-\frac{2\left(\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right.}{a\left(4ac-b^2\right)}}}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/4*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))-1/4*(-4*a*c+b^2)^(1/2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^
(1/2))/a/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b + 2cx^2 - \sqrt{-4ac + b^2}}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(c*x^4+b*x^2+a)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{b + 2\,c\,x^2 - \sqrt{b^2 - 4\,a\,c}}{\sqrt{c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.120 $\int (d+ex^2)^4 (a+cx^4)\,dx$

Optimal. Leaf size=106

$$ad^4x+\frac{4}{3}ad^3ex^3+\frac{1}{5}d^2(cd^2+6ae^2)x^5+\frac{4}{7}de(cd^2+ae^2)x^7+\frac{1}{9}e^2(6cd^2+ae^2)x^9+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

```
[Out] a*d^4*x+4/3*a*d^3*e*x^3+1/5*d^2*(6*a*e^2+c*d^2)*x^5+4/7*d*e*(a*e^2+c*d^2)*x
^7+1/9*e^2*(a*e^2+6*c*d^2)*x^9+4/11*c*d*e^3*x^11+1/13*c*e^4*x^13
```

Rubi [A]
time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1168}

$$\frac{1}{9}e^2x^9(ae^2+6cd^2)+\frac{4}{7}dex^7(ae^2+cd^2)+\frac{1}{5}d^2x^5(6ae^2+cd^2)+ad^4x+\frac{4}{3}ad^3ex^3+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + c*x^4),x]
```

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2
 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c
*e^4*x^13)/13
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int (d+ex^2)^4 (a+cx^4)\,dx = \int (ad^4+4ad^3ex^2+d^2(cd^2+6ae^2)x^4+4de(cd^2+ae^2)x^6+e^2(6cd^2+ae^2)x$$
$$= ad^4x+\frac{4}{3}ad^3ex^3+\frac{1}{5}d^2(cd^2+6ae^2)x^5+\frac{4}{7}de(cd^2+ae^2)x^7+\frac{1}{9}e^2(6cd^2+ae^2)$$

Mathematica [A]
time = 0.01, size = 106, normalized size = 1.00

$$ad^4x+\frac{4}{3}ad^3ex^3+\frac{1}{5}d^2(cd^2+6ae^2)x^5+\frac{4}{7}de(cd^2+ae^2)x^7+\frac{1}{9}e^2(6cd^2+ae^2)x^9+\frac{4}{11}cde^3x^{11}+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + c*x^4),x]
```

```
[Out] a*d^4*x + (4*a*d^3*e*x^3)/3 + (d^2*(c*d^2 + 6*a*e^2)*x^5)/5 + (4*d*e*(c*d^2
 + a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + a*e^2)*x^9)/9 + (4*c*d*e^3*x^11)/11 + (c
*e^4*x^13)/13
```

Maple [A]
time = 0.13, size = 97, normalized size = 0.92

method	result
norman	$\frac{c\,e^{4} x^{13}}{13}+\frac{4 c d\,e^{3} x^{11}}{11}+\left(\frac{1}{9} e^{4} a +\frac{2}{3} d^{2} e^{2} c \right) x^{9}+\left(\frac{4}{7} d\,e^{3} a +\frac{4}{7} d^{3} e c \right) x^{7}+\left(\frac{6}{5} d^{2} e^{2} a +\frac{1}{5} d^{4} c \right) x^{5}+\frac{4 a\,d^{3} e\,x^{3}}{3}+$
default	$\frac{c\,e^{4} x^{13}}{13}+\frac{4 c d\,e^{3} x^{11}}{11}+\frac{\left(e^{4} a +6 d^{2} e^{2} c \right) x^{9}}{9}+\frac{\left(4 d\,e^{3} a +4 d^{3} e c \right) x^{7}}{7}+\frac{\left(6 d^{2} e^{2} a +d^{4} c \right) x^{5}}{5}+\frac{4 a\,d^{3} e\,x^{3}}{3}+a\,d^{4} x$
gosper	$\frac{1}{13} c\,e^{4} x^{13}+\frac{4}{11} c d\,e^{3} x^{11}+\frac{1}{9} x^{9} e^{4} a +\frac{2}{3} x^{9} d^{2} e^{2} c +\frac{4}{7} x^{7} d\,e^{3} a +\frac{4}{7} x^{7} d^{3} e c +\frac{6}{5} x^{5} d^{2} e^{2} a +\frac{1}{5} x^{5} d^{4} c +\frac{4}{3} a\,d^{3}$
risch	$\frac{1}{13} c\,e^{4} x^{13}+\frac{4}{11} c d\,e^{3} x^{11}+\frac{1}{9} x^{9} e^{4} a +\frac{2}{3} x^{9} d^{2} e^{2} c +\frac{4}{7} x^{7} d\,e^{3} a +\frac{4}{7} x^{7} d^{3} e c +\frac{6}{5} x^{5} d^{2} e^{2} a +\frac{1}{5} x^{5} d^{4} c +\frac{4}{3} a\,d^{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c*e^4*x^13+4/11*c*d*e^3*x^11+1/9*(a*e^4+6*c*d^2*e^2)*x^9+1/7*(4*a*d*e^
3+4*c*d^3*e)*x^7+1/5*(6*a*d^2*e^2+c*d^4)*x^5+4/3*a*d^3*e*x^3+a*d^4*x
```

Maxima [A]
time = 0.29, size = 90, normalized size = 0.85

$$\frac{1}{13}\,cx^{13}e^{4}+\frac{4}{11}\,cdx^{11}e^{3}+\frac{1}{9}\,{\left(6\,cd^{2}e^{2}+ae^{4}\right)}x^{9}+\frac{4}{7}\,{\left(cd^{3}e+ade^{3}\right)}x^{7}+\frac{4}{3}\,ad^{3}x^{3}e+ad^{4}x+\frac{1}{5}\,{\left(cd^{4}+6\,ad^{2}e^{2}\right)}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*x^13*e^4 + 4/11*c*d*x^11*e^3 + 1/9*(6*c*d^2*e^2 + a*e^4)*x^9 + 4/7*(
c*d^3*e + a*d*e^3)*x^7 + 4/3*a*d^3*x^3*e + a*d^4*x + 1/5*(c*d^4 + 6*a*d^2*e
^2)*x^5
```

Fricas [A]
time = 0.33, size = 98, normalized size = 0.92

$$\frac{1}{5}\,cd^{4}x^{5}+ad^{4}x+\frac{1}{117}\,{\left(9\,cx^{13}+13\,ax^{9}\right)}e^{4}+\frac{4}{77}\,{\left(7\,cdx^{11}+11\,adx^{7}\right)}e^{3}+\frac{2}{15}\,{\left(5\,cd^{2}x^{9}+9\,ad^{2}x^{5}\right)}e^{2}+\frac{4}{21}\,{\left(3\,cd^{3}x^{7}+7\,ad^{3}x^{3}\right)}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/5*c*d^4*x^5 + a*d^4*x + 1/117*(9*c*x^13 + 13*a*x^9)*e^4 + 4/77*(7*c*d*x^1
1 + 11*a*d*x^7)*e^3 + 2/15*(5*c*d^2*x^9 + 9*a*d^2*x^5)*e^2 + 4/21*(3*c*d^3*
x^7 + 7*a*d^3*x^3)*e
```

Sympy [A]
time = 0.01, size = 110, normalized size = 1.04

$$ad^4x + \frac{4ad^3ex^3}{3} + \frac{4cde^3x^{11}}{11} + \frac{ce^4x^{13}}{13} + x^9\left(\frac{ae^4}{9} + \frac{2cd^2e^2}{3}\right) + x^7\cdot\left(\frac{4ade^3}{7} + \frac{4cd^3e}{7}\right) + x^5\cdot\left(\frac{6ad^2e^2}{5} + \frac{cd^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(c*x**4+a),x)
```

```
[Out] a*d**4*x + 4*a*d**3*e*x**3/3 + 4*c*d*e**3*x**11/11 + c*e**4*x**13/13 + x**9
*(a*e**4/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 4*c*d**3*e/7) + x**5*(
6*a*d**2*e**2/5 + c*d**4/5)
```

Giac [A]
time = 3.78, size = 94, normalized size = 0.89

$$\frac{1}{13}cx^{13}e^4 + \frac{4}{11}cdx^{11}e^3 + \frac{2}{3}cd^2x^9e^2 + \frac{4}{7}cd^3x^7e + \frac{1}{9}ax^9e^4 + \frac{1}{5}cd^4x^5 + \frac{4}{7}adx^7e^3 + \frac{6}{5}ad^2x^5e^2 + \frac{4}{3}ad^3x^3e + ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^4 + 4/11*c*d*x^11*e^3 + 2/3*c*d^2*x^9*e^2 + 4/7*c*d^3*x^7*e +
 1/9*a*x^9*e^4 + 1/5*c*d^4*x^5 + 4/7*a*d*x^7*e^3 + 6/5*a*d^2*x^5*e^2 + 4/3*
a*d^3*x^3*e + a*d^4*x
```

Mupad [B]
time = 4.35, size = 95, normalized size = 0.90

$$x^5\left(\frac{c\,d^4}{5} + \frac{6\,a\,d^2\,e^2}{5}\right) + x^9\left(\frac{2\,c\,d^2\,e^2}{3} + \frac{a\,e^4}{9}\right) + x^7\left(\frac{4\,c\,d^3\,e}{7} + \frac{4\,a\,d\,e^3}{7}\right) + \frac{c\,e^4\,x^{13}}{13} + a\,d^4\,x + \frac{4\,a\,d^3\,e\,x^3}{3} + \frac{4\,c\,d\,e^3\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)*(d + e*x^2)^4,x)
```

```
[Out] x^5*((c*d^4)/5 + (6*a*d^2*e^2)/5) + x^9*((a*e^4)/9 + (2*c*d^2*e^2)/3) + x^7
*((4*a*d*e^3)/7 + (4*c*d^3*e)/7) + (c*e^4*x^13)/13 + a*d^4*x + (4*a*d^3*e*x
^3)/3 + (4*c*d*e^3*x^11)/11
```

3.121 $\int (d+ex^2)^3 (a+cx^4) \, dx$

Optimal. Leaf size=79

$$ad^3x+ad^2ex^3+\frac{1}{5}d(cd^2+3ae^2)\,x^5+\frac{1}{7}e(3cd^2+ae^2)\,x^7+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

```
[Out] a*d^3*x+a*d^2*e*x^3+1/5*d*(3*a*e^2+c*d^2)*x^5+1/7*e*(a*e^2+3*c*d^2)*x^7+1/3
*c*d*e^2*x^9+1/11*c*e^3*x^11
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1168}

$$\frac{1}{7}ex^7(ae^2+3cd^2)+\frac{1}{5}dx^5(3ae^2+cd^2)+ad^3x+ad^2ex^3+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + c*x^4),x]
```

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*
x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex^2)^3\,(a+cx^4)\,dx &= \int \left(ad^3+3ad^2ex^2+d(cd^2+3ae^2)\,x^4+e(3cd^2+ae^2)\,x^6+3cde^2x^8+ce^3x^{10}\right)\\ &= ad^3x+ad^2ex^3+\frac{1}{5}d(cd^2+3ae^2)\,x^5+\frac{1}{7}e(3cd^2+ae^2)\,x^7+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 79, normalized size = 1.00

$$ad^3x+ad^2ex^3+\frac{1}{5}d(cd^2+3ae^2)\,x^5+\frac{1}{7}e(3cd^2+ae^2)\,x^7+\frac{1}{3}cde^2x^9+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + c*x^4),x]
```

```
[Out] a*d^3*x + a*d^2*e*x^3 + (d*(c*d^2 + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + a*e^2)*
x^7)/7 + (c*d*e^2*x^9)/3 + (c*e^3*x^11)/11
```

Maple [A]
time = 0.15, size = 72, normalized size = 0.91

method	result	size
default	$\frac{c\,e^3x^{11}}{11}+\frac{cd\,e^2x^9}{3}+\frac{(a\,e^3+3c\,d^2e)x^7}{7}+\frac{(3d\,e^2a+c\,d^3)x^5}{5}+a\,d^2e\,x^3+a\,d^3x$	72
norman	$\frac{c\,e^3x^{11}}{11}+\frac{cd\,e^2x^9}{3}+\left(\frac{1}{7}a\,e^3+\frac{3}{7}c\,d^2e\right)x^7+\left(\frac{3}{5}d\,e^2a+\frac{1}{5}c\,d^3\right)x^5+a\,d^2e\,x^3+a\,d^3x$	72
gosper	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{1}{5}x^5c\,d^3+a\,d^2e\,x^3+a\,d^3x$	74
risch	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{1}{5}x^5c\,d^3+a\,d^2e\,x^3+a\,d^3x$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c*e^3*x^11+1/3*c*d*e^2*x^9+1/7*(a*e^3+3*c*d^2*e)*x^7+1/5*(3*a*d*e^2+c*
d^3)*x^5+a*d^2*e*x^3+a*d^3*x
```

Maxima [A]
time = 0.28, size = 69, normalized size = 0.87

$$\frac{1}{11}\,cx^{11}e^3+\frac{1}{3}\,cdx^9e^2+\frac{1}{7}\,\left(3\,cd^2e+ae^3\right)x^7+ad^2x^3e+\frac{1}{5}\,\left(cd^3+3\,ade^2\right)x^5+ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/11*c*x^11*e^3 + 1/3*c*d*x^9*e^2 + 1/7*(3*c*d^2*e + a*e^3)*x^7 + a*d^2*x^3
*e + 1/5*(c*d^3 + 3*a*d*e^2)*x^5 + a*d^3*x
```

Fricas [A]
time = 0.31, size = 75, normalized size = 0.95

$$\frac{1}{5}\,cd^3x^5+ad^3x+\frac{1}{77}\,\left(7\,cx^{11}+11\,ax^7\right)e^3+\frac{1}{15}\,\left(5\,cdx^9+9\,adx^5\right)e^2+\frac{1}{7}\,\left(3\,cd^2x^7+7\,ad^2x^3\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/5*c*d^3*x^5 + a*d^3*x + 1/77*(7*c*x^11 + 11*a*x^7)*e^3 + 1/15*(5*c*d*x^9
+ 9*a*d*x^5)*e^2 + 1/7*(3*c*d^2*x^7 + 7*a*d^2*x^3)*e
```

Sympy [A]
time = 0.01, size = 78, normalized size = 0.99

$$ad^3x+ad^2ex^3+\frac{cde^2x^9}{3}+\frac{ce^3x^{11}}{11}+x^7\left(\frac{ae^3}{7}+\frac{3cd^2e}{7}\right)+x^5\cdot\left(\frac{3ade^2}{5}+\frac{cd^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+a),x)
```

```
[Out] a*d**3*x + a*d**2*e*x**3 + c*d*e**2*x**9/3 + c*e**3*x**11/11 + x**7*(a*e**3
/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + c*d**3/5)
```

Giac [A]
time = 4.20, size = 71, normalized size = 0.90

$$\frac{1}{11}cx^{11}e^3+\frac{1}{3}cdx^9e^2+\frac{3}{7}cd^2x^7e+\frac{1}{5}cd^3x^5+\frac{1}{7}ax^7e^3+\frac{3}{5}adx^5e^2+ad^2x^3e+ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^3 + 1/3*c*d*x^9*e^2 + 3/7*c*d^2*x^7*e + 1/5*c*d^3*x^5 + 1/7*a
*x^7*e^3 + 3/5*a*d*x^5*e^2 + a*d^2*x^3*e + a*d^3*x
```

Mupad [B]
time = 0.03, size = 71, normalized size = 0.90

$$x^5\left(\frac{c\,d^3}{5}+\frac{3\,a\,d\,e^2}{5}\right)+x^7\left(\frac{3\,c\,d^2\,e}{7}+\frac{a\,e^3}{7}\right)+\frac{c\,e^3\,x^{11}}{11}+a\,d^3\,x+a\,d^2\,e\,x^3+\frac{c\,d\,e^2\,x^9}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)*(d + e*x^2)^3,x)
```

```
[Out] x^5*((c*d^3)/5 + (3*a*d*e^2)/5) + x^7*((a*e^3)/7 + (3*c*d^2*e)/7) + (c*e^3*
x^11)/11 + a*d^3*x + a*d^2*e*x^3 + (c*d*e^2*x^9)/3
```

3.122 $\int \left(d+ex^2\right)^2 \left(a+cx^4\right) \, dx$

Optimal. Leaf size=56

$$ad^2x+\frac{2}{3}adex^3+\frac{1}{5}\left(cd^2+ae^2\right)x^5+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1168}

$$\frac{1}{5}x^5\left(ae^2+cd^2\right)+ad^2x+\frac{2}{3}adex^3+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + c*x^4),x]
```

```
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*
e^2*x^9)/9
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2\left(a+cx^4\right)\,dx &= \int \left(ad^2+2adex^2+\left(cd^2+ae^2\right)x^4+2cdex^6+ce^2x^8\right)\,dx\\ &= ad^2x+\frac{2}{3}adex^3+\frac{1}{5}\left(cd^2+ae^2\right)x^5+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.00

$$ad^2x+\frac{2}{3}adex^3+\frac{1}{5}\left(cd^2+ae^2\right)x^5+\frac{2}{7}cdex^7+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + c*x^4),x]
```

```
[Out] a*d^2*x + (2*a*d*e*x^3)/3 + ((c*d^2 + a*e^2)*x^5)/5 + (2*c*d*e*x^7)/7 + (c*
e^2*x^9)/9
```

Maple [A]
time = 0.16, size = 49, normalized size = 0.88

method	result	size
default	$a\,d^{2}x+\frac{2ade\,x^{3}}{3}+\frac{\left(a\,e^{2}+c\,d^{2}\right)x^{5}}{5}+\frac{2cde\,x^{7}}{7}+\frac{c\,e^{2}x^{9}}{9}$	49
norman	$\frac{c\,e^{2}x^{9}}{9}+\frac{2cde\,x^{7}}{7}+\left(\frac{a\,e^{2}}{5}+\frac{c\,d^{2}}{5}\right)x^{5}+\frac{2ade\,x^{3}}{3}+a\,d^{2}x$	50
gosper	$\frac{1}{9}c\,e^{2}x^{9}+\frac{2}{7}cde\,x^{7}+\frac{1}{5}x^{5}a\,e^{2}+\frac{1}{5}x^{5}c\,d^{2}+\frac{2}{3}ade\,x^{3}+a\,d^{2}x$	51
risch	$\frac{1}{9}c\,e^{2}x^{9}+\frac{2}{7}cde\,x^{7}+\frac{1}{5}x^{5}a\,e^{2}+\frac{1}{5}x^{5}c\,d^{2}+\frac{2}{3}ade\,x^{3}+a\,d^{2}x$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] a*d^2*x+2/3*a*d*e*x^3+1/5*(a*e^2+c*d^2)*x^5+2/7*c*d*e*x^7+1/9*c*e^2*x^9
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.86

$$\frac{1}{9}\,cx^{9}e^{2}+\frac{2}{7}\,cdx^{7}e+\frac{1}{5}\,\left(cd^{2}+ae^{2}\right)x^{5}+\frac{2}{3}\,adx^{3}e+ad^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/5*(c*d^2 + a*e^2)*x^5 + 2/3*a*d*x^3*e + a
*d^2*x
```

Fricas [A]
time = 0.39, size = 52, normalized size = 0.93

$$\frac{1}{5}\,cd^{2}x^{5}+ad^{2}x+\frac{1}{45}\,\left(5\,cx^{9}+9\,ax^{5}\right)e^{2}+\frac{2}{21}\,\left(3\,cdx^{7}+7\,adx^{3}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/5*c*d^2*x^5 + a*d^2*x + 1/45*(5*c*x^9 + 9*a*x^5)*e^2 + 2/21*(3*c*d*x^7 +
7*a*d*x^3)*e
```

Sympy [A]
time = 0.01, size = 56, normalized size = 1.00

$$ad^{2}x+\frac{2adex^{3}}{3}+\frac{2cdex^{7}}{7}+\frac{ce^{2}x^{9}}{9}+x^{5}\left(\frac{ae^{2}}{5}+\frac{cd^{2}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+a),x)
```

```
[Out] a*d**2*x + 2*a*d*e*x**3/3 + 2*c*d*e*x**7/7 + c*e**2*x**9/9 + x**5*(a*e**2/5
 + c*d**2/5)
```

Giac [A]
time = 5.04, size = 50, normalized size = 0.89

$$\frac{1}{9}\,cx^9e^2+\frac{2}{7}\,cdx^7e+\frac{1}{5}\,cd^2x^5+\frac{1}{5}\,ax^5e^2+\frac{2}{3}\,adx^3e+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/5*c*d^2*x^5 + 1/5*a*x^5*e^2 + 2/3*a*d*x^3
*e + a*d^2*x
```

Mupad [B]
time = 0.02, size = 49, normalized size = 0.88

$$x^5\left(\frac{c\,d^2}{5}+\frac{a\,e^2}{5}\right)+\frac{c\,e^2\,x^9}{9}+a\,d^2\,x+\frac{2\,a\,d\,e\,x^3}{3}+\frac{2\,c\,d\,e\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)*(d + e*x^2)^2,x)
```

```
[Out] x^5*((a*e^2)/5 + (c*d^2)/5) + (c*e^2*x^9)/9 + a*d^2*x + (2*a*d*e*x^3)/3 + (
2*c*d*e*x^7)/7
```

3.123 $\int (d+ex^2)(a+cx^4)\,dx$

Optimal. Leaf size=32

$$adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

```
[Out] a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.067$, Rules used = $\{1168\}$

$$adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex^2)(a+cx^4)\,dx&=\int (ad+aex^2+cdx^4+cex^6)\,dx\\&=adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7\end{aligned}$$

Mathematica [A]
time = 0.00, size = 32, normalized size = 1.00

$$adx+\frac{1}{3}aex^3+\frac{1}{5}cdx^5+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + c*x^4),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

Maple [A]
time = 0.04, size = 27, normalized size = 0.84

method	result	size
gosper	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
default	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
norman	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27
risch	$adx+\frac{1}{3}ae\,x^3+\frac{1}{5}cd\,x^5+\frac{1}{7}ce\,x^7$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a),x,method=_RETURNVERBOSE)
[Out] a*d*x+1/3*a*e*x^3+1/5*c*d*x^5+1/7*c*e*x^7
```

Maxima [A]
time = 0.28, size = 28, normalized size = 0.88

$$\frac{1}{7}\,cx^7e+\frac{1}{5}\,cdx^5+\frac{1}{3}\,ax^3e+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="maxima")
[Out] 1/7*c*x^7*e + 1/5*c*d*x^5 + 1/3*a*x^3*e + a*d*x
```

Fricas [A]
time = 0.33, size = 29, normalized size = 0.91

$$\frac{1}{5}\,cdx^5+adx+\frac{1}{21}\,\left(3\,cx^7+7\,ax^3\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="fricas")
[Out] 1/5*c*d*x^5 + a*d*x + 1/21*(3*c*x^7 + 7*a*x^3)*e
```

Sympy [A]
time = 0.01, size = 29, normalized size = 0.91

$$adx+\frac{aex^3}{3}+\frac{cdx^5}{5}+\frac{cex^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a),x)
```

```
[Out] a*d*x + a*e*x**3/3 + c*d*x**5/5 + c*e*x**7/7
```

Giac [A]
time = 4.91, size = 28, normalized size = 0.88

$$\frac{1}{7}\,cx^7e + \frac{1}{5}\,cdx^5 + \frac{1}{3}\,ax^3e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e + 1/5*c*d*x^5 + 1/3*a*x^3*e + a*d*x
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.81

$$\frac{c\,e\,x^7}{7} + \frac{c\,d\,x^5}{5} + \frac{a\,e\,x^3}{3} + a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)*(d + e*x^2),x)
```

```
[Out] a*d*x + (a*e*x^3)/3 + (c*d*x^5)/5 + (c*e*x^7)/7
```

3.124 $\int \frac{a+cx^4}{d+ex^2} \, dx$

Optimal. Leaf size=55

$$-\frac{cdx}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}$$

```
[Out] -c*d*x/e^2+1/3*c*x^3/e+(a*e^2+c*d^2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/d^(1
/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$,
Rules used = {1168, 211}

$$\frac{(ae^2+cd^2)\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}-\frac{cdx}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2),x]
```

```
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[d]*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+cx^4}{d+ex^2}\,dx &= \int \left(-\frac{cd}{e^2}+\frac{cx^2}{e}+\frac{cd^2+ae^2}{e^2\,(d+ex^2)}\right) dx\\
&= -\frac{cdx}{e^2}+\frac{cx^3}{3e}+\left(a+\frac{cd^2}{e^2}\right)\int \frac{1}{d+ex^2}\,dx\\
&= -\frac{cdx}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 1.00

$$-\frac{cdx}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2),x]
[Out] -((c*d*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[d]*e^(5/2))
```

Maple [A]
time = 0.18, size = 47, normalized size = 0.85

method	result	size
default	$-\frac{c\left(-\frac{1}{3}e\,x^3+dx\right)}{e^2}+\frac{\left(a\,e^2+c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e^2\sqrt{de}}$	47
risch	$\frac{c\,x^3}{3e}-\frac{cdx}{e^2}-\frac{\ln\left(ex+\sqrt{-de}\right)a}{2\sqrt{-de}}-\frac{\ln\left(ex+\sqrt{-de}\right)c\,d^2}{2e^2\sqrt{-de}}+\frac{\ln\left(-ex+\sqrt{-de}\right)a}{2\sqrt{-de}}+\frac{\ln\left(-ex+\sqrt{-de}\right)c\,d^2}{2e^2\sqrt{-de}}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d),x,method=_RETURNVERBOSE)
[Out] -c/e^2*(-1/3*e*x^3+d*x)+(a*e^2+c*d^2)/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2
))
```

Maxima [A]
time = 0.50, size = 42, normalized size = 0.76

$$\frac{(cd^2+ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}}+\frac{1}{3}\left(cx^3e-3\,cdx\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="maxima")
[Out] (c*d^2 + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + 1/3*(c*x^3*e -
 3*c*d*x)*e^(-2)
```

Fricas [A]
time = 0.34, size = 128, normalized size = 2.33

$$\left[\frac{\left(2\,cdx^3e^2-6\,cd^2xe-3\,(cd^2+ae^2)\sqrt{-de}\,\log\left(\frac{x^2e-2\sqrt{-de}\,x-d}{x^2e+d}\right)\right)e^{(-3)}}{6\,d},\frac{\left(cdx^3e^2-3\,cd^2xe+3\,(cd^2+ae^2)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}\right)e^{(-3)}}{3\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c*d*x^3*e^2 - 6*c*d^2*x*e - 3*(c*d^2 + a*e^2)*sqrt(-d*e)*log((x^2*e
 - 2*sqrt(-d*e)*x - d)/(x^2*e + d)))*e^(-3)/d, 1/3*(c*d*x^3*e^2 - 3*c*d^2*x
*e + 3*(c*d^2 + a*e^2)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2))*e^(-3)/d]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(49) = 98$.
time = 0.16, size = 104, normalized size = 1.89

$$-\frac{cdx}{e^2}+\frac{cx^3}{3e}-\frac{\sqrt{-\frac{1}{de^5}}\,(ae^2+cd^2)\log\left(-de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{de^5}}\,(ae^2+cd^2)\log\left(de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d),x)
```

```
[Out] -c*d*x/e**2 + c*x**3/(3*e) - sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(-d*e**
2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 + c*d**2)*log(d*e**2
*sqrt(-1/(d*e**5)) + x)/2
```

Giac [A]
time = 3.23, size = 44, normalized size = 0.80

$$\frac{(cd^2+ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}}+\frac{1}{3}\left(cx^3e^2-3\,cdxe\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c*d^2 + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + 1/3*(c*x^3*e^2
 - 3*c*d*x*e)*e^(-3)
```

Mupad [B]
time = 0.07, size = 45, normalized size = 0.82

$$\frac{c\,x^3}{3\,e}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+a\,e^2\right)}{\sqrt{d}\,e^{5/2}}-\frac{c\,d\,x}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/(d + e*x^2),x)
```

```
[Out] (c*x^3)/(3*e) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c*d^2))/(d^(1/2)*e^(5/2
)) - (c*d*x)/e^2
```

3.125 $\int \frac{a+cx^4}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=74

$$\frac{cx}{e^2}+\frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d\,(d+ex^2)}-\frac{(3cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[Out] c*x/e^2+1/2*(a+c*d^2/e^2)*x/d/(e*x^2+d)-1/2*(-a*e^2+3*c*d^2)*arctan(x*e^(1/
2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A]

time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1172, 396, 211}

$$-\frac{(3cd^2-ae^2)\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{2d\,(d+ex^2)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (c*d^2)/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
```

```
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+cx^4}{(d+ex^2)^2}\,dx = \frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d(d+ex^2)} - \frac{\int \frac{-a+\frac{cd^2}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}$$
$$= \frac{cx}{e^2} + \frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d(d+ex^2)} + \frac{\left(a-\frac{3cd^2}{e^2}\right)\int \frac{1}{d+ex^2}\,dx}{2d}$$
$$= \frac{cx}{e^2} + \frac{\left(a+\frac{cd^2}{e^2}\right)x}{2d(d+ex^2)} + \frac{\left(a-\frac{3cd^2}{e^2}\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 1.05

$$\frac{cx}{e^2} + \frac{(cd^2+ae^2)\,x}{2de^2(d+ex^2)} - \frac{(3cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^2,x]
[Out] (c*x)/e^2 + ((c*d^2 + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 - a*e^2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A]
time = 0.12, size = 70, normalized size = 0.95

method	result
default	$\frac{cx}{e^2} + \frac{\frac{(a e^2+c d^2)x}{2d(e x^2+d)} + \frac{(a e^2-3c d^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}}{e^2}$
risch	$\frac{cx}{e^2} + \frac{(a e^2+c d^2)x}{2d e^2(e x^2+d)} - \frac{\ln\left(ex+\sqrt{-de}\right)a}{4\sqrt{-de}\,d} + \frac{3d\ln\left(ex+\sqrt{-de}\right)c}{4e^2\sqrt{-de}} + \frac{\ln\left(-ex+\sqrt{-de}\right)a}{4\sqrt{-de}\,d} - \frac{3d\ln\left(-ex+\sqrt{-de}\right)c}{4e^2\sqrt{-de}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/e^2*(1/2*(a*e^2+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2-3*c*d^2)/d/(d*e)^
(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.49, size = 62, normalized size = 0.84

$$cxe^{(-2)} - \frac{(3\,cd^2 - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{(cd^2 + ae^2)x}{2\,(dx^2e^3 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(3/
2) + 1/2*(c*d^2 + a*e^2)*x/(d*x^2*e^3 + d^2*e^2)
```

Fricas [A]
time = 0.37, size = 215, normalized size = 2.91

$$\left[\frac{4\,cd^2x^3e^2 + 6\,cd^3xe + 2\,adxe^3 + (3\,cd^2x^2e + 3\,cd^3 - ax^2e^3 - ade^2)\sqrt{-de}\,\log\left(\frac{x^2e - 2\,\sqrt{-de}\,x - d}{x^2e + d}\right)}{4\,(d^2x^2e^4 + d^3e^3)}, \frac{2\,cd^2x^3e^2 + 3\,cd^3xe + adxe^3 - (3\,cd^2x^2e + 3\,cd^3 - ax^2e^3 - ade^2)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{2\,(d^2x^2e^4 + d^3e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*x^3*e^2 + 6*c*d^3*x*e + 2*a*d*x*e^3 + (3*c*d^2*x^2*e + 3*c*d^
3 - a*x^2*e^3 - a*d*e^2)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e
 + d)))/(d^2*x^2*e^4 + d^3*e^3), 1/2*(2*c*d^2*x^3*e^2 + 3*c*d^3*x*e + a*d*x
*e^3 - (3*c*d^2*x^2*e + 3*c*d^3 - a*x^2*e^3 - a*d*e^2)*sqrt(d)*arctan(x*e^(
1/2)/sqrt(d))*e^(1/2))/(d^2*x^2*e^4 + d^3*e^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(68) = 136$.
time = 0.25, size = 138, normalized size = 1.86

$$\frac{cx}{e^2} + \frac{x(ae^2 + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2 - 3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2 - 3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt(-1/(d**
3*e**5))*(a*e**2 - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e**5)) + x)/4 + s
qrt(-1/(d**3*e**5))*(a*e**2 - 3*c*d**2)*log(d**2*e**2*sqrt(-1/(d**3*e**5))
+ x)/4
```

Giac [A]
time = 3.49, size = 62, normalized size = 0.84

$$cxe^{(-2)} - \frac{(3\,cd^2 - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{(cd^2x + axe^2)e^{(-2)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(3/
2) + 1/2*(c*d^2*x + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

Mupad [B]
time = 4.44, size = 68, normalized size = 0.92

$$\frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(a\,e^2 - 3\,c\,d^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2 + a\,e^2\right)}{2\,d\,\left(e^3\,x^2 + d\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2))/(2*d^(3/2)*e^(5/2
)) + (x*(a*e^2 + c*d^2))/(2*d*(d*e^2 + e^3*x^2))
```

3.126 $\int \frac{a+cx^4}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=93

$$\frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\left(d+ex^2\right)^2}+\frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8\left(d+ex^2\right)}+\frac{3(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

```
[Out] 1/4*(a+c*d^2/e^2)*x/d/(e*x^2+d)^2+1/8*(3*a/d^2-5*c/e^2)*x/(e*x^2+d)+3/8*(a*
e^2+c*d^2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1172, 393, 211}

$$\frac{3(ae^2+cd^2)\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)}{8\left(d+ex^2\right)}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (c*d^2)/e^2)*x)/(4*d*(d + e*x^2)^2) + (((3*a)/d^2 - (5*c)/e^2)*x)/(8*
(d + e*x^2)) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e
^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
```

```
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+cx^4}{(d+ex^2)^3}\,dx &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\,(d+ex^2)^2} - \frac{\int \frac{-3a+\frac{cd^2}{e^2}-\frac{4cdx^2}{e}}{(d+ex^2)^2}\,dx}{4d}\\ &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\,(d+ex^2)^2} + \frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8\,(d+ex^2)} + \frac{1}{8}\left(3\left(\frac{a}{d^2}+\frac{c}{e^2}\right)\right)\int \frac{1}{d+ex^2}\,dx\\ &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{4d\,(d+ex^2)^2} + \frac{\left(\frac{3a}{d^2}-\frac{5c}{e^2}\right)x}{8\,(d+ex^2)} + \frac{3(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 92, normalized size = 0.99

$$\frac{ae^2x(5d+3ex^2)-cd^2x(3d+5ex^2)}{8d^2e^2\,(d+ex^2)^2} + \frac{3(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^3,x]
[Out] (a*e^2*x*(5*d + 3*e*x^2) - c*d^2*x*(3*d + 5*e*x^2))/(8*d^2*e^2*(d + e*x^2)^
2) + (3*(c*d^2 + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Maple [A]
time = 0.17, size = 92, normalized size = 0.99

method	result
default	$\frac{\frac{(3a\,e^2-5c\,d^2)x^3}{8d^2e}+\frac{(5a\,e^2-3c\,d^2)x}{8d\,e^2}}{(e\,x^2+d)^2}+\frac{3(a\,e^2+c\,d^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8d^2e^2\sqrt{de}}$
risch	$\frac{\frac{(3a\,e^2-5c\,d^2)x^3}{8d^2e}+\frac{(5a\,e^2-3c\,d^2)x}{8d\,e^2}}{(e\,x^2+d)^2}-\frac{3\ln\left(ex+\sqrt{-de}\right)a}{16\sqrt{-de}\,d^2}-\frac{3\ln\left(ex+\sqrt{-de}\right)c}{16\sqrt{-de}\,e^2}+\frac{3\ln\left(-ex+\sqrt{-de}\right)a}{16\sqrt{-de}\,d^2}+\frac{3\ln\left(-ex+\sqrt{\right.}}{16\sqrt{-d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a*e^2-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-3*c*d^2)/d/e^2*x)/(e*x^2+d)^2
+3/8*(a*e^2+c*d^2)/d^2/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [A]
time = 0.49, size = 90, normalized size = 0.97

$$\frac{3\left(cd^2+ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{(5\,cd^2e-3\,ae^3)x^3+(3\,cd^3-5\,ade^2)x}{8\left(d^2x^4e^4+2\,d^3x^2e^3+d^4e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 3/8*(c*d^2 + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2) - 1/8*((5*c*
d^2*e - 3*a*e^3)*x^3 + (3*c*d^3 - 5*a*d*e^2)*x)/(d^2*x^4*e^4 + 2*d^3*x^2*e^
3 + d^4*e^2)
```

Fricas [A]
time = 0.32, size = 295, normalized size = 3.17

$$\left[-\frac{10\,cd^3x^3e^2+6\,cd^4xe-6\,adx^3e^4-10\,ad^2xe^3+3\left(2\,cd^3x^2e+ax^4e^4+cd^4+2\,adx^2e^3+(cd^2x^4+ad^2)e^2\right)\sqrt{-de}\,\log\left(\frac{x^2e-2\sqrt{-de}\,x-d}{x^2e+d}\right)}{16\left(d^3x^4e^5+2\,d^4x^2e^4+d^5e^3\right)},-\frac{5\,cd^3x^3e^2+3\,cd^4xe-3\,adx^3e^4-5\,ad^2xe^3-3\left(2\,cd^3x^2e+ax^4e^4+cd^4+2\,adx^2e^3+(cd^2x^4+ad^2)e^2\right)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{8\left(d^3x^4e^5+2\,d^4x^2e^4+d^5e^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(10*c*d^3*x^3*e^2 + 6*c*d^4*x*e - 6*a*d*x^3*e^4 - 10*a*d^2*x*e^3 + 3
*(2*c*d^3*x^2*e + a*x^4*e^4 + c*d^4 + 2*a*d*x^2*e^3 + (c*d^2*x^4 + a*d^2)*e
^2)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)))/(d^3*x^4*e^5
+ 2*d^4*x^2*e^4 + d^5*e^3), -1/8*(5*c*d^3*x^3*e^2 + 3*c*d^4*x*e - 3*a*d*x^3
*e^4 - 5*a*d^2*x*e^3 - 3*(2*c*d^3*x^2*e + a*x^4*e^4 + c*d^4 + 2*a*d*x^2*e^3
 + (c*d^2*x^4 + a*d^2)*e^2)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2))/(d^3
*x^4*e^5 + 2*d^4*x^2*e^4 + d^5*e^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(90) = 180$.
time = 0.37, size = 219, normalized size = 2.35

$$-\frac{3\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)\log\left(-\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)}{3ae^2+3cd^2}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)\log\left(\frac{3d^3e^2\sqrt{-\frac{1}{d^5e^5}}\left(ae^2+cd^2\right)}{3ae^2+3cd^2}+x\right)}{16}+\frac{x^3\cdot\left(3ae^3-5cd^2e\right)+x\left(5ade^2-3cd^3\right)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d)**3,x)
```

```
[Out] -3*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)*log(-3*d**3*e**2*sqrt(-1/(d**5*e*
*5))*(a*e**2 + c*d**2)/(3*a*e**2 + 3*c*d**2) + x)/16 + 3*sqrt(-1/(d**5*e**5
```

```
))*(a*e**2 + c*d**2)*log(3*d**3*e**2*sqrt(-1/(d**5*e**5))*(a*e**2 + c*d**2)
/(3*a*e**2 + 3*c*d**2) + x)/16 + (x**3*(3*a*e**3 - 5*c*d**2*e) + x*(5*a*d*e
**2 - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e**3*x**2 + 8*d**2*e**4*x**4)
```

Giac [A]
time = 3.96, size = 77, normalized size = 0.83

$$\frac{3\left(cd^2+ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(5\,cd^2x^3e+3\,cd^3x-3\,ax^3e^3-5\,adxe^2\right)e^{(-2)}}{8\left(x^2e+d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 3/8*(c*d^2 + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2) - 1/8*(5*c*d
^2*x^3*e + 3*c*d^3*x - 3*a*x^3*e^3 - 5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2
)
```

Mupad [B]
time = 4.48, size = 97, normalized size = 1.04

$$\frac{\frac{x^3\,\left(3\,a\,e^2-5\,c\,d^2\right)}{8\,d^2\,e}+\frac{x\,\left(5\,a\,e^2-3\,c\,d^2\right)}{8\,d\,e^2}}{d^2+2\,d\,e\,x^2+e^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/(d + e*x^2)^3,x)
```

```
[Out] ((x^3*(3*a*e^2 - 5*c*d^2))/(8*d^2*e) + (x*(5*a*e^2 - 3*c*d^2))/(8*d*e^2))/(
d^2 + e^2*x^4 + 2*d*e*x^2) + (3*atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c*d^2))/
(8*d^(5/2)*e^(5/2))
```

3.127 $\int \frac{a+cx^4}{(d+ex^2)^4} dx$

Optimal. Leaf size=123

$$\frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\left(d+ex^2\right)^3}+\frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\left(d+ex^2\right)^2}+\frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d\left(d+ex^2\right)}+\frac{\left(cd^2+5ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

```
[Out] 1/6*(a+c*d^2/e^2)*x/d/(e*x^2+d)^3+1/24*(5*a/d^2-7*c/e^2)*x/(e*x^2+d)^2+1/16
*(5*a/d^2+c/e^2)*x/d/(e*x^2+d)+1/16*(5*a*e^2+c*d^2)*arctan(x*e^(1/2)/d^(1/2
))/d^(7/2)/e^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1172, 393, 205, 211}

$$\frac{\left(5ae^2+cd^2\right)\mathrm{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}+\frac{x\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)}{16d\left(d+ex^2\right)}+\frac{x\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)}{24\left(d+ex^2\right)^2}+\frac{x\left(a+\frac{cd^2}{e^2}\right)}{6d\left(d+ex^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] ((a + (c*d^2)/e^2)*x)/(6*d*(d + e*x^2)^3) + (((5*a)/d^2 - (7*c)/e^2)*x)/(24
*(d + e*x^2)^2) + (((5*a)/d^2 + c/e^2)*x)/(16*d*(d + e*x^2)) + ((c*d^2 + 5*
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+cx^4}{(d+ex^2)^4}\,dx &= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\,(d+ex^2)^3}-\frac{\int \frac{-5a+\frac{cd^2}{e^2}-\frac{6cdx^2}{e}}{(d+ex^2)^3}\,dx}{6d}\\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\,(d+ex^2)^3}+\frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\,(d+ex^2)^2}+\frac{1}{8}\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)\int \frac{1}{(d+ex^2)^2}\,dx\\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\,(d+ex^2)^3}+\frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\,(d+ex^2)^2}+\frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d\,(d+ex^2)}+\frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)\int \frac{1}{d+ex^2}\,dx}{16d}\\
&= \frac{\left(a+\frac{cd^2}{e^2}\right)x}{6d\,(d+ex^2)^3}+\frac{\left(\frac{5a}{d^2}-\frac{7c}{e^2}\right)x}{24\,(d+ex^2)^2}+\frac{\left(\frac{5a}{d^2}+\frac{c}{e^2}\right)x}{16d\,(d+ex^2)}+\frac{(cd^2+5ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 113, normalized size = 0.92

$$\frac{x(cd^2(-3d^2-8dex^2+3e^2x^4)+ae^2(33d^2+40dex^2+15e^2x^4))}{48d^3e^2\,(d+ex^2)^3}+\frac{(cd^2+5ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + a*e^2*(33*d^2 + 40*d*e*x^2 + 1
5*e^2*x^4)))/(48*d^3*e^2*(d + e*x^2)^3) + ((c*d^2 + 5*a*e^2)*ArcTan[(Sqrt[e
]*x)/Sqrt[d]])/(16*d^(7/2)*e^(5/2))
```

Maple [A]
time = 0.15, size = 113, normalized size = 0.92

method	result
default	$\frac{\frac{(5ae^2+cd^2)x^5}{16d^3}+\frac{(5ae^2-cd^2)x^3}{6d^2e}+\frac{(11ae^2-cd^2)x}{16de^2}}{(ex^2+d)^3}+\frac{(5ae^2+cd^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{16d^3e^2\sqrt{de}}$
risch	$\frac{\frac{(5ae^2+cd^2)x^5}{16d^3}+\frac{(5ae^2-cd^2)x^3}{6d^2e}+\frac{(11ae^2-cd^2)x}{16de^2}}{(ex^2+d)^3}-\frac{5\ln\left(ex+\sqrt{-de}\right)a}{32\sqrt{-de}\,d^3}-\frac{\ln\left(ex+\sqrt{-de}\right)c}{32\sqrt{-de}\,e^2d}+\frac{5\ln\left(-ex+\sqrt{-de}\right)a}{32\sqrt{-de}\,d^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*a*e^2+c*d^2)/d^3*x^5+1/6*(5*a*e^2-c*d^2)/d^2/e*x^3+1/16*(11*a*e^2-
c*d^2)/d/e^2*x)/(e*x^2+d)^3+1/16*(5*a*e^2+c*d^2)/d^3/e^2/(d*e)^(1/2)*arctan
(e*x/(d*e)^(1/2))
```

Maxima [A]
time = 0.52, size = 122, normalized size = 0.99

$$\frac{3\,(cd^2e^2+5\,ae^4)x^5-8\,(cd^3e-5\,ade^3)x^3-3\,(cd^4-11\,ad^2e^2)x}{48\,(d^3x^6e^5+3\,d^4x^4e^4+3\,d^5x^2e^3+d^6e^2)}+\frac{(cd^2+5\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{16\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(c*d^2*e^2 + 5*a*e^4)*x^5 - 8*(c*d^3*e - 5*a*d*e^3)*x^3 - 3*(c*d^4
- 11*a*d^2*e^2)*x)/(d^3*x^6*e^5 + 3*d^4*x^4*e^4 + 3*d^5*x^2*e^3 + d^6*e^2)
+ 1/16*(c*d^2 + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(7/2)
```

Fricas [A]
time = 0.36, size = 407, normalized size = 3.31

$$\left[-\frac{16\,cd^4x^3e^2-30\,adx^5e^5+6\,cd^5xe-80\,ad^2x^3e^4+3\,(5\,ax^6e^5+3\,cd^4x^2e+15\,adx^4e^4+cd^5+(cd^2x^6+15\,ad^2x^2)e^3+(3\,cd^3x^4+5\,ad^3)e^2)\sqrt{-de}\,\log\left(\frac{x^2e-2\sqrt{-de}\,x-d}{x^2e+d}\right)-6\,(cd^3x^5+11\,ad^3x)e^3}{96\,(d^4x^6e^6+3\,d^5x^4e^5+3\,d^6x^2e^4+d^7e^3)},\ -\frac{8\,cd^4x^3e^2-15\,adx^5e^5+3\,cd^5xe-40\,ad^2x^3e^4-3\,(5\,ax^6e^5+3\,cd^4x^2e+15\,adx^4e^4+cd^5+(cd^2x^6+15\,ad^2x^2)e^3+(3\,cd^3x^4+5\,ad^3)e^2)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-3\,(cd^3x^5+11\,ad^3x)e^3}{48\,(d^4x^6e^6+3\,d^5x^4e^5+3\,d^6x^2e^4+d^7e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*c*d^4*x^3*e^2 - 30*a*d*x^5*e^5 + 6*c*d^5*x*e - 80*a*d^2*x^3*e^4
+ 3*(5*a*x^6*e^5 + 3*c*d^4*x^2*e + 15*a*d*x^4*e^4 + c*d^5 + (c*d^2*x^6 + 15
*a*d^2*x^2)*e^3 + (3*c*d^3*x^4 + 5*a*d^3)*e^2)*sqrt(-d*e)*log((x^2*e - 2*sq
rt(-d*e)*x - d)/(x^2*e + d)) - 6*(c*d^3*x^5 + 11*a*d^3*x)*e^3)/(d^4*x^6*e^6
 + 3*d^5*x^4*e^5 + 3*d^6*x^2*e^4 + d^7*e^3), -1/48*(8*c*d^4*x^3*e^2 - 15*a*
```

```
d*x^5*e^5 + 3*c*d^5*x*e - 40*a*d^2*x^3*e^4 - 3*(5*a*x^6*e^5 + 3*c*d^4*x^2*e
 + 15*a*d*x^4*e^4 + c*d^5 + (c*d^2*x^6 + 15*a*d^2*x^2)*e^3 + (3*c*d^3*x^4 +
 5*a*d^3)*e^2)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - 3*(c*d^3*x^5 + 1
1*a*d^3*x)*e^3)/(d^4*x^6*e^6 + 3*d^5*x^4*e^5 + 3*d^6*x^2*e^4 + d^7*e^3)]
```

Sympy [A]
time = 0.47, size = 204, normalized size = 1.66

$$-\frac{\sqrt{-\frac{1}{d^7e^5}}\cdot(5ae^2+cd^2)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{d^7e^5}}\cdot(5ae^2+cd^2)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{x^5\cdot(15ae^4+3cd^2e^2)+x^3\cdot(40ade^3-8cd^3e)+x(33ad^2e^2-3cd^4)}{48d^6e^2+144d^5e^3x^2+144d^4e^4x^4+48d^3e^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)/(e*x**2+d)**4,x)
```

```
[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(-d**4*e**2*sqrt(-1/(d**7*e**5
)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + c*d**2)*log(d**4*e**2*sqrt(-1
/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*c*d**2*e**2) + x**3*(40*a*d*e*
*3 - 8*c*d**3*e) + x*(33*a*d**2*e**2 - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*
e**3*x**2 + 144*d**4*e**4*x**4 + 48*d**3*e**5*x**6)
```

Giac [A]
time = 3.16, size = 100, normalized size = 0.81

$$\frac{(cd^2+5\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{16\,d^{\frac{7}{2}}}+\frac{(3\,cd^2x^5e^2-8\,cd^3x^3e+15\,ax^5e^4-3\,cd^4x+40\,adx^3e^3+33\,ad^2xe^2)e^{(-2)}}{48\,(x^2e+d)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] 1/16*(c*d^2 + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(7/2) + 1/48*(3
*c*d^2*x^5*e^2 - 8*c*d^3*x^3*e + 15*a*x^5*e^4 - 3*c*d^4*x + 40*a*d*x^3*e^3
+ 33*a*d^2*x*e^2)*e^(-2)/((x^2*e + d)^3*d^3)
```

Mupad [B]
time = 4.48, size = 129, normalized size = 1.05

$$\frac{\frac{x^5\,\left(c\,d^2+5\,a\,e^2\right)}{16\,d^3}+\frac{x^3\,\left(5\,a\,e^2-c\,d^2\right)}{6\,d^2\,e}+\frac{x\,\left(11\,a\,e^2-c\,d^2\right)}{16\,d\,e^2}}{d^3+3\,d^2\,e\,x^2+3\,d\,e^2\,x^4+e^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+5\,a\,e^2\right)}{16\,d^{7/2}\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)/(d + e*x^2)^4,x)
```

```
[Out] ((x^5*(5*a*e^2 + c*d^2))/(16*d^3) + (x^3*(5*a*e^2 - c*d^2))/(6*d^2*e) + (x*
(11*a*e^2 - c*d^2))/(16*d*e^2))/(d^3 + e^3*x^6 + 3*d^2*e*x^2 + 3*d*e^2*x^4)
 + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2))/(16*d^(7/2)*e^(5/2))
```

3.128 $\int (d+ex^2)^3 (a+cx^4)^2 \, dx$

Optimal. Leaf size=133

$$a^2d^3x+a^2d^2ex^3+\frac{1}{5}ad(2cd^2+3ae^2)x^5+\frac{1}{7}ae(6cd^2+ae^2)x^7+\frac{1}{9}cd(cd^2+6ae^2)x^9+\frac{1}{11}ce(3cd^2+2ae^2)x^{11}+$$

```
[Out] a^2*d^3*x+a^2*d^2*e*x^3+1/5*a*d*(3*a*e^2+2*c*d^2)*x^5+1/7*a*e*(a*e^2+6*c*d^
2)*x^7+1/9*c*d*(6*a*e^2+c*d^2)*x^9+1/11*c*e*(2*a*e^2+3*c*d^2)*x^11+3/13*c^2
*d*e^2*x^13+1/15*c^2*e^3*x^15
```

Rubi [A]
time = 0.07, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1168}

$$a^2d^3x+a^2d^2ex^3+\frac{1}{11}cex^{11}(2ae^2+3cd^2)+\frac{1}{9}cdx^9(6ae^2+cd^2)+\frac{1}{7}aex^7(ae^2+6cd^2)+\frac{1}{5}adx^5(3ae^2+2cd^2)+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2
 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)
*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int (d+ex^2)^3 (a+cx^4)^2 \, dx = \int \left(a^2d^3+3a^2d^2ex^2+ad(2cd^2+3ae^2)x^4+ae(6cd^2+ae^2)x^6+cd(cd^2+6\right.$$

$$= a^2d^3x+a^2d^2ex^3+\frac{1}{5}ad(2cd^2+3ae^2)x^5+\frac{1}{7}ae(6cd^2+ae^2)x^7+\frac{1}{9}cd(cd^2+$$

Mathematica [A]
time = 0.01, size = 133, normalized size = 1.00

$$a^2d^3x+a^2d^2ex^3+\frac{1}{5}ad(2cd^2+3ae^2)x^5+\frac{1}{7}ae(6cd^2+ae^2)x^7+\frac{1}{9}cd(cd^2+6ae^2)x^9+\frac{1}{11}ce(3cd^2+2ae^2)x^{11}+\frac{3}{13}c^2de^2x^{13}+\frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3*(a + c*x^4)^2,x]

[Out] a^2*d^3*x + a^2*d^2*e*x^3 + (a*d*(2*c*d^2 + 3*a*e^2)*x^5)/5 + (a*e*(6*c*d^2 + a*e^2)*x^7)/7 + (c*d*(c*d^2 + 6*a*e^2)*x^9)/9 + (c*e*(3*c*d^2 + 2*a*e^2)*x^11)/11 + (3*c^2*d*e^2*x^13)/13 + (c^2*e^3*x^15)/15

Maple [A]
time = 0.16, size = 130, normalized size = 0.98

method	result
norman	$a^2d^3x+a^2d^2e\,x^3+\left(\frac{3}{5}d\,e^2a^2+\frac{2}{5}d^3ac\right)x^5+\left(\frac{1}{7}e^3a^2+\frac{6}{7}d^2eac\right)x^7+\left(\frac{2}{3}acd\,e^2+\frac{1}{9}c^2d^3\right)x^9+\left(\frac{2}{11}e^3ac\right.$
default	$\frac{c^2e^3x^{15}}{15}+\frac{3c^2d\,e^2x^{13}}{13}+\frac{(2e^3ac+3d^2e\,c^2)x^{11}}{11}+\frac{(6acd\,e^2+c^2d^3)x^9}{9}+\frac{(e^3a^2+6d^2eac)x^7}{7}+\frac{(3d\,e^2a^2+2d^3ac)x^5}{5}+a^2d^2e\,x^3$
gosper	$a^2d^3x+a^2d^2e\,x^3+\frac{3}{5}x^5d\,e^2a^2+\frac{2}{5}x^5d^3ac+\frac{1}{7}x^7e^3a^2+\frac{6}{7}x^7d^2eac+\frac{2}{3}x^9acd\,e^2+\frac{1}{9}x^9c^2d^3+\frac{2}{11}x^{11}e^3a$
risch	$a^2d^3x+a^2d^2e\,x^3+\frac{3}{5}x^5d\,e^2a^2+\frac{2}{5}x^5d^3ac+\frac{1}{7}x^7e^3a^2+\frac{6}{7}x^7d^2eac+\frac{2}{3}x^9acd\,e^2+\frac{1}{9}x^9c^2d^3+\frac{2}{11}x^{11}e^3a$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3*(c*x^4+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/15*c^2*e^3*x^15+3/13*c^2*d*e^2*x^13+1/11*(2*a*c*e^3+3*c^2*d^2*e)*x^11+1/9*(6*a*c*d*e^2+c^2*d^3)*x^9+1/7*(a^2*e^3+6*a*c*d^2*e)*x^7+1/5*(3*a^2*d*e^2+2*a*c*d^3)*x^5+a^2*d^2*e*x^3+a^2*d^3*x

Maxima [A]
time = 0.29, size = 126, normalized size = 0.95

$$\frac{1}{15}c^2x^{15}e^3+\frac{3}{13}c^2dx^{13}e^2+\frac{1}{11}\left(3\,c^2d^2e+2\,ace^3\right)x^{11}+\frac{1}{9}\left(c^2d^3+6\,acde^2\right)x^9+\frac{1}{7}\left(6\,acd^2e+a^2e^3\right)x^7+a^2d^2x^3e+a^2d^3x+\frac{1}{5}\left(2\,acd^3+3\,a^2de^2\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="maxima")

[Out] 1/15*c^2*x^15*e^3 + 3/13*c^2*d*x^13*e^2 + 1/11*(3*c^2*d^2*e + 2*a*c*e^3)*x^11 + 1/9*(c^2*d^3 + 6*a*c*d*e^2)*x^9 + 1/7*(6*a*c*d^2*e + a^2*e^3)*x^7 + a^2*d^2*x^3*e + a^2*d^3*x + 1/5*(2*a*c*d^3 + 3*a^2*d*e^2)*x^5

Fricas [A]
time = 0.38, size = 126, normalized size = 0.95

$$\frac{1}{9}c^2d^3x^9+\frac{2}{5}acd^3x^5+a^2d^3x+\frac{1}{1155}\left(77\,c^2x^{15}+210\,acx^{11}+165\,a^2x^7\right)e^3+\frac{1}{195}\left(45\,c^2dx^{13}+130\,acdx^9+117\,a^2dx^5\right)e^2+\frac{1}{77}\left(21\,c^2d^2x^{11}+66\,acd^2x^7+77\,a^2d^2x^3\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="fricas")

[Out] 1/9*c^2*d^3*x^9 + 2/5*a*c*d^3*x^5 + a^2*d^3*x + 1/1155*(77*c^2*x^15 + 210*a*c*x^11 + 165*a^2*x^7)*e^3 + 1/195*(45*c^2*d*x^13 + 130*a*c*d*x^9 + 117*a^2*d*x^5)*e^2 + 1/77*(21*c^2*d^2*x^11 + 66*a*c*d^2*x^7 + 77*a^2*d^2*x^3)*e

Sympy [A]
time = 0.02, size = 144, normalized size = 1.08

$$a^2d^3x + a^2d^2ex^3 + \frac{3c^2de^2x^{13}}{13} + \frac{c^2e^3x^{15}}{15} + x^{11}\cdot\left(\frac{2ace^3}{11} + \frac{3c^2d^2e}{11}\right) + x^9\cdot\left(\frac{2acde^2}{3} + \frac{c^2d^3}{9}\right) + x^7\left(\frac{a^2e^3}{7} + \frac{6acd^2e}{7}\right) + x^5\cdot\left(\frac{3a^2de^2}{5} + \frac{2acd^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**3*x + a**2*d**2*e*x**3 + 3*c**2*d*e**2*x**13/13 + c**2*e**3*x**15/1
5 + x**11*(2*a*c*e**3/11 + 3*c**2*d**2*e/11) + x**9*(2*a*c*d*e**2/3 + c**2*
d**3/9) + x**7*(a**2*e**3/7 + 6*a*c*d**2*e/7) + x**5*(3*a**2*d*e**2/5 + 2*a
*c*d**3/5)
```

Giac [A]
time = 3.30, size = 128, normalized size = 0.96

$$\frac{1}{15}c^2x^{15}e^3 + \frac{3}{13}c^2dx^{13}e^2 + \frac{3}{11}c^2d^2x^{11}e + \frac{1}{9}c^2d^3x^9 + \frac{2}{11}acx^{11}e^3 + \frac{2}{3}acdx^9e^2 + \frac{6}{7}acd^2x^7e + \frac{2}{5}acd^3x^5 + \frac{1}{7}a^2x^7e^3 + \frac{3}{5}a^2dx^5e^2 + a^2d^2x^3e + a^2d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*c^2*x^15*e^3 + 3/13*c^2*d*x^13*e^2 + 3/11*c^2*d^2*x^11*e + 1/9*c^2*d^3
*x^9 + 2/11*a*c*x^11*e^3 + 2/3*a*c*d*x^9*e^2 + 6/7*a*c*d^2*x^7*e + 2/5*a*c*
d^3*x^5 + 1/7*a^2*x^7*e^3 + 3/5*a^2*d*x^5*e^2 + a^2*d^2*x^3*e + a^2*d^3*x
```

Mupad [B]
time = 0.06, size = 127, normalized size = 0.95

$$x^5\left(\frac{3\,a^2\,d\,e^2}{5} + \frac{2\,c\,a\,d^3}{5}\right) + x^7\left(\frac{a^2\,e^3}{7} + \frac{6\,c\,a\,d^2\,e}{7}\right) + x^9\left(\frac{c^2\,d^3}{9} + \frac{2\,a\,c\,d\,e^2}{3}\right) + x^{11}\left(\frac{3\,c^2\,d^2\,e}{11} + \frac{2\,a\,c\,e^3}{11}\right) + a^2\,d^3\,x + \frac{c^2\,e^3\,x^{15}}{15} + a^2\,d^2\,e\,x^3 + \frac{3\,c^2\,d\,e^2\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2*(d + e*x^2)^3,x)
```

```
[Out] x^5*((3*a^2*d*e^2)/5 + (2*a*c*d^3)/5) + x^7*((a^2*e^3)/7 + (6*a*c*d^2*e)/7)
 + x^9*((c^2*d^3)/9 + (2*a*c*d*e^2)/3) + x^11*((3*c^2*d^2*e)/11 + (2*a*c*e^
3)/11) + a^2*d^3*x + (c^2*e^3*x^15)/15 + a^2*d^2*e*x^3 + (3*c^2*d*e^2*x^13)
/13
```

3.129 $\int (d+ex^2)^2 (a+cx^4)^2 \, dx$

Optimal. Leaf size=97

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{5}a\left(2cd^2+ae^2\right)x^5+\frac{4}{7}acdex^7+\frac{1}{9}c\left(cd^2+2ae^2\right)x^9+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

```
[Out] a^2*d^2*x+2/3*a^2*d*e*x^3+1/5*a*(a*e^2+2*c*d^2)*x^5+4/7*a*c*d*e*x^7+1/9*c*(
2*a*e^2+c*d^2)*x^9+2/11*c^2*d*e*x^11+1/13*c^2*e^2*x^13
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1168}

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{9}cx^9\left(2ae^2+cd^2\right)+\frac{1}{5}ax^5\left(ae^2+2cd^2\right)+\frac{4}{7}acdex^7+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x^
7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)/1
3
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2\left(a+cx^4\right)^2\,dx &= \int \left(a^2d^2+2a^2dex^2+a\left(2cd^2+ae^2\right)x^4+4acdex^6+c\left(cd^2+2ae^2\right)x^8+2c^2de\right.\\ &= a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{5}a\left(2cd^2+ae^2\right)x^5+\frac{4}{7}acdex^7+\frac{1}{9}c\left(cd^2+2ae^2\right)x^9+\frac{2}{11}c\end{aligned}$$

Mathematica [A]
time = 0.01, size = 97, normalized size = 1.00

$$a^2d^2x+\frac{2}{3}a^2dex^3+\frac{1}{5}a\left(2cd^2+ae^2\right)x^5+\frac{4}{7}acdex^7+\frac{1}{9}c\left(cd^2+2ae^2\right)x^9+\frac{2}{11}c^2dex^{11}+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a^2*d*e*x^3)/3 + (a*(2*c*d^2 + a*e^2)*x^5)/5 + (4*a*c*d*e*x^
7)/7 + (c*(c*d^2 + 2*a*e^2)*x^9)/9 + (2*c^2*d*e*x^11)/11 + (c^2*e^2*x^13)/1
3
```

Maple [A]
time = 0.12, size = 90, normalized size = 0.93

method	result
default	$\frac{c^2e^2x^{13}}{13}+\frac{2c^2de\,x^{11}}{11}+\frac{(2ac\,e^2+c^2d^2)x^9}{9}+\frac{4acde\,x^7}{7}+\frac{(e^2a^2+2ac\,d^2)x^5}{5}+\frac{2a^2de\,x^3}{3}+a^2d^2x$
norman	$\frac{c^2e^2x^{13}}{13}+\frac{2c^2de\,x^{11}}{11}+\left(\frac{2}{9}ac\,e^2+\frac{1}{9}c^2d^2\right)x^9+\frac{4acde\,x^7}{7}+\left(\frac{1}{5}e^2a^2+\frac{2}{5}ac\,d^2\right)x^5+\frac{2a^2de\,x^3}{3}+a^2d^2x$
gosper	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9ac\,e^2+\frac{1}{9}x^9c^2d^2+\frac{4}{7}acde\,x^7+\frac{1}{5}x^5e^2a^2+\frac{2}{5}x^5ac\,d^2+\frac{2}{3}a^2de\,x^3+a^2d$
risch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9ac\,e^2+\frac{1}{9}x^9c^2d^2+\frac{4}{7}acde\,x^7+\frac{1}{5}x^5e^2a^2+\frac{2}{5}x^5ac\,d^2+\frac{2}{3}a^2de\,x^3+a^2d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c^2*e^2*x^13+2/11*c^2*d*e*x^11+1/9*(2*a*c*e^2+c^2*d^2)*x^9+4/7*a*c*d*e
*x^7+1/5*(a^2*e^2+2*a*c*d^2)*x^5+2/3*a^2*d*e*x^3+a^2*d^2*x
```

Maxima [A]
time = 0.28, size = 89, normalized size = 0.92

$$\frac{1}{13}\,c^2x^{13}e^2+\frac{2}{11}\,c^2dx^{11}e+\frac{4}{7}\,acdx^7e+\frac{1}{9}\,\left(c^2d^2+2\,ace^2\right)x^9+\frac{2}{3}\,a^2dx^3e+\frac{1}{5}\,\left(2\,acd^2+a^2e^2\right)x^5+a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*x^13*e^2 + 2/11*c^2*d*x^11*e + 4/7*a*c*d*x^7*e + 1/9*(c^2*d^2 + 2*
a*c*e^2)*x^9 + 2/3*a^2*d*x^3*e + 1/5*(2*a*c*d^2 + a^2*e^2)*x^5 + a^2*d^2*x
```

Fricas [A]
time = 0.34, size = 89, normalized size = 0.92

$$\frac{1}{9}\,c^2d^2x^9+\frac{2}{5}\,acd^2x^5+a^2d^2x+\frac{1}{585}\,\left(45\,c^2x^{13}+130\,acx^9+117\,a^2x^5\right)e^2+\frac{2}{231}\,\left(21\,c^2dx^{11}+66\,acdx^7+77\,a^2dx^3\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*d^2*x^9 + 2/5*a*c*d^2*x^5 + a^2*d^2*x + 1/585*(45*c^2*x^13 + 130*a*
c*x^9 + 117*a^2*x^5)*e^2 + 2/231*(21*c^2*d*x^11 + 66*a*c*d*x^7 + 77*a^2*d*x
^3)*e
```

Sympy [A]
time = 0.01, size = 104, normalized size = 1.07

$$a^2d^2x + \frac{2a^2dex^3}{3} + \frac{4acdex^7}{7} + \frac{2c^2dex^{11}}{11} + \frac{c^2e^2x^{13}}{13} + x^9 \cdot \left(\frac{2ace^2}{9} + \frac{c^2d^2}{9}\right) + x^5\left(\frac{a^2e^2}{5} + \frac{2acd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+a)**2,x)
```

```
[Out] a**2*d**2*x + 2*a**2*d*e*x**3/3 + 4*a*c*d*e*x**7/7 + 2*c**2*d*e*x**11/11 +
c**2*e**2*x**13/13 + x**9*(2*a*c*e**2/9 + c**2*d**2/9) + x**5*(a**2*e**2/5
+ 2*a*c*d**2/5)
```

Giac [A]
time = 5.14, size = 91, normalized size = 0.94

$$\frac{1}{13}\,c^2x^{13}e^2 + \frac{2}{11}\,c^2dx^{11}e + \frac{1}{9}\,c^2d^2x^9 + \frac{2}{9}\,acx^9e^2 + \frac{4}{7}\,acdx^7e + \frac{2}{5}\,acd^2x^5 + \frac{1}{5}\,a^2x^5e^2 + \frac{2}{3}\,a^2dx^3e + a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13*e^2 + 2/11*c^2*d*x^11*e + 1/9*c^2*d^2*x^9 + 2/9*a*c*x^9*e^2 +
 4/7*a*c*d*x^7*e + 2/5*a*c*d^2*x^5 + 1/5*a^2*x^5*e^2 + 2/3*a^2*d*x^3*e + a^
2*d^2*x
```

Mupad [B]
time = 0.05, size = 89, normalized size = 0.92

$$x^5\left(\frac{a^2\,e^2}{5} + \frac{2\,c\,a\,d^2}{5}\right) + x^9\left(\frac{c^2\,d^2}{9} + \frac{2\,a\,c\,e^2}{9}\right) + a^2\,d^2\,x + \frac{c^2\,e^2\,x^{13}}{13} + \frac{2\,a^2\,d\,e\,x^3}{3} + \frac{2\,c^2\,d\,e\,x^{11}}{11} + \frac{4\,a\,c\,d\,e\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2*(d + e*x^2)^2,x)
```

```
[Out] x^5*((a^2*e^2)/5 + (2*a*c*d^2)/5) + x^9*((c^2*d^2)/9 + (2*a*c*e^2)/9) + a^2
*d^2*x + (c^2*e^2*x^13)/13 + (2*a^2*d*e*x^3)/3 + (2*c^2*d*e*x^11)/11 + (4*a
*c*d*e*x^7)/7
```

3.130 $\int (d+ex^2)(a+cx^4)^2\,dx$

Optimal. Leaf size=60

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

```
[Out] a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1/11*c^2*e*
x^11
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1168}

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9
 + (c^2*e*x^11)/11
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex^2)(a+cx^4)^2\,dx &= \int \left(a^2d+a^2ex^2+2acdx^4+2acex^6+c^2dx^8+c^2ex^{10}\right)dx\\ &= a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 60, normalized size = 1.00

$$a^2dx+\frac{1}{3}a^2ex^3+\frac{2}{5}acdx^5+\frac{2}{7}acex^7+\frac{1}{9}c^2dx^9+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a^2*e*x^3)/3 + (2*a*c*d*x^5)/5 + (2*a*c*e*x^7)/7 + (c^2*d*x^9)/9
 + (c^2*e*x^11)/11
```

Maple [A]
time = 0.15, size = 51, normalized size = 0.85

method	result	size
gosper	$a^{2}dx+\frac{1}{3}a^{2}e\,x^{3}+\frac{2}{5}acd\,x^{5}+\frac{2}{7}ace\,x^{7}+\frac{1}{9}c^{2}d\,x^{9}+\frac{1}{11}c^{2}e\,x^{11}$	51
default	$a^{2}dx+\frac{1}{3}a^{2}e\,x^{3}+\frac{2}{5}acd\,x^{5}+\frac{2}{7}ace\,x^{7}+\frac{1}{9}c^{2}d\,x^{9}+\frac{1}{11}c^{2}e\,x^{11}$	51
norman	$a^{2}dx+\frac{1}{3}a^{2}e\,x^{3}+\frac{2}{5}acd\,x^{5}+\frac{2}{7}ace\,x^{7}+\frac{1}{9}c^{2}d\,x^{9}+\frac{1}{11}c^{2}e\,x^{11}$	51
risch	$a^{2}dx+\frac{1}{3}a^{2}e\,x^{3}+\frac{2}{5}acd\,x^{5}+\frac{2}{7}ace\,x^{7}+\frac{1}{9}c^{2}d\,x^{9}+\frac{1}{11}c^{2}e\,x^{11}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*d*x+1/3*a^2*e*x^3+2/5*a*c*d*x^5+2/7*a*c*e*x^7+1/9*c^2*d*x^9+1/11*c^2*e*
x^11
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.88

$$\frac{1}{11}\,c^{2}x^{11}e+\frac{1}{9}\,c^{2}dx^{9}+\frac{2}{7}\,acx^{7}e+\frac{2}{5}\,acdx^{5}+\frac{1}{3}\,a^{2}x^{3}e+a^{2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11*e + 1/9*c^2*d*x^9 + 2/7*a*c*x^7*e + 2/5*a*c*d*x^5 + 1/3*a^2*x
^3*e + a^2*d*x
```

Fricas [A]
time = 0.33, size = 52, normalized size = 0.87

$$\frac{1}{9}\,c^{2}dx^{9}+\frac{2}{5}\,acdx^{5}+a^{2}dx+\frac{1}{231}\,\left(21\,c^{2}x^{11}+66\,acx^{7}+77\,a^{2}x^{3}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*d*x^9 + 2/5*a*c*d*x^5 + a^2*d*x + 1/231*(21*c^2*x^11 + 66*a*c*x^7 +
 77*a^2*x^3)*e
```

Sympy [A]
time = 0.01, size = 60, normalized size = 1.00

$$a^{2}dx+\frac{a^{2}ex^{3}}{3}+\frac{2acdx^{5}}{5}+\frac{2acex^{7}}{7}+\frac{c^{2}dx^{9}}{9}+\frac{c^{2}ex^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**2,x)
```

```
[Out] a**2*d*x + a**2*e*x**3/3 + 2*a*c*d*x**5/5 + 2*a*c*e*x**7/7 + c**2*d*x**9/9
+ c**2*e*x**11/11
```

Giac [A]
time = 4.97, size = 53, normalized size = 0.88

$$\frac{1}{11}\,c^2x^{11}e+\frac{1}{9}\,c^2dx^9+\frac{2}{7}\,acx^7e+\frac{2}{5}\,acdx^5+\frac{1}{3}\,a^2x^3e+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11*e + 1/9*c^2*d*x^9 + 2/7*a*c*x^7*e + 2/5*a*c*d*x^5 + 1/3*a^2*x
^3*e + a^2*d*x
```

Mupad [B]
time = 0.03, size = 50, normalized size = 0.83

$$\frac{e\,a^2\,x^3}{3}+d\,a^2\,x+\frac{2\,e\,a\,c\,x^7}{7}+\frac{2\,d\,a\,c\,x^5}{5}+\frac{e\,c^2\,x^{11}}{11}+\frac{d\,c^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2*(d + e*x^2),x)
```

```
[Out] (a^2*e*x^3)/3 + (c^2*d*x^9)/9 + (c^2*e*x^11)/11 + a^2*d*x + (2*a*c*d*x^5)/5
 + (2*a*c*e*x^7)/7
```

3.131 $\int (a+cx^4)^2 \, dx$

Optimal. Leaf size=25

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

```
[Out] a^2*x+2/5*a*c*x^5+1/9*c^2*x^9
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {200}

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int (a+cx^4)^2 \, dx &= \int (a^2+2acx^4+c^2x^8) \, dx \\ &= a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$a^2x + \frac{2}{5}acx^5 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^5)/5 + (c^2*x^9)/9
```

Maple [A]
time = 0.14, size = 22, normalized size = 0.88

method	result	size
gosper	$a^2x + \frac{2}{5}ac\,x^5 + \frac{1}{9}c^2x^9$	22
default	$a^2x + \frac{2}{5}ac\,x^5 + \frac{1}{9}c^2x^9$	22
norman	$a^2x + \frac{2}{5}ac\,x^5 + \frac{1}{9}c^2x^9$	22
risch	$a^2x + \frac{2}{5}ac\,x^5 + \frac{1}{9}c^2x^9$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*x+2/5*a*c*x^5+1/9*c^2*x^9
```

Maxima [A]
time = 0.28, size = 21, normalized size = 0.84

$$\frac{1}{9}\,c^2x^9 + \frac{2}{5}\,acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Fricas [A]
time = 0.31, size = 21, normalized size = 0.84

$$\frac{1}{9}\,c^2x^9 + \frac{2}{5}\,acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Sympy [A]
time = 0.01, size = 22, normalized size = 0.88

$$a^2x + \frac{2acx^5}{5} + \frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2,x)
```

```
[Out] a**2*x + 2*a*c*x**5/5 + c**2*x**9/9
```

Giac [A]
time = 6.38, size = 21, normalized size = 0.84

$$\frac{1}{9}c^2x^9 + \frac{2}{5}acx^5 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/5*a*c*x^5 + a^2*x
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.84

$$a^2\,x + \frac{2\,a\,c\,x^5}{5} + \frac{c^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2,x)
```

```
[Out] a^2*x + (c^2*x^9)/9 + (2*a*c*x^5)/5
```

3.132 $\int \frac{(a+cx^4)^2}{d+ex^2}\,dx$

Optimal. Leaf size=108

$$-\frac{cd(cd^2+2ae^2)\,x}{e^4}+\frac{c(cd^2+2ae^2)\,x^3}{3e^3}-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{(cd^2+ae^2)^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{9/2}}$$

```
[Out] -c*d*(2*a*e^2+c*d^2)*x/e^4+1/3*c*(2*a*e^2+c*d^2)*x^3/e^3-1/5*c^2*d*x^5/e^2+
1/7*c^2*x^7/e+(a*e^2+c*d^2)^2*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1168, 211}

$$\frac{(ae^2+cd^2)^2\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{9/2}}-\frac{cdx(2ae^2+cd^2)}{e^4}+\frac{cx^3(2ae^2+cd^2)}{3e^3}-\frac{c^2dx^5}{5e^2}+\frac{c^2x^7}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] -((c*d*(c*d^2 + 2*a*e^2)*x)/e^4) + (c*(c*d^2 + 2*a*e^2)*x^3)/(3*e^3) - (c^2
*d*x^5)/(5*e^2) + (c^2*x^7)/(7*e) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(Sqrt[d]*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(a+cx^4)^2}{d+ex^2}\,dx = \int \left(-\frac{cd(cd^2+2ae^2)}{e^4} + \frac{c(cd^2+2ae^2)\,x^2}{e^3} - \frac{c^2dx^4}{e^2} + \frac{c^2x^6}{e} + \frac{c^2d^4+2acd^2e^2+a^2e^4}{e^4\,(d+ex^2)}\right)dx$$
$$= -\frac{cd(cd^2+2ae^2)\,x}{e^4} + \frac{c(cd^2+2ae^2)\,x^3}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + \frac{(cd^2+ae^2)^2\int\frac{1}{d+ex^2}\,dx}{e^4}$$
$$= -\frac{cd(cd^2+2ae^2)\,x}{e^4} + \frac{c(cd^2+2ae^2)\,x^3}{3e^3} - \frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + \frac{(cd^2+ae^2)^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{9/2}}$$

Mathematica [A]
time = 0.05, size = 97, normalized size = 0.90

$$\frac{cx(70ae^2(-3d+ex^2)+c(-105d^3+35d^2ex^2-21de^2x^4+15e^3x^6))}{105e^4} + \frac{(cd^2+ae^2)^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] (c*x*(70*a*e^2*(-3*d + e*x^2) + c*(-105*d^3 + 35*d^2*e*x^2 - 21*d*e^2*x^4 +
 15*e^3*x^6)))/(105*e^4) + ((c*d^2 + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/
(Sqrt[d]*e^(9/2))
```

Maple [A]
time = 0.18, size = 104, normalized size = 0.96

method	result
default	$-\frac{c\left(-\frac{c\,x^7e^3}{7}+\frac{cd\,x^5e^2}{5}-\frac{\left(2a\,e^2+c\,d^2\right)x^3e}{3}+d\left(2a\,e^2+c\,d^2\right)x\right)}{e^4}+\frac{\left(a^2e^4+2ac\,d^2e^2+c^2d^4\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e^4\sqrt{de}}$
risch	$\frac{c^2x^7}{7e}-\frac{c^2d\,x^5}{5e^2}+\frac{2ca\,x^3}{3e}+\frac{c^2d^2x^3}{3e^3}-\frac{2cadx}{e^2}-\frac{c^2d^3x}{e^4}-\frac{\ln\left(ex+\sqrt{-de}\right)a^2}{2\sqrt{-de}}-\frac{\ln\left(ex+\sqrt{-de}\right)ac\,d^2}{e^2\sqrt{-de}}-\frac{\ln\left(ex+\sqrt{-\right.}{2e^4\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -c/e^4*(-1/7*c*x^7*e^3+1/5*c*d*x^5*e^2-1/3*(2*a*e^2+c*d^2)*x^3*e+d*(2*a*e^2
+c*d^2)*x)+(a^2*e^4+2*a*c*d^2*e^2+c^2*d^4)/e^4/(d*e)^(1/2)*arctan(e*x/(d*e)
^(1/2))
```

Maxima [A]
time = 0.50, size = 103, normalized size = 0.95

$$\frac{(c^2d^4+2\,acd^2e^2+a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} + \frac{1}{105}\left(15\,c^2x^7e^3-21\,c^2dx^5e^2+35\left(c^2d^2e+2\,ace^3\right)x^3-105\left(c^2d^3+2\,acde^2\right)x\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt
(d) + 1/105*(15*c^2*x^7*e^3 - 21*c^2*d*x^5*e^2 + 35*(c^2*d^2*e + 2*a*c*e^3)
*x^3 - 105*(c^2*d^3 + 2*a*c*d*e^2)*x)*e^(-4)
```

Fricas [A]
time = 0.34, size = 257, normalized size = 2.38

$$\left[\frac{\left(70\,c^2d^3x^3e^2 - 210\,c^2d^4xe - 105\,(c^2d^4 + 2\,acd^2e^2 + a^2e^4)\sqrt{-de}\,\log\left(\frac{x^2e - 2\,\sqrt{-de}\,x - d}{x^2e + d}\right) + 10\,(3\,c^2dx^7 + 14\,acdx^3)e^4 - 42\,(c^2d^2x^5 + 10\,acd^2x)e^3\right)e^{(-5)}}{210\,d}, \frac{\left(35\,c^2d^3x^3e^2 - 105\,c^2d^4xe + 105\,(c^2d^4 + 2\,acd^2e^2 + a^2e^4)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}} + 5\,(3\,c^2dx^7 + 14\,acdx^3)e^4 - 21\,(c^2d^2x^5 + 10\,acd^2x)e^3\right)e^{(-5)}}{105\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/210*(70*c^2*d^3*x^3*e^2 - 210*c^2*d^4*x*e - 105*(c^2*d^4 + 2*a*c*d^2*e^2
 + a^2*e^4)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) + 10*(
3*c^2*d*x^7 + 14*a*c*d*x^3)*e^4 - 42*(c^2*d^2*x^5 + 10*a*c*d^2*x)*e^3)*e^(-
5)/d, 1/105*(35*c^2*d^3*x^3*e^2 - 105*c^2*d^4*x*e + 105*(c^2*d^4 + 2*a*c*d^
2*e^2 + a^2*e^4)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) + 5*(3*c^2*d*x^7
 + 14*a*c*d*x^3)*e^4 - 21*(c^2*d^2*x^5 + 10*a*c*d^2*x)*e^3)*e^(-5)/d]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(100) = 200$.
time = 0.25, size = 236, normalized size = 2.19

$$-\frac{c^2dx^5}{5e^2} + \frac{c^2x^7}{7e} + x^3\cdot\left(\frac{2ac}{3e} + \frac{c^2d^2}{3e^3}\right) + x\left(-\frac{2acd}{e^2} - \frac{c^2d^3}{e^4}\right) - \frac{\sqrt{-\frac{1}{de^9}}\,(ae^2 + cd^2)^2\log\left(-\frac{de^4\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2}{a^2e^4+2acd^2e^2+c^2d^4} + x\right)}{2} + \frac{\sqrt{-\frac{1}{de^9}}\,(ae^2 + cd^2)^2\log\left(\frac{de^4\sqrt{-\frac{1}{de^9}}\,(ae^2+cd^2)^2}{a^2e^4+2acd^2e^2+c^2d^4} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d),x)
```

```
[Out] -c**2*d*x**5/(5*e**2) + c**2*x**7/(7*e) + x**3*(2*a*c/(3*e) + c**2*d**2/(3*
e**3)) + x*(-2*a*c*d/e**2 - c**2*d**3/e**4) - sqrt(-1/(d*e**9))*(a*e**2 + c
*d**2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2
*a*c*d**2*e**2 + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2
*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 + c*d**2)**2/(a**2*e**4 + 2*a*c*d**2*
e**2 + c**2*d**4) + x)/2
```

Giac [A]
time = 5.49, size = 105, normalized size = 0.97

$$\frac{(c^2d^4 + 2\,acd^2e^2 + a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} + \frac{1}{105}\left(15\,c^2x^7e^6 - 21\,c^2dx^5e^5 + 35\,c^2d^2x^3e^4 - 105\,c^2d^3xe^3 + 70\,acx^3e^6 - 210\,acdxe^5\right)e^{(-7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt
(d) + 1/105*(15*c^2*x^7*e^6 - 21*c^2*d*x^5*e^5 + 35*c^2*d^2*x^3*e^4 - 105*c
^2*d^3*x*e^3 + 70*a*c*x^3*e^6 - 210*a*c*d*x*e^5)*e^(-7)
```

Mupad [B]
time = 4.39, size = 141, normalized size = 1.31

$$x^3\left(\frac{c^2\,d^2}{3\,e^3}+\frac{2\,a\,c}{3\,e}\right)+\frac{c^2\,x^7}{7\,e}-\frac{c^2\,d\,x^5}{5\,e^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,{\left(c\,d^2+a\,e^2\right)}^2}{\sqrt{d}\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}\right)\,{\left(c\,d^2+a\,e^2\right)}^2}{\sqrt{d}\,e^{9/2}}-\frac{d\,x\,\left(\frac{c^2\,d^2}{e^3}+\frac{2\,a\,c}{e}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/(d + e*x^2),x)
```

```
[Out] x^3*((c^2*d^2)/(3*e^3) + (2*a*c)/(3*e)) + (c^2*x^7)/(7*e) - (c^2*d*x^5)/(5*
e^2) + (atan((e^(1/2)*x*(a*e^2 + c*d^2)^2)/(d^(1/2)*(a^2*e^4 + c^2*d^4 + 2*
a*c*d^2*e^2)))*(a*e^2 + c*d^2)^2)/(d^(1/2)*e^(9/2)) - (d*x*((c^2*d^2)/e^3 +
 (2*a*c)/e))/e
```

3.133 $\int \frac{(a+cx^4)^2}{(d+ex^2)^2} dx$

Optimal. Leaf size=131

$$\frac{c(3cd^2+2ae^2)\,x}{e^4}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}+\frac{(cd^2+ae^2)^2\,x}{2de^4\,(d+ex^2)}-\frac{(7cd^2-ae^2)\,(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

```
[Out] c*(2*a*e^2+3*c*d^2)*x/e^4-2/3*c^2*d*x^3/e^3+1/5*c^2*x^5/e^2+1/2*(a*e^2+c*d^
2)^2*x/d/e^4/(e*x^2+d)-1/2*(-a*e^2+7*c*d^2)*(a*e^2+c*d^2)*arctan(x*e^(1/2)/
d^(1/2))/d^(3/2)/e^(9/2)
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1172, 1824, 211}

$$-\frac{(7cd^2-ae^2)\,(ae^2+cd^2)\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}+\frac{x(ae^2+cd^2)^2}{2de^4\,(d+ex^2)}+\frac{cx(2ae^2+3cd^2)}{e^4}-\frac{2c^2dx^3}{3e^3}+\frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) +
 ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c*d^2 - a*e^2)*(c*d^2 +
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+cx^4)^2}{(d+ex^2)^2}\,dx &= \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{\int \frac{-a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{2cd\left(cd^2+2ae^2\right)x^2}{e^3}+\frac{2c^2d^2x^4}{e^2}-\frac{2c^2dx^6}{e}}{d+ex^2}\,dx}{2d} \\
&= \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{\int \left(-\frac{2cd(3cd^2+2ae^2)}{e^4}+\frac{4c^2d^2x^2}{e^3}-\frac{2c^2dx^4}{e^2}+\frac{7c^2d^4+6acd^2e^2-a^2e^4}{e^4(d+ex^2)}\right)dx}{2d} \\
&= \frac{c(3cd^2+2ae^2)x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{((7cd^2-ae^2)(cd^2+ae^2))\int \frac{1}{d+ex^2}}{2de^4} \\
&= \frac{c(3cd^2+2ae^2)x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{(7cd^2-ae^2)(cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{\ }}{\sqrt{\ }}\right.}{2d^{3/2}e^{9/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 134, normalized size = 1.02

$$
\frac{c(3cd^2+2ae^2)x}{e^4} - \frac{2c^2dx^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{(7c^2d^4+6acd^2e^2-a^2e^4)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] (c*(3*c*d^2 + 2*a*e^2)*x)/e^4 - (2*c^2*d*x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) +
 ((c*d^2 + a*e^2)^2*x)/(2*d*e^4*(d + e*x^2)) - ((7*c^2*d^4 + 6*a*c*d^2*e^2
- a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Maple [A]
time = 0.16, size = 129, normalized size = 0.98

method	result
default	$\frac{c\left(\frac{1}{5}cx^5e^2-\frac{2}{3}cdex^3+2ae^2x+3cd^2x\right)}{e^4}+\frac{\frac{\left(a^2e^4+2acd^2e^2+c^2d^4\right)x}{2d\left(ex^2+d\right)}+\frac{\left(a^2e^4-6acd^2e^2-7c^2d^4\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}}{e^4}$
risch	$\frac{c^2x^5}{5e^2}-\frac{2c^2dx^3}{3e^3}+\frac{2cax}{e^2}+\frac{3c^2d^2x}{e^4}+\frac{(a^2e^4+2acd^2e^2+c^2d^4)x}{2de^4(ex^2+d)}-\frac{\ln\left(ex+\sqrt{-de}\right)a^2}{4\sqrt{-de}\,d}+\frac{3d\ln\left(ex+\sqrt{-de}\right)ac}{2e^2\sqrt{-de}}+\frac{7d^3\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c/e^4*(1/5*c*x^5*e^2-2/3*c*d*e*x^3+2*a*e^2*x+3*c*d^2*x)+1/e^4*(1/2*(a^2*e^4
+2*a*c*d^2*e^2+c^2*d^4)/d*x/(e*x^2+d)+1/2*(a^2*e^4-6*a*c*d^2*e^2-7*c^2*d^4)
/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.51, size = 127, normalized size = 0.97

$$\frac{1}{15}\left(3\,c^{2}x^{5}e^{2}-10\,c^{2}dx^{3}e+15\left(3\,c^{2}d^{2}+2\,ace^{2}\right)x\right)e^{(-4)}-\frac{\left(7\,c^{2}d^{4}+6\,acd^{2}e^{2}-a^{2}e^{4}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,d^{\frac{3}{2}}}+\frac{\left(c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4}\right)x}{2\left(dx^{2}e^{5}+d^{2}e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*x^5*e^2 - 10*c^2*d*x^3*e + 15*(3*c^2*d^2 + 2*a*c*e^2)*x)*e^(-4)
 - 1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-
9/2)/d^(3/2) + 1/2*(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*x/(d*x^2*e^5 + d^2*e
^4)
```

Fricas [A]
time = 0.35, size = 385, normalized size = 2.94

$$\left[\frac{140\,c^{2}d^{4}x^{3}e^{2}+210\,c^{2}d^{5}xe+30\,a^{2}dxe^{5}+15\left(7\,c^{2}d^{4}x^{2}e+7\,c^{2}d^{5}+6\,acd^{2}x^{2}e^{3}+6\,acd^{3}e^{2}-a^{2}x^{2}e^{5}-a^{2}de^{4}\right)\sqrt{-de}\,\log\left(\frac{x^{2}e-2\,\sqrt{-de}\,x-d}{x^{2}e+d}\right)+12\left(c^{2}d^{2}x^{7}+10\,acd^{2}x^{3}\right)e^{4}-4\left(7\,c^{2}d^{3}x^{5}-45\,acd^{3}x\right)e^{3}}{60\left(d^{2}x^{2}e^{6}+d^{3}e^{5}\right)},\ \frac{70\,c^{2}d^{4}x^{3}e^{2}+105\,c^{2}d^{5}xe+15\,a^{2}dxe^{5}-15\left(7\,c^{2}d^{4}x^{2}e+7\,c^{2}d^{5}+6\,acd^{2}x^{2}e^{3}+6\,acd^{3}e^{2}-a^{2}x^{2}e^{5}-a^{2}de^{4}\right)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}+6\left(c^{2}d^{2}x^{7}+10\,acd^{2}x^{3}\right)e^{4}-2\left(7\,c^{2}d^{3}x^{5}-45\,acd^{3}x\right)e^{3}}{30\left(d^{2}x^{2}e^{6}+d^{3}e^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(140*c^2*d^4*x^3*e^2 + 210*c^2*d^5*x*e + 30*a^2*d*x*e^5 + 15*(7*c^2*d
^4*x^2*e + 7*c^2*d^5 + 6*a*c*d^2*x^2*e^3 + 6*a*c*d^3*e^2 - a^2*x^2*e^5 - a^
2*d*e^4)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) + 12*(c^2
*d^2*x^7 + 10*a*c*d^2*x^3)*e^4 - 4*(7*c^2*d^3*x^5 - 45*a*c*d^3*x)*e^3)/(d^2
*x^2*e^6 + d^3*e^5), 1/30*(70*c^2*d^4*x^3*e^2 + 105*c^2*d^5*x*e + 15*a^2*d*
x*e^5 - 15*(7*c^2*d^4*x^2*e + 7*c^2*d^5 + 6*a*c*d^2*x^2*e^3 + 6*a*c*d^3*e^2
 - a^2*x^2*e^5 - a^2*d*e^4)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) + 6*(
c^2*d^2*x^7 + 10*a*c*d^2*x^3)*e^4 - 2*(7*c^2*d^3*x^5 - 45*a*c*d^3*x)*e^3)/(
d^2*x^2*e^6 + d^3*e^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(122) = 244$.
time = 0.46, size = 314, normalized size = 2.40

$$-\frac{2c^{2}dx^{3}}{3e^{3}}+\frac{c^{2}x^{5}}{5e^{2}}+x\left(\frac{2ac}{e^{2}}+\frac{3c^{2}d^{2}}{e^{4}}\right)+\frac{x(a^{2}e^{4}+2acd^{2}e^{2}+c^{2}d^{4})}{2d^{2}e^{4}+2de^{5}x^{2}}-\frac{\sqrt{-\frac{1}{d^{3}e^{9}}}\,(ae^{2}-7cd^{2})\,(ae^{2}+cd^{2})\log\left(-\frac{d^{2}e^{4}\sqrt{-\frac{1}{d^{3}e^{9}}}\,(ae^{2}-7cd^{2})\,(ae^{2}+cd^{2})}{a^{2}e^{4}-6acd^{2}e^{2}-7c^{2}d^{4}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^{3}e^{9}}}\,(ae^{2}-7cd^{2})\,(ae^{2}+cd^{2})\log\left(\frac{d^{2}e^{4}\sqrt{-\frac{1}{d^{3}e^{9}}}\,(ae^{2}-7cd^{2})\,(ae^{2}+cd^{2})}{a^{2}e^{4}-6acd^{2}e^{2}-7c^{2}d^{4}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**2,x)
```

```
[Out] -2*c**2*d*x**3/(3*e**3) + c**2*x**5/(5*e**2) + x*(2*a*c/e**2 + 3*c**2*d**2/
e**4) + x*(a**2*e**4 + 2*a*c*d**2*e**2 + c**2*d**4)/(2*d**2*e**4 + 2*d*e**5
*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)*log(-d*
*2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7*c*d**2)*(a*e**2 + c*d**2)/(a**2*e*
*4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)/4 + sqrt(-1/(d**3*e**9))*(a*e**2 -
 7*c*d**2)*(a*e**2 + c*d**2)*log(d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - 7
*c*d**2)*(a*e**2 + c*d**2)/(a**2*e**4 - 6*a*c*d**2*e**2 - 7*c**2*d**4) + x)
/4
```

Giac [A]
time = 5.33, size = 128, normalized size = 0.98

$$\frac{1}{15}\left(3\,c^2x^5e^8-10\,c^2dx^3e^7+45\,c^2d^2xe^6+30\,acxe^8\right)e^{(-10)}-\frac{\left(7\,c^2d^4+6\,acd^2e^2-a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,d^{\frac{3}{2}}}+\frac{\left(c^2d^4x+2\,acd^2xe^2+a^2xe^4\right)e^{(-4)}}{2\,(x^2e+d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*c^2*x^5*e^8 - 10*c^2*d*x^3*e^7 + 45*c^2*d^2*x*e^6 + 30*a*c*x*e^8)*e
^(-10) - 1/2*(7*c^2*d^4 + 6*a*c*d^2*e^2 - a^2*e^4)*arctan(x*e^(1/2)/sqrt(d)
)*e^(-9/2)/d^(3/2) + 1/2*(c^2*d^4*x + 2*a*c*d^2*x*e^2 + a^2*x*e^4)*e^(-4)/(
(x^2*e + d)*d)
```

Mupad [B]
time = 4.40, size = 183, normalized size = 1.40

$$x\left(\frac{3\,c^2\,d^2}{e^4}+\frac{2\,a\,c}{e^2}\right)+\frac{c^2\,x^5}{5\,e^2}-\frac{2\,c^2\,d\,x^3}{3\,e^3}+\frac{x\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}{2\,d\,\left(e^5\,x^2+d\,e^4\right)}-\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,\left(c\,d^2+a\,e^2\right)\,\left(a\,e^2-7\,c\,d^2\right)}{\sqrt{d}\,\left(-a^2\,e^4+6\,a\,c\,d^2\,e^2+7\,c^2\,d^4\right)}\right)\,\left(c\,d^2+a\,e^2\right)\,\left(a\,e^2-7\,c\,d^2\right)}{2\,d^{3/2}\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/(d + e*x^2)^2,x)
```

```
[Out] x*((3*c^2*d^2)/e^4 + (2*a*c)/e^2) + (c^2*x^5)/(5*e^2) - (2*c^2*d*x^3)/(3*e^
3) + (x*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2))/(2*d*(d*e^4 + e^5*x^2)) - (ata
n((e^(1/2)*x*(a*e^2 + c*d^2)*(a*e^2 - 7*c*d^2))/(d^(1/2)*(7*c^2*d^4 - a^2*e
^4 + 6*a*c*d^2*e^2)))*(a*e^2 + c*d^2)*(a*e^2 - 7*c*d^2))/(2*d^(3/2)*e^(9/2)
)
```

3.134 $\int \frac{(a+cx^4)^2}{(d+ex^2)^3} dx$

Optimal. Leaf size=155

$$-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}+\frac{(cd^2+ae^2)^2 x}{4de^4(d+ex^2)^2}+\frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2(d+ex^2)}+\frac{(35c^2d^4+6acd^2e^2+3a^2e^4)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}$$

```
[Out] -3*c^2*d*x/e^4+1/3*c^2*x^3/e^3+1/4*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^2+1/8*
(3*a^2-13*c^2*d^4/e^4-10*a*c*d^2/e^2)*x/d^2/(e*x^2+d)+1/8*(3*a^2*e^4+6*a*c*
d^2*e^2+35*c^2*d^4)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(9/2)
```

Rubi [A]
time = 0.17, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {1172, 1828, 1167, 211}

$$\frac{(3a^2e^4+6acd^2e^2+35c^2d^4)\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}+\frac{x\left(3a^2-\frac{10acd^2}{e^2}-\frac{13c^2d^4}{e^4}\right)}{8d^2(d+ex^2)}+\frac{x(ae^2+cd^2)^2}{4de^4(d+ex^2)^2}-\frac{3c^2dx}{e^4}+\frac{c^2x^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (-3*c^2*d*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + a*e^2)^2*x)/(4*d*e^4*(d +
e*x^2)^2) + ((3*a^2 - (13*c^2*d^4)/e^4 - (10*a*c*d^2)/e^2)*x)/(8*d^2*(d + e
*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(8*d^(5/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
```

```
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+cx^4)^2}{(d+ex^2)^3}\,dx &= \frac{(cd^2+ae^2)^2 x}{4de^4(d+ex^2)^2} - \frac{\int \frac{-3a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{4cd(cd^2+2ae^2)x^2}{e^3}+\frac{4c^2d^2x^4}{e^2}-\frac{4c^2dx^6}{e}}{(d+ex^2)^2}\,dx}{4d} \\ &= \frac{(cd^2+ae^2)^2 x}{4de^4(d+ex^2)^2} + \frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2(d+ex^2)} + \frac{\int \frac{3a^2+\frac{11c^2d^4}{e^4}+\frac{6acd^2}{e^2}-\frac{16c^2d^3x^2}{e^3}+\frac{8c^2d^2x^4}{e^2}}{d+ex^2}\,dx}{8d^2} \\ &= \frac{(cd^2+ae^2)^2 x}{4de^4(d+ex^2)^2} + \frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2(d+ex^2)} + \frac{\int \left(-\frac{24c^2d^3}{e^4}+\frac{8c^2d^2x^2}{e^3}+\frac{35c^2d^4+6acd^2e^2+3a^2e^4}{e^4(d+ex^2)}\right.}{8d^2} \\ &= -\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2+ae^2)^2 x}{4de^4(d+ex^2)^2} + \frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2(d+ex^2)} + \frac{(35c^2d^4+6acd^2e^2+3a}{8d^2e^4} \\ &= -\frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2+ae^2)^2 x}{4de^4(d+ex^2)^2} + \frac{\left(3a^2-\frac{13c^2d^4}{e^4}-\frac{10acd^2}{e^2}\right)x}{8d^2(d+ex^2)} + \frac{(35c^2d^4+6acd^2e^2+3a}{8d^{5/2}e^9}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 154, normalized size = 0.99

$$\frac{x(3a^2e^4(5d+3ex^2)-6acd^2e^2(3d+5ex^2)-c^2d^2(105d^3+175d^2ex^2+56de^2x^4-8e^3x^6))}{24d^2e^4(d+ex^2)^2}+\frac{(35c^2d^4+6acd^2e^2+3a^2e^4)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (x*(3*a^2*e^4*(5*d + 3*e*x^2) - 6*a*c*d^2*e^2*(3*d + 5*e*x^2) - c^2*d^2*(10
5*d^3 + 175*d^2*e*x^2 + 56*d*e^2*x^4 - 8*e^3*x^6)))/(24*d^2*e^4*(d + e*x^2)
^2) + ((35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(8*d^(5/2)*e^(9/2))
```

Maple [A]
time = 0.17, size = 152, normalized size = 0.98

method	result
default	$-\frac{c^{2}\left(-\frac{1}{3}e\,x^{3}+3dx\right)}{e^{4}}+\frac{\frac{\frac{e\left(3a^{2}e^{4}-10ac\,d^{2}e^{2}-13c^{2}d^{4}\right)x^{3}}{8d^{2}}+\frac{\left(5a^{2}e^{4}-6ac\,d^{2}e^{2}-11c^{2}d^{4}\right)x}{8d}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(3a^{2}e^{4}+6ac\,d^{2}e^{2}+35c^{2}d^{4}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8d^{2}\sqrt{de}}}{e^{4}}$
risch	$\frac{c^{2}x^{3}}{3e^{3}}-\frac{3c^{2}dx}{e^{4}}+\frac{\frac{e\left(3a^{2}e^{4}-10ac\,d^{2}e^{2}-13c^{2}d^{4}\right)x^{3}}{8d^{2}}+\frac{\left(5a^{2}e^{4}-6ac\,d^{2}e^{2}-11c^{2}d^{4}\right)x}{8d}}{e^{4}\left(e\,x^{2}+d\right)^{2}}-\frac{3\ln\left(ex+\sqrt{-de}\right)a^{2}}{16\sqrt{-de}\,d^{2}}-\frac{3\ln\left(ex+\sqrt{-de}\right)}{8e^{2}\sqrt{-de}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c^2/e^4*(-1/3*e*x^3+3*d*x)+1/e^4*((1/8*e*(3*a^2*e^4-10*a*c*d^2*e^2-13*c^2*
d^4)/d^2*x^3+1/8*(5*a^2*e^4-6*a*c*d^2*e^2-11*c^2*d^4)/d*x)/(e*x^2+d)^2+1/8*
(3*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)
))
```

Maxima [A]
time = 0.52, size = 152, normalized size = 0.98

$$\frac{1}{3}\left(c^{2}x^{3}e-9\,c^{2}dx\right)e^{(-4)}+\frac{\left(35\,c^{2}d^{4}+6\,acd^{2}e^{2}+3\,a^{2}e^{4}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(13\,c^{2}d^{4}e+10\,acd^{2}e^{3}-3\,a^{2}e^{5}\right)x^{3}+\left(11\,c^{2}d^{5}+6\,acd^{3}e^{2}-5\,a^{2}de^{4}\right)x}{8\left(d^{2}x^{4}e^{6}+2\,d^{3}x^{2}e^{5}+d^{4}e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^3*e - 9*c^2*d*x)*e^(-4) + 1/8*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^
2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(5/2) - 1/8*((13*c^2*d^4*e + 10
*a*c*d^2*e^3 - 3*a^2*e^5)*x^3 + (11*c^2*d^5 + 6*a*c*d^3*e^2 - 5*a^2*d*e^4)*
x)/(d^2*x^4*e^6 + 2*d^3*x^2*e^5 + d^4*e^4)
```

Fricas [A]
time = 0.35, size = 509, normalized size = 3.28

$$\left[-\frac{350\,c^{2}d^{5}x^{3}e^{2}+210\,c^{2}d^{6}xe-18\,a^{2}dx^{3}e^{6}-30\,a^{2}d^{2}xe^{5}+3\left(70\,c^{2}d^{5}x^{2}e+35\,c^{2}d^{6}+12\,acd^{3}x^{2}e^{3}+3\,a^{2}x^{4}e^{6}+6\,a^{2}dx^{2}e^{5}+3\left(2\,acd^{2}x^{4}+a^{2}d^{2}\right)e^{4}+\left(35\,c^{2}d^{4}x^{4}+6\,acd^{4}\right)e^{2}\right)\sqrt{-de}\,\log\left(\frac{x^{2}e-2\sqrt{-de}\,x-d}{x^{2}e+d}\right)-4\left(4\,c^{2}d^{3}x^{7}-15\,acd^{3}x^{3}\right)e^{4}+4\left(28\,c^{2}d^{4}x^{5}+9\,acd^{4}x\right)e^{3}}{48\left(d^{3}x^{4}e^{7}+2\,d^{4}x^{2}e^{6}+d^{5}e^{5}\right)},\ -\frac{175\,c^{2}d^{5}x^{3}e^{2}+105\,c^{2}d^{6}xe-9\,a^{2}dx^{3}e^{6}-15\,a^{2}d^{2}xe^{5}-3\left(70\,c^{2}d^{5}x^{2}e+35\,c^{2}d^{6}+12\,acd^{3}x^{2}e^{3}+3\,a^{2}x^{4}e^{6}+6\,a^{2}dx^{2}e^{5}+3\left(2\,acd^{2}x^{4}+a^{2}d^{2}\right)e^{4}+\left(35\,c^{2}d^{4}x^{4}+6\,acd^{4}\right)e^{2}\right)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-2\left(4\,c^{2}d^{3}x^{7}-15\,acd^{3}x^{3}\right)e^{4}+2\left(28\,c^{2}d^{4}x^{5}+9\,acd^{4}x\right)e^{3}}{24\left(d^{3}x^{4}e^{7}+2\,d^{4}x^{2}e^{6}+d^{5}e^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(350*c^2*d^5*x^3*e^2 + 210*c^2*d^6*x*e - 18*a^2*d*x^3*e^6 - 30*a^2*d
^2*x*e^5 + 3*(70*c^2*d^5*x^2*e + 35*c^2*d^6 + 12*a*c*d^3*x^2*e^3 + 3*a^2*x^
4*e^6 + 6*a^2*d*x^2*e^5 + 3*(2*a*c*d^2*x^4 + a^2*d^2)*e^4 + (35*c^2*d^4*x^4
 + 6*a*c*d^4)*e^2)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d))
 - 4*(4*c^2*d^3*x^7 - 15*a*c*d^3*x^3)*e^4 + 4*(28*c^2*d^4*x^5 + 9*a*c*d^4*x
)*e^3)/(d^3*x^4*e^7 + 2*d^4*x^2*e^6 + d^5*e^5), -1/24*(175*c^2*d^5*x^3*e^2
+ 105*c^2*d^6*x*e - 9*a^2*d*x^3*e^6 - 15*a^2*d^2*x*e^5 - 3*(70*c^2*d^5*x^2*
e + 35*c^2*d^6 + 12*a*c*d^3*x^2*e^3 + 3*a^2*x^4*e^6 + 6*a^2*d*x^2*e^5 + 3*(
2*a*c*d^2*x^4 + a^2*d^2)*e^4 + (35*c^2*d^4*x^4 + 6*a*c*d^4)*e^2)*sqrt(d)*ar
ctan(x*e^(1/2)/sqrt(d))*e^(1/2) - 2*(4*c^2*d^3*x^7 - 15*a*c*d^3*x^3)*e^4 +
2*(28*c^2*d^4*x^5 + 9*a*c*d^4*x)*e^3)/(d^3*x^4*e^7 + 2*d^4*x^2*e^6 + d^5*e^
5)]
```

Sympy [A]
time = 0.83, size = 257, normalized size = 1.66

$$- \frac{3c^2dx}{e^4} + \frac{c^2x^3}{3e^3} - \frac{\sqrt{-\frac{1}{d^5e^9}} \cdot (3a^2e^4 + 6acd^2e^2 + 35c^2d^4)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^9}} \cdot (3a^2e^4 + 6acd^2e^2 + 35c^2d^4)\log\left(d^3e^4\sqrt{-\frac{1}{d^5e^9}} + x\right)}{16} + \frac{x^3 \cdot (3a^2e^5 - 10acd^2e^3 - 13c^2d^4e) + x(5a^2de^4 - 6acd^3e^2 - 11c^2d^5)}{8d^4e^4 + 16d^3e^5x^2 + 8d^2e^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**3,x)
```

```
[Out] -3*c**2*d*x/e**4 + c**2*x**3/(3*e**3) - sqrt(-1/(d**5*e**9))*(3*a**2*e**4 +
 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/1
6 + sqrt(-1/(d**5*e**9))*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log
(d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + (x**3*(3*a**2*e**5 - 10*a*c*d**2*
e**3 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 6*a*c*d**3*e**2 - 11*c**2*d**5)
)/(8*d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)
```

Giac [A]
time = 4.77, size = 145, normalized size = 0.94

$$\frac{1}{3}\left(c^2x^3e^6 - 9\,c^2dxe^5\right)e^{(-9)} + \frac{\left(35\,c^2d^4 + 6\,acd^2e^2 + 3\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,d^{\frac{5}{2}}} - \frac{\left(13\,c^2d^4x^3e + 11\,c^2d^5x + 10\,acd^2x^3e^3 + 6\,acd^3xe^2 - 3\,a^2x^3e^5 - 5\,a^2dxe^4\right)e^{(-4)}}{8\left(x^2e + d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^6 - 9*c^2*d*x*e^5)*e^(-9) + 1/8*(35*c^2*d^4 + 6*a*c*d^2*e^2
+ 3*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(5/2) - 1/8*(13*c^2*d^4*x
^3*e + 11*c^2*d^5*x + 10*a*c*d^2*x^3*e^3 + 6*a*c*d^3*x*e^2 - 3*a^2*x^3*e^5
- 5*a^2*d*x*e^4)*e^(-4)/((x^2*e + d)^2*d^2)
```

Mupad [B]
time = 4.41, size = 164, normalized size = 1.06

$$\frac{c^2\,x^3}{3\,e^3} - \frac{\frac{x^3\,\left(-3\,a^2\,e^5+10\,a\,c\,d^2\,e^3+13\,c^2\,d^4\,e\right)}{8\,d^2} + \frac{x\,\left(-5\,a^2\,e^4+6\,a\,c\,d^2\,e^2+11\,c^2\,d^4\right)}{8\,d}}{d^2\,e^4 + 2\,d\,e^5\,x^2 + e^6\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,a^2\,e^4 + 6\,a\,c\,d^2\,e^2 + 35\,c^2\,d^4\right)}{8\,d^{5/2}\,e^{9/2}} - \frac{3\,c^2\,d\,x}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/(d + e*x^2)^3,x)
```

```
[Out] (c^2*x^3)/(3*e^3) - ((x^3*(13*c^2*d^4*e - 3*a^2*e^5 + 10*a*c*d^2*e^3))/(8*d
^2) + (x*(11*c^2*d^4 - 5*a^2*e^4 + 6*a*c*d^2*e^2))/(8*d))/(d^2*e^4 + e^6*x^
4 + 2*d*e^5*x^2) + (atan((e^(1/2)*x)/d^(1/2))*(3*a^2*e^4 + 35*c^2*d^4 + 6*a
*c*d^2*e^2))/(8*d^(5/2)*e^(9/2)) - (3*c^2*d*x)/e^4
```

3.135 $\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx$

Optimal. Leaf size=184

$$\frac{c^2x}{e^4}+\frac{(cd^2+ae^2)^2x}{6de^4(d+ex^2)^3}+\frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2}+\frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)}-\frac{(35c^2d^4-2acd^2e^2-5a^2e^4)\tan^{-}}{16d^{7/2}e^{9/2}}$$

```
[Out] c^2*x/e^4+1/6*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^3+1/24*(5*a^2-19*c^2*d^4/e^
4-14*a*c*d^2/e^2)*x/d^2/(e*x^2+d)^2+1/16*(5*a^2+29*c^2*d^4/e^4+2*a*c*d^2/e^
2)*x/d^3/(e*x^2+d)-1/16*(-5*a^2*e^4-2*a*c*d^2*e^2+35*c^2*d^4)*arctan(x*e^(1
/2)/d^(1/2))/d^(7/2)/e^(9/2)
```

Rubi [A]
time = 0.19, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1172, 1828, 1171, 396, 211}

$$-\frac{(-5a^2e^4-2acd^2e^2+35c^2d^4)\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}+\frac{x\left(5a^2-\frac{14acd^2}{e^2}-\frac{19c^2d^4}{e^4}\right)}{24d^2(d+ex^2)^2}+\frac{x\left(5a^2+\frac{2acd^2}{e^2}+\frac{29c^2d^4}{e^4}\right)}{16d^3(d+ex^2)}+\frac{x(ae^2+cd^2)^2}{6de^4(d+ex^2)^3}+\frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) + ((5*a^2 - (19
*c^2*d^4)/e^4 - (14*a*c*d^2)/e^2)*x)/(24*d^2*(d + e*x^2)^2) + ((5*a^2 + (29
*c^2*d^4)/e^4 + (2*a*c*d^2)/e^2)*x)/(16*d^3*(d + e*x^2)) - ((35*c^2*d^4 - 2
*a*c*d^2*e^2 - 5*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} - \frac{\int \frac{-5a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{6cd(cd^2+2ae^2)x^2}{e^3}+\frac{6c^2d^2x^4}{e^2}-\frac{6c^2dx^6}{e}}{(d+ex^2)^3}\,dx}{6d}$$

$$= \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\int \frac{3\left(5a^2+\frac{5c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)-\frac{48c^2d^3x^2}{e^3}+\frac{24c^2d^2x^4}{e^2}}{(d+ex^2)^2}\,dx}{24d^2}$$

$$= \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)} - \frac{\int \frac{-3\left(5a^2-\frac{19c^2d}{e^4}\right.}{}}{}$$

$$= \frac{c^2x}{e^4} + \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)} - \frac{(35c^2d}{}$$

$$= \frac{c^2x}{e^4} + \frac{(cd^2+ae^2)^2 x}{6de^4(d+ex^2)^3} + \frac{\left(5a^2-\frac{19c^2d^4}{e^4}-\frac{14acd^2}{e^2}\right)x}{24d^2(d+ex^2)^2} + \frac{\left(5a^2+\frac{29c^2d^4}{e^4}+\frac{2acd^2}{e^2}\right)x}{16d^3(d+ex^2)} - \frac{(35c^2d}{}$$

Mathematica [A]
time = 0.09, size = 174, normalized size = 0.95

$$\frac{x(-2acd^2e^2(3d^2+8dex^2-3e^2x^4)+a^2e^4(33d^2+40dex^2+15e^2x^4)+c^2d^3(105d^3+280d^2ex^2+231de^2x^4+48e^3x^6))}{48d^3e^4\left(d+ex^2\right)^3}-\frac{(35c^2d^4-2acd^2e^2-5a^2e^4)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (x*(-2*a*c*d^2*e^2*(3*d^2 + 8*d*e*x^2 - 3*e^2*x^4) + a^2*e^4*(33*d^2 + 40*d
*e*x^2 + 15*e^2*x^4) + c^2*d^3*(105*d^3 + 280*d^2*e*x^2 + 231*d*e^2*x^4 + 4
8*e^3*x^6)))/(48*d^3*e^4*(d + e*x^2)^3) - ((35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*
a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))
```

Maple [A]
time = 0.19, size = 179, normalized size = 0.97

method	result
default	$\frac{c^2x}{e^4}+\frac{\frac{\frac{e^2\left(5a^2e^4+2ac\,d^2e^2+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4-2ac\,d^2e^2+17c^2d^4\right)x^3}{6d^2}+\frac{\left(11a^2e^4-2ac\,d^2e^2+19c^2d^4\right)x}{16d}}{\left(e\,x^2+d\right)^3}+\frac{\left(5a^2e^4+2ac\,d^2e^2-35c^2d^4\right)\arctan\left(\right.}{16d^3\sqrt{de}}}{e^4}$
risch	$\frac{c^2x}{e^4}+\frac{\frac{e^2\left(5a^2e^4+2ac\,d^2e^2+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4-2ac\,d^2e^2+17c^2d^4\right)x^3}{6d^2}+\frac{\left(11a^2e^4-2ac\,d^2e^2+19c^2d^4\right)x}{16d}}{e^4\left(e\,x^2+d\right)^3}-\frac{5\ln\left(ex+\sqrt{-de}\right)a^2}{32\sqrt{-de}\,d^3}-\frac{\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*x/e^4+1/e^4*((1/16*e^2*(5*a^2*e^4+2*a*c*d^2*e^2+29*c^2*d^4)/d^3*x^5+1/6
*e*(5*a^2*e^4-2*a*c*d^2*e^2+17*c^2*d^4)/d^2*x^3+1/16*(11*a^2*e^4-2*a*c*d^2*
e^2+19*c^2*d^4)/d*x)/(e*x^2+d)^3+1/16*(5*a^2*e^4+2*a*c*d^2*e^2-35*c^2*d^4)/
d^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.51, size = 185, normalized size = 1.01

$$c^2xe^{(-4)}+\frac{3\,(29\,c^2d^4e^2+2\,acd^2e^4+5\,a^2e^6)x^5+8\,(17\,c^2d^5e-2\,acd^3e^3+5\,a^2de^5)x^3+3\,(19\,c^2d^6-2\,acd^4e^2+11\,a^2d^2e^4)x}{48\,(d^3x^6e^7+3\,d^4x^4e^6+3\,d^5x^2e^5+d^6e^4)}-\frac{(35\,c^2d^4-2\,acd^2e^2-5\,a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{16\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] c^2*x*e^(-4) + 1/48*(3*(29*c^2*d^4*e^2 + 2*a*c*d^2*e^4 + 5*a^2*e^6)*x^5 + 8
*(17*c^2*d^5*e - 2*a*c*d^3*e^3 + 5*a^2*d*e^5)*x^3 + 3*(19*c^2*d^6 - 2*a*c*d
^4*e^2 + 11*a^2*d^2*e^4)*x)/(d^3*x^6*e^7 + 3*d^4*x^4*e^6 + 3*d^5*x^2*e^5 +
```

```
d^6*e^4) - 1/16*(35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*arctan(x*e^(1/2)/s
qrt(d))*e^(-9/2)/d^(7/2)
```

Fricas [A]
time = 0.37, size = 661, normalized size = 3.59

$$\left[\frac{560\,c^{2}d^{6}x^{3}e^{2}+210\,c^{2}d^{7}xe+30\,a^{2}dx^{5}e^{7}+80\,a^{2}d^{2}x^{3}e^{6}+3\left(105\,c^{2}d^{6}x^{2}e+35\,c^{2}d^{7}-5\,a^{2}x^{6}e^{7}-15\,a^{2}dx^{4}e^{6}-\left(2\,acd^{2}x^{6}+15\,a^{2}d^{2}x^{2}\right)e^{5}-\left(6\,acd^{3}x^{4}+5\,a^{2}d^{3}\right)e^{4}+\left(35\,c^{2}d^{4}x^{6}-6\,acd^{4}x^{2}\right)e^{3}+\left(105\,c^{2}d^{5}x^{4}-2\,acd^{5}\right)e^{2}\right)\sqrt{-de}\log\left(\frac{x^{2}e-2\sqrt{-de}\,x-d}{x^{2}e+d}\right)+6\left(2\,acd^{3}x^{5}+11\,a^{2}d^{3}x\right)e^{5}+32\left(3\,c^{2}d^{4}x^{7}-acd^{4}x^{3}\right)e^{4}+6\left(77\,c^{2}d^{5}x^{5}-2\,acd^{5}x\right)e^{3}}{96\left(d^{4}x^{6}e^{8}+3\,d^{5}x^{4}e^{7}+3\,d^{6}x^{2}e^{6}+d^{7}e^{5}\right)},\ \frac{280\,c^{2}d^{6}x^{3}e^{2}+105\,c^{2}d^{7}xe+15\,a^{2}dx^{5}e^{7}+40\,a^{2}d^{2}x^{3}e^{6}-3\left(105\,c^{2}d^{6}x^{2}e+35\,c^{2}d^{7}-5\,a^{2}x^{6}e^{7}-15\,a^{2}dx^{4}e^{6}-\left(2\,acd^{2}x^{6}+15\,a^{2}d^{2}x^{2}\right)e^{5}-\left(6\,acd^{3}x^{4}+5\,a^{2}d^{3}\right)e^{4}+\left(35\,c^{2}d^{4}x^{6}-6\,acd^{4}x^{2}\right)e^{3}+\left(105\,c^{2}d^{5}x^{4}-2\,acd^{5}\right)e^{2}\right)\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}+3\left(2\,acd^{3}x^{5}+11\,a^{2}d^{3}x\right)e^{5}+16\left(3\,c^{2}d^{4}x^{7}-acd^{4}x^{3}\right)e^{4}+3\left(77\,c^{2}d^{5}x^{5}-2\,acd^{5}x\right)e^{3}}{48\left(d^{4}x^{6}e^{8}+3\,d^{5}x^{4}e^{7}+3\,d^{6}x^{2}e^{6}+d^{7}e^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(560*c^2*d^6*x^3*e^2 + 210*c^2*d^7*x*e + 30*a^2*d*x^5*e^7 + 80*a^2*d^
2*x^3*e^6 + 3*(105*c^2*d^6*x^2*e + 35*c^2*d^7 - 5*a^2*x^6*e^7 - 15*a^2*d*x^
4*e^6 - (2*a*c*d^2*x^6 + 15*a^2*d^2*x^2)*e^5 - (6*a*c*d^3*x^4 + 5*a^2*d^3)*
e^4 + (35*c^2*d^4*x^6 - 6*a*c*d^4*x^2)*e^3 + (105*c^2*d^5*x^4 - 2*a*c*d^5)*
e^2)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) + 6*(2*a*c*d^
3*x^5 + 11*a^2*d^3*x)*e^5 + 32*(3*c^2*d^4*x^7 - a*c*d^4*x^3)*e^4 + 6*(77*c^
2*d^5*x^5 - 2*a*c*d^5*x)*e^3)/(d^4*x^6*e^8 + 3*d^5*x^4*e^7 + 3*d^6*x^2*e^6
+ d^7*e^5), 1/48*(280*c^2*d^6*x^3*e^2 + 105*c^2*d^7*x*e + 15*a^2*d*x^5*e^7
+ 40*a^2*d^2*x^3*e^6 - 3*(105*c^2*d^6*x^2*e + 35*c^2*d^7 - 5*a^2*x^6*e^7 -
15*a^2*d*x^4*e^6 - (2*a*c*d^2*x^6 + 15*a^2*d^2*x^2)*e^5 - (6*a*c*d^3*x^4 +
5*a^2*d^3)*e^4 + (35*c^2*d^4*x^6 - 6*a*c*d^4*x^2)*e^3 + (105*c^2*d^5*x^4 -
2*a*c*d^5)*e^2)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) + 3*(2*a*c*d^3*x^
5 + 11*a^2*d^3*x)*e^5 + 16*(3*c^2*d^4*x^7 - a*c*d^4*x^3)*e^4 + 3*(77*c^2*d^
5*x^5 - 2*a*c*d^5*x)*e^3)/(d^4*x^6*e^8 + 3*d^5*x^4*e^7 + 3*d^6*x^2*e^6 + d^
7*e^5)]
```

Sympy [A]
time = 1.89, size = 292, normalized size = 1.59

$$\frac{c^{2}x}{e^{4}}-\frac{\sqrt{-\frac{1}{d^{7}e^{9}}}\cdot\left(5a^{2}e^{4}+2acd^{2}e^{2}-35c^{2}d^{4}\right)\log\left(-d^{4}e^{4}\sqrt{-\frac{1}{d^{7}e^{9}}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{d^{7}e^{9}}}\cdot\left(5a^{2}e^{4}+2acd^{2}e^{2}-35c^{2}d^{4}\right)\log\left(d^{4}e^{4}\sqrt{-\frac{1}{d^{7}e^{9}}}+x\right)}{32}+\frac{x^{5}\cdot\left(15a^{2}e^{6}+6acd^{2}e^{4}+87c^{2}d^{4}e^{2}\right)+x^{3}\cdot\left(40a^{2}de^{5}-16acd^{3}e^{3}+136c^{2}d^{5}e\right)+x\left(33a^{2}d^{2}e^{4}-6acd^{4}e^{2}+57c^{2}d^{6}\right)}{48d^{6}e^{4}+144d^{5}e^{5}x^{2}+144d^{4}e^{6}x^{4}+48d^{3}e^{7}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**4,x)
```

```
[Out] c**2*x/e**4 - sqrt(-1/(d**7*e**9))*(5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2
*d**4)*log(-d**4*e**4*sqrt(-1/(d**7*e**9)) + x)/32 + sqrt(-1/(d**7*e**9))*(
5*a**2*e**4 + 2*a*c*d**2*e**2 - 35*c**2*d**4)*log(d**4*e**4*sqrt(-1/(d**7*e
**9)) + x)/32 + (x**5*(15*a**2*e**6 + 6*a*c*d**2*e**4 + 87*c**2*d**4*e**2)
+ x**3*(40*a**2*d*e**5 - 16*a*c*d**3*e**3 + 136*c**2*d**5*e) + x*(33*a**2*d
**2*e**4 - 6*a*c*d**4*e**2 + 57*c**2*d**6))/(48*d**6*e**4 + 144*d**5*e**5*x
**2 + 144*d**4*e**6*x**4 + 48*d**3*e**7*x**6)
```

Giac [A]
time = 4.62, size = 167, normalized size = 0.91

$$c^{2}xe^{(-4)}-\frac{\left(35\,c^{2}d^{4}-2\,acd^{2}e^{2}-5\,a^{2}e^{4}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{16\,d^{\frac{7}{2}}}+\frac{\left(87\,c^{2}d^{4}x^{5}e^{2}+136\,c^{2}d^{5}x^{3}e+6\,acd^{2}x^{5}e^{4}+57\,c^{2}d^{6}x-16\,acd^{3}x^{3}e^{3}+15\,a^{2}x^{5}e^{6}-6\,acd^{4}xe^{2}+40\,a^{2}dx^{3}e^{5}+33\,a^{2}d^{2}xe^{4}\right)e^{(-4)}}{48\,\left(x^{2}e+d\right)^{3}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] c^2*x*e^(-4) - 1/16*(35*c^2*d^4 - 2*a*c*d^2*e^2 - 5*a^2*e^4)*arctan(x*e^(1/
2)/sqrt(d))*e^(-9/2)/d^(7/2) + 1/48*(87*c^2*d^4*x^5*e^2 + 136*c^2*d^5*x^3*e
 + 6*a*c*d^2*x^5*e^4 + 57*c^2*d^6*x - 16*a*c*d^3*x^3*e^3 + 15*a^2*x^5*e^6 -
 6*a*c*d^4*x*e^2 + 40*a^2*d*x^3*e^5 + 33*a^2*d^2*x*e^4)*e^(-4)/((x^2*e + d)
^3*d^3)
```

Mupad [B]
time = 4.49, size = 199, normalized size = 1.08

$$\frac{\frac{x^3\,\left(5\,a^2\,e^5-2\,a\,c\,d^2\,e^3+17\,c^2\,d^4\,e\right)}{6\,d^2}+\frac{x\,\left(11\,a^2\,e^4-2\,a\,c\,d^2\,e^2+19\,c^2\,d^4\right)}{16\,d}+\frac{x^5\,\left(5\,a^2\,e^6+2\,a\,c\,d^2\,e^4+29\,c^2\,d^4\,e^2\right)}{16\,d^3}}{d^3\,e^4+3\,d^2\,e^5\,x^2+3\,d\,e^6\,x^4+e^7\,x^6}+\frac{c^2\,x}{e^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(5\,a^2\,e^4+2\,a\,c\,d^2\,e^2-35\,c^2\,d^4\right)}{16\,d^{7/2}\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/(d + e*x^2)^4,x)
```

```
[Out] ((x^3*(5*a^2*e^5 + 17*c^2*d^4*e - 2*a*c*d^2*e^3))/(6*d^2) + (x*(11*a^2*e^4
+ 19*c^2*d^4 - 2*a*c*d^2*e^2))/(16*d) + (x^5*(5*a^2*e^6 + 29*c^2*d^4*e^2 +
2*a*c*d^2*e^4))/(16*d^3))/(d^3*e^4 + e^7*x^6 + 3*d*e^6*x^4 + 3*d^2*e^5*x^2)
 + (c^2*x)/e^4 + (atan((e^(1/2)*x)/d^(1/2))*(5*a^2*e^4 - 35*c^2*d^4 + 2*a*c
*d^2*e^2))/(16*d^(7/2)*e^(9/2))
```

3.136 $\int \frac{(a+cx^4)^2}{(d+ex^2)^5} dx$

Optimal. Leaf size=223

$$\frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4}+\frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3}+\frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3(d+ex^2)^2}-\frac{(93c^2d^4-6acd^2e^2-35a^2e^4)x}{128d^4e^4(d+ex^2)}+\cdots$$

```
[Out] 1/8*(a*e^2+c*d^2)^2*x/d/e^4/(e*x^2+d)^4+1/48*(7*a^2-25*c^2*d^4/e^4-18*a*c*d
^2/e^2)*x/d^2/(e*x^2+d)^3+1/192*(35*a^2+163*c^2*d^4/e^4+6*a*c*d^2/e^2)*x/d^
3/(e*x^2+d)^2-1/128*(-35*a^2*e^4-6*a*c*d^2*e^2+93*c^2*d^4)*x/d^4/e^4/(e*x^2
+d)+1/128*(35*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)*arctan(x*e^(1/2)/d^(1/2))/d
^(9/2)/e^(9/2)
```

Rubi [A]

time = 0.22, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1172, 1828, 1171, 393, 211}

$$\frac{(35a^2e^4+6acd^2e^2+35c^2d^4)\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{128d^{9/2}e^{9/2}}-\frac{x(-35a^2e^4-6acd^2e^2+93c^2d^4)}{128d^4e^4(d+ex^2)}+\frac{x\left(7a^2-\frac{18acd^2}{e^2}-\frac{25c^2d^4}{e^4}\right)}{48d^2(d+ex^2)^3}+\frac{x\left(35a^2+\frac{6acd^2}{e^2}+\frac{163c^2d^4}{e^4}\right)}{192d^3(d+ex^2)^2}+\frac{x(ae^2+cd^2)^2}{8de^4(d+ex^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((c*d^2 + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) + ((7*a^2 - (25*c^2*d^4)/e^4
- (18*a*c*d^2)/e^2)*x)/(48*d^2*(d + e*x^2)^3) + ((35*a^2 + (163*c^2*d^4)/e^
4 + (6*a*c*d^2)/e^2)*x)/(192*d^3*(d + e*x^2)^2) - ((93*c^2*d^4 - 6*a*c*d^2*
e^2 - 35*a^2*e^4)*x)/(128*d^4*e^4*(d + e*x^2)) + ((35*c^2*d^4 + 6*a*c*d^2*e
^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Wi
th[{Qx = PolynomialQuotient[(a + c*x^4)^p, d + e*x^2, x], R = Coeff[Polynom
ialRemainder[(a + c*x^4)^p, d + e*x^2, x], x, 0]}, Simp[(-R)*x*((d + e*x^2)
^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)
*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+cx^4)^2}{(d+ex^2)^5}\,dx = \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} - \frac{\int \frac{-7a^2+\frac{c^2d^4}{e^4}+\frac{2acd^2}{e^2}-\frac{8cd(cd^2+2ae^2)x^2}{e^3}+\frac{8c^2d^2x^4}{e^2}-\frac{8c^2dx^6}{e}}{(d+ex^2)^4}dx}{8d}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\int \frac{35a^2+\frac{19c^2d^4}{e^4}+\frac{6acd^2}{e^2}-\frac{96c^2d^3x^2}{e^3}+\frac{48c^2d^2x^4}{e^2}}{(d+ex^2)^3}dx}{48d^2}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3(d+ex^2)^2} - \frac{\int \frac{-3(35a^2-2}{}}{}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3(d+ex^2)^2} - \frac{(93c^2d^4-6}{128d}$$

$$= \frac{(cd^2+ae^2)^2 x}{8de^4(d+ex^2)^4} + \frac{\left(7a^2-\frac{25c^2d^4}{e^4}-\frac{18acd^2}{e^2}\right)x}{48d^2(d+ex^2)^3} + \frac{\left(35a^2+\frac{163c^2d^4}{e^4}+\frac{6acd^2}{e^2}\right)x}{192d^3(d+ex^2)^2} - \frac{(93c^2d^4-6}{128d}$$

Mathematica [A]
time = 0.12, size = 200, normalized size = 0.90

$$\frac{\frac{\sqrt{d}\sqrt{e}\,x(-6acd^2e^2(3d^3+11d^2ex^2-11de^2x^4-3e^3x^6)+a^2e^4(279d^3+511d^2ex^2+385de^2x^4+105e^3x^6)-c^2d^4(105d^3+385d^2ex^2+511de^2x^4+279e^3x^6))}{(d+ex^2)^4}+3(35c^2d^4+6acd^2e^2+35a^2e^4)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{384d^{9/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((Sqrt[d]*Sqrt[e]*x*(-6*a*c*d^2*e^2*(3*d^3 + 11*d^2*e*x^2 - 11*d*e^2*x^4 -
3*e^3*x^6) + a^2*e^4*(279*d^3 + 511*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6
) - c^2*d^4*(105*d^3 + 385*d^2*e*x^2 + 511*d*e^2*x^4 + 279*e^3*x^6)))/(d +
e*x^2)^4 + 3*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(384*d^(9/2)*e^(9/2))
```

Maple [A]
time = 0.16, size = 212, normalized size = 0.95

method	result
default	$\frac{\frac{(35a^2e^4+6ac\,d^2e^2-93c^2d^4)x^7}{128d^4e}+\frac{(385a^2e^4+66ac\,d^2e^2-511c^2d^4)x^5}{384d^3e^2}+\frac{(511a^2e^4-66ac\,d^2e^2-385c^2d^4)x^3}{384d^2e^3}+\frac{(93a^2e^4-6ac\,d^2e^2-35c^2d^4)x}{128d\,e^4}}{(e\,x^2+d)^4}+$
risch	$\frac{\frac{(35a^2e^4+6ac\,d^2e^2-93c^2d^4)x^7}{128d^4e}+\frac{(385a^2e^4+66ac\,d^2e^2-511c^2d^4)x^5}{384d^3e^2}+\frac{(511a^2e^4-66ac\,d^2e^2-385c^2d^4)x^3}{384d^2e^3}+\frac{(93a^2e^4-6ac\,d^2e^2-35c^2d^4)x}{128d\,e^4}}{(e\,x^2+d)^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^2/(e*x^2+d)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (1/128*(35*a^2*e^4+6*a*c*d^2*e^2-93*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+6
6*a*c*d^2*e^2-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4-66*a*c*d^2*e^2-38
5*c^2*d^4)/d^2/e^3*x^3+1/128*(93*a^2*e^4-6*a*c*d^2*e^2-35*c^2*d^4)/d/e^4*x)
/(e*x^2+d)^4+1/128*(35*a^2*e^4+6*a*c*d^2*e^2+35*c^2*d^4)/d^4/e^4/(d*e)^(1/2
)*arctan(e*x/(d*e)^(1/2))
```

Maxima [A]
time = 0.52, size = 221, normalized size = 0.99

$$-\frac{3\,(93\,c^2d^4e^3-6\,acd^2e^5-35\,a^2e^7)x^7+(511\,c^2d^5e^2-66\,acd^3e^4-385\,a^2de^6)x^5+(385\,c^2d^6e+66\,acd^4e^3-511\,a^2d^2e^5)x^3+3\,(35\,c^2d^7+6\,acd^5e^2-93\,a^2d^3e^4)x}{384\,(d^4x^8e^8+4\,d^5x^6e^7+6\,d^6x^4e^6+4\,d^7x^2e^5+d^8e^4)}+\frac{(35\,c^2d^4+6\,acd^2e^2+35\,a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{128\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="maxima")
```

```
[Out] -1/384*(3*(93*c^2*d^4*e^3 - 6*a*c*d^2*e^5 - 35*a^2*e^7)*x^7 + (511*c^2*d^5*
e^2 - 66*a*c*d^3*e^4 - 385*a^2*d*e^6)*x^5 + (385*c^2*d^6*e + 66*a*c*d^4*e^3
 - 511*a^2*d^2*e^5)*x^3 + 3*(35*c^2*d^7 + 6*a*c*d^5*e^2 - 93*a^2*d^3*e^4)*x
)/(d^4*x^8*e^8 + 4*d^5*x^6*e^7 + 6*d^6*x^4*e^6 + 4*d^7*x^2*e^5 + d^8*e^4) +
 1/128*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*
e^(-9/2)/d^(9/2)
```

Fricas [A]
time = 0.37, size = 804, normalized size = 3.61

$$\left[-\frac{770\,c^2d^7x^3e^2+210\,c^2d^8xe-210\,a^2dx^7e^8-770\,a^2d^2x^5e^7+3\,(140\,c^2d^7x^2e+35\,a^2x^8e^8+35\,c^2d^8+140\,a^2dx^6e^7+6\,(acd^2x^8+35\,a^2d^2x^4)e^6+4\,(6\,acd^3x^6+35\,a^2d^3x^2)e^5+(35\,c^2d^4x^8+36\,acd^4x^4+35\,a^2d^4)e^4+4\,(35\,c^2d^5x^6+6\,acd^5x^2)e^3+6\,(35\,c^2d^6x^4+acd^6)e^2)\sqrt{-de}\log\left(\frac{x^2e-2\sqrt{-de}\,x-d}{x^2e+d}\right)-2\,(18\,acd^3x^7+511\,a^2d^3x^3)e^6-6\,(22\,acd^4x^5+93\,a^2d^4x)e^5+6\,(93\,c^2d^5x^7+22\,acd^5x^3)e^4+2\,(511\,c^2d^6x^5+18\,acd^6x)e^3}{768\,(d^5x^8e^9+4\,d^6x^6e^8+6\,d^7x^4e^7+4\,d^8x^2e^6+d^9e^5)},-\frac{385\,c^2d^7x^3e^2+105\,c^2d^8xe-105\,a^2dx^7e^8-385\,a^2d^2x^5e^7-3\,(140\,c^2d^7x^2e+35\,a^2x^8e^8+35\,c^2d^8+140\,a^2dx^6e^7+6\,(acd^2x^8+35\,a^2d^2x^4)e^6+4\,(6\,acd^3x^6+35\,a^2d^3x^2)e^5+(35\,c^2d^4x^8+36\,acd^4x^4+35\,a^2d^4)e^4+4\,(35\,c^2d^5x^6+6\,acd^5x^2)e^3+6\,(35\,c^2d^6x^4+acd^6)e^2)\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-(18\,acd^3x^7+511\,a^2d^3x^3)e^6-3\,(22\,acd^4x^5+93\,a^2d^4x)e^5+3\,(93\,c^2d^5x^7+22\,acd^5x^3)e^4+(511\,c^2d^6x^5+18\,acd^6x)e^3}{384\,(d^5x^8e^9+4\,d^6x^6e^8+6\,d^7x^4e^7+4\,d^8x^2e^6+d^9e^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="fricas")
```

```
[Out] [-1/768*(770*c^2*d^7*x^3*e^2 + 210*c^2*d^8*x*e - 210*a^2*d*x^7*e^8 - 770*a^
2*d^2*x^5*e^7 + 3*(140*c^2*d^7*x^2*e + 35*a^2*x^8*e^8 + 35*c^2*d^8 + 140*a^
2*d*x^6*e^7 + 6*(a*c*d^2*x^8 + 35*a^2*d^2*x^4)*e^6 + 4*(6*a*c*d^3*x^6 + 35*
a^2*d^3*x^2)*e^5 + (35*c^2*d^4*x^8 + 36*a*c*d^4*x^4 + 35*a^2*d^4)*e^4 + 4*(
35*c^2*d^5*x^6 + 6*a*c*d^5*x^2)*e^3 + 6*(35*c^2*d^6*x^4 + a*c*d^6)*e^2)*sqr
t(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) - 2*(18*a*c*d^3*x^7 +
 511*a^2*d^3*x^3)*e^6 - 6*(22*a*c*d^4*x^5 + 93*a^2*d^4*x)*e^5 + 6*(93*c^2*d
^5*x^7 + 22*a*c*d^5*x^3)*e^4 + 2*(511*c^2*d^6*x^5 + 18*a*c*d^6*x)*e^3)/(d^5
*x^8*e^9 + 4*d^6*x^6*e^8 + 6*d^7*x^4*e^7 + 4*d^8*x^2*e^6 + d^9*e^5), -1/384
*(385*c^2*d^7*x^3*e^2 + 105*c^2*d^8*x*e - 105*a^2*d*x^7*e^8 - 385*a^2*d^2*x
^5*e^7 - 3*(140*c^2*d^7*x^2*e + 35*a^2*x^8*e^8 + 35*c^2*d^8 + 140*a^2*d*x^6
```

```
*e^7 + 6*(a*c*d^2*x^8 + 35*a^2*d^2*x^4)*e^6 + 4*(6*a*c*d^3*x^6 + 35*a^2*d^3
*x^2)*e^5 + (35*c^2*d^4*x^8 + 36*a*c*d^4*x^4 + 35*a^2*d^4)*e^4 + 4*(35*c^2*
d^5*x^6 + 6*a*c*d^5*x^2)*e^3 + 6*(35*c^2*d^6*x^4 + a*c*d^6)*e^2)*sqrt(d)*ar
ctan(x*e^(1/2)/sqrt(d))*e^(1/2) - (18*a*c*d^3*x^7 + 511*a^2*d^3*x^3)*e^6 -
3*(22*a*c*d^4*x^5 + 93*a^2*d^4*x)*e^5 + 3*(93*c^2*d^5*x^7 + 22*a*c*d^5*x^3)
*e^4 + (511*c^2*d^6*x^5 + 18*a*c*d^6*x)*e^3)/(d^5*x^8*e^9 + 4*d^6*x^6*e^8 +
 6*d^7*x^4*e^7 + 4*d^8*x^2*e^6 + d^9*e^5)]
```

Sympy [A]
time = 13.66, size = 335, normalized size = 1.50

$$-\frac{\sqrt{-\frac{1}{d^9e^9}}\cdot(35a^2e^4+6acd^2e^2+35c^2d^4)\log\left(-d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256}+\frac{\sqrt{-\frac{1}{d^9e^9}}\cdot(35a^2e^4+6acd^2e^2+35c^2d^4)\log\left(d^5e^4\sqrt{-\frac{1}{d^9e^9}}+x\right)}{256}+\frac{x^7\cdot(105a^2e^7+18acd^2e^5-279c^2d^4e^3)+x^5\cdot(385a^2de^6+66acd^3e^4-511c^2d^5e^2)+x^3\cdot(511a^2d^2e^5-66acd^4e^3-385c^2d^6e)+x(279a^2d^3e^4-18acd^5e^2-105c^2d^7)}{384d^8e^4+1536d^7e^5x^2+2304d^6e^6x^4+1536d^5e^7x^6+384d^4e^8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**2/(e*x**2+d)**5,x)
```

```
[Out] -sqrt(-1/(d**9*e**9))*(35*a**2*e**4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(-
d**5*e**4*sqrt(-1/(d**9*e**9)) + x)/256 + sqrt(-1/(d**9*e**9))*(35*a**2*e**
4 + 6*a*c*d**2*e**2 + 35*c**2*d**4)*log(d**5*e**4*sqrt(-1/(d**9*e**9)) + x)
/256 + (x**7*(105*a**2*e**7 + 18*a*c*d**2*e**5 - 279*c**2*d**4*e**3) + x**5
*(385*a**2*d*e**6 + 66*a*c*d**3*e**4 - 511*c**2*d**5*e**2) + x**3*(511*a**2
*d**2*e**5 - 66*a*c*d**4*e**3 - 385*c**2*d**6*e) + x*(279*a**2*d**3*e**4 -
18*a*c*d**5*e**2 - 105*c**2*d**7))/(384*d**8*e**4 + 1536*d**7*e**5*x**2 + 2
304*d**6*e**6*x**4 + 1536*d**5*e**7*x**6 + 384*d**4*e**8*x**8)
```

Giac [A]
time = 5.29, size = 198, normalized size = 0.89

$$\frac{(35\,c^2d^4+6\,acd^2e^2+35\,a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{128\,d^{\frac{9}{2}}}-\frac{(279\,c^2d^4x^7e^3+511\,c^2d^5x^5e^2-18\,acd^2x^7e^5+385\,c^2d^6x^3e-66\,acd^3x^5e^4+105\,c^2d^7x-105\,a^2x^7e^7+66\,acd^4x^3e^3-385\,a^2dx^5e^6+18\,acd^5xe^2-511\,a^2d^2x^3e^5-279\,a^2d^3xe^4)e^{(-4)}}{384\,(x^2e+d)^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^2/(e*x^2+d)^5,x, algorithm="giac")
```

```
[Out] 1/128*(35*c^2*d^4 + 6*a*c*d^2*e^2 + 35*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e
^(-9/2)/d^(9/2) - 1/384*(279*c^2*d^4*x^7*e^3 + 511*c^2*d^5*x^5*e^2 - 18*a*c
*d^2*x^7*e^5 + 385*c^2*d^6*x^3*e - 66*a*c*d^3*x^5*e^4 + 105*c^2*d^7*x - 105
*a^2*x^7*e^7 + 66*a*c*d^4*x^3*e^3 - 385*a^2*d*x^5*e^6 + 18*a*c*d^5*x*e^2 -
511*a^2*d^2*x^3*e^5 - 279*a^2*d^3*x*e^4)*e^(-4)/((x^2*e + d)^4*d^4)
```

Mupad [B]
time = 4.49, size = 240, normalized size = 1.08

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(35\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4\right)}{128\,d^{9/2}\,e^{9/2}}-\frac{\frac{x\,\left(-93\,a^2\,e^4+6\,a\,c\,d^2\,e^2+35\,c^2\,d^4\right)}{128\,d\,e^4}-\frac{x^7\,\left(35\,a^2\,e^4+6\,a\,c\,d^2\,e^2-93\,c^2\,d^4\right)}{128\,d^4\,e}+\frac{x^3\,\left(-511\,a^2\,e^4+66\,a\,c\,d^2\,e^2+385\,c^2\,d^4\right)}{384\,d^2\,e^3}-\frac{x^5\,\left(385\,a^2\,e^4+66\,a\,c\,d^2\,e^2-511\,c^2\,d^4\right)}{384\,d^3\,e^2}}{d^4+4\,d^3\,e\,x^2+6\,d^2\,e^2\,x^4+4\,d\,e^3\,x^6+e^4\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^2/(d + e*x^2)^5,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(35*a^2*e^4 + 35*c^2*d^4 + 6*a*c*d^2*e^2))/(128*
d^(9/2)*e^(9/2)) - ((x*(35*c^2*d^4 - 93*a^2*e^4 + 6*a*c*d^2*e^2))/(128*d*e^
4) - (x^7*(35*a^2*e^4 - 93*c^2*d^4 + 6*a*c*d^2*e^2))/(128*d^4*e) + (x^3*(38
5*c^2*d^4 - 511*a^2*e^4 + 66*a*c*d^2*e^2))/(384*d^2*e^3) - (x^5*(385*a^2*e^
4 - 511*c^2*d^4 + 66*a*c*d^2*e^2))/(384*d^3*e^2))/(d^4 + e^4*x^8 + 4*d^3*e*
x^2 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4)
```

3.137 $\int \frac{(d+ex^2)^4}{a+cx^4}\,dx$

Optimal. Leaf size=437

$$\frac{e^2(6cd^2-ae^2)\,x}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}-\frac{\left(c^2d^4-6acd^2e^2+a^2e^4+4\sqrt{a}\,\sqrt{c}\,de(cd^2-ae^2)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{9/4}}$$

```
[Out] e^2*(-a*e^2+6*c*d^2)*x/c^2+4/3*d*e^3*x^3/c+1/5*e^4*x^5/c-1/8*ln(-a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4-4*d*e*(
-a*e^2+c*d^2)*a^(1/2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4-4*d*e*(-a*
e^2+c*d^2)*a^(1/2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x
*2^(1/2)/a^(1/4))*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4+4*d*e*(-a*e^2+c*d^2)*a^(1/
2)*c^(1/2))/a^(3/4)/c^(9/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))
*(c^2*d^4-6*a*c*d^2*e^2+a^2*e^4+4*d*e*(-a*e^2+c*d^2)*a^(1/2)*c^(1/2))/a^(3/
4)/c^(9/4)*2^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1185, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(a^2e^4-6acd^2e^2+4\sqrt{a}\,\sqrt{c}\,de(cd^2-ae^2)+c^2d^4\right)}{2\sqrt{2}\,a^{3/4}c^{9/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(a^2e^4-6acd^2e^2+4\sqrt{a}\,\sqrt{c}\,de(cd^2-ae^2)+c^2d^4\right)}{2\sqrt{2}\,a^{3/4}c^{9/4}}-\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\,\sqrt{c}\,de(cd^2-ae^2)+c^2d^4\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{9/4}}+\frac{\left(a^2e^4-6acd^2e^2-4\sqrt{a}\,\sqrt{c}\,de(cd^2-ae^2)+c^2d^4\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{9/4}}+\frac{e^2x(6cd^2-ae^2)}{c^2}+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(a + c*x^4),x]
```

```
[Out] (e^2*(6*c*d^2 - a*e^2)*x)/c^2 + (4*d*e^3*x^3)/(3*c) + (e^4*x^5)/(5*c) - ((c
^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) + ((c^2
*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 + 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(9/4)) - ((c^2*d
^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 - a*e^2))*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(9/
4)) + ((c^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4 - 4*Sqrt[a]*Sqrt[c]*d*e*(c*d^2 -
a*e^2))*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*
a^(3/4)*c^(9/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^4}{a+cx^4}\,dx = \int \left(\frac{e^2(6cd^2-ae^2)}{c^2} + \frac{4de^3x^2}{c} + \frac{e^4x^4}{c} + \frac{c^2d^4-6acd^2e^2+a^2e^4+4cde(cd^2-ae^2)\,x^2}{c^2\,(a+cx^4)}\right) d$$

$$= \frac{e^2(6cd^2-ae^2)\,x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} + \frac{\int \frac{c^2d^4-6acd^2e^2+a^2e^4+4cde(cd^2-ae^2)x^2}{a+cx^4}\,dx}{c^2}$$

$$= \frac{e^2(6cd^2-ae^2)\,x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} - \frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\,\sqrt{c}}\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx}{a+cx^4}}{2c^2}$$

$$= \frac{e^2(6cd^2-ae^2)\,x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} + \frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\,\sqrt{c}}\right)\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}}{}}}{4\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}}$$

$$= \frac{e^2(6cd^2-ae^2)\,x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} + \frac{\left(4cd^3e-4ade^3-\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\,\sqrt{c}}\right)\log\left(\sqrt{a}-\right.}{4\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}}$$

$$= \frac{e^2(6cd^2-ae^2)\,x}{c^2} + \frac{4de^3x^3}{3c} + \frac{e^4x^5}{5c} - \frac{\left(4cd^3e-4ade^3+\frac{c^2d^4-6acd^2e^2+a^2e^4}{\sqrt{a}\,\sqrt{c}}\right)\tan^{-1}\left(1-\right.}{2\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}}$$

Mathematica [A]
time = 0.22, size = 444, normalized size = 1.02

$$\frac{-120a^{3/4}\sqrt[4]{c}\,e^2(-6cd^2+ae^2)x+160a^{3/4}c^{5/4}de^3x^3+24a^{3/4}c^{5/4}e^4x^5-30\sqrt{2}\left(c^2d^4+4\sqrt{a}\,c^{3/2}d^3e-6acd^2e^2-4a^{3/2}\sqrt{c}\,de^3+a^2e^4\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+30\sqrt{2}\left(c^2d^4+4\sqrt{a}\,c^{3/2}d^3e-6acd^2e^2-4a^{3/2}\sqrt{c}\,de^3+a^2e^4\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-15\sqrt{2}\left(c^2d^4-4\sqrt{a}\,c^{3/2}d^3e-6acd^2e^2+4a^{3/2}\sqrt{c}\,de^3+a^2e^4\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+15\sqrt{2}\left(c^2d^4-4\sqrt{a}\,c^{3/2}d^3e-6acd^2e^2+4a^{3/2}\sqrt{c}\,de^3+a^2e^4\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{120a^{3/4}c^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(a + c*x^4),x]
```

```
[Out] (-120*a^(3/4)*c^(1/4)*e^2*(-6*c*d^2 + a*e^2)*x + 160*a^(3/4)*c^(5/4)*d*e^3*
x^3 + 24*a^(3/4)*c^(5/4)*e^4*x^5 - 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2)*
d^3*e - 6*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 - (Sqrt
[2]*c^(1/4)*x)/a^(1/4)] + 30*Sqrt[2]*(c^2*d^4 + 4*Sqrt[a]*c^(3/2)*d^3*e - 6
*a*c*d^2*e^2 - 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)] - 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*c*d^2*
e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2] + 15*Sqrt[2]*(c^2*d^4 - 4*Sqrt[a]*c^(3/2)*d^3*e - 6*a*
c*d^2*e^2 + 4*a^(3/2)*Sqrt[c]*d*e^3 + a^2*e^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*c^(1/4)*x + Sqrt[c]*x^2])/(120*a^(3/4)*c^(9/4))
```

Maple [A]
time = 0.16, size = 291, normalized size = 0.67

method	result
risch	$\frac{e^4x^5}{5c}+\frac{4d\,e^3x^3}{3c}-\frac{e^4ax}{c^2}+\frac{6e^2d^2x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(4cde\left(-a\,e^2+c\,d^2\right)\_R^2+a^2e^4-6ac\,d^2e^2+c^2d^4\right)\ln\left(x-\_R\right)}{\_R^3}}{4c^3}$
default	$-\frac{e^2\left(-\frac{1}{5}c\,x^5e^2-\frac{4}{3}cde\,x^3+a\,e^2x-6c\,d^2x\right)}{c^2}+\frac{\frac{\left(a^2e^4-6ac\,d^2e^2+c^2d^4\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)\right.}{8a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/c^2*(-1/5*c*x^5*e^2-4/3*c*d*e*x^3+a*e^2*x-6*c*d^2*x)+1/c^2*(1/8*(a^2*e
^4-6*a*c*d^2*e^2+c^2*d^4)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1
/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/
(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(-4*a*c*d*e^3+4*c^2
*d^3*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(
x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2
*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.51, size = 420, normalized size = 0.96

$$\frac{3\,cx^5e^4+20\,cdx^3e^3+15\,(6\,cd^2e^2-ae^4)x}{15\,c^2}+\frac{\frac{2\sqrt{2}\left(c^{\frac{5}{2}}d^4+4\sqrt{a}\,c^2d^3e-6\,ac^{\frac{3}{2}}d^2e^2-4\,a^{\frac{3}{2}}cde^3+a^2\sqrt{c}\,e^4\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(c^{\frac{5}{2}}d^4+4\sqrt{a}\,c^2d^3e-6\,ac^{\frac{3}{2}}d^2e^2-4\,a^{\frac{3}{2}}cde^3+a^2\sqrt{c}\,e^4\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(c^{\frac{5}{2}}d^4-4\sqrt{a}\,c^2d^3e-6\,ac^{\frac{3}{2}}d^2e^2+4\,a^{\frac{3}{2}}cde^3+a^2\sqrt{c}\,e^4\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(c^{\frac{5}{2}}d^4-4\sqrt{a}\,c^2d^3e-6\,ac^{\frac{3}{2}}d^2e^2+4\,a^{\frac{3}{2}}cde^3+a^2\sqrt{c}\,e^4\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c*x^5*e^4 + 20*c*d*x^3*e^3 + 15*(6*c*d^2*e^2 - a*e^4)*x)/c^2 + 1/8*
(2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 - 4*a^(
3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a
^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(
c)) + 2*sqrt(2)*(c^(5/2)*d^4 + 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2 -
4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*
sqrt(c)) + sqrt(2)*(c^(5/2)*d^4 - 4*sqrt(a)*c^2*d^3*e - 6*a*c^(3/2)*d^2*e^2
 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c
^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(5/2)*d^4 - 4*sqrt(a)*c^
2*d^3*e - 6*a*c^(3/2)*d^2*e^2 + 4*a^(3/2)*c*d*e^3 + a^2*sqrt(c)*e^4)*log(sq
rt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2728 vs. $2(340) = 680$.

time = 1.61, size = 2728, normalized size = 6.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/60*(80*c*d*x^3*e^3 + 360*c*d^2*x*e^2 + 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a*c
^2*d^5*e^3 + 56*a^2*c*d^3*e^5 + a*c^4*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 +
 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976
*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a
^3*c^9)) - 8*a^3*d*e^7)/(a*c^4))*log(c^8*d^16*x - 24*a*c^7*d^14*x*e^2 - 36*
a^2*c^6*d^12*x*e^4 + 88*a^3*c^5*d^10*x*e^6 + 198*a^4*c^4*d^8*x*e^8 + 88*a^5
*c^3*d^6*x*e^10 - 36*a^6*c^2*d^4*x*e^12 - 24*a^7*c*d^2*x*e^14 + a^8*x*e^16
+ (a*c^8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6
*e^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 + 4*(a^3*c^
8*d^3*e - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*
d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*
e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqr
t(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 + a*c^4*sqrt(-(c^8*d^
16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 647
0*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c
*d^2*e^14 + a^8*e^16)/(a^3*c^9)) - 8*a^3*d*e^7)/(a*c^4))) - 15*c^2*sqrt(-(8
*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 + a*c^4*sqrt(-(c^8*d^16 -
56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4
*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*
e^14 + a^8*e^16)/(a^3*c^9)) - 8*a^3*d*e^7)/(a*c^4))*log(c^8*d^16*x - 24*a*c
^7*d^14*x*e^2 - 36*a^2*c^6*d^12*x*e^4 + 88*a^3*c^5*d^10*x*e^6 + 198*a^4*c^4
*d^8*x*e^8 + 88*a^5*c^3*d^6*x*e^10 - 36*a^6*c^2*d^4*x*e^12 - 24*a^7*c*d^2*x
*e^14 + a^8*x*e^16 - (a*c^8*d^12 - 34*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^
4 - 476*a^4*c^5*d^6*e^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c
^2*e^12 + 4*(a^3*c^8*d^3*e - a^4*c^7*d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14
*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8
- 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^
16)/(a^3*c^9)))*sqrt(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 +
a*c^4*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3
*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*
d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)) - 8*a^3*d*e^7)/(a*c^4))
) + 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - a*c^4
*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*
d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e
^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)) - 8*a^3*d*e^7)/(a*c^4))*log(
c^8*d^16*x - 24*a*c^7*d^14*x*e^2 - 36*a^2*c^6*d^12*x*e^4 + 88*a^3*c^5*d^10*
x*e^6 + 198*a^4*c^4*d^8*x*e^8 + 88*a^5*c^3*d^6*x*e^10 - 36*a^6*c^2*d^4*x*e^
12 - 24*a^7*c*d^2*x*e^14 + a^8*x*e^16 + (a*c^8*d^12 - 34*a^2*c^7*d^10*e^2 +
```

```
 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a^5*c^4*d^4*e^8 - 34*a^6*c
^3*d^2*e^10 + a^7*c^2*e^12 - 4*(a^3*c^8*d^3*e - a^4*c^7*d*e^3)*sqrt(-(c^8*d
^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 64
70*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*
c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 +
56*a^2*c*d^3*e^5 - a*c^4*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*
d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*
e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)) - 8*
a^3*d*e^7)/(a*c^4))) - 15*c^2*sqrt(-(8*c^3*d^7*e - 56*a*c^2*d^5*e^3 + 56*a^
2*c*d^3*e^5 - a*c^4*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*
e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10
+ 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)) - 8*a^3*d
*e^7)/(a*c^4))*log(c^8*d^16*x - 24*a*c^7*d^14*x*e^2 - 36*a^2*c^6*d^12*x*e^4
 + 88*a^3*c^5*d^10*x*e^6 + 198*a^4*c^4*d^8*x*e^8 + 88*a^5*c^3*d^6*x*e^10 -
36*a^6*c^2*d^4*x*e^12 - 24*a^7*c*d^2*x*e^14 + a^8*x*e^16 - (a*c^8*d^12 - 34
*a^2*c^7*d^10*e^2 + 239*a^3*c^6*d^8*e^4 - 476*a^4*c^5*d^6*e^6 + 239*a^5*c^4
*d^4*e^8 - 34*a^6*c^3*d^2*e^10 + a^7*c^2*e^12 - 4*(a^3*c^8*d^3*e - a^4*c^7*
d*e^3)*sqrt(-(c^8*d^16 - 56*a*c^7*d^14*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^
3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8 - 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2
*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^16)/(a^3*c^9)))*sqrt(-(8*c^3*d^7*e -
56*a*c^2*d^5*e^3 + 56*a^2*c*d^3*e^5 - a*c^4*sqrt(-(c^8*d^16 - 56*a*c^7*d^14
*e^2 + 924*a^2*c^6*d^12*e^4 - 3976*a^3*c^5*d^10*e^6 + 6470*a^4*c^4*d^8*e^8
- 3976*a^5*c^3*d^6*e^10 + 924*a^6*c^2*d^4*e^12 - 56*a^7*c*d^2*e^14 + a^8*e^
16)/(a^3*c^9)) - 8*a^3*d*e^7)/(a*c^4))) + 12*(c*x^5 - 5*a*x)*e^4)/c^2
```

Sympy [A]
time = 8.94, size = 500, normalized size = 1.14

$$x\left(-\frac{ae^4}{c^2}+\frac{6d^2e^2}{c}\right)+\operatorname{RootSum}\left(256t^4a^3c^9+t^2(-256a^5c^5de^7+1792a^4c^6d^3e^5-1792a^3c^7d^5e^3+256a^2c^8d^7e)+a^8e^{16}+8a^7cd^2e^{14}+28a^6c^2d^4e^{12}+56a^5c^3d^6e^{10}+70a^4c^4d^8e^8+56a^3c^5d^{10}e^6+28a^2c^6d^{12}e^4+8ac^7d^{14}e^2+c^8d^{16},\left(t\mapsto t\log\left(x+\frac{256t^3a^4c^7de^3-256t^3a^3c^8d^3e+4ta^7c^2e^{12}-264ta^6c^3d^2e^{10}+1980ta^5c^4d^4e^8-3696ta^4c^5d^6e^6+1980ta^3c^6d^8e^4-264ta^2c^7d^{10}e^2+4tac^8d^{12}}{a^8e^{16}-24a^7cd^2e^{14}-36a^6c^2d^4e^{12}+88a^5c^3d^6e^{10}+198a^4c^4d^8e^8+88a^3c^5d^{10}e^6-36a^2c^6d^{12}e^4-24ac^7d^{14}e^2+c^8d^{16}}\right)\right)\right)+\frac{4de^3x^3}{3c}+\frac{e^4x^5}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*x**4+a),x)
```

```
[Out] x*(-a*e**4/c**2 + 6*d**2*e**2/c) + RootSum(256*_t**4*a**3*c**9 + _t**2*(-25
6*a**5*c**5*d*e**7 + 1792*a**4*c**6*d**3*e**5 - 1792*a**3*c**7*d**5*e**3 +
256*a**2*c**8*d**7*e) + a**8*e**16 + 8*a**7*c*d**2*e**14 + 28*a**6*c**2*d**
4*e**12 + 56*a**5*c**3*d**6*e**10 + 70*a**4*c**4*d**8*e**8 + 56*a**3*c**5*d
**10*e**6 + 28*a**2*c**6*d**12*e**4 + 8*a*c**7*d**14*e**2 + c**8*d**16, Lam
bda(_t, _t*log(x + (256*_t**3*a**4*c**7*d*e**3 - 256*_t**3*a**3*c**8*d**3*e
 + 4*_t*a**7*c**2*e**12 - 264*_t*a**6*c**3*d**2*e**10 + 1980*_t*a**5*c**4*d
**4*e**8 - 3696*_t*a**4*c**5*d**6*e**6 + 1980*_t*a**3*c**6*d**8*e**4 - 264*
_t*a**2*c**7*d**10*e**2 + 4*_t*a*c**8*d**12)/(a**8*e**16 - 24*a**7*c*d**2*e
**14 - 36*a**6*c**2*d**4*e**12 + 88*a**5*c**3*d**6*e**10 + 198*a**4*c**4*d*
*8*e**8 + 88*a**3*c**5*d**10*e**6 - 36*a**2*c**6*d**12*e**4 - 24*a*c**7*d**
14*e**2 + c**8*d**16)))) + 4*d*e**3*x**3/(3*c) + e**4*x**5/(5*c)
```

Giac [A]
time = 7.95, size = 498, normalized size = 1.14

$$\frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4-6\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2+4\,(ac^3)^{\frac{3}{4}}cd^3e+(ac^3)^{\frac{1}{4}}a^2ce^4-4\,(ac^3)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4}+\frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4-6\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2+4\,(ac^3)^{\frac{3}{4}}cd^3e+(ac^3)^{\frac{1}{4}}a^2ce^4-4\,(ac^3)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4}+\frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4-6\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2-4\,(ac^3)^{\frac{3}{4}}cd^3e+(ac^3)^{\frac{1}{4}}a^2ce^4+4\,(ac^3)^{\frac{3}{4}}ade^3\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^4}-\frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^3d^4-6\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2-4\,(ac^3)^{\frac{3}{4}}cd^3e+(ac^3)^{\frac{1}{4}}a^2ce^4+4\,(ac^3)^{\frac{3}{4}}ade^3\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac^4}+\frac{3\,c^4x^5e^4+20\,c^4dx^3e^3+90\,c^4d^2xe^2-15\,ac^3xe^4}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 4*(a*c
^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3/4)*a*d*e^3)*arct
an(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/4*sqrt(
2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 4*(a*c^3)^(3/4)
*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^4 - 4*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^
3)^(1/4)*c^3*d^4 - 6*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 4*(a*c^3)^(3/4)*c*d^3*e
+ (a*c^3)^(1/4)*a^2*c*e^4 + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 + sqrt(2)*x*(a
/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^4 - 6*(a*
c^3)^(1/4)*a*c^2*d^2*e^2 - 4*(a*c^3)^(3/4)*c*d^3*e + (a*c^3)^(1/4)*a^2*c*e^
4 + 4*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(
a*c^4) + 1/15*(3*c^4*x^5*e^4 + 20*c^4*d*x^3*e^3 + 90*c^4*d^2*x*e^2 - 15*a*c
^3*x*e^4)/c^5
```

Mupad [B]
time = 5.08, size = 2500, normalized size = 5.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4/(a + c*x^4),x)
```

```
[Out] atan((((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a
^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*
((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8
*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2
*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*
(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^
4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5
*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^
2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))
^(1/2)*1i + ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6
+ 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2
*e^2)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7
*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d
^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^
4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9)^(1/2
```

```
) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c
^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^
3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3
*c^9))^(1/2)*1i)/(((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^
2*e^6 + 70*a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c
^5*d^2*e^2)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c
^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a
*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*
c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*c^9
)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56
*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) -
 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(
16*a^3*c^9))^(1/2) - ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c
*d^2*e^6 + 70*a^2*c^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^
2*c^5*d^2*e^2)*((a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^
2*c^8*d^7*e + 8*a^5*c^5*d*e^7 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 2
8*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a
^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*((a^4*e^8*(-a^3*
c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7 +
 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2
) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2)
)/(16*a^3*c^9))^(1/2) + (8*(a^5*d*e^11 - c^5*d^11*e - 3*a*c^4*d^9*e^3 + 3*a
^4*c*d^3*e^9 - 2*a^2*c^3*d^7*e^5 + 2*a^3*c^2*d^5*e^7))/c^3))*((a^4*e^8*(-a^
3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) - 8*a^2*c^8*d^7*e + 8*a^5*c^5*d*e^7
 + 56*a^3*c^7*d^5*e^3 - 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1
/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/
2))/(16*a^3*c^9))^(1/2)*2i - x*((a*e^4)/c^2 - (6*d^2*e^2)/c) + atan((((4*x*
(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c^2*d^4*e
^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*(-(a^4*e^8*(
-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*
e^7 - 56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)
^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^
(1/2))/(16*a^3*c^9))^(1/2))/c^3)*(-(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^
3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 - 56*a^3*c^7*d^5*e^3 + 56*
a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^
3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2)*1i
+ ((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 + 70*a^2*c
^2*d^4*e^4))/c + (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e^2)*(-(a
^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^
5*c^5*d*e^7 - 56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-
a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a
^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*(-(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*
d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e - 8*a^5*c^5*d*e^7 - 56*a^3*c^7*d^5*e
^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*
e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(
```

```
1/2)*1i)/(((4*x*(a^4*e^8 + c^4*d^8 - 28*a*c^3*d^6*e^2 - 28*a^3*c*d^2*e^6 +
70*a^2*c^2*d^4*e^4))/c - (4*(4*a*c^6*d^4 + 4*a^3*c^4*e^4 - 24*a^2*c^5*d^2*e
^2)*(-(a^4*e^8*(-a^3*c^9)^(1/2) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*
e - 8*a^5*c^5*d*e^7 - 56*a^3*c^7*d^5*e^3 + 56*a^4*c^6*d^3*e^5 - 28*a*c^3*d^
6*e^2*(-a^3*c^9)^(1/2) - 28*a^3*c*d^2*e^6*(-a^3*c^9)^(1/2) + 70*a^2*c^2*d^4
*e^4*(-a^3*c^9)^(1/2))/(16*a^3*c^9))^(1/2))/c^3)*(-(a^4*e^8*(-a^3*c^9)^(1/2
) + c^4*d^8*(-a^3*c^9)^(1/2) + 8*a^2*c^8*d^7*e ...
```

3.138 $\int \frac{(d+ex^2)^3}{a+cx^4}\,dx$

Optimal. Leaf size=370

$$\frac{3de^2x}{c}+\frac{e^3x^3}{3c}-\frac{\left(\sqrt{c}\,d(cd^2-3ae^2)+\sqrt{a}\,e(3cd^2-ae^2)\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}\,d(cd^2-3ae^2)+\sqrt{a}\,e\right.}{2}$$

```
[Out] 3*d*e^2*x/c+1/3*e^3*x^3/c-1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(-e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^(7
/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*(-a*e
^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^(7/4)*2^(1/2)+1/4
*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e
^2+c*d^2)*c^(1/2))/a^(3/4)/c^(7/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a
^(1/4))*(e*(-a*e^2+3*c*d^2)*a^(1/2)+d*(-3*a*e^2+c*d^2)*c^(1/2))/a^(3/4)/c^(
7/4)*2^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1185, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{c}\,d(cd^2-3ae^2)+\sqrt{a}\,e(3cd^2-ae^2)\right)}{2\sqrt{2}\,a^{3/4}c^{7/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}\,d(cd^2-3ae^2)+\sqrt{a}\,e(3cd^2-ae^2)\right)}{2\sqrt{2}\,a^{3/4}c^{7/4}}-\frac{\left(\sqrt{c}\,d(cd^2-3ae^2)-\sqrt{a}\,e(3cd^2-ae^2)\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{7/4}}+\frac{\left(\sqrt{c}\,d(cd^2-3ae^2)-\sqrt{a}\,e(3cd^2-ae^2)\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{7/4}}+\frac{3de^2x}{c}+\frac{e^3x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + c*x^4),x]
```

```
[Out] (3*d*e^2*x)/c + (e^3*x^3)/(3*c) - ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e
*(3*c*d^2 - a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(
3/4)*c^(7/4)) + ((Sqrt[c]*d*(c*d^2 - 3*a*e^2) + Sqrt[a]*e*(3*c*d^2 - a*e^2)
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(7/4)) - ((
Sqrt[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4)) + ((Sqr
t[c]*d*(c*d^2 - 3*a*e^2) - Sqrt[a]*e*(3*c*d^2 - a*e^2))*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{a+cx^4}\,dx = \int \left(\frac{3de^2}{c} + \frac{e^3x^2}{c} + \frac{cd^3-3ade^2+e(3cd^2-ae^2)\,x^2}{c\,(a+cx^4)}\right) dx$$

$$= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} + \frac{\int \frac{cd^3-3ade^2+e(3cd^2-ae^2)x^2}{a+cx^4}\,dx}{c}$$

$$= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} - \frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}\,d(cd^2-3ae^2)}{\sqrt{a}}\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c^2} + \left(3cd^2e-ae^3+\cdots\right.$$

$$= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} + \frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}\,d(cd^2-3ae^2)}{\sqrt{a}}\right)\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}} + \left(3cd^2e-\cdots\right.$$

$$= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} + \frac{\left(3cd^2e-ae^3-\frac{\sqrt{c}\,d(cd^2-3ae^2)}{\sqrt{a}}\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}} - \cdots$$

$$= \frac{3de^2x}{c} + \frac{e^3x^3}{3c} - \frac{\left(3cd^2e-ae^3+\frac{\sqrt{c}\,d(cd^2-3ae^2)}{\sqrt{a}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,c^{7/4}} + \left(3cd^2e-ae^3\cdots\right.$$

<u>Mathematica</u> [A]
time = 0.17, size = 360, normalized size = 0.97

$$\frac{72a^{3/4}c^{3/4}de^2x+8a^{3/4}c^{3/4}e^3x^3+6\sqrt{2}\left(-c^{3/2}d^3-3\sqrt{a}\,cd^2e+3a\sqrt{c}\,de^2+a^{3/2}e^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+6\sqrt{2}\left(c^{3/2}d^3+3\sqrt{a}\,cd^2e-3a\sqrt{c}\,de^2-a^{3/2}e^3\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-3\sqrt{2}\left(c^{3/2}d^3-3\sqrt{a}\,cd^2e-3a\sqrt{c}\,de^2+a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+3\sqrt{2}\left(c^{3/2}d^3-3\sqrt{a}\,cd^2e-3a\sqrt{c}\,de^2+a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{24a^{3/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + c*x^4),x]
```

```
[Out] (72*a^(3/4)*c^(3/4)*d*e^2*x + 8*a^(3/4)*c^(3/4)*e^3*x^3 + 6*Sqrt[2]*(-(c^(3
/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 -
(Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e -
 3*a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] -
 3*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e
^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*(c^(
3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^(3/4)*c^(7/4))
```

<u>Maple</u> [A]
time = 0.15, size = 254, normalized size = 0.69

method	result

risch	$\frac{e^{3} x^{3}}{3 c}+\frac{3 d \,e^{2} x}{c}+\frac{\sum_{\_R =\operatorname{RootOf}\left(c \,\_Z^{4}+a \right)}\frac{\left(e \left(-a \,e^{2}+3 c \,d^{2}\right) \_R^{2}-3 d \,e^{2} a +c \,d^{3}\right) \ln \left(x -\_R \right)}{\_R^{3}}}{4 c^{2}}$
default	$\frac{e^{2} \left(\frac{1}{3} e \,x^{3}+3 d x \right)}{c}+\frac{\frac{\left(-3 d \,e^{2} a +c \,d^{3}\right) \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8 a}+\frac{(-a \cdots}{\cdots}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/c*(1/3*e*x^3+3*d*x)+1/c*(1/8*(-3*a*d*e^2+c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(
ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)
^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1
))+1/8*(-a*e^3+3*c*d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1
/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/
(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.50, size = 336, normalized size = 0.91

$$\frac{x^{3} e^{3}+9 \, d x e^{2}}{3 \, c}+\frac{\frac{2 \, \sqrt{2} \left(c^{\frac{3}{2}} d^{3}+3 \, \sqrt{a} c d^{2} e-3 \, a \sqrt{c} d e^{2}-a^{\frac{3}{2}} e^{3}\right) \arctan\left(\frac{\sqrt{2} \left(2 \, \sqrt{c} x+\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \, \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}+\frac{2 \, \sqrt{2} \left(c^{\frac{3}{2}} d^{3}+3 \, \sqrt{a} c d^{2} e-3 \, a \sqrt{c} d e^{2}-a^{\frac{3}{2}} e^{3}\right) \arctan\left(\frac{\sqrt{2} \left(2 \, \sqrt{c} x-\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \, \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}+\frac{\sqrt{2} \left(c^{\frac{3}{2}} d^{3}-3 \, \sqrt{a} c d^{2} e-3 \, a \sqrt{c} d e^{2}+a^{\frac{3}{2}} e^{3}\right) \log\left(\sqrt{c} x^{2}+\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}-\frac{\sqrt{2} \left(c^{\frac{3}{2}} d^{3}-3 \, \sqrt{a} c d^{2} e-3 \, a \sqrt{c} d e^{2}+a^{\frac{3}{2}} e^{3}\right) \log\left(\sqrt{c} x^{2}-\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}}{8 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/3*(x^3*e^3 + 9*d*x*e^2)/c + 1/8*(2*sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2
*e - 3*a*sqrt(c)*d*e^2 - a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqr
t(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))
*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2
- a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/s
qrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(c^
(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c
^(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*log(sqrt(
c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2023 vs. 2(281) = 562.
time = 0.78, size = 2023, normalized size = 5.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="fricas")

[Out] 1/12*(4*x^3*e^3 + 36*d*x*e^2 - 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + a*
c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3
*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)) +
 6*a^2*d*e^5)/(a*c^3))*log(-c^6*d^12*x + 12*a*c^5*d^10*x*e^2 + 27*a^2*c^4*d
^8*x*e^4 - 27*a^4*c^2*d^4*x*e^8 - 12*a^5*c*d^2*x*e^10 + a^6*x*e^12 + (a*c^6
*d^9 - 18*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^5
*c^2*d*e^8 + (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^1
0*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 3
0*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^
3 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a
^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c
^7)) + 6*a^2*d*e^5)/(a*c^3))) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 + a
*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^
3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7))
+ 6*a^2*d*e^5)/(a*c^3))*log(-c^6*d^12*x + 12*a*c^5*d^10*x*e^2 + 27*a^2*c^4*
d^8*x*e^4 - 27*a^4*c^2*d^4*x*e^8 - 12*a^5*c*d^2*x*e^10 + a^6*x*e^12 - (a*c^
6*d^9 - 18*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a^
5*c^2*d*e^8 + (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d^
10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 -
30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e
^3 + a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*
a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*
c^7)) + 6*a^2*d*e^5)/(a*c^3))) - 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 -
a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c
^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7))
 + 6*a^2*d*e^5)/(a*c^3))*log(-c^6*d^12*x + 12*a*c^5*d^10*x*e^2 + 27*a^2*c^4
*d^8*x*e^4 - 27*a^4*c^2*d^4*x*e^8 - 12*a^5*c*d^2*x*e^10 + a^6*x*e^12 + (a*c
^6*d^9 - 18*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*a
^5*c^2*d*e^8 - (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*d
^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 -
 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*
e^3 - a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452
*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3
*c^7)) + 6*a^2*d*e^5)/(a*c^3))) + 3*c*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3*e^3 -
 a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*
c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)
) + 6*a^2*d*e^5)/(a*c^3))*log(-c^6*d^12*x + 12*a*c^5*d^10*x*e^2 + 27*a^2*c^
4*d^8*x*e^4 - 27*a^4*c^2*d^4*x*e^8 - 12*a^5*c*d^2*x*e^10 + a^6*x*e^12 - (a*
c^6*d^9 - 18*a^2*c^5*d^7*e^2 + 60*a^3*c^4*d^5*e^4 - 46*a^4*c^3*d^3*e^6 + 3*
a^5*c^2*d*e^8 - (3*a^3*c^6*d^2*e - a^4*c^5*e^3)*sqrt(-(c^6*d^12 - 30*a*c^5*
d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8
- 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^7)))*sqrt(-(6*c^2*d^5*e - 20*a*c*d^3
*e^3 - a*c^3*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 45
```

```
2*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^
3*c^7)) + 6*a^2*d*e^5)/(a*c^3))))/c
```

Sympy [A]
time = 1.53, size = 350, normalized size = 0.95

$$\mathrm{RootSum}\left(256t^4a^3c^7 + t^2\cdot\left(192a^4c^4de^5 - 640a^3c^5d^3e^3 + 192a^2c^6d^5e\right) + a^6e^{12} + 6a^5cd^2e^{10} + 15a^4c^2d^4e^8 + 20a^3c^3d^6e^6 + 15a^2c^4d^8e^4 + 6ac^5d^{10}e^2 + c^6d^{12}, \left(t \mapsto t\log\left(x + \frac{-64t^3a^4c^5e^3 + 192t^3a^3c^6d^2e - 36ta^5c^2de^8 + 336ta^4c^3d^3e^6 - 504ta^3c^4d^5e^4 + 144ta^2c^5d^7e^2 - 4tac^6d^9}{a^6e^{12} - 12a^5cd^2e^{10} - 27a^4c^2d^4e^8 + 27a^2c^4d^8e^4 + 12ac^5d^{10}e^2 - c^6d^{12}}\right)\right)\right) + \frac{3de^2x}{c} + \frac{e^3x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**7 + _t**2*(192*a**4*c**4*d*e**5 - 640*a**3*c**5*d
**3*e**3 + 192*a**2*c**6*d**5*e) + a**6*e**12 + 6*a**5*c*d**2*e**10 + 15*a*
*4*c**2*d**4*e**8 + 20*a**3*c**3*d**6*e**6 + 15*a**2*c**4*d**8*e**4 + 6*a*c
**5*d**10*e**2 + c**6*d**12, Lambda(_t, _t*log(x + (-64*_t**3*a**4*c**5*e**
3 + 192*_t**3*a**3*c**6*d**2*e - 36*_t*a**5*c**2*d*e**8 + 336*_t*a**4*c**3*
d**3*e**6 - 504*_t*a**3*c**4*d**5*e**4 + 144*_t*a**2*c**5*d**7*e**2 - 4*_t*
a*c**6*d**9)/(a**6*e**12 - 12*a**5*c*d**2*e**10 - 27*a**4*c**2*d**4*e**8 +
27*a**2*c**4*d**8*e**4 + 12*a*c**5*d**10*e**2 - c**6*d**12)))) + 3*d*e**2*x
/c + e**3*x**3/(3*c)
```

Giac [A]
time = 7.95, size = 405, normalized size = 1.09

$$\frac{c^2x^3e^3 + 9\,c^2dxe^2}{3\,c^3} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3 - 3\left(ac^3\right)^{\frac{1}{4}}ac^2de^2 + 3\left(ac^3\right)^{\frac{3}{4}}cd^2e - \left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3 - 3\left(ac^3\right)^{\frac{1}{4}}ac^2de^2 + 3\left(ac^3\right)^{\frac{3}{4}}cd^2e - \left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac^4} + \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3 - 3\left(ac^3\right)^{\frac{1}{4}}ac^2de^2 - 3\left(ac^3\right)^{\frac{3}{4}}cd^2e + \left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^4} - \frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^3d^3 - 3\left(ac^3\right)^{\frac{1}{4}}ac^2de^2 - 3\left(ac^3\right)^{\frac{3}{4}}cd^2e + \left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{8\,ac^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^3 + 9*c^2*d*x*e^2)/c^3 + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3
- 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e
^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1
/4*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 + 3*(a*c^3)
^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(a*c^4) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*
c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*log
(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4) - 1/8*sqrt(2)*((a*c^3)^(1
/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^
3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^4)
```

Mupad [B]
time = 4.88, size = 2712, normalized size = 7.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + c*x^4),x)

[Out] (e^3*x^3)/(3*c) - atan((a^3*e^6*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (5*d^3
*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c^7)^(
1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^4*e^2
*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 +
 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^3*c^7)^
(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*(-a^3*c
^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-a^3*c^7
)^(1/2))/(a*c^2)) - (c^3*d^6*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (5*d^3*e^
3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c^7)^(1/2
))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^4*e^2*(-
a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 12
0*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^3*c^7)^(1/
2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*(-a^3*c^7)
^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-a^3*c^7)^(
1/2))/(a*c^2)) + (a*c^2*d^4*e^2*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (5*d^3
*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c^7)^(
1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^4*e^2
*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2
 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^3*c^7
)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*(-a^3
*c^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-a^3*c
^7)^(1/2))/(a*c^2)) - (a^2*c*d^2*e^4*x*((e^6*(-a^3*c^7)^(1/2))/(16*c^7) + (
5*d^3*e^3)/(4*c^2) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) - (d^6*(-a^3*c
^7)^(1/2))/(16*a^3*c^4) - (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) + (15*d^
4*e^2*(-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9
)/c^2 + 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 + (2*d^9*(-a^
3*c^7)^(1/2))/(a^2*c) + (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 - (92*a*d^3*e^6*
(-a^3*c^7)^(1/2))/c^4 + (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 - (36*d^7*e^2*(-
a^3*c^7)^(1/2))/(a*c^2)))*(-(c^3*d^6*(-a^3*c^7)^(1/2) - a^3*e^6*(-a^3*c^7)^
(1/2) + 6*a^2*c^6*d^5*e + 6*a^4*c^4*d*e^5 - 20*a^3*c^5*d^3*e^3 - 15*a*c^2*d
^4*e^2*(-a^3*c^7)^(1/2) + 15*a^2*c*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a^3*c^7))^
(1/2)*2i - atan((a^3*e^6*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2))/(1
6*c^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/2))/(
16*a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(-a^3*
c^7)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 120*a^
2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^(1/2))/
(a^2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c^7)^(1/
2))/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7)^(1/2)
)/(a*c^2)) - (c^3*d^6*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2))/(16*c
^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/2))/(16*
a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(-a^3*c^7
)^(1/2))/(16*a^2*c^5))^(1/2)*8i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 120*a^2*d
^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^(1/2))/(a^
```

```
2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c^7)^(1/2))
/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7)^(1/2))/(
a*c^2)) + (a*c^2*d^4*e^2*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2))/(1
6*c^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/2))/(
16*a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(-a^3*
c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 + 120*
a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^(1/2)
)/(a^2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c^7)^(
1/2))/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7)^(1/
2))/(a*c^2)) - (a^2*c*d^2*e^4*x*((5*d^3*e^3)/(4*c^2) - (e^6*(-a^3*c^7)^(1/2
))/(16*c^7) - (3*d^5*e)/(8*a*c) - (3*a*d*e^5)/(8*c^3) + (d^6*(-a^3*c^7)^(1/
2))/(16*a^3*c^4) + (15*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a*c^6) - (15*d^4*e^2*(
-a^3*c^7)^(1/2))/(16*a^2*c^5))^(1/2)*120i)/(6*c^2*d^8*e + (2*a^4*e^9)/c^2 +
 120*a^2*d^4*e^5 - (36*a^3*d^2*e^7)/c - 92*a*c*d^6*e^3 - (2*d^9*(-a^3*c^7)^
(1/2))/(a^2*c) - (120*d^5*e^4*(-a^3*c^7)^(1/2))/c^3 + (92*a*d^3*e^6*(-a^3*c
^7)^(1/2))/c^4 - (6*a^2*d*e^8*(-a^3*c^7)^(1/2))/c^5 + (36*d^7*e^2*(-a^3*c^7
)^(1/2))/(a*c^2)))*(-(a^3*e^6*(-a^3*c^7)^(1/2) - c^3*d^6*(-a^3*c^7)^(1/2) +
 6*a^2*c^6*d^5*e + 6*a^4*c^4*d*e^5 - 20*a^3*c^5*d^3*e^3 + 15*a*c^2*d^4*e^2*
(-a^3*c^7)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^7)^(1/2))/(16*a^3*c^7))^(1/2)*2
i + (3*d*e^2*x)/c
```

3.139 $\int \frac{(d+ex^2)^2}{a+cx^4}\,dx$

Optimal. Leaf size=297

$$\frac{e^2x}{c}-\frac{\left(cd^2+2\sqrt{a}\,\sqrt{c}\,de-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{\left(cd^2+2\sqrt{a}\,\sqrt{c}\,de-ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{5/4}}$$

```
[Out] e^2*x/c-1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c*d^2-a*e^2
-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(c*d^2-a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c^(5
/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*d^2-a*e^2+2*d*e*a^(
1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(c*d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/c^(5/4)*2^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1185, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(2\sqrt{a}\,\sqrt{c}\,de-ae^2+cd^2\right)}{2\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(2\sqrt{a}\,\sqrt{c}\,de-ae^2+cd^2\right)}{2\sqrt{2}\,a^{3/4}c^{5/4}}-\frac{\left(-2\sqrt{a}\,\sqrt{c}\,de-ae^2+cd^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{\left(-2\sqrt{a}\,\sqrt{c}\,de-ae^2+cd^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{5/4}}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + c*x^4),x]
```

```
[Out] (e^2*x)/c - ((c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 + 2*Sqrt[a]*Sqrt[c
]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
c^(5/4)) - ((c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4)) + ((c*d^2 - 2*
Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{a+cx^4}\,dx = \int \left(\frac{e^2}{c} + \frac{cd^2 - ae^2 + 2cdex^2}{c(a+cx^4)}\right) dx$$

$$= \frac{e^2x}{c} + \frac{\int \frac{cd^2-ae^2+2cdex^2}{a+cx^4}\,dx}{c}$$

$$= \frac{e^2x}{c} + \frac{\left(cd^2 - 2\sqrt{a}\sqrt{c}\,de - ae^2\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\sqrt{a}\,c^{3/2}} + \frac{\left(cd^2 + 2\sqrt{a}\sqrt{c}\,de - ae^2\right)\int \frac{\sqrt{a}\sqrt{}}{a+}}{2\sqrt{a}\,c^{3/2}}$$

$$= \frac{e^2x}{c} - \frac{\left(cd^2 - 2\sqrt{a}\sqrt{c}\,de - ae^2\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,a^{3/4}c^{5/4}} - \frac{\left(cd^2 - 2\sqrt{a}\sqrt{c}\,de - ae^2\right)}{4\sqrt{2}\,a^{3}}$$

$$= \frac{e^2x}{c} - \frac{\left(cd^2 - 2\sqrt{a}\sqrt{c}\,de - ae^2\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{5/4}} + \frac{\left(cd^2 - 2\sqrt{a}\sqrt{c}\right.}{}$$

$$= \frac{e^2x}{c} - \frac{\left(cd^2 + 2\sqrt{a}\sqrt{c}\,de - ae^2\right)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{5/4}} + \frac{\left(cd^2 + 2\sqrt{a}\sqrt{c}\,de - ae^2\right)\tan}{2\sqrt{2}\,a^{3/4}c}$$

Mathematica [A]
time = 0.16, size = 269, normalized size = 0.91

$$\frac{8a^{3/4}\sqrt[4]{c}\,e^2x - 2\sqrt{2}\left(cd^2+2\sqrt{a}\sqrt{c}\,de-ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right) + 2\sqrt{2}\left(cd^2+2\sqrt{a}\sqrt{c}\,de-ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right) + \sqrt{2}\left(-cd^2+2\sqrt{a}\sqrt{c}\,de+ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right) + \sqrt{2}\left(cd^2-2\sqrt{a}\sqrt{c}\,de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{8a^{3/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + c*x^4),x]
```

```
[Out] (8*a^(3/4)*c^(1/4)*e^2*x - 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(c*d^2 + 2*Sqrt[a]*Sq
rt[c]*d*e - a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*(-(c*d
^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2] + Sqrt[2]*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(8*a^(3/4)*c^(5/4))
```

Maple [A]
time = 0.12, size = 228, normalized size = 0.77

method	result

risch	$\frac{e^2x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+a)}\frac{(2\_R^2cde-a\,e^2+c\,d^2)\ln(x-\_R)}{\_R^3}}{4c^2}$
default	$\frac{e^2x}{c}+\frac{\frac{(-a\,e^2+c\,d^2)(\frac{a}{c})^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+(\frac{a}{c})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-(\frac{a}{c})^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(\frac{a}{c})^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{(\frac{a}{c})^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{de\sqrt{2}\left(\ln\left(\frac{x^2-(\ldots}{x^2+(\ldots}\right.\right.}{\ldots}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*x/c+1/c*(1/8*(-a*e^2+c*d^2)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*
x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^
(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/4*d*e/(a/c)^(1/
4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2
^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/
c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.51, size = 287, normalized size = 0.97

$$\frac{xe^2}{c}+\frac{\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^2+2\sqrt{a}\,cde-a\sqrt{c}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^2+2\sqrt{a}\,cde-a\sqrt{c}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(c^{\frac{3}{2}}d^2-2\sqrt{a}\,cde-a\sqrt{c}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(c^{\frac{3}{2}}d^2-2\sqrt{a}\,cde-a\sqrt{c}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] x*e^2/c + 1/8*(2*sqrt(2)*(c^(3/2)*d^2 + 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*ar
ctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(
c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^2 + 2*s
qrt(a)*c*d*e - a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(
1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)
) + sqrt(2)*(c^(3/2)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sqrt(c)*x^2
 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(3/2
)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*
c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1415 vs. $2(215) = 430$.
time = 0.41, size = 1415, normalized size = 4.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(c*sqrt(-(4*c*d^3*e + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*
c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)) - 4*a*d*e^3)/(a*c^2))*
log(c^4*d^8*x - 4*a*c^3*d^6*x*e^2 - 10*a^2*c^2*d^4*x*e^4 - 4*a^3*c*d^2*x*e^
6 + a^4*x*e^8 + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 + 2*a^3*c^4*d*sqrt(-(c^4*d^8
 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3
*c^5))*e + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6)*sqrt(-(4*c*d^3*e + a*c^2*sqrt(-(c
^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8
)/(a^3*c^5)) - 4*a*d*e^3)/(a*c^2))) - c*sqrt(-(4*c*d^3*e + a*c^2*sqrt(-(c^4
*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/
(a^3*c^5)) - 4*a*d*e^3)/(a*c^2))*log(c^4*d^8*x - 4*a*c^3*d^6*x*e^2 - 10*a^2
*c^2*d^4*x*e^4 - 4*a^3*c*d^2*x*e^6 + a^4*x*e^8 - (a*c^4*d^6 - 7*a^2*c^3*d^4
*e^2 + 2*a^3*c^4*d*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 -
 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5))*e + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6)*
sqrt(-(4*c*d^3*e + a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4
*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)) - 4*a*d*e^3)/(a*c^2))) + c*sq
rt(-(4*c*d^3*e - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e
^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)) - 4*a*d*e^3)/(a*c^2))*log(c^4*d
^8*x - 4*a*c^3*d^6*x*e^2 - 10*a^2*c^2*d^4*x*e^4 - 4*a^3*c*d^2*x*e^6 + a^4*x
*e^8 + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 - 2*a^3*c^4*d*sqrt(-(c^4*d^8 - 12*a*c
^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5))*e
+ 7*a^3*c^2*d^2*e^4 - a^4*c*e^6)*sqrt(-(4*c*d^3*e - a*c^2*sqrt(-(c^4*d^8 -
12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^
5)) - 4*a*d*e^3)/(a*c^2))) - c*sqrt(-(4*c*d^3*e - a*c^2*sqrt(-(c^4*d^8 - 12
*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)
) - 4*a*d*e^3)/(a*c^2))*log(c^4*d^8*x - 4*a*c^3*d^6*x*e^2 - 10*a^2*c^2*d^4*
x*e^4 - 4*a^3*c*d^2*x*e^6 + a^4*x*e^8 - (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2 - 2*
a^3*c^4*d*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12*a^3*c
*d^2*e^6 + a^4*e^8)/(a^3*c^5))*e + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6)*sqrt(-(4*
c*d^3*e - a*c^2*sqrt(-(c^4*d^8 - 12*a*c^3*d^6*e^2 + 38*a^2*c^2*d^4*e^4 - 12
*a^3*c*d^2*e^6 + a^4*e^8)/(a^3*c^5)) - 4*a*d*e^3)/(a*c^2))) + 4*x*e^2)/c
```

Sympy [A]
time = 0.79, size = 238, normalized size = 0.80

$$\mathrm{RootSum}\left(256t^4a^3c^5+t^2\left(-128a^3c^3de^3+128a^2c^4d^3e\right)+a^4e^8+4a^3cd^2e^6+6a^2c^2d^4e^4+4ac^3d^6e^2+c^4d^8,\left(t\mapsto t\log\left(x+\frac{-128t^3a^3c^4de-4ta^4ce^6+60ta^3c^2d^2e^4-60ta^2c^3d^4e^2+4tac^4d^6}{a^4e^8-4a^3cd^2e^6-10a^2c^2d^4e^4-4ac^3d^6e^2+c^4d^8}\right)\right)\right)+\frac{e^2x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c**5 + _t**2*(-128*a**3*c**3*d*e**3 + 128*a**2*c**4*
d**3*e) + a**4*e**8 + 4*a**3*c*d**2*e**6 + 6*a**2*c**2*d**4*e**4 + 4*a*c**3
*d**6*e**2 + c**4*d**8, Lambda(_t, _t*log(x + (-128*_t**3*a**3*c**4*d*e - 4
*_t*a**4*c*e**6 + 60*_t*a**3*c**2*d**2*e**4 - 60*_t*a**2*c**3*d**4*e**2 + 4
*_t*a*c**4*d**6)/(a**4*e**8 - 4*a**3*c*d**2*e**6 - 10*a**2*c**2*d**4*e**4 -
 4*a*c**3*d**6*e**2 + c**4*d**8)))) + e**2*x/c
```

Giac [A]
time = 8.58, size = 318, normalized size = 1.07

$$\frac{xe^2}{c}+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2+2\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3}+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2+2\left(ac^3\right)^{\frac{3}{4}}de\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3}+\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2-2\left(ac^3\right)^{\frac{3}{4}}de\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac^3}-\frac{\sqrt{2}\left(\left(ac^3\right)^{\frac{1}{4}}c^2d^2-\left(ac^3\right)^{\frac{1}{4}}ace^2-2\left(ac^3\right)^{\frac{3}{4}}de\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] x*e^2/c + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*(a
*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4)
)/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 + 2*
(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/
4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 -
2*(a*c^3)^(3/4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) -
 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/
4)*d*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3)
```

Mupad [B]
time = 4.79, size = 1479, normalized size = 4.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + c*x^4),x)
```

```
[Out] (e^2*x)/c - 2*atanh((8*c^3*d^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*
(-a^3*c^5)^(1/2))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2
*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)
^(1/2))/a^2 + 4*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3
 - (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c))
 + (8*a^2*c*e^4*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*(-a^3*c^5)^(1/2
))/(16*a^3*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2*e^2*(-a^3*c^5)
^(1/2))/(8*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)^(1/2))/a^2 + 4
*c^2*d^5*e + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 - (14*d^2*e^4*
(-a^3*c^5)^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)) - (48*a*c^2*d^
2*e^2*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a*c) + (d^4*(-a^3*c^5)^(1/2))/(16*a^3
*c^3) + (e^4*(-a^3*c^5)^(1/2))/(16*a*c^5) - (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8
*a^2*c^4))^(1/2))/(4*a^2*d*e^5 - (2*d^6*(-a^3*c^5)^(1/2))/a^2 + 4*c^2*d^5*e
 + (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 - (14*d^2*e^4*(-a^3*c^5)
^(1/2))/c^2 + (14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)))*((a^2*e^4*(-a^3*c^5)^(1
/2) + c^2*d^4*(-a^3*c^5)^(1/2) - 4*a^2*c^4*d^3*e + 4*a^3*c^3*d*e^3 - 6*a*c*
d^2*e^2*(-a^3*c^5)^(1/2))/(16*a^3*c^5))^(1/2) - 2*atanh((8*c^3*d^4*x*((d*e^
3)/(4*c^2) - (d^3*e)/(4*a*c) - (d^4*(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (e^4*(
-a^3*c^5)^(1/2))/(16*a*c^5) + (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/
2))/((2*d^6*(-a^3*c^5)^(1/2))/a^2 + 4*a^2*d*e^5 + 4*c^2*d^5*e - (2*a*e^6*(-
```

```
a^3*c^5)^(1/2))/c^3 - 24*a*c*d^3*e^3 + (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 -
(14*d^4*e^2*(-a^3*c^5)^(1/2))/(a*c)) + (8*a^2*c*e^4*x*((d*e^3)/(4*c^2) - (d
^3*e)/(4*a*c) - (d^4*(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (e^4*(-a^3*c^5)^(1/2)
)/(16*a*c^5) + (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/((2*d^6*(-a
^3*c^5)^(1/2))/a^2 + 4*a^2*d*e^5 + 4*c^2*d^5*e - (2*a*e^6*(-a^3*c^5)^(1/2))
/c^3 - 24*a*c*d^3*e^3 + (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 - (14*d^4*e^2*(-a
^3*c^5)^(1/2))/(a*c)) - (48*a*c^2*d^2*e^2*x*((d*e^3)/(4*c^2) - (d^3*e)/(4*a
*c) - (d^4*(-a^3*c^5)^(1/2))/(16*a^3*c^3) - (e^4*(-a^3*c^5)^(1/2))/(16*a*c^
5) + (3*d^2*e^2*(-a^3*c^5)^(1/2))/(8*a^2*c^4))^(1/2))/((2*d^6*(-a^3*c^5)^(1
/2))/a^2 + 4*a^2*d*e^5 + 4*c^2*d^5*e - (2*a*e^6*(-a^3*c^5)^(1/2))/c^3 - 24*
a*c*d^3*e^3 + (14*d^2*e^4*(-a^3*c^5)^(1/2))/c^2 - (14*d^4*e^2*(-a^3*c^5)^(1
/2))/(a*c)))*(-(a^2*e^4*(-a^3*c^5)^(1/2) + c^2*d^4*(-a^3*c^5)^(1/2) + 4*a^2
*c^4*d^3*e - 4*a^3*c^3*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^5)^(1/2))/(16*a^3*c^5)
)^(1/2)
```

3.140 $\int \frac{d+ex^2}{a+cx^4}\,dx$

Optimal. Leaf size=247

$$-\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{\right.}}{4\sqrt{}}$$

```
[Out] -1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/
2))/a^(3/4)/c^(3/4)*2^(1/2)+1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^
(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arctan(-1+c^(1/4)
*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2)+1/4*arcta
n(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/c^(3/4)*2^(1/2
)
```

Rubi [A]
time = 0.10, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}+\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + c*x^4),x]
```

```
[Out] -1/2*((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(Sqr
t[2]*a^(3/4)*c^(3/4)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*c^(3/4)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/
4)) + ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rubi steps

$$\int \frac{d+ex^2}{a+cx^4}\,dx = \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c}$$

$$= \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c} - \frac{\left(\sqrt{c}\,d-\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{\cdots}\right.}{4\sqrt{2}\,a^{3/4}c^{3/4}}$$

$$= -\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}} + \frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}c^{3/4}} - \frac{\left(\sqrt{c}\cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.03, size = 183, normalized size = 0.74

$$\frac{-2(\sqrt{c}\,d+\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2(\sqrt{c}\,d+\sqrt{a}\,e)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-(\sqrt{c}\,d-\sqrt{a}\,e)\left(\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)-\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)\right)}{4\sqrt{2}\,a^{3/4}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + c*x^4),x]
```

```
[Out] (-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d -
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]))/(4*Sqrt[2]*a^(3/4)*c^(3
/4))
```

Maple [A]
time = 0.13, size = 206, normalized size = 0.83

method	result
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\left(\_R^2 e+d\right)\ln\left(x-\_R\right)}{\_R^3}}{4c}$
default	$\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{e\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}}{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^
2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*a
rctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*e/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^
(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))
```

Maxima [A]
time = 0.50, size = 225, normalized size = 0.91

$$\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8\,a^{\frac{3}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

[Out] 1/4*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(
2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*s
qrt(c)) + 1/4*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)
*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(c))*sqrt(c)) + 1/8*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 + s
qrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - 1/8*sqrt(2)*(sqrt(c
)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^
(3/4)*c^(3/4))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(170) = 340$.
time = 0.35, size = 739, normalized size = 2.99

$$-\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}+2de}{ac}}\log\left(-c^2d^4x+a^2xe^4+\left(a^3c^2\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}e+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}+2de}{ac}}\right)+\frac{1}{4}\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}+2de}{ac}}\log\left(-c^2d^4x+a^2xe^4-\left(a^3c^2\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}e+ac^2d^3-a^2cde^2\right)\sqrt{-\frac{ac\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}+2de}{ac}}\right)+\frac{1}{4}\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}-2de}{ac}}\log\left(-c^2d^4x+a^2xe^4+\left(a^3c^2\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}e-ac^2d^3+a^2cde^2\right)\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}-2de}{ac}}\right)-\frac{1}{4}\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}-2de}{ac}}\log\left(-c^2d^4x+a^2xe^4-\left(a^3c^2\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}e-ac^2d^3+a^2cde^2\right)\sqrt{\frac{ac\sqrt{-\frac{c^2d^4-2acd^2e^2+a^2e^4}{a^3c^3}}-2de}{ac}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out] -1/4*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*
e)/(a*c))*log(-c^2*d^4*x + a^2*x*e^4 + (a^3*c^2*sqrt(-(c^2*d^4 - 2*a*c*d^2*
e^2 + a^2*e^4)/(a^3*c^3))*e + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^
2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt(-(a
*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))*log
(-c^2*d^4*x + a^2*x*e^4 - (a^3*c^2*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4
)/(a^3*c^3))*e + a*c^2*d^3 - a^2*c*d*e^2)*sqrt(-(a*c*sqrt(-(c^2*d^4 - 2*a*c
*d^2*e^2 + a^2*e^4)/(a^3*c^3)) + 2*d*e)/(a*c))) + 1/4*sqrt((a*c*sqrt(-(c^2*
d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-c^2*d^4*x +
a^2*x*e^4 + (a^3*c^2*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3))*e
 - a*c^2*d^3 + a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*
e^4)/(a^3*c^3)) - 2*d*e)/(a*c))) - 1/4*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^3*c^3)) - 2*d*e)/(a*c))*log(-c^2*d^4*x + a^2*x*e^4 - (a^
3*c^2*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3))*e - a*c^2*d^3 +
a^2*c*d*e^2)*sqrt((a*c*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^3*c^3))
 - 2*d*e)/(a*c)))

Sympy [A]
time = 0.35, size = 109, normalized size = 0.44

$$\operatorname{RootSum}\left(256t^4a^3c^3+64t^2a^2c^2de+a^2e^4+2acd^2e^2+c^2d^4,\left(t\mapsto t\log\left(x+\frac{64t^3a^3c^2e+12ta^2cde^2-4tac^2d^3}{a^2e^4-c^2d^4}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)/(c*x**4+a),x)

[Out] RootSum(256*_t**4*a**3*c**3 + 64*_t**2*a**2*c**2*d*e + a**2*e**4 + 2*a*c*d*
*2*e**2 + c**2*d**4, Lambda(_t, _t*log(x + (64*_t**3*a**3*c**2*e + 12*_t*a*
*2*c*d*e**2 - 4*_t*a*c**2*d**3)/(a**2*e**4 - c**2*d**4))))

Giac [A]
time = 7.19, size = 245, normalized size = 0.99

$$\frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3}+\frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d+(ac^3)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4ac^3}+\frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac^3}-\frac{\sqrt{2}\left((ac^3)^{\frac{1}{4}}c^2d-(ac^3)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8ac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x
 + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c^3) + 1/4*sqrt(2)*((a*c^3)^(1/4)*c
^2*d + (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(a*c^3) + 1/8*sqrt(2)*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(
x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3) - 1/8*sqrt(2)*((a*c^3)^(1/
4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
*c^3)
```

Mupad [B]
time = 4.68, size = 599, normalized size = 2.43

$$-2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}-\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}-\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}-\frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}-\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3+\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}-\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}\right)\,\sqrt{-\frac{c\,d^2\,\sqrt{-a^3\,c^3}-a\,e^2\,\sqrt{-a^3\,c^3}+2\,a^2\,c^2\,d\,e}{16\,a^3\,c^3}}-2\,\mathrm{atanh}\left(\frac{8\,c^3\,d^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}-\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}+\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}-\frac{8\,a\,c^2\,e^2\,x\,\sqrt{\frac{d^2\,\sqrt{-a^3\,c^3}}{16\,a^3\,c^2}-\frac{d\,e}{8\,a\,c}-\frac{e^2\,\sqrt{-a^3\,c^3}}{16\,a^2\,c^3}}}{2\,c^2\,d^2\,e-2\,a\,c\,e^3-\frac{2\,c\,d^3\,\sqrt{-a^3\,c^3}}{a^2}+\frac{2\,d\,e^2\,\sqrt{-a^3\,c^3}}{a}}\right)\,\sqrt{-\frac{a\,e^2\,\sqrt{-a^3\,c^3}-c\,d^2\,\sqrt{-a^3\,c^3}+2\,a^2\,c^2\,d\,e}{16\,a^3\,c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + c*x^4),x)
```

```
[Out] - 2*atanh((8*c^3*d^2*x*((e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3) - (d^2*(-a^3*c^
3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 + (
2*c*d^3*(-a^3*c^3)^(1/2))/a^2 - (2*d*e^2*(-a^3*c^3)^(1/2))/a) - (8*a*c^2*e^
2*x*((e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3) - (d^2*(-a^3*c^3)^(1/2))/(16*a^3*c
^2) - (d*e)/(8*a*c))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 + (2*c*d^3*(-a^3*c^3)^
(1/2))/a^2 - (2*d*e^2*(-a^3*c^3)^(1/2))/a))*(-(c*d^2*(-a^3*c^3)^(1/2) - a*e
^2*(-a^3*c^3)^(1/2) + 2*a^2*c^2*d*e)/(16*a^3*c^3))^(1/2) - 2*atanh((8*c^3*d
^2*x*((d^2*(-a^3*c^3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c) - (e^2*(-a^3*c^3)
^(1/2))/(16*a^2*c^3))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 - (2*c*d^3*(-a^3*c^3)
^(1/2))/a^2 + (2*d*e^2*(-a^3*c^3)^(1/2))/a) - (8*a*c^2*e^2*x*((d^2*(-a^3*c^
3)^(1/2))/(16*a^3*c^2) - (d*e)/(8*a*c) - (e^2*(-a^3*c^3)^(1/2))/(16*a^2*c^3
))^(1/2))/(2*c^2*d^2*e - 2*a*c*e^3 - (2*c*d^3*(-a^3*c^3)^(1/2))/a^2 + (2*d*
e^2*(-a^3*c^3)^(1/2))/a))*(-(a*e^2*(-a^3*c^3)^(1/2) - c*d^2*(-a^3*c^3)^(1/2
) + 2*a^2*c^2*d*e)/(16*a^3*c^3))^(1/2)
```

3.141 $\int \frac{1}{a+cx^4} \, dx$

Optimal. Leaf size=185

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{a}+\sqrt{2}\right.}{4\sqrt{2}}$$

```
[Out] 1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)+1/4*arctan
(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/c^(1/4)*2^(1/2)-1/8*ln(-a^(1/4)*c^(1/
4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)+1/8*ln(a^(1/4)*c^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(3/4)/c^(1/4)*2^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-1),x]
```

```
[Out] -1/2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(Sqrt[2]*a^(3/4)*c^(1/4)) + Ar
cTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)]/(2*Sqrt[2]*a^(3/4)*c^(1/4)) - Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4)*c^(1/4))
 + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]/(4*Sqrt[2]*a^(3/4
)*c^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{a+cx^4}\,dx = \frac{\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{a}} + \frac{\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{2\sqrt{a}}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\,\sqrt{c}} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4\sqrt{a}\,\sqrt{c}} - \frac{\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} - \frac{\int \frac{\cdots}{-\frac{\sqrt{a}}{\sqrt{c}}\cdots}}{4\cdots}$$

$$= -\frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}} + \frac{\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{c}} - \frac{\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

Mathematica [A]
time = 0.01, size = 134, normalized size = 0.72

$$\frac{-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)] - Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(
1/4))
```

Maple [A]
time = 0.13, size = 102, normalized size = 0.55

method	result	size
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}}{4c}$	27
default	$\frac{\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x-1))
```

Maxima [A]
time = 0.51, size = 169, normalized size = 0.91

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{4\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{8a^{\frac{3}{4}}c^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt
(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/4*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
```

```
rt(a)*sqrt(sqrt(a)*sqrt(c))) + 1/8*sqrt(2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4
)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - 1/8*sqrt(2)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4))
```

Fricas [A]
time = 0.34, size = 121, normalized size = 0.65

$$\left(-\frac{1}{a^{3}c}\right)^{\frac{1}{4}}\arctan\left(-a^{2}cx\left(-\frac{1}{a^{3}c}\right)^{\frac{3}{4}}+\sqrt{a^{2}\sqrt{-\frac{1}{a^{3}c}}+x^{2}}\,a^{2}c\left(-\frac{1}{a^{3}c}\right)^{\frac{3}{4}}\right)+\frac{1}{4}\left(-\frac{1}{a^{3}c}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^{3}c}\right)^{\frac{1}{4}}+x\right)-\frac{1}{4}\left(-\frac{1}{a^{3}c}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^{3}c}\right)^{\frac{1}{4}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] (-1/(a^3*c))^(1/4)*arctan(-a^2*c*x*(-1/(a^3*c))^(3/4) + sqrt(a^2*sqrt(-1/(a
^3*c)) + x^2)*a^2*c*(-1/(a^3*c))^(3/4)) + 1/4*(-1/(a^3*c))^(1/4)*log(a*(-1/
(a^3*c))^(1/4) + x) - 1/4*(-1/(a^3*c))^(1/4)*log(-a*(-1/(a^3*c))^(1/4) + x)
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.11

$$\operatorname{RootSum}\left(256t^{4}a^{3}c+1,\left(t\mapsto t\log\left(4ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*c + 1, Lambda(_t, _t*log(4*_t*a + x)))
```

Giac [A]
time = 5.94, size = 179, normalized size = 0.97

$$\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac}+\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{4\,ac}+\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}-\frac{\sqrt{2}\left(ac^{3}\right)^{\frac{1}{4}}\log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a
/c)^(1/4))/(a*c) + 1/4*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x - sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a*c) + 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2 + s
qrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c) - 1/8*sqrt(2)*(a*c^3)^(1/4)*log(x^2
 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c)
```

Mupad [B]
time = 4.41, size = 33, normalized size = 0.18

$$-\frac{\operatorname{atan}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)+\operatorname{atanh}\left(\frac{c^{1/4}\,x}{{\left(-a\right)}^{1/4}}\right)}{2\,{\left(-a\right)}^{3/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4),x)
```

```
[Out] -(atan((c^(1/4)*x)/(-a)^(1/4)) + atanh((c^(1/4)*x)/(-a)^(1/4)))/(2*(-a)^(3/
4)*c^(1/4))
```

3.142 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)}-\frac{\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\,(cd^2+ae^2)}+\frac{\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{2\sqrt{2}\,a^{3/4}\,(cd^2+ae^2)}$$

```
[Out] 1/4*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(
3/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*
(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*c^(1/4)*ln(-a^(1/4
)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e
^2+c*d^2)*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1
/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)+e^(3/2)*arctan(x*e
^(1/2)/d^(1/2))/(a*e^2+c*d^2)/d^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1185, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,a^{3/4}\,(ae^2+cd^2)}+\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,a^{3/4}\,(ae^2+cd^2)}-\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\,(ae^2+cd^2)}+\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\,(ae^2+cd^2)}+\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx = \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)}\right)dx$$

$$= \frac{c\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\,\sqrt{c}}{a+c}}{2(cd^2+ae^2)}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\int}{4(c}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}\,(\sqrt{c}\,d+\sqrt{a}\,e)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(cd^2+ae^2)}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}\,(\sqrt{c}\,d-\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}\,(\sqrt{c}}{}$$

Mathematica [A]
time = 0.10, size = 234, normalized size = 0.70

$$\frac{8a^{3/4}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{2}\,\sqrt[4]{c}\,\sqrt{d}\left(\left(-2\sqrt{c}\,d+2\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\left(\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)-\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)\right)\right)}{8a^{3/4}\sqrt{d}\,(cd^2+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((
-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d]*(c*
d^2 + a*e^2))
```

Maple [A]
time = 0.19, size = 253, normalized size = 0.75

method	result

default	$\frac{e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(ae^2+cd^2)\sqrt{de}}+\frac{c\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\left(\ln\left(\cdots\right.\right.}{\cdots}\right)}{ae^2+cd^2}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^5e^4+2ce^2a^4d^2+a^3c^2d^4\right)\_Z^4-4a^2cde\_Z^2+c\right)}\_R\ln\left(\left(\left(-2a^5e^7-2a^4cd^2e^5+2a^3c^2d^4e^3+2a^2c^3d^6e\right)\_R^4+\left(15a^2cde^4\cdots\right.\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+c/(a*e^2+c*d^2)*(1/8*
d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a
/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arcta
n(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4
)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(
2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.53, size = 266, normalized size = 0.79

$$\frac{c\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\,(cd^2+ae^2)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}}{(cd^2+ae^2)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*c*(2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
c))*sqrt(c)) + 2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt
(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(c))*sqrt(c)) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d +
 sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)
*c^(3/4)))/(c*d^2 + a*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*d^2 + a*
e^2)*sqrt(d))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1904 vs. 2(244) = 488.
time = 0.76, size = 3842, normalized size = 11.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a
^2*c*d^2*e^2 + a^3*e^4))*log(-c^2*d^2*x + a*c*x*e^2 + (a*c^2*d^3 - a^2*c*d*
e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*
d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2
+ a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^
4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d
^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^
4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
 a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-c^2*d^2*x + a*c*x
*e^2 - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^
5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (
a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a
^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^
2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e
^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4
- 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2
*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4))*log(-c^2*d^2*x + a*c*x*e^2 + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*
e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4
)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c
^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*
a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-c^2*d^2*x + a*c*x*e^2 - (a*c^2*d^3 -
 a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - 2*sqrt(-e/d)*e*log((x^2*e + 2*d
*x*sqrt(-e/d) - d)/(x^2*e + d)))/(c*d^2 + a*e^2), -1/4*((c*d^2 + a*e^2)*sqr
t((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c
^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^
```

```
4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*l
og(-c^2*d^2*x + a*c*x*e^2 + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a
^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*
c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4
- 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2
*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^
3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2
*a^2*c*d^2*e^2 + a^3*e^4))*log(-c^2*d^2*x + a*c*x*e^2 - (a*c^2*d^3 - a^2*c*
d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^
2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4
 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2
*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*
e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6
 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-c^2*d^2*x + a*c
*x*e^2 + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*
e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3
*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e -
 (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.69, size = 339, normalized size = 1.01

$$\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^3d^2+\sqrt{2}\,a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^3d^2+\sqrt{2}\,a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}\,ac^3d^2+\sqrt{2}\,a^2c^2e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}\,ac^3d^2+\sqrt{2}\,a^2c^2e^2\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}}{\left(cd^2+ae^2\right)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/
2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*
((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^
2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2
)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*
d^2 + a*e^2)*sqrt(d))
```

Mupad [B]
time = 5.71, size = 2500, normalized size = 7.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^
5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-
a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
 c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d
^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^
5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3
*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*
((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
 a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*
(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)
))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) +
2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4
*c^4*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(
16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512
*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^
6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32
*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c
)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2
) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^
(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*
1i)/((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5
*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a
^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
```

```
c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5
 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*
c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*(
(a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((a
*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^
3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^
3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(
1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^
2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4
*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2
*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^
2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c
^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) +
20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2
) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*2i + atan(((((c*d^2*(-a^3*c)^(1/2) -
a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(2
56*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*
d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9
 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*
c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^
2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(
-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))
)^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^
(1/2)*1i - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(1
6*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d
^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*
c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1
/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d
^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 44
8*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c...
```

3.143 $\int \frac{1}{(d+ex^2)^2(a+cx^4)}\,dx$

Optimal. Leaf size=453

$$\frac{e^2x}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{2c\sqrt{d}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)}-\frac{c^{3/4}\left(cd^2-2\sqrt{a}\,\sqrt{c}\,de-ae^2\right)\tan}{2\sqrt{2}\,a^{3/4}\left(cd^2+\right.}$$

```
[Out] 1/2*e^2*x/d/(a*e^2+c*d^2)/(e*x^2+d)+1/2*e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/d
^(3/2)/(a*e^2+c*d^2)+1/4*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*d^
2-a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*c^(3/4)*
arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(c*d^2-a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(
3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*c^(1/2))*(c*d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^
2*2^(1/2)+1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(c*
d^2-a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+2*c*e^(3/2
)*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(a*e^2+c*d^2)^2
```

Rubi [A]
time = 0.25, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1185, 205, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{c^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(-2\sqrt{a}\,\sqrt{c}\,de-ae^2+cd^2\right)}{2\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(-2\sqrt{a}\,\sqrt{c}\,de-ae^2+cd^2\right)}{2\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(2\sqrt{a}\,\sqrt{c}\,de-ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{c^{3/4}\left(2\sqrt{a}\,\sqrt{c}\,de-ae^2+cd^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{2c\sqrt{d}\,e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(ae^2+cd^2\right)^2}+\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\left(ae^2+cd^2\right)}+\frac{e^2x}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

```
[Out] (e^2*x)/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (2*c*Sqrt[d]*e^(3/2)*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 + a*e^2)) - (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a
*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 +
a*e^2)^2) + (c^(3/4)*(c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)
*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*(c*d
^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x
 + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
```

```
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2(a+cx^4)}\,dx = \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)^2} + \frac{2cde^2}{(cd^2+ae^2)^2(d+ex^2)} + \frac{c(cd^2-ae^2-2cdex^2)}{(cd^2+ae^2)^2(a+cx^4)}\right.$$

$$= \frac{c\int \frac{cd^2-ae^2-2cdex^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2} + \frac{(2cde^2)\int \frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2} + \frac{e^2\int \frac{1}{(d+ex^2)^2}\,dx}{cd^2+ae^2}$$

$$= \frac{e^2x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\left(\sqrt{c}\left(cd^2-2\sqrt{a}\sqrt{c}\right.\right.}{2\sqrt{a}}$$

$$= \frac{e^2x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} +$$

$$= \frac{e^2x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} -$$

$$= \frac{e^2x}{2d(cd^2+ae^2)(d+ex^2)} + \frac{2c\sqrt{d}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}(cd^2+ae^2)} -$$

Mathematica [A]
time = 0.30, size = 362, normalized size = 0.80

$$\frac{\frac{4e^2(cd^2+ae^2)x}{d(d+ex^2)} + \frac{4e^{3/2}(5cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2\sqrt{2}\,c^{3/4}\left(-cd^2+2\sqrt{a}\sqrt{c}\,de+ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}} - \frac{2\sqrt{2}\,c^{3/4}\left(-cd^2+2\sqrt{a}\sqrt{c}\,de+ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \frac{\sqrt{2}\,c^{3/4}\left(-cd^2-2\sqrt{a}\sqrt{c}\,de+ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{3/4}} + \frac{\sqrt{2}\,c^{3/4}\left(cd^2+2\sqrt{a}\sqrt{c}\,de-ae^2\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{3/4}}}{8(cd^2+ae^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + c*x^4)),x]
```

```
[Out] ((4*e^2*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (4*e^(3/2)*(5*c*d^2 + a*e^2)*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sqrt
```

```
[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) -
 (2*Sqrt[2]*c^(3/4)*(-(c*d^2) + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(3/4) + (Sqrt[2]*c^(3/4)*(-(c*d^2) - 2*Sqrt[
a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/a^(3/4) + (Sqrt[2]*c^(3/4)*(c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(3/4))/(8*(c*d^2 + a
*e^2)^2)
```

Maple [A]
time = 0.19, size = 307, normalized size = 0.68

method	result
default	$\frac{e^{2}\left(\frac{\left(a\,e^{2}+c\,d^{2}\right)x}{2d\left(e\,x^{2}+d\right)}+\frac{\left(a\,e^{2}+5c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{2}}-\frac{c\left(\frac{\left(a\,e^{2}-c\,d^{2}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1 \right.\right.}{8a}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/(a*e^2+c*d^2)^2*(1/2*(a*e^2+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+5*c*d^2)/d/
(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-c/(a*e^2+c*d^2)^2*(1/8*(a*e^2-c*d^2)*(
a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)
^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2
^(1/2)/(a/c)^(1/4)*x-1))+1/4*d*e/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x
*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.52, size = 394, normalized size = 0.87

$$\frac{\left(5\,cd^{2}e^{2}+ae^{4}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\left(c^{2}d^{5}+2\,acd^{3}e^{2}+a^{2}de^{4}\right)\sqrt{d}}+\frac{c\left(\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^{2}-2\sqrt{a}\,cde-a\sqrt{c}\,e^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(c^{\frac{3}{2}}d^{2}-2\sqrt{a}\,cde-a\sqrt{c}\,e^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(c^{\frac{3}{2}}d^{2}+2\sqrt{a}\,cde-a\sqrt{c}\,e^{2}\right)\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(c^{\frac{3}{2}}d^{2}+2\sqrt{a}\,cde-a\sqrt{c}\,e^{2}\right)\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\left(c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4}\right)}+\frac{xe^{2}}{2\left(cd^{4}+ad^{2}e^{2}+\left(cd^{3}e+ade^{3}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*(5*c*d^2*e^2 + a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^2*d^5 + 2*
a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) + 1/8*c*(2*sqrt(2)*(c^(3/2)*d^2 - 2*sqrt(
a)*c*d*e - a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)
*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) +
2*sqrt(2)*(c^(3/2)*d^2 - 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*arctan(1/2*sqrt(2
)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*s
```

```
qrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(c^(3/2)*d^2 + 2*sqrt(a)*c*d*e - a*
sqrt(c)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4
)*c^(3/4)) - sqrt(2)*(c^(3/2)*d^2 + 2*sqrt(a)*c*d*e - a*sqrt(c)*e^2)*log(sq
rt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c^2*d^
4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*x*e^2/(c*d^4 + a*d^2*e^2 + (c*d^3*e + a*
d*e^3)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3951 vs. $2(341) = 682$.
time = 9.59, size = 7933, normalized size = 17.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*c*d^2*x*e^2 + 2*a*x*e^4 + (c^2*d^5*x^2*e + c^2*d^6 + 2*a*c*d^3*x^2*
e^3 + 2*a*c*d^4*e^2 + a^2*d*x^2*e^5 + a^2*d^2*e^4)*sqrt((4*c^3*d^3*e - 4*a*
c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^
2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 1
2*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^
5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*
e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 +
4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log(c^4
*d^4*x - 6*a*c^3*d^2*x*e^2 + a^2*c^2*x*e^4 + (a*c^4*d^6 - 7*a^2*c^3*d^4*e^2
 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3 + 6
*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8 - 12*a*c^6*d
^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^
16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^
7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^
14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^
3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 -
 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/
(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*
e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^
10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4
*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) - (c^2*d^5*x^2*e + c^2*d^6 + 2*a*c*d^3*
x^2*e^3 + 2*a*c*d^4*e^2 + a^2*d*x^2*e^5 + a^2*d^2*e^4)*sqrt((4*c^3*d^3*e -
4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*
c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4
 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 2
8*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*
d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^
8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))*log
(c^4*d^4*x - 6*a*c^3*d^2*x*e^2 + a^2*c^2*x*e^4 - (a*c^4*d^6 - 7*a^2*c^3*d^4
```

```
*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 + 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7*e^3
 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8 - 12*a*c
^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^
8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 7
0*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^
2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 + (a*c^4*d^8 + 4*a^
2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d
^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e
^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d
^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 +
8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2
*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) + (c^2*d^5*x^2*e + c^2*d^6 + 2*a*c*
d^3*x^2*e^3 + 2*a*c*d^4*e^2 + a^2*d*x^2*e^5 + a^2*d^2*e^4)*sqrt((4*c^3*d^3*
e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*
a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4
*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2
 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*
c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^
4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))
*log(c^4*d^4*x - 6*a*c^3*d^2*x*e^2 + a^2*c^2*x*e^4 + (a*c^4*d^6 - 7*a^2*c^3
*d^4*e^2 + 7*a^3*c^2*d^2*e^4 - a^4*c*e^6 - 2*(a^3*c^4*d^9*e + 4*a^4*c^3*d^7
*e^3 + 6*a^5*c^2*d^5*e^5 + 4*a^6*c*d^3*e^7 + a^7*d*e^9)*sqrt(-(c^7*d^8 - 12
*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^
3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6
 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10*
c*d^2*e^14 + a^11*e^16)))*sqrt((4*c^3*d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 +
4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c
^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c
^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c
^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^1
2 + 8*a^10*c*d^2*e^14 + a^11*e^16)))/(a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3
*c^2*d^4*e^4 + 4*a^4*c*d^2*e^6 + a^5*e^8))) - (c^2*d^5*x^2*e + c^2*d^6 + 2*
a*c*d^3*x^2*e^3 + 2*a*c*d^4*e^2 + a^2*d*x^2*e^5 + a^2*d^2*e^4)*sqrt((4*c^3*
d^3*e - 4*a*c^2*d*e^3 - (a*c^4*d^8 + 4*a^2*c^3*d^6*e^2 + 6*a^3*c^2*d^4*e^4
+ 4*a^4*c*d^2*e^6 + a^5*e^8)*sqrt(-(c^7*d^8 - 12*a*c^6*d^6*e^2 + 38*a^2*c^5
*d^4*e^4 - 12*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)/(a^3*c^8*d^16 + 8*a^4*c^7*d^14
*e^2 + 28*a^5*c^6*d^12*e^4 + 56*a^6*c^5*d^10*e^6 + 70*a^7*c^4*d^8*e^8 + 56*
a^8*c^3*d^6*e^10 + 28*a^9*c^2*d^4*e^12 + 8*a^10...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.61, size = 517, normalized size = 1.14

$$\frac{(5\,cd^2e^2+ae^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,(c^2d^5+2\,acd^3e^2+a^2de^4)\sqrt{d}}+\frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{1}{4}}\,ace^2-2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^3d^4+2\,\sqrt{2}\,a^2c^2d^2e^2+\sqrt{2}\,a^3ce^4\right)}+\frac{\left((ac^3)^{\frac{1}{4}}\,c^2d^2-(ac^3)^{\frac{1}{4}}\,ace^2-2\,(ac^3)^{\frac{3}{4}}\,de\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ac^3d^4+2\,\sqrt{2}\,a^2c^2d^2e^2+\sqrt{2}\,a^3ce^4\right)}+\frac{\left(\sqrt{2}\,(ac^3)^{\frac{1}{4}}\,c^2d^2-\sqrt{2}\,(ac^3)^{\frac{1}{4}}\,ace^2+2\,\sqrt{2}\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4)}-\frac{\left(\sqrt{2}\,(ac^3)^{\frac{1}{4}}\,c^2d^2-\sqrt{2}\,(ac^3)^{\frac{1}{4}}\,ace^2+2\,\sqrt{2}\,(ac^3)^{\frac{3}{4}}\,de\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{8\,(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4)}+\frac{xe^2}{2\,(cd^3+ade^2)(x^2e+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 + a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^2*d^5 + 2*
a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1
/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^
(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2
)*a^3*c*e^4) + 1/2*((a*c^3)^(1/4)*c^2*d^2 - (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^
3)^(3/4)*d*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(
sqrt(2)*a*c^3*d^4 + 2*sqrt(2)*a^2*c^2*d^2*e^2 + sqrt(2)*a^3*c*e^4) + 1/8*(s
qrt(2)*(a*c^3)^(1/4)*c^2*d^2 - sqrt(2)*(a*c^3)^(1/4)*a*c*e^2 + 2*sqrt(2)*(a
*c^3)^(3/4)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a*c^3*d^4 +
2*a^2*c^2*d^2*e^2 + a^3*c*e^4) - 1/8*(sqrt(2)*(a*c^3)^(1/4)*c^2*d^2 - sqrt(
2)*(a*c^3)^(1/4)*a*c*e^2 + 2*sqrt(2)*(a*c^3)^(3/4)*d*e)*log(x^2 - sqrt(2)*x
*(a/c)^(1/4) + sqrt(a/c))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4) + 1/2
*x*e^2/((c*d^3 + a*d*e^2)*(x^2*e + d))
```

Mupad [B]
time = 6.55, size = 2500, normalized size = 5.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x^2)^2),x)
```

```
[Out] (e^2*x)/(2*d*(d + e*x^2)*(a*e^2 + c*d^2)) - atan((((((((256*a^8*c^4*d*e^16 -
 128*a*c^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960
*a^4*c^8*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^
7*c^5*d^3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*
e^6 + 6*a^2*c^2*d^6*e^4)) + (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c
^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1
/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a
^5*c^2*d^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4
608*a^4*c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608
*a^7*c^6*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^1
0 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*(
(a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*
```

```
a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8
+ 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(32
*a^6*c^5*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*
e^6 + 2208*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))
/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^
6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d
^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3
*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)
 + (480*a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*
d^4*e^9 + 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2
 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*
d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-
a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11
*a*c^8*d^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*
e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^
2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*
(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2)*1i - (((((256*a^8*c^4*d*e^16 - 128*a*c
^11*d^15*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960*a^4*c^8
*d^9*e^8 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^7*c^5*d^
3*e^14)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*
a^2*c^2*d^6*e^4)) - (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2
) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16
*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d
^4*e^4)))^(1/2)*(512*a^2*c^11*d^16*e^3 + 2560*a^3*c^10*d^14*e^5 + 4608*a^4*
c^9*d^12*e^7 + 2560*a^5*c^8*d^10*e^9 - 2560*a^6*c^7*d^8*e^11 - 4608*a^7*c^6
*d^6*e^13 - 2560*a^8*c^5*d^4*e^15 - 512*a^9*c^4*d^2*e^17))/(c^4*d^10 + a^4*
d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4
*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*
d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*
c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(32*a^6*c^5
*d*e^14 - 48*a*c^10*d^11*e^4 - 16*c^11*d^13*e^2 + 1024*a^2*c^9*d^9*e^6 + 22
08*a^3*c^8*d^7*e^8 + 1264*a^4*c^7*d^5*e^10 + 144*a^5*c^6*d^3*e^12))/(c^4*d^
10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*
((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4
*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8
 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4)))^(1/2) + (480*
a^2*c^8*d^6*e^7 - 200*a*c^9*d^8*e^5 - 8*a^5*c^5*e^13 + 784*a^3*c^7*d^4*e^9
+ 96*a^4*c^6*d^2*e^11)/(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3
*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4)))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^
3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)
^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4)))^(1/2) - (x*(a^3*c^6*e^11 - 27*c^9*d^6*e^5 + 11*a*c^8*d
^4*e^7 + 7*a^2*c^7*d^2*e^9))/(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*
```

```
a^3*c*d^4*e^6 + 6*a^2*c^2*d^6*e^4))*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-
a^3*c^3)^(1/2) + 4*a^2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^
3)^(1/2))/(16*(a^7*e^8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d^6*e^2
+ 6*a^5*c^2*d^4*e^4)))^(1/2)*1i)/((((((256*a^8*c^4*d*e^16 - 128*a*c^11*d^15
*e^2 + 256*a^2*c^10*d^13*e^4 + 3456*a^3*c^9*d^11*e^6 + 8960*a^4*c^8*d^9*e^8
 + 10880*a^5*c^7*d^7*e^10 + 6912*a^6*c^6*d^5*e^12 + 2176*a^7*c^5*d^3*e^14)/
(2*(c^4*d^10 + a^4*d^2*e^8 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 + 6*a^2*c^2*
d^6*e^4)) + (x*((a^2*e^4*(-a^3*c^3)^(1/2) + c^2*d^4*(-a^3*c^3)^(1/2) + 4*a^
2*c^3*d^3*e - 4*a^3*c^2*d*e^3 - 6*a*c*d^2*e^2*(-a^3*c^3)^(1/2))/(16*(a^7*e^
8 + a^3*c^4*d^8 + 4*a^6*c*d^2*e^6 + 4*a^4*c^3*d...
```

3.144 $\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=363

$$-\frac{e^3x^3}{c\left(a+cx^4\right)}+\frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)\,x^2)}{4ac\left(a+cx^4\right)}-\frac{3\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\left(cd^2+ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{7/4}}+\text{-}\ 3$$

```
[Out] -e^3*x^3/c/(c*x^4+a)+1/4*x*(d*(-3*a*e^2+c*d^2)+3*e*(a*e^2+c*d^2)*x^2)/a/c/(
c*x^4+a)-3/32*(a*e^2+c*d^2)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/
2))*(-e*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2)+3/32*(a*e^2+c*d^2)*ln(a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(7/4)
/c^(7/4)*2^(1/2)+3/16*(a*e^2+c*d^2)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e
*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2)+3/16*(a*e^2+c*d^2)*arctan(1+c^(
1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/c^(7/4)*2^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1221, 1872, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)}{8\sqrt{2}\,a^{7/4}c^{7/4}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(ae^2+cd^2\right)}{8\sqrt{2}\,a^{7/4}c^{7/4}}-\frac{3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(ae^2+cd^2\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{7/4}}+\frac{3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(ae^2+cd^2\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{7/4}}+\frac{x(3ex^2(ae^2+cd^2)+d(cd^2-3ae^2))}{4ac\left(a+cx^4\right)}-\frac{e^3x^3}{c\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

```
[Out] -((e^3*x^3)/(c*(a + c*x^4))) + (x*(d*(c*d^2 - 3*a*e^2) + 3*e*(c*d^2 + a*e^2
)*x^2))/(4*a*c*(a + c*x^4)) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*Ar
cTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(7/4)) + (3*(Sq
rt[c]*d + Sqrt[a]*e)*(c*d^2 + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(8*Sqrt[2]*a^(7/4)*c^(7/4)) - (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4
)*c^(7/4)) + (3*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(7/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1872

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] + Simp[(-x)*R*((
```

```
a + b*x^n)^(p + 1)/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1))), x]] /; GeQ[q, n
]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{(a+cx^4)^2}\,dx = -\frac{e^3x^3}{c\,(a+cx^4)} - \frac{\int \frac{-cd^3-3e(cd^2+ae^2)x^2-3cde^2x^4}{(a+cx^4)^2}\,dx}{c}$$

$$= -\frac{e^3x^3}{c\,(a+cx^4)} + \frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)\,x^2)}{4ac\,(a+cx^4)} + \frac{\int \frac{3cd(cd^2+ae^2)+3ce(cd^2+ae^2)x^2}{a+cx^4}\,dx}{4ac^2}$$

$$= -\frac{e^3x^3}{c\,(a+cx^4)} + \frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)\,x^2)}{4ac\,(a+cx^4)} + \frac{\left(3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)(cd^2+ae^2)\right)\int}{8a^{3/2}c^2}$$

$$= -\frac{e^3x^3}{c\,(a+cx^4)} + \frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)\,x^2)}{4ac\,(a+cx^4)} - \frac{\left(3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)(cd^2+ae^2)\right)\int}{16\sqrt{2}\,a^{7/4}c^{7}}$$

$$= -\frac{e^3x^3}{c\,(a+cx^4)} + \frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)\,x^2)}{4ac\,(a+cx^4)} - \frac{3\left(\sqrt{c}\,d-\sqrt{a}\,e\right)(cd^2+ae^2)\log\Big(}{16\sqrt{2}\,a}$$

$$= -\frac{e^3x^3}{c\,(a+cx^4)} + \frac{x(d(cd^2-3ae^2)+3e(cd^2+ae^2)\,x^2)}{4ac\,(a+cx^4)} - \frac{3\left(\sqrt{c}\,d+\sqrt{a}\,e\right)(cd^2+ae^2)\tan^{-}}{8\sqrt{2}\,a^{7/4}c^{7/4}}$$

Mathematica [A]

time = 0.16, size = 371, normalized size = 1.02

$$\frac{-\frac{8a^{3/4}c^{3/4}\left(ae^2x(3d+ex^2)-cd^2x(d+3ex^2)\right)}{a+cx^4}-6\sqrt{2}\left(c^{3/2}d^3+\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+a^{3/2}e^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+6\sqrt{2}\left(c^{3/2}d^3+\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+a^{3/2}e^3\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+3\sqrt{2}\left(-c^{3/2}d^3+\sqrt{a}\,cd^2e-a\sqrt{c}\,de^2+a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+3\sqrt{2}\left(c^{3/2}d^3-\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2-a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{32a^{7/4}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*c^(3/4)*(a*e^2*x*(3*d + e*x^2) - c*d^2*x*(d + 3*e*x^2)))/(a +
c*x^4) - 6*Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(3/
2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*Sqrt[2]*(c^(3/2)*d^3 +
Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)] + 3*Sqrt[2]*(-(c^(3/2)*d^3) + Sqrt[a]*c*d^2*e - a*Sqrt[c]*d*e
^2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] +
3*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - a^(3/2)*e^3)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(32*a^(7/4)*c^(7/4))
```

Maple [A]
time = 0.12, size = 284, normalized size = 0.78

method	result
risch	$\frac{-\frac{e\left(a\,e^{2}-3c\,d^{2}\right)x^{3}}{4ac}-\frac{d\left(3a\,e^{2}-c\,d^{2}\right)x}{4ac}}{c\,x^{4}+a}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+a\right)}\frac{\left(e\left(a\,e^{2}+c\,d^{2}\right)\_R^{2}+d\left(a\,e^{2}+c\,d^{2}\right)\right)\ln\left(x-\_R\right)}{\_R^{3}}\right)}{16a\,c^{2}}$
default	$\frac{-\frac{e\left(a\,e^{2}-3c\,d^{2}\right)x^{3}}{4ac}-\frac{d\left(3a\,e^{2}-c\,d^{2}\right)x}{4ac}}{c\,x^{4}+a}+\frac{3(a\,e^{2}+c\,d^{2})\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\right.\right.}{8a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4*e*(a*e^2-3*c*d^2)/a/c*x^3-1/4*d*(3*a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+3/4*
(a*e^2+c*d^2)/a/c*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(
a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*e/c/(a/c)^(1/4)*2^(1
/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+
(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4
)*x-1)))
```

Maxima [A]
time = 0.52, size = 294, normalized size = 0.81

$$\frac{(3\,cd^2e-ae^3)x^3+(cd^3-3\,ade^2)x}{4\,(ac^2x^4+a^2c)}+\frac{3\,(cd^2+ae^2)\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{32\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((3*c*d^2*e - a*e^3)*x^3 + (c*d^3 - 3*a*d*e^2)*x)/(a*c^2*x^4 + a^2*c) +
 3/32*(c*d^2 + a*e^2)*(2*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2*sqrt(2)
*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sq
rt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*arctan(1/2
*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sq
rt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*log(
sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt
(2)*(sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + s
qrt(a))/(a^(3/4)*c^(3/4)))/(a*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2018 vs. $2(278) = 556$.
time = 0.38, size = 2018, normalized size = 5.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(12*c*d^2*x^3*e + 4*c*d^3*x - 4*a*x^3*e^3 - 12*a*d*x*e^2 - 3*(a*c^2*x^
4 + a^2*c)*sqrt(-(2*c^2*d^5*e + a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2
- a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10
+ a^6*e^12)/(a^7*c^7)) + 4*a*c*d^3*e^3 + 2*a^2*d*e^5)/(a^3*c^3))*log(-27*c^
5*d^10*x - 81*a*c^4*d^8*x*e^2 - 54*a^2*c^3*d^6*x*e^4 + 54*a^3*c^2*d^4*x*e^6
 + 81*a^4*c*d^2*x*e^8 + 27*a^5*x*e^10 + 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 +
 a^6*c^5*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d
^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7))*e - a^4*
c^3*d^3*e^4 - a^5*c^2*d*e^6)*sqrt(-(2*c^2*d^5*e + a^3*c^3*sqrt(-(c^6*d^12 +
 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 +
 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)) + 4*a*c*d^3*e^3 + 2*a^2*d*e^5)/(a^
3*c^3))) + 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2*d^5*e + a^3*c^3*sqrt(-(c^6*d^
12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e
^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)) + 4*a*c*d^3*e^3 + 2*a^2*d*e^5)
/(a^3*c^3))*log(-27*c^5*d^10*x - 81*a*c^4*d^8*x*e^2 - 54*a^2*c^3*d^6*x*e^4
+ 54*a^3*c^2*d^4*x*e^6 + 81*a^4*c*d^2*x*e^8 + 27*a^5*x*e^10 - 27*(a^2*c^5*d
^7 + a^3*c^4*d^5*e^2 + a^6*c^5*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4
*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^1
2)/(a^7*c^7))*e - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6)*sqrt(-(2*c^2*d^5*e + a^3
*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e
^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)) + 4*a*c*d^3*
e^3 + 2*a^2*d*e^5)/(a^3*c^3))) - 3*(a*c^2*x^4 + a^2*c)*sqrt(-(2*c^2*d^5*e -
 a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d
^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7)) + 4*a*c*
d^3*e^3 + 2*a^2*d*e^5)/(a^3*c^3))*log(-27*c^5*d^10*x - 81*a*c^4*d^8*x*e^2 -
 54*a^2*c^3*d^6*x*e^4 + 54*a^3*c^2*d^4*x*e^6 + 81*a^4*c*d^2*x*e^8 + 27*a^5*
x*e^10 + 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^6*c^5*sqrt(-(c^6*d^12 + 2*a*
c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^
5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7))*e - a^4*c^3*d^3*e^4 - a^5*c^2*d*e^6)*sq
rt(-(2*c^2*d^5*e - a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8
*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(
a^7*c^7)) + 4*a*c*d^3*e^3 + 2*a^2*d*e^5)/(a^3*c^3))) + 3*(a*c^2*x^4 + a^2*c
)*sqrt(-(2*c^2*d^5*e - a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4
*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^1
2)/(a^7*c^7)) + 4*a*c*d^3*e^3 + 2*a^2*d*e^5)/(a^3*c^3))*log(-27*c^5*d^10*x
- 81*a*c^4*d^8*x*e^2 - 54*a^2*c^3*d^6*x*e^4 + 54*a^3*c^2*d^4*x*e^6 + 81*a^4
```

```
*c*d^2*x*e^8 + 27*a^5*x*e^10 - 27*(a^2*c^5*d^7 + a^3*c^4*d^5*e^2 - a^6*c^5*
sqrt(-(c^6*d^12 + 2*a*c^5*d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 -
a^4*c^2*d^4*e^8 + 2*a^5*c*d^2*e^10 + a^6*e^12)/(a^7*c^7))*e - a^4*c^3*d^3*e
^4 - a^5*c^2*d*e^6)*sqrt(-(2*c^2*d^5*e - a^3*c^3*sqrt(-(c^6*d^12 + 2*a*c^5*
d^10*e^2 - a^2*c^4*d^8*e^4 - 4*a^3*c^3*d^6*e^6 - a^4*c^2*d^4*e^8 + 2*a^5*c*
d^2*e^10 + a^6*e^12)/(a^7*c^7)) + 4*a*c*d^3*e^3 + 2*a^2*d*e^5)/(a^3*c^3))))
/(a*c^2*x^4 + a^2*c)
```

Sympy [A]
time = 1.81, size = 352, normalized size = 0.97

$$\operatorname{RootSum}\left(65536t^4a^7c^7 + t^2\cdot\left(9216a^6c^4de^5 + 18432a^5c^5d^3e^3 + 9216a^4c^6d^5e\right) + 81a^6e^{12} + 486a^5cd^2e^{10} + 1215a^4c^2d^4e^8 + 1620a^3c^3d^6e^6 + 1215a^2c^4d^8e^4 + 486ac^5d^{10}e^2 + 81c^6d^{12}, \left(t\mapsto t\log\left(x + \frac{4096t^3a^6c^5e + 432ta^5c^2de^6 + 720ta^4c^3d^3e^4 + 144ta^3c^4d^5e^2 - 144ta^2c^5d^7}{27a^5e^{10} + 81a^4cd^2e^8 + 54a^3c^2d^4e^6 - 54a^2c^3d^6e^4 - 81ac^4d^8e^2 - 27c^5d^{10}}\right)\right)\right) + \frac{x^3(-ae^3 + 3cd^2e) + x(-3ade^2 + cd^3)}{4a^2c + 4ac^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**7 + _t**2*(9216*a**6*c**4*d*e**5 + 18432*a**5*c
**5*d**3*e**3 + 9216*a**4*c**6*d**5*e) + 81*a**6*e**12 + 486*a**5*c*d**2*e*
*10 + 1215*a**4*c**2*d**4*e**8 + 1620*a**3*c**3*d**6*e**6 + 1215*a**2*c**4*
d**8*e**4 + 486*a*c**5*d**10*e**2 + 81*c**6*d**12, Lambda(_t, _t*log(x + (4
096*_t**3*a**6*c**5*e + 432*_t*a**5*c**2*d*e**6 + 720*_t*a**4*c**3*d**3*e**
4 + 144*_t*a**3*c**4*d**5*e**2 - 144*_t*a**2*c**5*d**7)/(27*a**5*e**10 + 81
*a**4*c*d**2*e**8 + 54*a**3*c**2*d**4*e**6 - 54*a**2*c**3*d**6*e**4 - 81*a*
c**4*d**8*e**2 - 27*c**5*d**10)))) + (x**3*(-a*e**3 + 3*c*d**2*e) + x*(-3*a
*d*e**2 + c*d**3))/(4*a**2*c + 4*a*c**2*x**4)
```

Giac [A]
time = 9.01, size = 425, normalized size = 1.17

$$\frac{3\,cd^2x^3e + cd^3x - ax^3e^3 - 3\,adxe^2}{4\,(cx^4+a)ac} + \frac{3\,\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 + (ac^3)^{\frac{3}{4}}\,cd^2e + (ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^4} + \frac{3\,\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 + (ac^3)^{\frac{3}{4}}\,cd^2e + (ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\,\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c^4} + \frac{3\,\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 - (ac^3)^{\frac{3}{4}}\,cd^2e - (ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2 + \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^4} - \frac{3\,\sqrt{2}\left((ac^3)^{\frac{1}{4}}\,c^3d^3 + (ac^3)^{\frac{1}{4}}\,ac^2de^2 - (ac^3)^{\frac{3}{4}}\,cd^2e - (ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2 - \sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{32\,a^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(3*c*d^2*x^3*e + c*d^3*x - a*x^3*e^3 - 3*a*d*x*e^2)/((c*x^4 + a)*a*c) +
 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^
(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/16*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*
c^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arcta
n(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^4) + 3/32*sqr
t(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*c*d
^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a
^2*c^4) - 3/32*sqrt(2)*((a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 -
 (a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/
4) + sqrt(a/c))/(a^2*c^4)
```

Mupad [B]
time = 4.94, size = 2560, normalized size = 7.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + c*x^4)^2,x)
```

```
[Out] - ((d*x*(3*a*e^2 - c*d^2))/(4*a*c) + (e*x^3*(a*e^2 - 3*c*d^2))/(4*a*c))/(a
+ c*x^4) - 2*atanh((9*c^3*d^6*x*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (
9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^7)^(1/2))/
(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a
^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*c*d^6*
e^3)/16 - (27*a^3*e^9)/(32*c^2) + (27*c^2*d^8*e)/(32*a) - (27*a^2*d^2*e^7)/
(16*c) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^5*c) - (27*d*e^8*(-a^7*c^7)^(1/2))
/(32*a*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^2*c^4) + (27*d^7*e^2*(-a^
7*c^7)^(1/2))/(16*a^4*c^2))) + (9*a*e^6*x*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^
4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^
7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/
2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*(
(27*a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a^2) - (27*c
^2*d^8*e)/(32*a^3) - (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c) + (27*d*e^8*(-a^7
*c^7)^(1/2))/(32*a^3*c^5) + (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^4*c^4) - (2
7*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) + (9*c*d^2*e^4*x*((9*e^6*(-a^7*c
^7)^(1/2))/(256*a^4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2)
 - (9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(128*a*c^3) + (9*d^2*
e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6
*c^5))^(1/2))/(2*((27*a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3
)/(16*a^2) - (27*c^2*d^8*e)/(32*a^3) - (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c)
 + (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^3*c^5) + (27*d^3*e^6*(-a^7*c^7)^(1/2))
/(16*a^4*c^4) - (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) - (9*c^2*d^4*e
^2*x*((9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^5*e)/(128*a^3*c) - (9*d
^3*e^3)/(64*a^2*c^2) - (9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d*e^5)/(
128*a*c^3) + (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) - (9*d^4*e^2*(-a^7*
c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*a^2*e^9)/(32*c^2) + (27*a*d^2*e^7
)/(16*c) - (27*c*d^6*e^3)/(16*a) - (27*c^2*d^8*e)/(32*a^2) - (27*d^9*(-a^7*
c^7)^(1/2))/(32*a^6*c) + (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^2*c^5) + (27*d^3
*e^6*(-a^7*c^7)^(1/2))/(16*a^3*c^4) - (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^5
*c^2))))*(-(9*(c^3*d^6*(-a^7*c^7)^(1/2) - a^3*e^6*(-a^7*c^7)^(1/2) + 2*a^4*
c^6*d^5*e + 2*a^6*c^4*d*e^5 + 4*a^5*c^5*d^3*e^3 + a*c^2*d^4*e^2*(-a^7*c^7)^
(1/2) - a^2*c*d^2*e^4*(-a^7*c^7)^(1/2)))/(256*a^7*c^7))^(1/2) - 2*atanh((9*
c^3*d^6*x*((9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d^5*e)/(128*a^3*c) -
 (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*c^7)^(1/2)
)/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) + (9*d^4*e^2*(
-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*c*d^6*e^3)/16 - (27*a^3*e^9)
```

/(32*c^2) + (27*c^2*d^8*e)/(32*a) - (27*a^2*d^2*e^7)/(16*c) - (27*d^9*(-a^7*c^7)^(1/2))/(32*a^5*c) + (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a*c^5) + (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^2*c^4) - (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^4*c^2))) + (9*a*e^6*x*((9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) + (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a^2) - (27*c^2*d^8*e)/(32*a^3) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c) - (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^3*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^4*c^4) + (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) + (9*c*d^2*e^4*x*((9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) + (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*a*e^9)/(32*c^2) + (27*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a^2) - (27*c^2*d^8*e)/(32*a^3) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^7*c) - (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^3*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^4*c^4) + (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^6*c^2))) - (9*c^2*d^4*e^2*x*((9*d^6*(-a^7*c^7)^(1/2))/(256*a^7*c^4) - (9*d^5*e)/(128*a^3*c) - (9*d^3*e^3)/(64*a^2*c^2) - (9*d*e^5)/(128*a*c^3) - (9*e^6*(-a^7*c^7)^(1/2))/(256*a^4*c^7) - (9*d^2*e^4*(-a^7*c^7)^(1/2))/(256*a^5*c^6) + (9*d^4*e^2*(-a^7*c^7)^(1/2))/(256*a^6*c^5))^(1/2))/(2*((27*a^2*e^9)/(32*c^2) + (27*a*d^2*e^7)/(16*c) - (27*c*d^6*e^3)/(16*a) - (27*c^2*d^8*e)/(32*a^2) + (27*d^9*(-a^7*c^7)^(1/2))/(32*a^6*c) - (27*d*e^8*(-a^7*c^7)^(1/2))/(32*a^2*c^5) - (27*d^3*e^6*(-a^7*c^7)^(1/2))/(16*a^3*c^4) + (27*d^7*e^2*(-a^7*c^7)^(1/2))/(16*a^5*c^2))))*(-(9*(a^3*e^6*(-a^7*c^7)^(1/2) - c^3*d^6*(-a^7*c^7)^(1/2) + 2*a^4*c^6*d^5*e + 2*a^6*c^4*d*e^5 + 4*a^5*c^5*d^3*e^3 - a*c^2*d^4*e^2*(-a^7*c^7)^(1/2) + a^2*c*d^2*e^4*(-a^7*c^7)^(1/2)))/(256*a^7*c^7))^(1/2)

3.145 $\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=349

$$-\frac{e^2x}{3c\,(a+cx^4)}+\frac{x(3cd^2+ae^2+6cdex^2)}{12ac\,(a+cx^4)}-\frac{(3cd^2+2\sqrt{a}\,\sqrt{c}\,de+ae^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{5/4}}+\frac{(3cd^2+2\sqrt{a}$$

```
[Out] -1/3*e^2*x/c/(c*x^4+a)+1/12*x*(6*c*d*e*x^2+a*e^2+3*c*d^2)/a/c/(c*x^4+a)-1/3
2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2+a*e^2-2*d*e*a
^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)+1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(3*c*d^2+a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2
^(1/2)+1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2+a*e^2+2*d*e*a^(1/
2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4)
)*(3*c*d^2+a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/c^(5/4)*2^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1221, 1193, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)(2\sqrt{a}\,\sqrt{c}\,de+ae^2+3cd^2)}{8\sqrt{2}\,a^{7/4}c^{5/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)(2\sqrt{a}\,\sqrt{c}\,de+ae^2+3cd^2)}{8\sqrt{2}\,a^{7/4}c^{5/4}}-\frac{(-2\sqrt{a}\,\sqrt{c}\,de+ae^2+3cd^2)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{5/4}}+\frac{(-2\sqrt{a}\,\sqrt{c}\,de+ae^2+3cd^2)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{5/4}}+\frac{x(ae^2+3cd^2+6cdex^2)}{12ac\,(a+cx^4)}-\frac{e^2x}{3c\,(a+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

```
[Out] -1/3*(e^2*x)/(c*(a + c*x^4)) + (x*(3*c*d^2 + a*e^2 + 6*c*d*e*x^2))/(12*a*c*
(a + c*x^4)) - ((3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[
2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(5/4)) + ((3*c*d^2 + 2*Sqrt[a]
*Sqrt[c]*d*e + a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a
^(7/4)*c^(5/4)) - ((3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4)) + ((
3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1
/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(5/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{(a+cx^4)^2}\,dx = -\frac{e^2x}{3c\,(a+cx^4)} - \frac{\int \frac{-3cd^2-ae^2-6cdex^2}{(a+cx^4)^2}\,dx}{3c}$$

$$= -\frac{e^2x}{3c\,(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac\,(a+cx^4)} + \frac{\int \frac{3(3cd^2+ae^2)+6cdex^2}{a+cx^4}\,dx}{12ac}$$

$$= -\frac{e^2x}{3c\,(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac\,(a+cx^4)} + \frac{\left(3cd^2-2\sqrt{a}\,\sqrt{c}\,de+ae^2\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a^{3/2}c^{3/2}}$$

$$= -\frac{e^2x}{3c\,(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac\,(a+cx^4)} - \frac{\left(3cd^2-2\sqrt{a}\,\sqrt{c}\,de+ae^2\right)\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}+2}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}}}{16\sqrt{2}\,a^{7/4}c^{5/4}}$$

$$= -\frac{e^2x}{3c\,(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac\,(a+cx^4)} - \frac{\left(3cd^2-2\sqrt{a}\,\sqrt{c}\,de+ae^2\right)\log\left(\sqrt{a}-\sqrt{2}\right.}{16\sqrt{2}\,a^{7/4}c^{5/4}}$$

$$= -\frac{e^2x}{3c\,(a+cx^4)} + \frac{x(3cd^2+ae^2+6cdex^2)}{12ac\,(a+cx^4)} - \frac{\left(3cd^2+2\sqrt{a}\,\sqrt{c}\,de+ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}}{\sqrt[4]{}}\right.}{8\sqrt{2}\,a^{7/4}c^{5/4}}$$

Mathematica [A]
time = 0.11, size = 295, normalized size = 0.85

$$\frac{-\frac{8a^{3/4}\sqrt[4]{c}\,(ae^2x-cdx(d+2ex^2))}{a+cx^4}-2\sqrt{2}\,(3cd^2+2\sqrt{a}\,\sqrt{c}\,de+ae^2)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\sqrt{2}\,(3cd^2+2\sqrt{a}\,\sqrt{c}\,de+ae^2)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-\sqrt{2}\,(3cd^2-2\sqrt{a}\,\sqrt{c}\,de+ae^2)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+\sqrt{2}\,(3cd^2-2\sqrt{a}\,\sqrt{c}\,de+ae^2)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{32a^{7/4}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + c*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*c^(1/4)*(a*e^2*x - c*d*x*(d + 2*e*x^2)))/(a + c*x^4) - 2*Sqrt[
2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)
/a^(1/4)] + 2*Sqrt[2]*(3*c*d^2 + 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a
*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*(3*c
*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(32*a^(7/4)*c^(5/4))
```

Maple [A]
time = 0.16, size = 266, normalized size = 0.76

method	result
risch	$\frac{\frac{de\,x^3}{2a}-\frac{(a\,e^2-c\,d^2)x}{4ac}}{c\,x^4+a}+\frac{\sum_{\_R=\text{RootOf}(c\_Z^4+a)}\frac{\left(2de\_R^2+\frac{a\,e^2+3c\,d^2}{c}\right)\ln(x-\_R)}{\_R^3}}{16ac}$
default	$\frac{\frac{de\,x^3}{2a}-\frac{(a\,e^2-c\,d^2)x}{4ac}}{c\,x^4+a}+\frac{\frac{(a\,e^2+3c\,d^2)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}+\frac{de}{\cdots}}{4ac}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*d*e/a*x^3-1/4*(a*e^2-c*d^2)/a/c*x)/(c*x^4+a)+1/4/a/c*(1/8*(a*e^2+3*c*d
^2)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x-1))+1/4*d*e/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1
/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arcta
n(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.52, size = 324, normalized size = 0.93

$$\frac{2\,cdx^3e+(cd^2-ae^2)x}{4\,(ac^2x^4+a^2c)}+\frac{\frac{2\sqrt{2}\left(3c^{\frac{3}{2}}d^2+2\sqrt{a}\,cde+a\sqrt{c}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(3c^{\frac{3}{2}}d^2+2\sqrt{a}\,cde+a\sqrt{c}\,e^2\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(3c^{\frac{3}{2}}d^2-2\sqrt{a}\,cde+a\sqrt{c}\,e^2\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3c^{\frac{3}{2}}d^2-2\sqrt{a}\,cde+a\sqrt{c}\,e^2\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{32\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*c*d*x^3*e + (c*d^2 - a*e^2)*x)/(a*c^2*x^4 + a^2*c) + 1/32*(2*sqrt(2)
*(3*c^(3/2)*d^2 + 2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*arctan(1/2*sqrt(2)*(2*sq
rt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*c^(3/2)*d^2 + 2*sqrt(a)*c*d*e + a*sqr
t(c)*e^2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(s
qrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*c^(3/
2)*d^2 - 2*sqrt(a)*c*d*e + a*sqrt(c)*e^2)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)
*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*c^(3/2)*d^2 - 2*sqrt(a
)*c*d*e + a*sqrt(c)*e^2)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt
(a))/(a^(3/4)*c^(3/4)))/(a*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1531 vs. $2(263) = 526$.
time = 0.39, size = 1531, normalized size = 4.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*c*d*x^3*e + 4*c*d^2*x - 4*a*x*e^2 + (a*c^2*x^4 + a^2*c)*sqrt(-(a^3*
c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2
*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log(81*c^4*
d^8*x + 108*a*c^3*d^6*x*e^2 + 38*a^2*c^2*d^4*x*e^4 + 12*a^3*c*d^2*x*e^6 + a
^4*x*e^8 + (2*a^6*c^4*d*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d
^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5))*e + 27*a^2*c^4*d^6 + 15*a^3*
c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c^4*d
^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^
7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqrt(-(
a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c
*d^2*e^6 + a^4*e^8)/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))*log(81*
c^4*d^8*x + 108*a*c^3*d^6*x*e^2 + 38*a^2*c^2*d^4*x*e^4 + 12*a^3*c*d^2*x*e^6
 + a^4*x*e^8 - (2*a^6*c^4*d*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c
^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5))*e + 27*a^2*c^4*d^6 + 15*
a^3*c^3*d^4*e^2 + 5*a^4*c^2*d^2*e^4 + a^5*c*e^6)*sqrt(-(a^3*c^2*sqrt(-(81*c
^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)
/(a^7*c^5)) + 12*c*d^3*e + 4*a*d*e^3)/(a^3*c^2))) - (a*c^2*x^4 + a^2*c)*sqr
t((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^
3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))*log(
81*c^4*d^8*x + 108*a*c^3*d^6*x*e^2 + 38*a^2*c^2*d^4*x*e^4 + 12*a^3*c*d^2*x*
e^6 + a^4*x*e^8 + (2*a^6*c^4*d*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^
2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5))*e - 27*a^2*c^4*d^6 -
15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a^5*c*e^6)*sqrt((a^3*c^2*sqrt(-(81
*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^
8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))) + (a*c^2*x^4 + a^2*c)*s
qrt((a^3*c^2*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*
a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))*lo
g(81*c^4*d^8*x + 108*a*c^3*d^6*x*e^2 + 38*a^2*c^2*d^4*x*e^4 + 12*a^3*c*d^2*
x*e^6 + a^4*x*e^8 - (2*a^6*c^4*d*sqrt(-(81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*
a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*e^8)/(a^7*c^5))*e - 27*a^2*c^4*d^6
- 15*a^3*c^3*d^4*e^2 - 5*a^4*c^2*d^2*e^4 - a^5*c*e^6)*sqrt((a^3*c^2*sqrt(-(
81*c^4*d^8 + 36*a*c^3*d^6*e^2 + 22*a^2*c^2*d^4*e^4 + 4*a^3*c*d^2*e^6 + a^4*
e^8)/(a^7*c^5)) - 12*c*d^3*e - 4*a*d*e^3)/(a^3*c^2))))/(a*c^2*x^4 + a^2*c)
```

Sympy [A]
time = 1.07, size = 275, normalized size = 0.79

$$\operatorname{RootSum}\left(65536t^4a^7c^5 + t^2\cdot\left(2048a^5c^3de^3 + 6144a^4c^4d^3e\right) + a^4e^8 + 20a^3cd^2e^6 + 118a^2c^2d^4e^4 + 180ac^3d^6e^2 + 81c^4d^8, \left(t \mapsto t\log\left(x + \frac{-8192t^3a^6c^4de + 16ta^5ce^6 - 48ta^4c^2d^2e^4 - 144ta^3c^3d^4e^2 + 432ta^2c^4d^6}{a^4e^8 + 12a^3cd^2e^6 + 38a^2c^2d^4e^4 + 108ac^3d^6e^2 + 81c^4d^8}\right)\right)\right) + \frac{2cdex^3 + x(-ae^2 + cd^2)}{4a^2c + 4ac^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+a)**2,x)
```

[Out] RootSum(65536*_t**4*a**7*c**5 + _t**2*(2048*a**5*c**3*d*e**3 + 6144*a**4*c*
*4*d**3*e) + a**4*e**8 + 20*a**3*c*d**2*e**6 + 118*a**2*c**2*d**4*e**4 + 18
0*a*c**3*d**6*e**2 + 81*c**4*d**8, Lambda(_t, _t*log(x + (-8192*_t**3*a**6*
c**4*d*e + 16*_t*a**5*c*e**6 - 48*_t*a**4*c**2*d**2*e**4 - 144*_t*a**3*c**3
*d**4*e**2 + 432*_t*a**2*c**4*d**6)/(a**4*e**8 + 12*a**3*c*d**2*e**6 + 38*a
2*c2*d**4*e**4 + 108*a*c**3*d**6*e**2 + 81*c**4*d**8)))) + (2*c*d*e*x**
3 + x*(-a*e**2 + c*d**2))/(4*a**2*c + 4*a*c**2*x**4)

Giac [A]
time = 4.30, size = 350, normalized size = 1.00

$$\frac{2\,cdx^3e+cd^2x-axe^2}{4\,(cx^4+a)ac}+\frac{\sqrt{2}\left(3\,(ac^3)^{\frac14}c^2d^2+(ac^3)^{\frac14}ace^2+2\,(ac^3)^{\frac34}de\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{16\,a^2c^3}+\frac{\sqrt{2}\left(3\,(ac^3)^{\frac14}c^2d^2+(ac^3)^{\frac14}ace^2+2\,(ac^3)^{\frac34}de\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{16\,a^2c^3}+\frac{\sqrt{2}\left(3\,(ac^3)^{\frac14}c^2d^2+(ac^3)^{\frac14}ace^2-2\,(ac^3)^{\frac34}de\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}-\frac{\sqrt{2}\left(3\,(ac^3)^{\frac14}c^2d^2+(ac^3)^{\frac14}ace^2-2\,(ac^3)^{\frac34}de\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/4*(2*c*d*x^3*e + c*d^2*x - a*x*e^2)/((c*x^4 + a)*a*c) + 1/16*sqrt(2)*(3*(
a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*arctan(
1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/16*sqrt(
2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 + 2*(a*c^3)^(3/4)*d*e)*
arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/3
2*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4
)*d*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2
)*(3*(a*c^3)^(1/4)*c^2*d^2 + (a*c^3)^(1/4)*a*c*e^2 - 2*(a*c^3)^(3/4)*d*e)*l
og(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)

Mupad [B]
time = 4.79, size = 1565, normalized size = 4.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^2)^2/(a + c*x^4)^2,x)

[Out] 2*atanh((9*c^3*d^4*x*((9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (3*d^3*e)/(6
4*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) + (d
^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/(2*((27*d^6*(-a^7*c^5)^(1/2)
)/(32*a^5) - (c*d^3*e^3)/8 - (a*d*e^5)/16 - (9*c^2*d^5*e)/(16*a) + (e^6*(-a
^7*c^5)^(1/2))/(32*a^2*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^3*c^2) + (
15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^4*c))) + (c*e^4*x*((9*d^4*(-a^7*c^5)^(1/
2))/(256*a^7*c^3) - (3*d^3*e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^
7*c^5)^(1/2))/(256*a^5*c^5) + (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/
2))/(2*((27*d^6*(-a^7*c^5)^(1/2))/(32*a^7) - (d*e^5)/(16*a) - (c*d^3*e^3)/(
8*a^2) - (9*c^2*d^5*e)/(16*a^3) + (e^6*(-a^7*c^5)^(1/2))/(32*a^4*c^3) + (5*
d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^5*c^2) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*
a^6*c))) + (c^2*d^2*e^2*x*((9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (3*d^3*

```
e)/(64*a^3*c) - (d*e^3)/(64*a^2*c^2) + (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5)
 + (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/((27*d^6*(-a^7*c^5)^(1/
2))/(32*a^6) - (d*e^5)/16 - (c*d^3*e^3)/(8*a) - (9*c^2*d^5*e)/(16*a^2) + (e
^6*(-a^7*c^5)^(1/2))/(32*a^3*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^4*c^
2) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^5*c)))*((a^2*e^4*(-a^7*c^5)^(1/2)
+ 9*c^2*d^4*(-a^7*c^5)^(1/2) - 12*a^4*c^4*d^3*e - 4*a^5*c^3*d*e^3 + 2*a*c*d
^2*e^2*(-a^7*c^5)^(1/2))/(256*a^7*c^5))^(1/2) - 2*atanh((9*c^3*d^4*x*(- (d*
e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c) - (9*d^4*(-a^7*c^5)^(1/2))/(256*a^
7*c^3) - (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) - (d^2*e^2*(-a^7*c^5)^(1/2))/
(128*a^6*c^4))^(1/2))/(2*((27*d^6*(-a^7*c^5)^(1/2))/(32*a^5) + (c*d^3*e^3)/
8 + (a*d*e^5)/16 + (9*c^2*d^5*e)/(16*a) + (e^6*(-a^7*c^5)^(1/2))/(32*a^2*c^
3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^3*c^2) + (15*d^4*e^2*(-a^7*c^5)^(1/
2))/(32*a^4*c))) + (c*e^4*x*(- (d*e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c)
- (9*d^4*(-a^7*c^5)^(1/2))/(256*a^7*c^3) - (e^4*(-a^7*c^5)^(1/2))/(256*a^5*
c^5) - (d^2*e^2*(-a^7*c^5)^(1/2))/(128*a^6*c^4))^(1/2))/(2*((27*d^6*(-a^7*c
^5)^(1/2))/(32*a^7) + (d*e^5)/(16*a) + (c*d^3*e^3)/(8*a^2) + (9*c^2*d^5*e)/
(16*a^3) + (e^6*(-a^7*c^5)^(1/2))/(32*a^4*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2
))/(32*a^5*c^2) + (15*d^4*e^2*(-a^7*c^5)^(1/2))/(32*a^6*c))) + (c^2*d^2*e^2
*x*(- (d*e^3)/(64*a^2*c^2) - (3*d^3*e)/(64*a^3*c) - (9*d^4*(-a^7*c^5)^(1/2)
)/(256*a^7*c^3) - (e^4*(-a^7*c^5)^(1/2))/(256*a^5*c^5) - (d^2*e^2*(-a^7*c^5
)^(1/2))/(128*a^6*c^4))^(1/2))/((d*e^5)/16 + (27*d^6*(-a^7*c^5)^(1/2))/(32*
a^6) + (c*d^3*e^3)/(8*a) + (9*c^2*d^5*e)/(16*a^2) + (e^6*(-a^7*c^5)^(1/2))/
(32*a^3*c^3) + (5*d^2*e^4*(-a^7*c^5)^(1/2))/(32*a^4*c^2) + (15*d^4*e^2*(-a^
7*c^5)^(1/2))/(32*a^5*c)))*(-(a^2*e^4*(-a^7*c^5)^(1/2) + 9*c^2*d^4*(-a^7*c^
5)^(1/2) + 12*a^4*c^4*d^3*e + 4*a^5*c^3*d*e^3 + 2*a*c*d^2*e^2*(-a^7*c^5)^(1
/2))/(256*a^7*c^5))^(1/2) + ((d*e*x^3)/(2*a) - (x*(a*e^2 - c*d^2))/(4*a*c))
/(a + c*x^4)
```

3.146 $\int \frac{d+ex^2}{(a+cx^4)^2}\,dx$

Optimal. Leaf size=275

$$\frac{x(d+ex^2)}{4a\,(a+cx^4)}-\frac{(3\sqrt{c}\,d+\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{(3\sqrt{c}\,d+\sqrt{a}\,e)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{(3\sqrt{c}\,d}{}$$

```
[Out] 1/4*x*(e*x^2+d)/a/(c*x^4+a)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)+1/32*ln(a^(1/4)*c
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/c^(3
/4)*2^(1/2)+1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+3*d*c^(1/2
))/a^(7/4)/c^(3/4)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1
/2)+3*d*c^(1/2))/a^(7/4)/c^(3/4)*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1193, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)(\sqrt{a}\,e+3\sqrt{c}\,d)}{8\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)(\sqrt{a}\,e+3\sqrt{c}\,d)}{8\sqrt{2}\,a^{7/4}c^{3/4}}-\frac{(3\sqrt{c}\,d-\sqrt{a}\,e)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{(3\sqrt{c}\,d-\sqrt{a}\,e)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}}+\frac{x(d+ex^2)}{4a\,(a+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + c*x^4)^2,x]
```

```
[Out] (x*(d + e*x^2))/(4*a*(a + c*x^4)) - ((3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d +
Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*c^(3
/4)) - ((3*Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(3/4)) + ((3*Sqrt[c]*d - Sqrt[a]*e)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c
^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rubi steps

$$\int \frac{d+ex^2}{(a+cx^4)^2}\,dx = \frac{x(d+ex^2)}{4a\,(a+cx^4)} - \frac{\int \frac{-3d-ex^2}{a+cx^4}\,dx}{4a}$$

$$= \frac{x(d+ex^2)}{4a\,(a+cx^4)} + \frac{\left(\frac{3\sqrt{c}\,d}{\sqrt{a}} - e\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{8ac} + \frac{\left(\frac{3\sqrt{c}\,d}{\sqrt{a}} + e\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{8ac}$$

$$= \frac{x(d+ex^2)}{4a\,(a+cx^4)} + \frac{\left(\frac{3\sqrt{c}\,d}{\sqrt{a}} + e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{16ac} + \frac{\left(\frac{3\sqrt{c}\,d}{\sqrt{a}} + e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} + \frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}}}{16ac}$$

$$= \frac{x(d+ex^2)}{4a\,(a+cx^4)} - \frac{(3\sqrt{c}\,d - \sqrt{a}\,e)\log\left(\sqrt{a} - \sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}\,d - \sqrt{a}}{}$$

$$= \frac{x(d+ex^2)}{4a\,(a+cx^4)} - \frac{(3\sqrt{c}\,d + \sqrt{a}\,e)\tan^{-1}\left(1 - \frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}c^{3/4}} + \frac{(3\sqrt{c}\,d + \sqrt{a}\,e)\tan^{-1}\left(1 +\right.}{8\sqrt{2}\,a^{7/4}c^{3/4}}$$

Mathematica [A]
time = 0.18, size = 267, normalized size = 0.97

$$\frac{\frac{8ax(d+ex^2)}{a+cx^4} - \frac{2\sqrt{2}\,\sqrt[4]{a}\left(3\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{2\sqrt{2}\,\sqrt[4]{a}\left(3\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{3/4}} + \frac{\sqrt{2}\left(-3\sqrt[4]{a}\,\sqrt{c}\,d+a^{3/4}e\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{c^{3/4}} + \frac{\sqrt{2}\left(3\sqrt[4]{a}\,\sqrt{c}\,d-a^{3/4}e\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{c^{3/4}}}{32a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + c*x^4)^2,x]
```

```
[Out] ((8*a*x*(d + e*x^2))/(a + c*x^4) - (2*Sqrt[2]*a^(1/4)*(3*Sqrt[c]*d + Sqrt[a
]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4) + (2*Sqrt[2]*a^(1/4)*
(3*Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(3/4)
+ (Sqrt[2]*(-3*a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4) + (Sqrt[2]*(3*a^(1/4)*Sqrt[c]*d - a^(3/4
)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(3/4))/(32*a
^2)
```

Maple [A]
time = 0.15, size = 245, normalized size = 0.89

method	result

risch	$\frac{\frac{e\,x^3}{4a}+\frac{dx}{4a}}{c\,x^4+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\,\_Z^4+a\right)}\frac{\left(e\,\_R^2+3d\right)\ln\left(x-\_R\right)}{\_R^3}}{16ac}$
default	$d\left(\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}\right)+e\left(\frac{}{4a(}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d*(1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^
(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2
)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))+e*(1/4*x^3/a/(c*x^4+
a)+1/32/a/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))
/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)
+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.51, size = 258, normalized size = 0.94

$$\frac{x^3e+dx}{4\left(acx^4+a^2\right)}+\frac{\frac{2\sqrt{2}\left(3\sqrt{c}\,d+\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(3\sqrt{c}\,d+\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x^3*e + d*x)/(a*c*x^4 + a^2) + 1/32*(2*sqrt(2)*(3*sqrt(c)*d + sqrt(a)*
e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*
sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*sqrt(c)*d
+ sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*sq
rt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))
/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*sqrt(c)*d - sqrt(a)*e)*log(sqrt(c)*x^2 - sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 842 vs. 2(197) = 394.
time = 0.34, size = 842, normalized size = 3.06

$$\frac{4x^3e-\left(acx^4+a^2\right)\sqrt{-\frac{a^3c\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}+6de}{a^3c}}\log\left(-81c^2d^4x+a^2xe^4+\left(a^6c^2\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}\,e+27a^2c^2d^3-3a^3cde^2\right)\sqrt{-\frac{a^3c\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}+6de}{a^3c}}\right)+\left(acx^4+a^2\right)\sqrt{-\frac{a^3c\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}+6de}{a^3c}}\log\left(-81c^2d^4x+a^2xe^4-\left(a^6c^2\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}\,e+27a^2c^2d^3-3a^3cde^2\right)\sqrt{-\frac{a^3c\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}+6de}{a^3c}}\right)+\left(acx^4+a^2\right)\sqrt{\frac{a^3c\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}-6de}{a^3c}}\log\left(-81c^2d^4x+a^2xe^4+\left(a^6c^2\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}\,e-27a^2c^2d^3+3a^3cde^2\right)\sqrt{\frac{a^3c\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}-6de}{a^3c}}\right)-\left(acx^4+a^2\right)\sqrt{\frac{a^3c\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}-6de}{a^3c}}\log\left(-81c^2d^4x+a^2xe^4-\left(a^6c^2\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}\,e-27a^2c^2d^3+3a^3cde^2\right)\sqrt{\frac{a^3c\sqrt{-\frac{81c^2d^4-18acd^2e^2+a^2e^4}{a^7c^3}}-6de}{a^3c}}\right)+4dx}{16\left(acx^4+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*x^3*e - (a*c*x^4 + a^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2
*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*log(-81*c^2*d^4*x + a^2*x*e^4
+ (a^6*c^2*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3))*e + 27*
a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^
2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (a*c*x^4 + a^2)*sqrt(-(a^3*c*s
qrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))*l
og(-81*c^2*d^4*x + a^2*x*e^4 - (a^6*c^2*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2
+ a^2*e^4)/(a^7*c^3))*e + 27*a^2*c^2*d^3 - 3*a^3*c*d*e^2)*sqrt(-(a^3*c*sqrt
(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) + 6*d*e)/(a^3*c))) + (
a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a
^7*c^3)) - 6*d*e)/(a^3*c))*log(-81*c^2*d^4*x + a^2*x*e^4 + (a^6*c^2*sqrt(-(
81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3))*e - 27*a^2*c^2*d^3 + 3*a^
3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c
^3)) - 6*d*e)/(a^3*c))) - (a*c*x^4 + a^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 1
8*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))*log(-81*c^2*d^4*x + a
^2*x*e^4 - (a^6*c^2*sqrt(-(81*c^2*d^4 - 18*a*c*d^2*e^2 + a^2*e^4)/(a^7*c^3)
)*e - 27*a^2*c^2*d^3 + 3*a^3*c*d*e^2)*sqrt((a^3*c*sqrt(-(81*c^2*d^4 - 18*a*
c*d^2*e^2 + a^2*e^4)/(a^7*c^3)) - 6*d*e)/(a^3*c))) + 4*d*x)/(a*c*x^4 + a^2)
```

Sympy [A]
time = 0.49, size = 136, normalized size = 0.49

$$\operatorname{RootSum}\left(65536t^4a^7c^3+3072t^2a^4c^2de+a^2e^4+18acd^2e^2+81c^2d^4,\ \left(t\mapsto t\log\left(x+\frac{4096t^3a^6c^2e+144ta^3cde^2-432ta^2c^2d^3}{a^2e^4-81c^2d^4}\right)\right)\right)+\frac{dx+ex^3}{4a^2+4acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] RootSum(65536*_t**4*a**7*c**3 + 3072*_t**2*a**4*c**2*d*e + a**2*e**4 + 18*a
*c*d**2*e**2 + 81*c**2*d**4, Lambda(_t, _t*log(x + (4096*_t**3*a**6*c**2*e
+ 144*_t*a**3*c*d*e**2 - 432*_t*a**2*c**2*d**3)/(a**2*e**4 - 81*c**2*d**4))
)) + (d*x + e*x**3)/(4*a**2 + 4*a*c*x**4)
```

Giac [A]
time = 3.51, size = 273, normalized size = 0.99

$$\frac{x^3e+dx}{4\,(cx^4+a)a}+\frac{\sqrt{2}\left(3\,(ac^3)^{\frac14}\,c^2d+(ac^3)^{\frac34}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{16\,a^2c^3}+\frac{\sqrt{2}\left(3\,(ac^3)^{\frac14}\,c^2d+(ac^3)^{\frac34}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac14}\right)}{2\left(\frac{a}{c}\right)^{\frac14}}\right)}{16\,a^2c^3}+\frac{\sqrt{2}\left(3\,(ac^3)^{\frac14}\,c^2d-(ac^3)^{\frac34}\,e\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}-\frac{\sqrt{2}\left(3\,(ac^3)^{\frac14}\,c^2d-(ac^3)^{\frac34}\,e\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac14}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^3*e + d*x)/((c*x^4 + a)*a) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (
a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))
/(a^2*c^3) + 1/16*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c^3) + 1/32*sqrt(
```

```
2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4
) + sqrt(a/c))/(a^2*c^3) - 1/32*sqrt(2)*(3*(a*c^3)^(1/4)*c^2*d - (a*c^3)^(3
/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^3)
```

Mupad [B]
time = 0.40, size = 637, normalized size = 2.32

$$\frac{\frac{e\,x^3}{4\,a}+\frac{d\,x}{4\,a}}{c\,x^4+a}-2\,\mathrm{atanh}\left(\frac{c^2\,e^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}-\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}}}{2\,\left(\frac{c\,e^3}{32\,a}-\frac{9\,c^2\,d^2\,e}{32\,a^2}-\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^6}+\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^5}\right)}-\frac{9\,c^3\,d^2\,x\,\sqrt{\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}-\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}}}{2\,\left(\frac{c\,e^3}{32}-\frac{9\,c^2\,d^2\,e}{32\,a}-\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^5}+\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^4}\right)}\right)\,\sqrt{-\frac{9\,c\,d^2\,\sqrt{-a^7\,c^3}-a\,e^2\,\sqrt{-a^7\,c^3}+6\,a^4\,c^2\,d\,e}{256\,a^7\,c^3}}-2\,\mathrm{atanh}\left(\frac{c^2\,e^2\,x\,\sqrt{\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}-\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}}}{2\,\left(\frac{c\,e^3}{32\,a}-\frac{9\,c^2\,d^2\,e}{32\,a^2}+\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^6}-\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^5}\right)}-\frac{9\,c^3\,d^2\,x\,\sqrt{\frac{9\,d^2\,\sqrt{-a^7\,c^3}}{256\,a^7\,c^2}-\frac{3\,d\,e}{128\,a^3\,c}-\frac{e^2\,\sqrt{-a^7\,c^3}}{256\,a^6\,c^3}}}{2\,\left(\frac{c\,e^3}{32}-\frac{9\,c^2\,d^2\,e}{32\,a}+\frac{27\,c\,d^3\,\sqrt{-a^7\,c^3}}{32\,a^5}-\frac{3\,d\,e^2\,\sqrt{-a^7\,c^3}}{32\,a^4}\right)}\right)\,\sqrt{-\frac{a\,e^2\,\sqrt{-a^7\,c^3}-9\,c\,d^2\,\sqrt{-a^7\,c^3}+6\,a^4\,c^2\,d\,e}{256\,a^7\,c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + c*x^4)^2,x)
```

```
[Out] ((e*x^3)/(4*a) + (d*x)/(4*a))/(a + c*x^4) - 2*atanh((c^2*e^2*x*((e^2*(-a^7*
c^3)^(1/2))/(256*a^6*c^3) - (9*d^2*(-a^7*c^3)^(1/2))/(256*a^7*c^2) - (3*d*e
)/(128*a^3*c))^(1/2))/(2*((c*e^3)/(32*a) - (9*c^2*d^2*e)/(32*a^2) - (27*c*d
^3*(-a^7*c^3)^(1/2))/(32*a^6) + (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^5))) - (9*
c^3*d^2*x*((e^2*(-a^7*c^3)^(1/2))/(256*a^6*c^3) - (9*d^2*(-a^7*c^3)^(1/2))/
(256*a^7*c^2) - (3*d*e)/(128*a^3*c))^(1/2))/(2*((c*e^3)/32 - (9*c^2*d^2*e)/
(32*a) - (27*c*d^3*(-a^7*c^3)^(1/2))/(32*a^5) + (3*d*e^2*(-a^7*c^3)^(1/2))/
(32*a^4))))*(-(9*c*d^2*(-a^7*c^3)^(1/2) - a*e^2*(-a^7*c^3)^(1/2) + 6*a^4*c^
2*d*e)/(256*a^7*c^3))^(1/2) - 2*atanh((c^2*e^2*x*((9*d^2*(-a^7*c^3)^(1/2))/
(256*a^7*c^2) - (3*d*e)/(128*a^3*c) - (e^2*(-a^7*c^3)^(1/2))/(256*a^6*c^3))
^(1/2))/(2*((c*e^3)/(32*a) - (9*c^2*d^2*e)/(32*a^2) + (27*c*d^3*(-a^7*c^3)^
(1/2))/(32*a^6) - (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^5))) - (9*c^3*d^2*x*((9*
d^2*(-a^7*c^3)^(1/2))/(256*a^7*c^2) - (3*d*e)/(128*a^3*c) - (e^2*(-a^7*c^3)
^(1/2))/(256*a^6*c^3))^(1/2))/(2*((c*e^3)/32 - (9*c^2*d^2*e)/(32*a) + (27*c
*d^3*(-a^7*c^3)^(1/2))/(32*a^5) - (3*d*e^2*(-a^7*c^3)^(1/2))/(32*a^4))))*(-
(a*e^2*(-a^7*c^3)^(1/2) - 9*c*d^2*(-a^7*c^3)^(1/2) + 6*a^4*c^2*d*e)/(256*a^
7*c^3))^(1/2)
```

3.147 $\int \frac{1}{(a+cx^4)^2} dx$

Optimal. Leaf size=202

$$\frac{x}{4a(a+cx^4)}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}$$

```
[Out] 1/4*x/a/(c*x^4+a)+3/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)
*2^(1/2)+3/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/c^(1/4)*2^(1/2)-3
/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*2^(1
/2)+3/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))/a^(7/4)/c^(1/4)*
2^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {205, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{c}}-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{7/4}\sqrt[4]{c}}+\frac{x}{4a(a+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-2),x]
```

```
[Out] x/(4*a*(a + c*x^4)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2
]*a^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]
*a^(7/4)*c^(1/4)) - (3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(16*Sqrt[2]*a^(7/4)*c^(1/4)) + (3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*c^(1/4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(a+cx^4)^2}\,dx = \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{1}{a+cx^4}\,dx}{4a}$$

$$= \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{\sqrt{a}-\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}} + \frac{3\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{a+cx^4}\,dx}{8a^{3/2}}$$

$$= \frac{x}{4a(a+cx^4)} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} + \frac{3\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{16a^{3/2}\sqrt{c}} - \frac{3\int \frac{\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}}{}}}{16\sqrt{2}\,a}$$

$$= \frac{x}{4a(a+cx^4)} - \frac{3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\right.}{16\sqrt{2}\,a^{7/4}\sqrt[4]{c}}$$

$$= \frac{x}{4a(a+cx^4)} - \frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} + \frac{3\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{7/4}\sqrt[4]{c}} - \frac{3\log\left(\sqrt{a}-\sqrt{\right.}}{16\sqrt{}}$$

Mathematica [A]
time = 0.07, size = 183, normalized size = 0.91

$$\frac{\frac{8a^{3/4}x}{a+cx^4} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} + \frac{6\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{c}} - \frac{3\sqrt{2}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}} + \frac{3\sqrt{2}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{c}}}{32a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-2),x]
```

```
[Out] ((8*a^(3/4)*x)/(a + c*x^4) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1
/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/c^(1/4)
 - (3*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(1/
4) + (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/c^(
1/4))/(32*a^(7/4))
```

Maple [A]
time = 0.14, size = 118, normalized size = 0.58

method	result	size
risch	$\frac{x}{4a(c\,x^4+a)} + \frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+a\right)} \frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16ac}$	46

default	$\frac{x}{4a(c\,x^4+a)}+\frac{3\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{32a^2}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/a/(c*x^4+a)+3/32/a^2*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/
2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(
a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))
```

Maxima [A]
time = 0.51, size = 189, normalized size = 0.94

$$\frac{x}{4\,(acx^4+a^2)}+\frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}}+\frac{\sqrt{2}\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{1}{4}}}\right)}{32\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(a*c*x^4 + a^2) + 3/32*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + s
qrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c
))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/
sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))) + sqrt(2)*log(sqrt(c
)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)) - sqrt(2)*lo
g(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(1/4)))/a
```

Fricas [A]
time = 0.34, size = 173, normalized size = 0.86

$$\frac{12\,(acx^4+a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\arctan\left(-a^5cx\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}+\sqrt{a^4\sqrt{-\frac{1}{a^7c}}+x^2}\,a^5c\left(-\frac{1}{a^7c}\right)^{\frac{3}{4}}\right)+3\,(acx^4+a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)-3\,(acx^4+a^2)\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}\log\left(-a^2\left(-\frac{1}{a^7c}\right)^{\frac{1}{4}}+x\right)+4\,x}{16\,(acx^4+a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(12*(a*c*x^4 + a^2)*(-1/(a^7*c))^(1/4)*arctan(-a^5*c*x*(-1/(a^7*c))^(3
/4) + sqrt(a^4*sqrt(-1/(a^7*c)) + x^2)*a^5*c*(-1/(a^7*c))^(3/4)) + 3*(a*c*x
^4 + a^2)*(-1/(a^7*c))^(1/4)*log(a^2*(-1/(a^7*c))^(1/4) + x) - 3*(a*c*x^4 +
 a^2)*(-1/(a^7*c))^(1/4)*log(-a^2*(-1/(a^7*c))^(1/4) + x) + 4*x)/(a*c*x^4 +
 a^2)
```

Sympy [A]
time = 0.13, size = 39, normalized size = 0.19

$$\frac{x}{4a^2+4acx^4}+\mathrm{RootSum}\left(65536t^4a^7c+81,\left(t\mapsto t\log\left(\frac{16ta^2}{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**2,x)
```

```
[Out] x/(4*a**2 + 4*a*c*x**4) + RootSum(65536*_t**4*a**7*c + 81, Lambda(_t, _t*lo
g(16*_t*a**2/3 + x)))
```

Giac [A]
time = 3.97, size = 194, normalized size = 0.96

$$\frac{x}{4\,(cx^4+a)a}+\frac{3\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c}+\frac{3\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\,\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{16\,a^2c}+\frac{3\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}-\frac{3\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/((c*x^4 + a)*a) + 3/16*sqrt(2)*(a*c^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x
+ sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/16*sqrt(2)*(a*c^3)^(1/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(a^2*c) + 3/32*sq
rt(2)*(a*c^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c) -
3/32*sqrt(2)*(a*c^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^
2*c)
```

Mupad [B]
time = 0.08, size = 58, normalized size = 0.29

$$\frac{x}{4\,a\,\left(c\,x^4+a\right)}+\frac{3\,\mathrm{atan}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)}{8\,{(-a)}^{7/4}\,c^{1/4}}+\frac{3\,\mathrm{atanh}\left(\frac{c^{1/4}\,x}{{(-a)}^{1/4}}\right)}{8\,{(-a)}^{7/4}\,c^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^2,x)
```

```
[Out] x/(4*a*(a + c*x^4)) + (3*atan((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4
)) + (3*atanh((c^(1/4)*x)/(-a)^(1/4)))/(8*(-a)^(7/4)*c^(1/4))
```

3.148 $\int \frac{1}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=689

$$\frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2}-\frac{\sqrt[4]{c}\,e^2(\sqrt{c}\,d-\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(cd^2+ae^2)^2}-\frac{\sqrt[4]{c}\,(3\sqrt{c}\,d-}{8}$$

```
[Out] 1/4*c*x*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/4*c^(1/4)*e^2*arctan(-1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/
2)+1/4*c^(1/4)*e^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2
))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*e^2*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^
(1/2)+1/8*c^(1/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*
a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/16*c^(1/4)*arctan(-1+c
^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^
(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1
/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/
2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2
)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+e^(7/2)*arctan(x*e^(1/2)/d^(1/
2))/(a*e^2+c*d^2)^2/d^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1253, 211, 1193, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{c}\,e^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)(\sqrt{c}\,d-\sqrt{a}\,e)}{2\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}\,e^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)(\sqrt{c}\,d-\sqrt{a}\,e)}{2\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)(3\sqrt{c}\,d-\sqrt{a}\,e)}{8\sqrt{2}\,a^{7/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)(3\sqrt{c}\,d-\sqrt{a}\,e)}{8\sqrt{2}\,a^{7/4}(ae^2+cd^2)}-\frac{\sqrt[4]{c}\,e^2(\sqrt{a}\,e+\sqrt{c}\,d)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}\,e^2(\sqrt{a}\,e+\sqrt{c}\,d)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}\,(\sqrt{a}\,e+3\sqrt{c}\,d)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}\,(\sqrt{a}\,e+3\sqrt{c}\,d)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(ae^2+cd^2)}+\frac{e^{7/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2+cd^2)^2}+\frac{cx(d-ex^2)}{4a(a+cx^4)(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
```

```
7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1253

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \int \left(\frac{e^4}{(cd^2+ae^2)^2(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)^2} - \frac{ce^2(-d+ex^2)}{(cd^2+ae^2)^2(a+cx^4)}\right.$$

$$= -\frac{(ce^2)\int \frac{-d+ex^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2} + \frac{e^4\int \frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2} + \frac{c\int \frac{d-ex^2}{(a+cx^4)^2}\,dx}{cd^2+ae^2}$$

$$= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} + \frac{\left(\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)e^2\right)\int \frac{\sqrt{a}\,\sqrt{}}{a+c}}{2(cd^2+ae^2)^2}$$

$$= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} + \frac{\left(\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)e^2\right)\int \frac{}{\frac{\sqrt{a}}{\sqrt{c}}-}}{4(cd^2+ae^2)^2}$$

$$= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\right.}{4\sqrt{2}\,a^{3/4}}$$

$$= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}}{2\sqrt{2}\,a^{3/4}(cd^2+}$$

$$= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}}{2\sqrt{2}\,a^{3/4}(cd^2+}$$

Mathematica [A]
time = 0.19, size = 429, normalized size = 0.62

$$\frac{\frac{8c(cd^2+ae^2)x(d-ex^2)}{a(a+cx^4)} + \frac{32e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\sqrt[4]{c}\left(-3c^{3/2}d^3+\sqrt{a}\,cd^2e-7a\sqrt{c}\,de^2+5a^{3/2}e^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}} - \frac{2\sqrt{2}\sqrt[4]{c}\left(-3c^{3/2}d^3+\sqrt{a}\,cd^2e-7a\sqrt{c}\,de^2+5a^{3/2}e^3\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}} - \frac{\sqrt{2}\sqrt[4]{c}\left(3c^{3/2}d^3+\sqrt{a}\,cd^2e+7a\sqrt{c}\,de^2+5a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{7/4}} + \frac{\sqrt{2}\sqrt[4]{c}\left(3c^{3/2}d^3+\sqrt{a}\,cd^2e+7a\sqrt{c}\,de^2+5a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{7/4}}}{32(cd^2+ae^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]
*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e
- 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c
```

```
]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqr
t[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A]
time = 0.20, size = 334, normalized size = 0.48

method	result
default	$\frac{e^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(a e^2+c d^2)^2 \sqrt{de}} + \frac{c\left(\frac{-\frac{e(a e^2+c d^2)x^3}{4a}+\frac{d(a e^2+c d^2)x}{4a}}{c x^4+a} + \frac{\frac{(7d e^2 a+3c d^3)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\right.\right.}{8a}}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+c/(a*e^2+c*d^2)^2*(
(-1/4*e*(a*e^2+c*d^2)/a*x^3+1/4*d*(a*e^2+c*d^2)/a*x)/(c*x^4+a)+1/4/a*(1/8*(
7*a*d*e^2+3*c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/
c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(-5*a*e^3-c*d^2*e)/c/(a/c)
^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)
*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)
/(a/c)^(1/4)*x-1))))
```

Maxima [A]
time = 0.52, size = 489, normalized size = 0.71

$$\frac{c\left(\frac{2\sqrt{2}\left(3c^{\frac{3}{2}}d^3-\sqrt{a}cd^2e+7a\sqrt{c}de^2-5a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(3c^{\frac{3}{2}}d^3-\sqrt{a}cd^2e+7a\sqrt{c}de^2-5a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(3c^{\frac{3}{2}}d^3+\sqrt{a}cd^2e+7a\sqrt{c}de^2+5a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3c^{\frac{3}{2}}d^3+\sqrt{a}cd^2e+7a\sqrt{c}de^2+5a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{32\left(ac^2d^4+2a^2cd^2e^2+a^3e^4\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{\left(c^2d^4+2acd^2e^2+a^2e^4\right)\sqrt{d}}-\frac{(c^2d^2e+ace^3)x^3-(c^2d^3+acde^2)x}{4\left(a^2c^2d^4+2a^3cd^2e^2+(ac^3d^4+2a^2c^2d^2e^2+a^3ce^4)x^4+a^4e^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/32*c*(2*sqrt(2)*(3*c^(3/2)*d^3 - sqrt(a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 - 5*
a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*
c^(3/2)*d^3 - sqrt(a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 - 5*a^(3/2)*e^3)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(
```

```
sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*c^(3/2)*d^3 + sqrt(a)*c
*d^2*e + 7*a*sqrt(c)*d*e^2 + 5*a^(3/2)*e^3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/
4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*c^(3/2)*d^3 + sqrt(a
)*c*d^2*e + 7*a*sqrt(c)*d*e^2 + 5*a^(3/2)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
+ a^3*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4)*sqrt(d)) - 1/4*((c^2*d^2*e + a*c*e^3)*x^3 - (c^2*d^3 + a*c*d*e^2)*
x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*
e^4)*x^4 + a^4*e^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4630 vs. $2(511) = 1022$.
time = 11.87, size = 9294, normalized size = 13.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*c^2*d^2*x^3*e - 4*c^2*d^3*x + 4*a*c*x^3*e^3 - 4*a*c*d*x*e^2 - 8*(
a*c*x^4 + a^2)*sqrt(-e/d)*e^3*log((x^2*e + 2*d*x*sqrt(-e/d) - d)/(x^2*e + d
)) + (a*c^3*d^4*x^4 + a^2*c^2*d^4 + (a^3*c*x^4 + a^4)*e^4 + 2*(a^2*c^2*d^2*
x^4 + a^3*c*d^2)*e^2)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5
 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
 a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 +
2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^
6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^1
0*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d
^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2
 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-81*c^5*d^8*x - 594*
a*c^4*d^6*x*e^2 - 1376*a^2*c^3*d^4*x*e^4 - 750*a^3*c^2*d^2*x*e^6 + 625*a^4*
c*x*e^8 + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198
*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 +
26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*
a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*
c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*
```

```
c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a*c^3*d^4
*x^4 + a^2*c^2*d^4 + (a^3*c*x^4 + a^4)*e^4 + 2*(a^2*c^2*d^2*x^4 + a^3*c*d^2
)*e^2)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(
-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^
6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*
c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6
+ 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^1
4*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^
4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-81*c^5*d^8*x - 594*a*c^4*d^6*x*e^2
 - 1376*a^2*c^3*d^4*x*e^4 - 750*a^3*c^2*d^2*x*e^6 + 625*a^4*c*x*e^8 - (27*a
^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^
6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*
e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d
^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 52
9*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 +
 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*
c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^
14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (
a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7
*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748
*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*
e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^
5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e
^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a*c^3*d^4*x^4 + a^2*c^2*
d^4 + (a^3*c*x^4 + a^4)*e^4 + 2*(a^2*c^2*d^2*x^4 + a^3*c*d^2)*e^2)*sqrt((6*
c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 +
 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4
*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^
8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d
^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 +
a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c
*d^2*e^6 + a^7*e^8))*log(-81*c^5*d^8*x - 594*a*c^4*d^6*x*e^2 - 1376*a^2*c^3
*d^4*x*e^4 - 750*a^3*c^2*d^2*x*e^6 + 625*a^4*c*x*e^8 + (27*a^2*c^5*d^9 + 18
6*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d
*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^
2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6
*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e
^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.93, size = 603, normalized size = 0.88

$$\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-\left(ac^3\right)^{\frac{3}{4}}cd^2e-5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}+\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2-\left(ac^3\right)^{\frac{3}{4}}cd^2e-5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}+\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2+\left(ac^3\right)^{\frac{3}{4}}cd^2e+5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}-\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}c^3d^3+7\left(ac^3\right)^{\frac{1}{4}}ac^2de^2+\left(ac^3\right)^{\frac{3}{4}}cd^2e+5\left(ac^3\right)^{\frac{3}{4}}ae^3\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{\left(c^2d^4+2acd^2e^2+a^2e^4\right)\sqrt{d}}-\frac{cx^3e-cdx}{4\left(cx^4+a\right)\left(acd^2+a^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*
c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2
)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2
 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3
*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)
^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*
c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c
^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(
x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a
^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/(
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d)) - 1/4*(c*x^3*e - c*d*x)/((c*x^
4 + a)*(a*c*d^2 + a^2*e^2))
```

Mupad [B]
time = 6.78, size = 2500, normalized size = 3.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^
4) - atan((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c
^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8
*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a
^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*
e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c
^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^
(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a
```

^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*
c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*
c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a
^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 +
4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a
^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*
e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(
-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a
^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c
^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e
^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)
))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
+ 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) +
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e
^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 +
21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8
+ 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8
*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(
128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^
2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4
*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c
)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4
*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i - (((((
65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 3
6864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 +
663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4
*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c
^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^
5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c
*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536
*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 3
27680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13
+ 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
+ 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25
*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d
*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(
256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*
c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 +

```
7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 666
88*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c
^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^
5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e
^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) -
25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c
*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))
/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^
9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 ...
```

3.149 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=864

$$\frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}+\frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}+\frac{4c\sqrt{d}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^3}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\left(cd^2+ae^2\right)^2}-\frac{c^{3/4}e^{\cdots}}{\cdots}$$

```
[Out] 1/2*e^4*x/d/(a*e^2+c*d^2)^2/(e*x^2+d)+1/4*c*x*(-2*c*d*e*x^2-a*e^2+c*d^2)/a/
(a*e^2+c*d^2)^2/(c*x^4+a)+1/2*e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*
e^2+c*d^2)^2+1/4*c^(3/4)*e^2*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2-
a*e^2-4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^3*2^(1/2)+1/4*c^(3/4)*e^
2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2-a*e^2-4*d*e*a^(1/2)*c^(1/2))
/a^(3/4)/(a*e^2+c*d^2)^3*2^(1/2)+1/16*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a
^(1/4))*(3*c*d^2-3*a*e^2-2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)^2*2^(
1/2)+1/16*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*c*d^2-3*a*e^2-2*d*
e*a^(1/2)*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/32*c^(3/4)*ln(-a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2-3*a*e^2+2*d*e*a^(1/2)*c^(1
/2))/a^(7/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/32*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1
/2)+a^(1/2)+x^2*c^(1/2))*(3*c*d^2-3*a*e^2+2*d*e*a^(1/2)*c^(1/2))/a^(7/4)/(a
*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*e^2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(3*c*d^2-a*e^2+4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^3
*2^(1/2)+1/8*c^(3/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*
(3*c*d^2-a*e^2+4*d*e*a^(1/2)*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^3*2^(1/2)+4*c*e
^(7/2)*arctan(x*e^(1/2)/d^(1/2))*d^(1/2)/(a*e^2+c*d^2)^3
```

Rubi [A]

time = 0.59, antiderivative size = 864, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1253, 205, 211, 1193, 1182, 1176, 631, 210, 1179, 642}

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 + a*e^2)^2*(d + e*x^2)) + (c*x*(c*d^2 - a*e^2 - 2*c*d*e
*x^2))/(4*a*(c*d^2 + a*e^2)^2*(a + c*x^4)) + (4*c*Sqrt[d]*e^(7/2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^3 + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*d^(3/2)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c
]*d*e - a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^
2) + (c^(3/4)*e^2*(3*c*d^2 - 4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*ArcTan[1 + (Sqr
```

```
t[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*
(3*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*e^2*(3*c*d^2 + 4
*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) - (c^(3/4)*(3*c*d^2 + 2*Sq
rt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2) + (c^(3/4)*e^2*(3*c*d^2 +
4*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^3) + (c^(3/4)*(3*c*d^2 + 2*S
qrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqr
t[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
```

```
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1253

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+cx^4\right)^2}\,dx = \int\left(\frac{e^4}{\left(cd^2+ae^2\right)^2\left(d+ex^2\right)^2} + \frac{4cde^4}{\left(cd^2+ae^2\right)^3\left(d+ex^2\right)} + \frac{c(cd^2-ae^2-2cdex}{\left(cd^2+ae^2\right)^2\left(a+cx^4\right.}\right.$$

$$= -\frac{\left(ce^2\right)\int\frac{-3cd^2+ae^2+4cdex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^3} + \frac{\left(4cde^4\right)\int\frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^3} + \frac{c\int\frac{cd^2-ae^2-2cdex^2}{\left(a+cx^4\right)^2}\,dx}{\left(cd^2+ae^2\right)^2} + \ldots$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}\,e^{7/2}\tan^{-1}\left(\ldots\right.}{\left(cd^2+ae^2\right)^3}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}\,e^{7/2}\tan^{-1}\left(\ldots\right.}{\left(cd^2+ae^2\right)^3}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}\,e^{7/2}\tan^{-1}\left(\ldots\right.}{\left(cd^2+ae^2\right)^3}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}\,e^{7/2}\tan^{-1}\left(\ldots\right.}{\left(cd^2+ae^2\right)^3}$$

$$= \frac{e^4x}{2d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{cx\left(cd^2-ae^2-2cdex^2\right)}{4a\left(cd^2+ae^2\right)^2\left(a+cx^4\right)} + \frac{4c\sqrt{d}\,e^{7/2}\tan^{-1}\left(\ldots\right.}{\left(cd^2+ae^2\right)^3}$$

Mathematica [A]
time = 0.37, size = 540, normalized size = 0.62

$$\frac{\frac{16e^4\left(cd^2+ae^2\right)x}{d\left(d+ex^2\right)} + \frac{8c\left(cd^2+ae^2\right)x\left(-ae^2+cd\left(d-2ex^2\right)\right)}{a\left(a+cx^4\right)} + \frac{16e^{7/2}\left(9cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}} + \frac{2\sqrt{2}\,c^{3/4}\left(-3c^2d^4+2\sqrt{a}\,c^{3/2}d^3e-12acd^2e^2+18a^{3/2}\sqrt{c}\,de^3+7a^2e^4\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}} - \frac{2\sqrt{2}\,c^{3/4}\left(-3c^2d^4+2\sqrt{a}\,c^{3/2}d^3e-12acd^2e^2+18a^{3/2}\sqrt{c}\,de^3+7a^2e^4\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}} - \frac{\sqrt{2}\,c^{3/4}\left(3c^2d^4+2\sqrt{a}\,c^{3/2}d^3e+12acd^2e^2+18a^{3/2}\sqrt{c}\,de^3-7a^2e^4\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{7/4}} + \frac{\sqrt{2}\,c^{3/4}\left(3c^2d^4+2\sqrt{a}\,c^{3/2}d^3e+12acd^2e^2+18a^{3/2}\sqrt{c}\,de^3-7a^2e^4\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{7/4}}}{32\left(cd^2+ae^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + c*x^4)^2),x]
```

```
[Out] ((16*e^4*(c*d^2 + a*e^2)*x)/(d*(d + e*x^2)) + (8*c*(c*d^2 + a*e^2)*x*(-(a*e
^2) + c*d*(d - 2*e*x^2)))/(a*(a + c*x^4)) + (16*e^(7/2)*(9*c*d^2 + a*e^2)*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + (2*Sqrt[2]*c^(3/4)*(-3*c^2*d^4 + 2*Sq
rt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 + 7*a^2*e^4
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(3/4)*(-3
*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e - 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*
e^3 + 7*a^2*e^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/a^(7/4) - (Sqrt[2
```

```
]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e + 12*a*c*d^2*e^2 + 18*a^(3/2
)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/a^(7/4) + (Sqrt[2]*c^(3/4)*(3*c^2*d^4 + 2*Sqrt[a]*c^(3/2)*d^3*e +
 12*a*c*d^2*e^2 + 18*a^(3/2)*Sqrt[c]*d*e^3 - 7*a^2*e^4)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^3)
```

Maple [A]
time = 0.25, size = 402, normalized size = 0.47

method	result
default	$\frac{e^{4}\left(\frac{\left(a\,e^{2}+c\,d^{2}\right)x}{2d\left(e\,x^{2}+d\right)}+\frac{\left(a\,e^{2}+9c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)^{3}}-c\left(\frac{\frac{cde\left(a\,e^{2}+c\,d^{2}\right)x^{3}}{2a}+\frac{\left(a^{2}e^{4}-c^{2}d^{4}\right)x}{4a}}{c\,x^{4}+a}+\frac{\left(7a^{2}e^{4}-12ac\,d^{2}e^{2}-3c^{2}d^{4}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\right.}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e^4/(a*e^2+c*d^2)^3*(1/2*(a*e^2+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+9*c*d^2)/d/
(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-c/(a*e^2+c*d^2)^3*((1/2*c*d*e*(a*e^2+c
*d^2)/a*x^3+1/4*(a^2*e^4-c^2*d^4)/a*x)/(c*x^4+a)+1/4/a*(1/8*(7*a^2*e^4-12*a
*c*d^2*e^2-3*c^2*d^4)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+
(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c
)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(18*a*c*d*e^3+2*c^2*d^3
*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x-1))))
```

Maxima [A]
time = 0.54, size = 705, normalized size = 0.82

$$\frac{\left(9\,cd^{2}e^{4}+ae^{6}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\left(c^{3}d^{7}+3\,ac^{2}d^{5}e^{2}+3\,a^{2}cd^{3}e^{4}+a^{3}de^{6}\right)\sqrt{d}}+\frac{c\left(\frac{2\sqrt{2}\left(3\,c^{\frac{5}{2}}d^{4}-2\sqrt{a}\,c^{2}d^{3}e+12\,ac^{\frac{3}{2}}d^{2}e^{2}-18\,a^{\frac{3}{2}}cde^{3}-7\,a^{2}\sqrt{c}\,e^{4}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(3\,c^{\frac{5}{2}}d^{4}-2\sqrt{a}\,c^{2}d^{3}e+12\,ac^{\frac{3}{2}}d^{2}e^{2}-18\,a^{\frac{3}{2}}cde^{3}-7\,a^{2}\sqrt{c}\,e^{4}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(3\,c^{\frac{5}{2}}d^{4}+2\sqrt{a}\,c^{2}d^{3}e+12\,ac^{\frac{3}{2}}d^{2}e^{2}+18\,a^{\frac{3}{2}}cde^{3}-7\,a^{2}\sqrt{c}\,e^{4}\right)\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(3\,c^{\frac{5}{2}}d^{4}+2\sqrt{a}\,c^{2}d^{3}e+12\,ac^{\frac{3}{2}}d^{2}e^{2}+18\,a^{\frac{3}{2}}cde^{3}-7\,a^{2}\sqrt{c}\,e^{4}\right)\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{32\left(ac^{3}d^{6}+3\,a^{2}c^{2}d^{4}e^{2}+3\,a^{3}cd^{2}e^{4}+a^{4}e^{6}\right)}-\frac{2\left(c^{2}d^{2}e^{2}-ace^{4}\right)x^{5}+\left(c^{2}d^{3}e+acde^{3}\right)x^{3}-\left(c^{2}d^{4}-acd^{2}e^{2}+2\,a^{2}e^{4}\right)x}{4\left(a^{2}c^{2}d^{6}+2\,a^{3}cd^{4}e^{2}+\left(ac^{3}d^{5}e+2\,a^{2}c^{2}d^{3}e^{3}+a^{3}cde^{5}\right)x^{6}+a^{4}d^{2}e^{4}+\left(ac^{3}d^{6}+2\,a^{2}c^{2}d^{4}e^{2}+a^{3}cd^{2}e^{4}\right)x^{4}+\left(a^{2}c^{2}d^{5}e+2\,a^{3}cd^{3}e^{3}+a^{4}de^{5}\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(9*c*d^2*e^4 + a*e^6)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^3*d^7 + 3*
a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*sqrt(d)) + 1/32*c*(2*sqrt(2)*(
3*c^(5/2)*d^4 - 2*sqrt(a)*c^2*d^3*e + 12*a*c^(3/2)*d^2*e^2 - 18*a^(3/2)*c*d
*e^3 - 7*a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)
```

```
*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) +
2*sqrt(2)*(3*c^(5/2)*d^4 - 2*sqrt(a)*c^2*d^3*e + 12*a*c^(3/2)*d^2*e^2 - 18*
a^(3/2)*c*d*e^3 - 7*a^2*sqrt(c)*e^4)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt
(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*
sqrt(c)) + sqrt(2)*(3*c^(5/2)*d^4 + 2*sqrt(a)*c^2*d^3*e + 12*a*c^(3/2)*d^2*
e^2 + 18*a^(3/2)*c*d*e^3 - 7*a^2*sqrt(c)*e^4)*log(sqrt(c)*x^2 + sqrt(2)*a^(
1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*c^(5/2)*d^4 + 2*sq
rt(a)*c^2*d^3*e + 12*a*c^(3/2)*d^2*e^2 + 18*a^(3/2)*c*d*e^3 - 7*a^2*sqrt(c)
*e^4)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/
4)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6) - 1/4*(2*(
c^2*d^2*e^2 - a*c*e^4)*x^5 + (c^2*d^3*e + a*c*d*e^3)*x^3 - (c^2*d^4 - a*c*d
^2*e^2 + 2*a^2*e^4)*x)/(a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + (a*c^3*d^5*e + 2*a^
2*c^2*d^3*e^3 + a^3*c*d*e^5)*x^6 + a^4*d^2*e^4 + (a*c^3*d^6 + 2*a^2*c^2*d^4
*e^2 + a^3*c*d^2*e^4)*x^4 + (a^2*c^2*d^5*e + 2*a^3*c*d^3*e^3 + a^4*d*e^5)*x
^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7251 vs. $2(660) = 1320$.
time = 97.76, size = 14534, normalized size = 16.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*c^3*d^4*x^5*e^2 + 4*c^3*d^5*x^3*e - 4*c^3*d^6*x + 8*a*c^2*d^3*x^3
*e^3 + 4*a^2*c*d*x^3*e^5 - 4*a^2*c*d^2*x*e^4 - (a*c^4*d^8*x^4 + a^2*c^3*d^8
 + (a^4*c*d*x^6 + a^5*d*x^2)*e^7 + (a^4*c*d^2*x^4 + a^5*d^2)*e^6 + 3*(a^3*c
^2*d^3*x^6 + a^4*c*d^3*x^2)*e^5 + 3*(a^3*c^2*d^4*x^4 + a^4*c*d^4)*e^4 + 3*(
a^2*c^3*d^5*x^6 + a^3*c^2*d^5*x^2)*e^3 + 3*(a^2*c^3*d^6*x^4 + a^3*c^2*d^6)*
e^2 + (a*c^4*d^7*x^6 + a^2*c^3*d^7*x^2)*e)*sqrt((12*c^5*d^7*e + 156*a*c^4*d
^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*e^7 + (a^3*c^6*d^12 + 6*a^4*c^
5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 +
 6*a^8*c*d^2*e^10 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 +
5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 228
56*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 24
01*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e
^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10
 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16
+ 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*
e^24)))/(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^
3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))*log(81*c^7*d
^10*x + 1053*a*c^6*d^8*x*e^2 + 3602*a^2*c^5*d^6*x*e^4 - 2958*a^3*c^4*d^4*x*
e^6 - 23667*a^4*c^3*d^2*x*e^8 + 2401*a^5*c^2*x*e^10 + (27*a^2*c^7*d^12 + 31
2*a^3*c^6*d^10*e^2 + 843*a^4*c^5*d^8*e^4 - 1592*a^5*c^4*d^6*e^6 - 5967*a^6*
```

c^3*d^4*e^8 + 4032*a^7*c^2*d^2*e^10 - 343*a^8*c*e^12 + 2*(a^6*c^7*d^15*e + 15*a^7*c^6*d^13*e^3 + 69*a^8*c^5*d^11*e^5 + 155*a^9*c^4*d^9*e^7 + 195*a^10*c^3*d^7*e^9 + 141*a^11*c^2*d^5*e^11 + 55*a^12*c*d^3*e^13 + 9*a^13*d*e^15)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))*sqrt((12*c^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*e^7 + (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))) + (a*c^4*d^8*x^4 + a^2*c^3*d^8 + (a^4*c*d*x^6 + a^5*d*x^2)*e^7 + (a^4*c*d^2*x^4 + a^5*d^2)*e^6 + 3*(a^3*c^2*d^3*x^6 + a^4*c*d^3*x^2)*e^5 + 3*(a^3*c^2*d^4*x^4 + a^4*c*d^4)*e^4 + 3*(a^2*c^3*d^5*x^6 + a^3*c^2*d^5*x^2)*e^3 + 3*(a^2*c^3*d^6*x^4 + a^3*c^2*d^6)*e^2 + (a*c^4*d^7*x^6 + a^2*c^3*d^7*x^2)*e)*sqrt((12*c^5*d^7*e + 156*a*c^4*d^5*e^3 + 404*a^2*c^3*d^3*e^5 - 252*a^3*c^2*d*e^7 + (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4 + 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8 + 792*a^12*c^7*d^14*e^10 + 924*a^13*c^6*d^12*e^12 + 792*a^14*c^5*d^10*e^14 + 495*a^15*c^4*d^8*e^16 + 220*a^16*c^3*d^6*e^18 + 66*a^17*c^2*d^4*e^20 + 12*a^18*c*d^2*e^22 + a^19*e^24)))/(a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^10 + a^9*e^12))*log(81*c^7*d^10*x + 1053*a*c^6*d^8*x*e^2 + 3602*a^2*c^5*d^6*x*e^4 - 2958*a^3*c^4*d^4*x*e^6 - 23667*a^4*c^3*d^2*x*e^8 + 2401*a^5*c^2*x*e^10 - (27*a^2*c^7*d^12 + 312*a^3*c^6*d^10*e^2 + 843*a^4*c^5*d^8*e^4 - 1592*a^5*c^4*d^6*e^6 - 5967*a^6*c^3*d^4*e^8 + 4032*a^7*c^2*d^2*e^10 - 343*a^8*c*e^12 + 2*(a^6*c^7*d^15*e + 15*a^7*c^6*d^13*e^3 + 69*a^8*c^5*d^11*e^5 + 155*a^9*c^4*d^9*e^7 + 195*a^10*c^3*d^7*e^9 + 141*a^11*c^2*d^5*e^11 + 55*a^12*c*d^3*e^13 + 9*a^13*d*e^15)*sqrt(-(81*c^11*d^16 + 1224*a*c^10*d^14*e^2 + 5164*a^2*c^9*d^12*e^4 - 4776*a^3*c^8*d^10*e^6 - 65130*a^4*c^7*d^8*e^8 - 22856*a^5*c^6*d^6*e^10 + 245004*a^6*c^5*d^4*e^12 - 48216*a^7*c^4*d^2*e^14 + 2401*

```
a^8*c^3*e^16)/(a^7*c^12*d^24 + 12*a^8*c^11*d^22*e^2 + 66*a^9*c^10*d^20*e^4
+ 220*a^10*c^9*d^18*e^6 + 495*a^11*c^8*d^16*e^8...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.30, size = 855, normalized size = 0.99

$$\frac{(9\,cd^2e^4+ae^6)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,(c^3d^7+3\,ac^2d^5e^2+3\,a^2cd^3e^4+a^3de^6)\sqrt{d}}+\frac{\left(3\,(ac^3)^{\frac{1}{4}}c^3d^4+12\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2-2\,(ac^3)^{\frac{3}{4}}cd^3e-7\,(ac^3)^{\frac{1}{4}}a^2ce^4-18\,(ac^3)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^2c^4d^6+3\,\sqrt{2}\,a^3c^3d^4e^2+3\,\sqrt{2}\,a^4c^2d^2e^4+\sqrt{2}\,a^5ce^6\right)}+\frac{\left(3\,(ac^3)^{\frac{1}{4}}c^3d^4+12\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2-2\,(ac^3)^{\frac{3}{4}}cd^3e-7\,(ac^3)^{\frac{1}{4}}a^2ce^4-18\,(ac^3)^{\frac{3}{4}}ade^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^2c^4d^6+3\,\sqrt{2}\,a^3c^3d^4e^2+3\,\sqrt{2}\,a^4c^2d^2e^4+\sqrt{2}\,a^5ce^6\right)}+\frac{\left(3\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}c^3d^4+12\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2+2\,\sqrt{2}\,(ac^3)^{\frac{3}{4}}cd^3e-7\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}a^2ce^4+18\,\sqrt{2}\,(ac^3)^{\frac{3}{4}}ade^3\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,(a^2c^4d^6+3\,a^3c^3d^4e^2+3\,a^4c^2d^2e^4+a^5ce^6)}-\frac{\left(3\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}c^3d^4+12\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}ac^2d^2e^2+2\,\sqrt{2}\,(ac^3)^{\frac{3}{4}}cd^3e-7\,\sqrt{2}\,(ac^3)^{\frac{1}{4}}a^2ce^4+18\,\sqrt{2}\,(ac^3)^{\frac{3}{4}}ade^3\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{32\,(a^2c^4d^6+3\,a^3c^3d^4e^2+3\,a^4c^2d^2e^4+a^5ce^6)}-\frac{2\,c^2d^2x^5e^2+c^2d^3x^3e-2\,acx^5e^4-c^2d^4x+acdx^3e^3+acd^2xe^2-2\,a^2xe^4}{4\,(ac^2d^5+2\,a^2cd^3e^2+a^3de^4)(cx^6e+cdx^4+ax^2e+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*c*d^2*e^4 + a*e^6)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^3*d^7 + 3*
a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*sqrt(d)) + 1/8*(3*(a*c^3)^(1/4
)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 2*(a*c^3)^(3/4)*c*d^3*e - 7*(a
*c^3)^(1/4)*a^2*c*e^4 - 18*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sqrt(2)*(2*x +
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^6 + 3*sqrt(2)*a^3*c^3
*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6) + 1/8*(3*(a*c^3)^
(1/4)*c^3*d^4 + 12*(a*c^3)^(1/4)*a*c^2*d^2*e^2 - 2*(a*c^3)^(3/4)*c*d^3*e -
7*(a*c^3)^(1/4)*a^2*c*e^4 - 18*(a*c^3)^(3/4)*a*d*e^3)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^6 + 3*sqrt(2)*a^3
*c^3*d^4*e^2 + 3*sqrt(2)*a^4*c^2*d^2*e^4 + sqrt(2)*a^5*c*e^6) + 1/32*(3*sqr
t(2)*(a*c^3)^(1/4)*c^3*d^4 + 12*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 2*sqr
t(2)*(a*c^3)^(3/4)*c*d^3*e - 7*sqrt(2)*(a*c^3)^(1/4)*a^2*c*e^4 + 18*sqrt(2)
*(a*c^3)^(3/4)*a*d*e^3)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c
^4*d^6 + 3*a^3*c^3*d^4*e^2 + 3*a^4*c^2*d^2*e^4 + a^5*c*e^6) - 1/32*(3*sqrt(
2)*(a*c^3)^(1/4)*c^3*d^4 + 12*sqrt(2)*(a*c^3)^(1/4)*a*c^2*d^2*e^2 + 2*sqrt(
2)*(a*c^3)^(3/4)*c*d^3*e - 7*sqrt(2)*(a*c^3)^(1/4)*a^2*c*e^4 + 18*sqrt(2)*(
a*c^3)^(3/4)*a*d*e^3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(a^2*c^4
*d^6 + 3*a^3*c^3*d^4*e^2 + 3*a^4*c^2*d^2*e^4 + a^5*c*e^6) - 1/4*(2*c^2*d^2*
x^5*e^2 + c^2*d^3*x^3*e - 2*a*c*x^5*e^4 - c^2*d^4*x + a*c*d*x^3*e^3 + a*c*d
^2*x*e^2 - 2*a^2*x*e^4)/((a*c^2*d^5 + 2*a^2*c*d^3*e^2 + a^3*d*e^4)*(c*x^6*e
 + c*d*x^4 + a*x^2*e + a*d))
```

Mupad [**B**]
time = 8.33, size = 2500, normalized size = 2.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x^2)^2),x)
```

```
[Out] ((x*(2*a^2*e^4 + c^2*d^4 - a*c*d^2*e^2))/(4*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c*
d^2*e^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)) + (c*e^2*x^5*(a*e^2 - c*d^2))/(
2*a*d*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)))/(a*d + a*e*x^2 + c*d*x^4 + c*e*
x^6) + atan(((((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*
d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5
*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254
784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^
2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*
a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*
d^6*e^12)) - (((65536*a^15*c^4*d*e^24 - 24576*a^4*c^15*d^23*e^2 - 212992*a^
5*c^14*d^21*e^4 - 352256*a^6*c^13*d^19*e^6 + 1966080*a^7*c^12*d^17*e^8 + 10
960896*a^8*c^11*d^15*e^10 + 25460736*a^9*c^10*d^13*e^12 + 34750464*a^10*c^9
*d^11*e^14 + 30081024*a^11*c^8*d^9*e^16 + 16588800*a^12*c^7*d^7*e^18 + 5554
176*a^13*c^6*d^5*e^20 + 991232*a^14*c^5*d^3*e^22)/(512*(a^4*c^8*d^18 + a^12
*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 +
56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c
^2*d^6*e^12)) - (x*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1
/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*
c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c
^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*
d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^1
0*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*(65536*a^6*c^15*d^24*e^3 + 589
824*a^7*c^14*d^22*e^5 + 2293760*a^8*c^13*d^20*e^7 + 4915200*a^9*c^12*d^18*e
^9 + 5898240*a^10*c^11*d^16*e^11 + 2752512*a^11*c^10*d^14*e^13 - 2752512*a^
12*c^9*d^12*e^15 - 5898240*a^13*c^8*d^10*e^17 - 4915200*a^14*c^7*d^8*e^19 -
 2293760*a^15*c^6*d^6*e^21 - 589824*a^16*c^5*d^4*e^23 - 65536*a^17*c^4*d^2*
e^25))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d
^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 +
 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/
2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 15
6*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2)
 - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2)
)/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 +
 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) -
(x*(4096*a^12*c^5*d*e^22 - 1152*a^2*c^15*d^21*e^2 - 15232*a^3*c^14*d^19*e^4
 - 78336*a^4*c^13*d^17*e^6 - 140800*a^5*c^12*d^15*e^8 + 489728*a^6*c^11*d^1
3*e^10 + 2219776*a^7*c^10*d^11*e^12 + 3155456*a^8*c^9*d^9*e^14 + 1901056*a^
9*c^8*d^7*e^16 + 362368*a^10*c^7*d^5*e^18 - 32640*a^11*c^6*d^3*e^20))/(128*
(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28
*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*
d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d
```

^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2) - (x*(81*c^13*d^14*e^5 - 392*a^7*c^6*e^19 + 1206*a*c^12*d^12*e^7 + 12247*a^2*c^11*d^10*e^9 + 58636*a^3*c^10*d^8*e^11 + 114927*a^4*c^9*d^6*e^13 - 1306*a^5*c^8*d^4*e^15 - 3575*a^6*c^7*d^2*e^17))/(128*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*d^12*e^6 + 70*a^8*c^4*d^10*e^8 + 56*a^9*c^3*d^8*e^10 + 28*a^10*c^2*d^6*e^12)))*(-(49*a^4*e^8*(-a^7*c^3)^(1/2) + 9*c^4*d^8*(-a^7*c^3)^(1/2) - 12*a^4*c^5*d^7*e + 252*a^7*c^2*d*e^7 - 156*a^5*c^4*d^5*e^3 - 404*a^6*c^3*d^3*e^5 + 68*a*c^3*d^6*e^2*(-a^7*c^3)^(1/2) - 492*a^3*c*d^2*e^6*(-a^7*c^3)^(1/2) + 30*a^2*c^2*d^4*e^4*(-a^7*c^3)^(1/2))/(256*(a^13*e^12 + a^7*c^6*d^12 + 6*a^12*c*d^2*e^10 + 6*a^8*c^5*d^10*e^2 + 15*a^9*c^4*d^8*e^4 + 20*a^10*c^3*d^6*e^6 + 15*a^11*c^2*d^4*e^8)))^(1/2)*1i - (((3584*a^10*c^5*e^21 + 1152*a*c^14*d^18*e^3 + 13184*a^2*c^13*d^16*e^5 + 54912*a^3*c^12*d^14*e^7 + 296832*a^4*c^11*d^12*e^9 + 1282432*a^5*c^10*d^10*e^11 + 769152*a^6*c^9*d^8*e^13 - 1421440*a^7*c^8*d^6*e^15 - 1254784*a^8*c^7*d^4*e^17 - 89088*a^9*c^6*d^2*e^19)/(512*(a^4*c^8*d^18 + a^12*d^2*e^16 + 8*a^11*c*d^4*e^14 + 8*a^5*c^7*d^16*e^2 + 28*a^6*c^6*d^14*e^4 + 56*a^7*c^5*...

3.150 $\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=388

$$\frac{e^2(42cd^2-5ae^2)\,x\sqrt{a+cx^4}}{21c^2}+\frac{4de^3x^3\sqrt{a+cx^4}}{5c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}+\frac{4de(5cd^2-3ae^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{4\sqrt[4]{a}\,de(5$$

```
[Out] 1/21*e^2*(-5*a*e^2+42*c*d^2)*x*(c*x^4+a)^(1/2)/c^2+4/5*d*e^3*x^3*(c*x^4+a)^
(1/2)/c+1/7*e^4*x^5*(c*x^4+a)^(1/2)/c+4/5*d*e*(-3*a*e^2+5*c*d^2)*x*(c*x^4+a
)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-4/5*a^(1/4)*d*e*(-3*a*e^2+5*c*d^2)*(c
os(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*E
llipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2)
)*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)+1/210*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(105*c^2*d^4-210*a*
c*d^2*e^2+25*a^2*e^4+420*c^(3/2)*d^3*e*a^(1/2)-252*a^(3/2)*d*e^3*c^(1/2))*(
a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(9
/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 386, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1221, 1902, 1212, 226, 1210}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(5(5a^2e^4-42acd^2e^2+21c^2d^4)+84\sqrt{a}\sqrt{c}\,de(5cd^2-3ae^2)\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{210\sqrt[4]{a}\,c^{9/4}\sqrt{a+cx^4}}-\frac{4\sqrt[4]{a}\,de\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}(5cd^2-3ae^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}+\frac{4dex\sqrt{a+cx^4}\,(5cd^2-3ae^2)}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{e^2x\sqrt{a+cx^4}\,(42cd^2-5ae^2)}{21c^2}+\frac{4de^3x^3\sqrt{a+cx^4}}{5c}+\frac{e^4x^5\sqrt{a+cx^4}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*(42*c*d^2 - 5*a*e^2)*x*Sqrt[a + c*x^4])/(21*c^2) + (4*d*e^3*x^3*Sqrt[a
 + c*x^4])/(5*c) + (e^4*x^5*Sqrt[a + c*x^4])/(7*c) + (4*d*e*(5*c*d^2 - 3*a*
e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (4*a^(1/4)*d*
e*(5*c*d^2 - 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)*Sq
rt[a + c*x^4]) + ((84*Sqrt[a]*Sqrt[c]*d*e*(5*c*d^2 - 3*a*e^2) + 5*(21*c^2*d
^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
210*a^(1/4)*c^(9/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex^2)^4}{\sqrt{a+cx^4}}\,dx = \frac{e^4x^5\sqrt{a+cx^4}}{7c} + \frac{\int \frac{7cd^4+28cd^3ex^2+e^2(42cd^2-5ae^2)x^4+28cde^3x^6}{\sqrt{a+cx^4}}\,dx}{7c}$$

$$= \frac{4de^3x^3\sqrt{a+cx^4}}{5c} + \frac{e^4x^5\sqrt{a+cx^4}}{7c} + \frac{\int \frac{35c^2d^4+28cde(5cd^2-3ae^2)x^2+5ce^2(42cd^2-5ae^2)x^4}{\sqrt{a+cx^4}}\,dx}{35c^2}$$

$$= \frac{e^2(42cd^2-5ae^2)\,x\sqrt{a+cx^4}}{21c^2} + \frac{4de^3x^3\sqrt{a+cx^4}}{5c} + \frac{e^4x^5\sqrt{a+cx^4}}{7c} + \frac{\int \frac{5c(21c^2d^4-42acd^2e^2 \cdots}{\cdots}}{\cdots}$$

$$= \frac{e^2(42cd^2-5ae^2)\,x\sqrt{a+cx^4}}{21c^2} + \frac{4de^3x^3\sqrt{a+cx^4}}{5c} + \frac{e^4x^5\sqrt{a+cx^4}}{7c} - \frac{(4\sqrt{a}\,de(5cd^2-\cdots}{\cdots}$$

$$= \frac{e^2(42cd^2-5ae^2)\,x\sqrt{a+cx^4}}{21c^2} + \frac{4de^3x^3\sqrt{a+cx^4}}{5c} + \frac{e^4x^5\sqrt{a+cx^4}}{7c} + \frac{4de(5cd^2-3ae^2)\cdots}{5c^{3/2}\left(\sqrt{a}+\cdots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.14, size = 203, normalized size = 0.52

$$\frac{5(21c^2d^4-42acd^2e^2+5a^2e^4)\,x\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex\left(-25a^2e^3+2ace(105d^2+42dex^2-5e^2x^4)+3c^2ex^4(70d^2+28dex^2+5e^2x^4)+28cd(5cd^2-3ae^2)\,x^2\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)}{105c^2\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/Sqrt[a + c*x^4],x]
```

```
[Out] (5*(21*c^2*d^4 - 42*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + e*x*(-25*a^2*e^3 + 2*a*c*e*(105*d^
2 + 42*d*e*x^2 - 5*e^2*x^4) + 3*c^2*e*x^4*(70*d^2 + 28*d*e*x^2 + 5*e^2*x^4)
 + 28*c*d*(5*c*d^2 - 3*a*e^2)*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2
, 3/4, 7/4, -((c*x^4)/a)]))/(105*c^2*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 506, normalized size = 1.30

method	result
elliptic	$\frac{e^4x^5\sqrt{c\,x^4+a}}{7c} + \frac{4d\,e^3x^3\sqrt{c\,x^4+a}}{5c} + \frac{\left(6d^2e^2-\frac{5a\,e^4}{7c}\right)x\sqrt{c\,x^4+a}}{3c} + \frac{\left(d^4-\frac{a\left(6d^2e^2-\frac{5a\,e^4}{7c}\right)}{3c}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\cdots}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\cdots}$

risch	$-\frac{e^{2} x \left(-15 e^{2} x^{4} c-84 c d e \,x^{2}+25 a \,e^{2}-210 c \,d^{2}\right) \sqrt{c \,x^{4}+a}}{105 c^{2}}+\frac{-\frac{i \left(252 a c d \,e^{3}-420 c^{2} d^{3} e \right) \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c x^{4}}}}{}$
default	$e^{4} \left(\frac{x^{5} \sqrt{c \,x^{4}+a}}{7 c}-\frac{5 a x \sqrt{c \,x^{4}+a}}{21 c^{2}}+\frac{5 a^{2} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{21 c^{2} \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c \,x^{4}+a}}\right)+4$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^4*(1/7/c*x^5*(c*x^4+a)^(1/2)-5/21*a/c^2*x*(c*x^4+a)^(1/2)+5/21*a^2/c^2/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2
)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+4*d*
e^3*(1/5/c*x^3*(c*x^4+a)^(1/2)-3/5*I*a^(3/2)/c^(3/2)/(I/a^(1/2)*c^(1/2))^(1
/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+
a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c
^(1/2))^(1/2),I)))+6*d^2*e^2*(1/3*x*(c*x^4+a)^(1/2)/c-1/3*a/c/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+4*I*d^3*e*a^(1/
2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^
(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+d^4/(I/a^(1/2)*c^(1/2))^
(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^4/sqrt(c*x^4 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.61, size = 214, normalized size = 0.55

$$\frac{d^4x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{d^3ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{3d^2e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{de^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}+\frac{e^4x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*x**4+a)**(1/2),x)
```

```
[Out] d**4*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + d**3*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4
*exp_polar(I*pi)/a)/(sqrt(a)*gamma(7/4)) + 3*d**2*e**2*x**5*gamma(5/4)*hype
r((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(9/4)) + d*
e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(
sqrt(a)*gamma(11/4)) + e**4*x**9*gamma(9/4)*hyper((1/2, 9/4), (13/4,), c*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(13/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^4/sqrt(c*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^4}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^4/(a + c*x^4)^(1/2), x)
```

3.151 $\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=326

$$\frac{de^2x\sqrt{a+cx^4}}{c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}+\frac{3e(5cd^2-ae^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{3\sqrt[4]{a}\,e(5cd^2-ae^2)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a}{(\sqrt{a}}}}{5c^{7/4}\sqrt{a+}}$$

```
[Out] d*e^2*x*(c*x^4+a)^(1/2)/c+1/5*e^3*x^3*(c*x^4+a)^(1/2)/c+3/5*e*(-a*e^2+5*c*d
^2)*x*(c*x^4+a)^(1/2)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-3/5*a^(1/4)*e*(-a*e^2+5
*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a
^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x
^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/
2)+1/10*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*c^(1/2))*(15*c*d^2*e-3*a*e^3+5*d*(-a*e^2+c*d^2)*c^(1/2)/a^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1221, 1902, 1212, 226, 1210}

$$-\frac{3\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,(5cd^2-ae^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{a+cx^4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{5\sqrt{c}\,d(cd^2-ae^2)}{\sqrt{a}}-3ae^3+15cd^2e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10c^{7/4}\sqrt{a+cx^4}}+\frac{3ex\sqrt{a+cx^4}\,(5cd^2-ae^2)}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{de^2x\sqrt{a+cx^4}}{c}+\frac{e^3x^3\sqrt{a+cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (d*e^2*x*Sqrt[a + c*x^4])/c + (e^3*x^3*Sqrt[a + c*x^4])/(5*c) + (3*e*(5*c*d
^2 - a*e^2)*x*Sqrt[a + c*x^4])/(5*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (3*a^(
1/4)*e*(5*c*d^2 - a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(7/4)
*Sqrt[a + c*x^4]) + (a^(1/4)*(15*c*d^2*e - 3*a*e^3 + (5*Sqrt[c]*d*(c*d^2 -
a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(10*c^(7/4)*Sqrt[a
 + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{\sqrt{a+cx^4}}\,dx = \frac{e^3x^3\sqrt{a+cx^4}}{5c} + \frac{\int \frac{5cd^3+3e(5cd^2-ae^2)x^2+15cde^2x^4}{\sqrt{a+cx^4}}\,dx}{5c}$$

$$= \frac{de^2x\sqrt{a+cx^4}}{c} + \frac{e^3x^3\sqrt{a+cx^4}}{5c} + \frac{\int \frac{15cd(cd^2-ae^2)+9ce(5cd^2-ae^2)x^2}{\sqrt{a+cx^4}}\,dx}{15c^2}$$

$$= \frac{de^2x\sqrt{a+cx^4}}{c} + \frac{e^3x^3\sqrt{a+cx^4}}{5c} - \frac{\left(3\sqrt{a}\,e(5cd^2-ae^2)\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{5c^{3/2}} + \frac{\left(5\sqrt{c}\,d(c\right.}{}$$

$$= \frac{de^2x\sqrt{a+cx^4}}{c} + \frac{e^3x^3\sqrt{a+cx^4}}{5c} + \frac{3e(5cd^2-ae^2)\,x\sqrt{a+cx^4}}{5c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{3\sqrt[4]{a}\,e(5cd^2-ae^2)\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 140, normalized size = 0.43

$$\frac{5d(cd^2-ae^2)\,x\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex\left(e(5d+ex^2)\,(a+cx^4)+(5cd^2-ae^2)\,x^2\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)}{5c\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (5*d*(c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
 -((c*x^4)/a)] + e*x*(e*(5*d + e*x^2)*(a + c*x^4) + (5*c*d^2 - a*e^2)*x^2*S
qrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(5*c*Sq
rt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 388, normalized size = 1.19

method	result
elliptic	$\frac{e^3x^3\sqrt{c\,x^4+a}}{5c}+\frac{d\,e^2x\sqrt{c\,x^4+a}}{c}+\frac{\left(d^3-\frac{a\,d\,e^2}{c}\right)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$ −

risch	$\frac{e^2x(ex^2+5d)\sqrt{cx^4+a}}{5c} - \frac{\frac{i(3ae^3-15cd^2e)\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(\right.\right.}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}\sqrt{c}}}{}$
default	$e^3\left(\frac{x^3\sqrt{cx^4+a}}{5c} - \frac{3ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{5c^{\frac{3}{2}}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{cx^4+a}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(1/5/c*x^3*(c*x^4+a)^(1/2)-3/5*I*a^(3/2)/c^(3/2)/(I/a^(1/2)*c^(1/2))^(1
/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+
a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c
^(1/2))^(1/2),I)))+3*d*e^2*(1/3*x*(c*x^4+a)^(1/2)/c-1/3*a/c/(I/a^(1/2)*c^(1
/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/
(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+3*I*d^2*e*a^(1/2)
/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(
1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1
/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+d^3/(I/a^(1/2)*c^(1/2))^(1
/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+
a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^3/sqrt(c*x^4 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.04, size = 173, normalized size = 0.53

$$\frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x*
*4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)*hyp
er((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4)) + e
**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(I*pi)/a)/(4
*sqrt(a)*gamma(11/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^3/sqrt(c*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^3}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^3/(a + c*x^4)^(1/2), x)
```

3.152 $\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=264

$$\frac{e^2x\sqrt{a+cx^4}}{3c}+\frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2\sqrt[4]{a}\,de\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+$$

```
[Out] 1/3*e^2*x*(c*x^4+a)^(1/2)/c+2*d*e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^
(1/2))-2*a^(1/4)*d*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/
(c*x^4+a)^(1/2)+1/6*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*(3*c*d^2-a*e^2+6*d*e*a^(1/2)*c^(1/2))*((c*x^4+a)/(a
^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(5/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1221, 1212, 226, 1210}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(6\sqrt{a}\sqrt{c}\,de-ae^2+3cd^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}\,c^{5/4}\sqrt{a+cx^4}}-\frac{2\sqrt[4]{a}\,de\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(3*c) + (2*d*e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a]
 + Sqrt[c]*x^2)) - (2*a^(1/4)*d*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
c^(3/4)*Sqrt[a + c*x^4]) + ((3*c*d^2 + 6*Sqrt[a]*Sqrt[c]*d*e - a*e^2)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*c^(5/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2}{\sqrt{a+cx^4}}\,dx &= \frac{e^2x\sqrt{a+cx^4}}{3c}+\frac{\int \frac{3cd^2-ae^2+6cdex^2}{\sqrt{a+cx^4}}\,dx}{3c}\\ &= \frac{e^2x\sqrt{a+cx^4}}{3c}-\frac{\left(2\sqrt{a}\,de\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}}+\frac{\left(3cd^2+6\sqrt{a}\,\sqrt{c}\,de-ae^2\right)\int \frac{1}{\sqrt{a+cx^4}}}{3c}\\ &= \frac{e^2x\sqrt{a+cx^4}}{3c}+\frac{2dex\sqrt{a+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2\sqrt[4]{a}\,de\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E}{c^{3/4}\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 120, normalized size = 0.45

$$\frac{(3cd^2-ae^2)\,x\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex\left(e(a+cx^4)+2cdx^2\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)}{3c\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + c*x^4],x]
```

```
[Out] ((3*c*d^2 - a*e^2)*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + e*x*(e*(a + c*x^4) + 2*c*d*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*c*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 266, normalized size = 1.01

method	result
elliptic	$\frac{e^{2} x \sqrt{c\,x^{4}+a}}{3 c}+\frac{\left(d^{2}-\frac{a\,e^{2}}{3 c}\right) \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+\frac{2 i d e \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}}{\sqrt{a}}}}{}$
default	$e^{2} \left(\frac{x \sqrt{c\,x^{4}+a}}{3 c}-\frac{a \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{3 c \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}\right)+\frac{2 i d e \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}}{}$
	$-\frac{6 i \sqrt{c}\, d e \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \left(\mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-\mathrm{EllipticE}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}$
risch	$\frac{e^{2} x \sqrt{c\,x^{4}+a}}{3 c}-\frac{}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(1/3*x*(c*x^4+a)^(1/2)/c-1/3*a/c/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+2*I*d*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^2/sqrt(c*x^4 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.59, size = 124, normalized size = 0.47

$$\frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+a)**(1/2),x)
[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*ex
p_polar(I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1/2,
5/4), (9/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)^2/sqrt(c*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^2}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^2/(a + c*x^4)^(1/2), x)
```

3.153 $\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=226

$$\frac{ex\sqrt{a+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{\sqrt[4]{a}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)}{}$$

```
[Out] e*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*a
rctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipti
cF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d
*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+
a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1212, 226, 1210}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{ex\sqrt{a+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*e*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sq
rt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sq
rt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+cx^4}}\,dx = -\frac{\left(\sqrt{a}\,e\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{ex\sqrt{a+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\right.}{c^{3/4}\sqrt{a+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 77, normalized size = 0.34

$$\frac{\sqrt{1+\frac{cx^4}{a}}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
+ e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[a + c*x^4]
)
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 169, normalized size = 0.75

method	result

default	$\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}+\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{}$
elliptic	$\frac{ie\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}+\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*e*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*
c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+d/(I/a^(1/2)*c^
(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/sqrt(c*x^4 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.95, size = 78, normalized size = 0.35

$$\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(
a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_pol
ar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)/sqrt(c*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(a + c*x^4)^(1/2), x)
```

3.154 $\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=334

$$\frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,\sqrt{cd^2+ae^2}}+\frac{\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+cx^4}}-\frac{a^{3/4}\left(\right.}{}$$

```
[Out] 1/2*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^(1/2))*e^(1/2)/d
^(1/2)/(a*e^2+c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/a^(1/4)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)-1/4*a^(3/4)*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellip
ticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^
(1/2)/c^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((
c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(c*x^4+a)^
(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1231, 226, 1721}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,de};2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}\,d\sqrt{a+cx^4}\,\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,\sqrt{ae^2+cd^2}}+\frac{\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt{a+cx^4}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])
/(2*Sqrt[d]*Sqrt[c*d^2 + a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + c*x^4]) - (a^(3/4)*((S
qrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c
]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 - a*e^2)*S
qrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+cx^4}}\,dx = \frac{\sqrt{c}\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e} - \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e}$$

$$= \frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+cx^4}}\right)}{2\sqrt{d}\,\sqrt{cd^2+ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\cdots}{2\sqrt[4]{a}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.11, size = 95, normalized size = 0.28

$$-\frac{i\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSin
h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[a +
c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 107, normalized size = 0.32

method	result	size
default	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	107
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I
*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(x^2*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/(c*d*x^4 + a*d + (c*x^6 + a*x^2)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^4}\,(d + ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(x^2*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^4 + a}\,\left(e\,x^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)),x)
```

```
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)), x)
```

3.155 $\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=581

$$-\frac{\sqrt{c}\,ex\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{2d\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{\sqrt{e}\left(3cd^2+ae^2\right)\tan^{-1}\left(\frac{\sqrt{cd^2+ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+cx^4}}\right)}{4d^{3/2}\left(cd^2+ae^2\right)^{3/2}}+$$

```
[Out] 1/4*(a*e^2+3*c*d^2)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+a)^
(1/2))*e^(1/2)/d^(3/2)/(a*e^2+c*d^2)^(3/2)+1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e
^2+c*d^2)/(e*x^2+d)-1/2*e*x*c^(1/2)*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(a^(1/2
)+x^2*c^(1/2))+1/2*a^(1/4)*c^(1/4)*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(
1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2
)^(1/2)/d/(a*e^2+c*d^2)/(c*x^4+a)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x
/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan
(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)
+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)-1/8
*(a*e^2+3*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1
/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^
(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4
)/d^2/(a*e^2+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1238, 1729, 1210, 1723, 226, 1721}

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2d\sqrt{a+cx^4}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(ae^2+3cd^2\right)\Pi\left(-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\sqrt{c}\,de};2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\sqrt[4]{c}\,d^2\sqrt{a+cx^4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\left(ae^2+cd^2\right)}+\frac{\sqrt{e}\left(ae^2+3cd^2\right)\text{ArcTan}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+cx^4}}\right)}{4d^{3/2}\left(ae^2+cd^2\right)^{3/2}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,d\sqrt{a+cx^4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}+\frac{e^2x\sqrt{a+cx^4}}{2d\left(d+ex^2\right)\left(ae^2+cd^2\right)}-\frac{\sqrt{c}\,ex\sqrt{a+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -1/2*(Sqrt[c]*e*x*Sqrt[a + c*x^4])/(d*(c*d^2 + a*e^2)*(Sqrt[a] + Sqrt[c]*x^
2)) + (e^2*x*Sqrt[a + c*x^4])/(2*d*(c*d^2 + a*e^2)*(d + e*x^2)) + (Sqrt[e]*
(3*c*d^2 + a*e^2)*ArcTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a +
c*x^4])])/(4*d^(3/2)*(c*d^2 + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] +
 Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*d*(c*d^2 + a*e^2)*Sqrt[a + c*x^4]) + (c^(1
/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[
a]*e)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 + a*e^2)*(Sqrt[a
```

```
] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/
4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(8*a^(1/4)*c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)
*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx = \frac{e^2x\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)\,(d+ex^2)} - \frac{\int \frac{-2cd^2-ae^2+2cdex^2+ce^2x^4}{(d+ex^2)\sqrt{a+cx^4}}\,dx}{2d\,(cd^2+ae^2)}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)\,(d+ex^2)} - \frac{\int \frac{\sqrt{a}\,c^{3/2}de^2+ce(-2cd^2-ae^2)+\left(2c^2de^2-ce^2\left(cd-\sqrt{a}\,\sqrt{c}\,e\right)\right)}{(d+ex^2)\sqrt{a+cx^4}}}{2cde\,(cd^2+ae^2)}$$

$$= -\frac{\sqrt{c}\,ex\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)\,(d+ex^2)} + \frac{\sqrt[4]{a}\,\sqrt[4]{c}\,e\big(\sqrt{a}}{}$$

$$= -\frac{\sqrt{c}\,ex\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{2d\,(cd^2+ae^2)\,(d+ex^2)} + \frac{\sqrt{e}\,(3cd^2+ae^2}{4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.38, size = 522, normalized size = 0.90

$$\frac{a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,de^2x+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,cde^2x^5-\sqrt{a}\,\sqrt{c}\,de(d+ex^2)\sqrt{1+\frac{cx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+\sqrt{c}\,d(i\sqrt{c}\,d+\sqrt{a}\,e)\,(d+ex^2)\sqrt{1+\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)-3icd^3\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)-iade^2\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)-3icd^2ex^2\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)-iae^3x^2\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d^2\,(cd^2+ae^2)\,(d+ex^2)\,\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] (a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*e^2*x + Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d*e^2*x^
5 - Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh
[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[c]*d*(I*Sqrt[c]*d + Sqrt[a]*e)*(d
 + e*x^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]
*x], -1] - (3*I)*c*d^3*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqr
t[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a*d*e^2*Sqrt[1 + (
c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c
])/Sqrt[a]]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I
```

```
)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - I*a
*e^3*x^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*Arc
Sinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*d^2*(c
*d^2 + a*e^2)*(d + e*x^2)*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 556, normalized size = 0.96

method	result
default	$\frac{e^2 x\sqrt{c\,x^4+a}}{2d(a\,e^2+c\,d^2)(e\,x^2+d)}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2(a\,e^2+c\,d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}-\frac{i\sqrt{c}\,e\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x}{\sqrt{a}}}}{2d(a\,e^2+c}$
elliptic	$\frac{e^2 x\sqrt{c\,x^4+a}}{2d(a\,e^2+c\,d^2)(e\,x^2+d)}-\frac{c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2(a\,e^2+c\,d^2)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}-\frac{i\sqrt{c}\,e\sqrt{a}\sqrt{1-\frac{i\sqrt{c}\,x}{\sqrt{a}}}}{2d(a\,e^2+c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)-1/2*c/(a*e^2+c*d^2)/(I/
a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)
*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-1/2*I*
c^(1/2)*e/d/(a*e^2+c*d^2)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*I*c^(1/2)*e/d/(a*e^2+c*d^2)*a^(1/2)/(I/a
^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2/d^2
/(a*e^2+c*d^2)*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c
^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c
^(1/2))^(1/2))*a+3/2/(a*e^2+c*d^2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticP
i(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1
/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(x^2*e + d)^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^4}\,(d+ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(x^2*e + d)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^4+a}\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^2),x)
```

```
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

3.156 $\int \frac{1}{(d+ex^2)^3 \sqrt{a+cx^4}} \, dx$

Optimal. Leaf size=729

$$-\frac{3\sqrt{c}\, e(3cd^2+ae^2)\, x\sqrt{a+cx^4}}{8d^2\,(cd^2+ae^2)^2\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{e^2x\sqrt{a+cx^4}}{4d\,(cd^2+ae^2)\,(d+ex^2)^2}+\frac{3e^2(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\,(cd^2+ae^2)^2\,(d+ex^2)}+\frac{3\sqrt{e}\,(5c^2d^4+}{}$$

```
[Out] 3/16*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*arctan(x*(a*e^2+c*d^2)^(1/2)/d^(1/2)
/e^(1/2)/(c*x^4+a)^(1/2))*e^(1/2)/d^(5/2)/(a*e^2+c*d^2)^(5/2)+1/4*e^2*x*(c*
x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^2)*x*(c*x^4+a
)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)-3/8*e*(a*e^2+3*c*d^2)*x*c^(1/2)*(c*x^
4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(a^(1/2)+x^2*c^(1/2))+3/8*a^(1/4)*c^(1/4)*e*
(a*e^2+3*c*d^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/d^2/(a*e^2+c*
d^2)^2/(c*x^4+a)^(1/2)-3/32*(a^2*e^4+2*a*c*d^2*e^2+5*c^2*d^4)*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(s
in(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c
^(1/2),1/2*2^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^3/(a*e^2+c*d^2)^2/(-e*a^(1
/2)+d*c^(1/2))/(c*x^4+a)^(1/2)+1/8*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4))
)^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*
x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(4*c*d^2+3*a*e^2-d*e*a^(1/2)
*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d^2/(a*e^2+c*d^
2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.78, antiderivative size = 729, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1238, 1711, 1729, 1210, 1723, 226, 1721}

$$\frac{3\sqrt{e}\,(a^2e^4+2acd^2e^2+5c^2d^4)\,\mathrm{ArcTan}\left(\frac{x\sqrt{ae^2+cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+cx^4}}\right)}{16d^{5/2}\,(ae^2+cd^2)^{5/2}}-\frac{3\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\left(\sqrt{a}\,e+\sqrt{c}\,d\right)(a^2e^4+2acd^2e^2+5c^2d^4)\,\Pi\left(-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,de};2\mathrm{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{32\sqrt[4]{a}\,\sqrt[4]{c}\,d^3\sqrt{a+cx^4}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,\left(-\sqrt{a}\,\sqrt{c}\,de+3ae^2+4cd^2\right)F\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8\sqrt[4]{a}\,d^2\sqrt{a+cx^4}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)(ae^2+cd^2)}+\frac{3\sqrt[4]{a}\,\sqrt[4]{c}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,(ae^2+3cd^2)\,E\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{8d^2\sqrt{a+cx^4}\,(ae^2+cd^2)^2}+\frac{3e^2x\sqrt{a+cx^4}\,(ae^2+3cd^2)}{8d^2\,(d+ex^2)\,(ae^2+cd^2)^2}+\frac{e^2x\sqrt{a+cx^4}}{4d\,(d+ex^2)^2\,(ae^2+cd^2)}-\frac{3\sqrt{c}\,ex\sqrt{a+cx^4}\,(ae^2+3cd^2)}{8d^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] (-3*Sqrt[c]*e*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 + a*e^2)^2
*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + c*x^4])/(4*d*(c*d^2 + a*e^2)*(d
 + e*x^2)^2) + (3*e^2*(3*c*d^2 + a*e^2)*x*Sqrt[a + c*x^4])/(8*d^2*(c*d^2 +
a*e^2)^2*(d + e*x^2)) + (3*Sqrt[e]*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*Ar
cTan[(Sqrt[c*d^2 + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + c*x^4])])/(16*d^(5/2
)*(c*d^2 + a*e^2)^(5/2)) + (3*a^(1/4)*c^(1/4)*e*(3*c*d^2 + a*e^2)*(Sqrt[a]
```

```
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(8*d^2*(c*d^2 + a*e^2)^2*Sqrt[a + c*x^4]) +
(c^(1/4)*(4*c*d^2 - Sqrt[a]*Sqrt[c]*d*e + 3*a*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(8*a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + a*e^2)*Sqrt
[a + c*x^4]) - (3*(Sqrt[c]*d + Sqrt[a]*e)*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*
e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(32*a^(1/4)*c^(1/4)*d^3*(Sqrt[c]*d - Sqrt[a]*e)*(c
*d^2 + a*e^2)^2*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1711

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), I
nt[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2
*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C
*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
&& PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILt
Q[q, -1]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^3\sqrt{a+cx^4}}\,dx = \frac{e^2x\sqrt{a+cx^4}}{4d\,(cd^2+ae^2)\,(d+ex^2)^2} - \frac{\int \frac{-4cd^2-3ae^2+4cdex^2-ce^2x^4}{(d+ex^2)^2\sqrt{a+cx^4}}\,dx}{4d\,(cd^2+ae^2)}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{4d\,(cd^2+ae^2)\,(d+ex^2)^2} + \frac{3e^2(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\,(cd^2+ae^2)^2\,(d+ex^2)} + \frac{\int \frac{8c^2d^4+5acd^2e^2+3a^2e^{4}\cdots}{(\cdots}}{8\cdots}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{4d\,(cd^2+ae^2)\,(d+ex^2)^2} + \frac{3e^2(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\,(cd^2+ae^2)^2\,(d+ex^2)} + \frac{\int \frac{-3\sqrt{a}\,c^{3/2}de^2(3cd^2\cdots}{\cdots}}{\cdots}$$

$$= -\frac{3\sqrt{c}\,e(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\,(cd^2+ae^2)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{4d\,(cd^2+ae^2)\,(d+ex^2)^2} + \frac{3e^2(3cd^2+a\cdots}{8d^2\,(cd^2+\cdots}$$

$$= -\frac{3\sqrt{c}\,e(3cd^2+ae^2)\,x\sqrt{a+cx^4}}{8d^2\,(cd^2+ae^2)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+cx^4}}{4d\,(cd^2+ae^2)\,(d+ex^2)^2} + \frac{3e^2(3cd^2+a\cdots}{8d^2\,(cd^2+\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.59, size = 332, normalized size = 0.46

$$\frac{\frac{de^2x(a+cx^4)(ae^2(5d+3ex^2)+cd^2(11d+9ex^2))}{(d+ex^2)^2} + \frac{\sqrt{1+\frac{cx^4}{a}}\left(-3\sqrt{a}\sqrt{c}\,de(3cd^2+ae^2)E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+i\left(\sqrt{c}\,d\left(7c^{3/2}d^3-9i\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2-3ia^{3/2}e^3\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)-3(5c^2d^4+2acd^2e^2+a^2e^4)\Pi\left(-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{8d^3\,(cd^2+ae^2)^2\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] ((d*e^2*x*(a + c*x^4)*(a*e^2*(5*d + 3*e*x^2) + c*d^2*(11*d + 9*e*x^2)))/(d
+ e*x^2)^2 + (Sqrt[1 + (c*x^4)/a]*(-3*Sqrt[a]*Sqrt[c]*d*e*(3*c*d^2 + a*e^2)
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*(Sqrt[c]*d*(7*c^
(3/2)*d^3 - (9*I)*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2 - (3*I)*a^(3/2)*e^3)*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - 3*(5*c^2*d^4 + 2*a*c*
d^2*e^2 + a^2*e^4)*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSinh[Sqrt[
(I*Sqrt[c])/Sqrt[a]]*x], -1])))/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(8*d^3*(c*d^2 +
a*e^2)^2*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 1018, normalized size = 1.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*e^2*x*(c*x^4+a)^(1/2)/d/(a*e^2+c*d^2)/(e*x^2+d)^2+3/8*e^2*(a*e^2+3*c*d^
2)*x*(c*x^4+a)^(1/2)/d^2/(a*e^2+c*d^2)^2/(e*x^2+d)-1/8*c/d/(a*e^2+c*d^2)^2/
(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1
/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)*a*e
^2-7/8*c^2*d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)
*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/
a^(1/2)*c^(1/2))^(1/2),I)+9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a^(1/2
)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^
(1/2)/(c*x^4+a)^(1/2)*EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/8*I*c^(1/2
)*e^3/d^2/(a*e^2+c*d^2)^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticE(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)-9/8*I*c^(3/2)*e/(a*e^2+c*d^2)^2*a^(1/2)/(I/a
^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-3/8*I*c
^(1/2)*e^3/d^2/(a*e^2+c*d^2)^2*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellip
ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3/8/d^3/(a*e^2+c*d^2)^2*e^4/(I/a^(1/2)*
c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2
)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*a^2+3/4/(a*e^2+
c*d^2)^2/d*e^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/
2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/
2))^(1/2))*a*c+15/8*d/(a*e^2+c*d^2)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icPi(x*(I/a^(1/2)*c^(1/2))^(1/2),I*a^(1/2)/c^(1/2)*e/d,(-I/a^(1/2)*c^(1/2))
^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2))*c^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(x^2*e + d)^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^4}\,(d + ex^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x**2)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(x^2*e + d)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^4 + a}\,{\left(e\,x^2 + d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^3),x)
```

```
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

3.157 $\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}} dx$

Optimal. Leaf size=213

$$-\frac{de^2x\sqrt{a-cx^4}}{c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}+\frac{3a^{3/4}e(5cd^2+ae^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{a^{3/4}\left(\frac{5\sqrt{c}\,d(c}{\sqrt{}}\right.}{}$$

```
[Out] -d*e^2*x*(-c*x^4+a)^(1/2)/c-1/5*e^3*x^3*(-c*x^4+a)^(1/2)/c+3/5*a^(3/4)*e*(a
*e^2+5*c*d^2)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(7/4)/(-c*
x^4+a)^(1/2)+1/5*a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-3*e*(a*e^2+5*c*d^
2)+5*d*(a*e^2+c*d^2)*c^(1/2)/a^(1/2))*(1-c*x^4/a)^(1/2)/c^(7/4)/(-c*x^4+a)^
(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1221, 1902, 1215, 230, 227, 1214, 1213, 435}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{5\sqrt{c}\,d(ae^2+cd^2)}{\sqrt{a}}-3e(ae^2+5cd^2)\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}+\frac{3a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,(ae^2+5cd^2)\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{5c^{7/4}\sqrt{a-cx^4}}-\frac{de^2x\sqrt{a-cx^4}}{c}-\frac{e^3x^3\sqrt{a-cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

```
[Out] -((d*e^2*x*Sqrt[a - c*x^4])/c) - (e^3*x^3*Sqrt[a - c*x^4])/(5*c) + (3*a^(3/
4)*e*(5*c*d^2 + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(
1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((5*Sqrt[c]*d*(c*d^2 + a
*e^2))/Sqrt[a] - 3*e*(5*c*d^2 + a*e^2))*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(5*c^(7/4)*Sqrt[a - c*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{\sqrt{a-cx^4}}\,dx = -\frac{e^3x^3\sqrt{a-cx^4}}{5c} - \frac{\int \frac{-5cd^3-3e(5cd^2+ae^2)x^2-15cde^2x^4}{\sqrt{a-cx^4}}\,dx}{5c}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\int \frac{15cd(cd^2+ae^2)+9ce(5cd^2+ae^2)x^2}{\sqrt{a-cx^4}}\,dx}{15c^2}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\left(3\sqrt{a}\,e(5cd^2+ae^2)\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{5c^{3/2}} + \frac{(5\sqrt{c}\,d\ \cdots}{\cdots}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\left(3\sqrt{a}\,e(5cd^2+ae^2)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}\ \cdots}}{5c^{3/2}\sqrt{a-cx^4}\ \cdots}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{\sqrt[4]{a}\left(5\sqrt{c}\,d(cd^2+ae^2)-3\sqrt{a}\,e(5cd^2+ae^2)\right)\sqrt{\cdots}}{5c^{7/4}\sqrt{a-cx^4}\ \cdots}$$

$$= -\frac{de^2x\sqrt{a-cx^4}}{c} - \frac{e^3x^3\sqrt{a-cx^4}}{5c} + \frac{3a^{3/4}e(5cd^2+ae^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\cdots\right.}{5c^{7/4}\sqrt{a-cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.11, size = 141, normalized size = 0.66

$$\frac{5d(cd^2+ae^2)\,x\sqrt{1-\frac{cx^4}{a}}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex\left(e(5d+ex^2)\left(-a+cx^4\right)+\left(5cd^2+ae^2\right)x^2\sqrt{1-\frac{cx^4}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{5c\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a - c*x^4],x]
```

```
[Out] (5*d*(c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4,
 (c*x^4)/a] + e*x*(e*(5*d + e*x^2)*(-a + c*x^4) + (5*c*d^2 + a*e^2)*x^2*Sqr
t[1 - (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(5*c*Sqrt[a
- c*x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(175) = 350$.

time = 0.13, size = 360, normalized size = 1.69

method	result
elliptic	$-\frac{e^{3} x^{3} \sqrt{-c\,x^{4}+a}}{5c}-\frac{d\,e^{2} x \sqrt{-c\,x^{4}+a}}{c}+\frac{\left(d^{3}+\frac{a d\,e^{2}}{c}\right) \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}$
risch	$-\frac{e^{2} x \left(e\,x^{2}+5d\right) \sqrt{-c\,x^{4}+a}}{5c}+\frac{-\frac{\left(3a\,e^{3}+15c\,d^{2} e\right) \sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)-\mathrm{Ellipti}\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}\,\sqrt{c}}}{}$
default	$e^{3} \left(-\frac{x^{3} \sqrt{-c\,x^{4}+a}}{5c}-\frac{3 a^{\frac{3}{2}} \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)-\mathrm{EllipticE}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)\right)}{5 c^{\frac{3}{2}} \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^{4}+a}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-1/5/c*x^3*(-c*x^4+a)^(1/2)-3/5*a^(3/2)/c^(3/2)/(1/a^(1/2)*c^(1/2))^(1
/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4
+a)^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*
c^(1/2))^(1/2),I)))+3*d*e^2*(-1/3/c*x*(-c*x^4+a)^(1/2)+1/3*a/c/(1/a^(1/2)*c
^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I))-3*d^2*e*a^(1/
2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))
^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))+d^3/(1/a^(1/2)*c^(1/2))
^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*
x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^3/sqrt(-c*x^4 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 2.21, size = 180, normalized size = 0.85

$$\frac{d^3x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{3d^2ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{3de^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}+\frac{e^3x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
[Out] d**3*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + 3*d**2*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*
x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4)) + 3*d*e**2*x**5*gamma(5/4)
*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4
)) + e**3*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c*x**4*exp_polar(2*I*p
i)/a)/(4*sqrt(a)*gamma(11/4))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^3}{\sqrt{a-c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^3/(a - c*x^4)^(1/2), x)
```

3.158 $\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=162

$$-\frac{e^2x\sqrt{a-cx^4}}{3c}+\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\sqrt{c}\,de+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,F}{3c^{5/4}\sqrt{a-cx^4}}$$

```
[Out] -1/3*e^2*x*(-c*x^4+a)^(1/2)/c+2*a^(3/4)*d*e*EllipticE(c^(1/4)*x/a^(1/4),I)*
(1-c*x^4/a)^(1/2)/c^(3/4)/(-c*x^4+a)^(1/2)+1/3*a^(1/4)*EllipticF(c^(1/4)*x/
a^(1/4),I)*(3*c*d^2+a*e^2-6*d*e*a^(1/2)*c^(1/2))*(1-c*x^4/a)^(1/2)/c^(5/4)/
(-c*x^4+a)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1221, 1215, 230, 227, 1214, 1213, 435}

$$\frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-6\sqrt{a}\sqrt{c}\,de+ae^2+3cd^2\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{5/4}\sqrt{a-cx^4}}-\frac{e^2x\sqrt{a-cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

```
[Out] -1/3*(e^2*x*Sqrt[a - c*x^4])/c + (2*a^(3/4)*d*e*Sqrt[1 - (c*x^4)/a]*Ellipti
cE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4]) + (a^(1/4)*(
3*c*d^2 - 6*Sqrt[a]*Sqrt[c]*d*e + a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcS
in[(c^(1/4)*x)/a^(1/4)], -1])/(3*c^(5/4)*Sqrt[a - c*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{3c} - \frac{\int \frac{-3cd^2-ae^2-6cdex^2}{\sqrt{a-cx^4}}\,dx}{3c}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c} + \frac{\left(2\sqrt{a}\,de\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{\sqrt{c}} - \frac{\left(-3cd^2+6\sqrt{a}\,\sqrt{c}\,de-ae^2\right)\int \frac{1}{\sqrt{a-c}}}{3c}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c} + \frac{\left(2\sqrt{a}\,de\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{c}\,\sqrt{a-cx^4}} - \frac{\left(\left(-3cd^2+6\sqrt{a}\,\sqrt{c}\,de-\right.\right.}{3}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c} + \frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\,\sqrt{c}\,de+ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3c^{5/4}\sqrt{a-cx^4}}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{3c} + \frac{2a^{3/4}de\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(3cd^2-6\sqrt{a}\,\sqrt{c}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.06, size = 121, normalized size = 0.75

$$\frac{\left(3cd^2+ae^2\right)x\sqrt{1-\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex\left(-ae+cex^4+2cdx^2\sqrt{1-\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{3c\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a - c*x^4],x]
```

```
[Out] ((3*c*d^2 + a*e^2)*x*Sqrt[1 - (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, (
c*x^4)/a] + e*x*(-(a*e) + c*e*x^4 + 2*c*d*x^2*Sqrt[1 - (c*x^4)/a]*Hypergeom
etric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*c*Sqrt[a - c*x^4])
```

Maple [A]
time = 0.13, size = 246, normalized size = 1.52

method	result
elliptic	$-\frac{e^2 x\sqrt{-c\,x^4+a}}{3c}+\frac{\left(d^2+\frac{a\,e^2}{3c}\right)\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}-\frac{2de\sqrt{a}\,\sqrt{1-\frac{\sqrt{}}{}}}{}$
default	$e^2\left(-\frac{x\sqrt{-c\,x^4+a}}{3c}+\frac{a\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{3c\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}\right)-\frac{2de\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}}{\sqrt{a}}}}{}$
risch	$-\frac{e^2 x\sqrt{-c\,x^4+a}}{3c}+\frac{-\frac{6\sqrt{c}\,de\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\right.\right.}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(-1/3/c*x*(-c*x^4+a)^(1/2)+1/3*a/c/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/
2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elli
pticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I))-2*d*e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/
2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+
a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(1/a
^(1/2)*c^(1/2))^(1/2),I))+d^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2
)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(
1/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^2/sqrt(-c*x^4 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 1.70, size = 129, normalized size = 0.80

$$\frac{d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{dex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{2\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}+\frac{e^2x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
```

```
[Out] d**2*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*
sqrt(a)*gamma(5/4)) + d*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*
exp_polar(2*I*pi)/a)/(2*sqrt(a)*gamma(7/4)) + e**2*x**5*gamma(5/4)*hyper((1
/2, 5/4), (9/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqrt(a)*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^2/sqrt(-c*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,x^2+d\right)}^2}{\sqrt{a-c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(a - c*x^4)^(1/2), x)
```

3.159 $\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=124

$$\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$

```
[Out] a^(3/4)*e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(3/4)/(-c*x^4+
a)^(1/2)+a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-e+d*c^(1/2)/a^(1/2))*(1-c
*x^4/a)^(1/2)/c^(3/4)/(-c*x^4+a)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1215, 230, 227, 1214, 1213, 435}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(c^(3/4)*Sqrt[a - c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c*
x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[a - c*x^4
])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a-cx^4}}\,dx = \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\,dx}{\sqrt{c}} + \left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a-cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}\,e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{c}\,\sqrt{a-cx^4}} + \frac{\left(\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{a-cx^4}}$$

$$= \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,\sqrt{a-cx^4}} + \frac{\left(\sqrt{a}\,e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1+\cdots}}{\sqrt{1-\cdots}}}{\sqrt{c}\,\sqrt{a-cx^4}}$$

$$= \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{a-cx^4}} + \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\cdots}{\sqrt[4]{a}\cdots}\right.\right.}{\sqrt[4]{c}\,\sqrt{a-cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 77, normalized size = 0.62

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a - c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[a - c*x^4])
```

Maple [A]
time = 0.11, size = 154, normalized size = 1.24

method	result
default	$-\frac{e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}+\frac{d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{}$
elliptic	$-\frac{e\sqrt{a}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}\,\sqrt{c}}+\frac{d\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -e*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a
^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(1/a^(1/2)*
c^(1/2))^(1/2),I)-EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I))+d/(1/a^(1/2)*c^
(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2
)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

[Out] integrate((x^2*e + d)/sqrt(-c*x^4 + a), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 0.99, size = 82, normalized size = 0.66

$$\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{ex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{cx^4e^{2i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(2*I*pi)/a)/(4*sqr
t(a)*gamma(5/4)) + e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_p
olar(2*I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)/sqrt(-c*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{\sqrt{a-c\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(a - c*x^4)^(1/2), x)
```

3.160 $\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{a-cx^4}}$$

```
[Out] a^(1/4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1
/2)/c^(1/4)/d/(-c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1233, 1232}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[a - c*x^4])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a-cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{(d+ex^2)\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{a-cx^4}}$$
$$= \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{a-cx^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 91, normalized size = 1.26

$$-\frac{i\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[a - c*x^
4])
```

Maple [A]
time = 0.16, size = 97, normalized size = 1.35

method	result	size
default	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{e\sqrt{a}}{d\sqrt{c}},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}$	97

elliptic	$\frac{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{\sqrt{c}}{\sqrt{a}}},-\frac{e\sqrt{a}}{d\sqrt{c}},\frac{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4+a}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),
-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(x^2*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 + a)/(c*d*x^4 - a*d + (c*x^6 - a*x^2)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a-cx^4}\,(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(x^2*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a-c\,x^4}\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)),x)
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

3.161 $\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx$

Optimal. Leaf size=299

$$-\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)\left(d+ex^2\right)}-\frac{a^{3/4}\sqrt[4]{c}\,e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\left(cd^2-ae^2\right)\sqrt{a-cx^4}}-\frac{\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\right.\right.}{2d\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{a-}}$$

```
[Out] -1/2*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)-1/2*a^(3/4)*c^(1/4)*
e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d/(-a*e^2+c*d^2)/(-c*x^4
+a)^(1/2)+1/2*a^(1/4)*(-a*e^2+3*c*d^2)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1
/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/d^2/(-a*e^2+c*d^2)/(-c*x^4+a)^(1
/2)-1/2*a^(1/4)*c^(1/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d/
(e*a^(1/2)+d*c^(1/2))/(-c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1238, 1731, 1215, 230, 227, 1214, 1213, 435, 1233, 1232}

$$-\frac{a^{3/4}\sqrt[4]{c}\,e\sqrt{1-\frac{cx^4}{a}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\,\left(cd^2-ae^2\right)}+\frac{\sqrt[4]{a}\,\sqrt{1-\frac{cx^4}{a}}\,\left(3cd^2-ae^2\right)\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{c}\,d^2\sqrt{a-cx^4}\,\left(cd^2-ae^2\right)}-\frac{\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{1-\frac{cx^4}{a}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\sqrt{a-cx^4}\,\left(\sqrt{a}\,e+\sqrt{c}\,d\right)}-\frac{e^2x\sqrt{a-cx^4}}{2d\left(d+ex^2\right)\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]
```

```
[Out] -1/2*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)) - (a^(3/4)*c^(
1/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d
*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*Sqrt[1 - (c*x^4)/a]*El
lipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*d*(Sqrt[c]*d + Sqrt[a]*e)*Sqrt
[a - c*x^4]) + (a^(1/4)*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-(
(Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(2*c^(1/4)*d^2*
(c*d^2 - a*e^2)*Sqrt[a - c*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1731

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Di
st[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)(d+ex^2)} + \frac{\int \frac{2cd^2-ae^2-2cdex^2-ce^2x^4}{(d+ex^2)\sqrt{a-cx^4}}\,dx}{2d\left(cd^2-ae^2\right)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)(d+ex^2)} - \frac{\int \frac{cde^2+ce^3x^2}{\sqrt{a-cx^4}}\,dx}{2de^2\left(cd^2-ae^2\right)} + \frac{\left(3cd^2-ae^2\right)\int \frac{1}{(d+ex^2)\sqrt{a-\cdots}}}{2d\left(cd^2-ae^2\right)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)(d+ex^2)} - \frac{\sqrt{c}\int \frac{1}{\sqrt{a-cx^4}}\,dx}{2d\left(\sqrt{c}\,d+\sqrt{a}\,e\right)} - \frac{\left(\sqrt{a}\,\sqrt{c}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a-cx^4}}\cdots}{2d\left(cd^2-ae^2\right)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)(d+ex^2)} + \frac{\sqrt[4]{a}\left(3cd^2-ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\cdots\right.\right.}{2\sqrt[4]{c}\,d^2\left(cd^2-ae^2\right)\sqrt{a-cx^4}}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)(d+ex^2)} - \frac{\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\sqrt{a-cx^4}} + \cdots$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{2d\left(cd^2-ae^2\right)(d+ex^2)} - \frac{a^{3/4}\sqrt[4]{c}\,e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2d\left(cd^2-ae^2\right)\sqrt{a-cx^4}} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.51, size = 508, normalized size = 1.70

$$\frac{-a\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,de^2x+\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,cde^2x^5+i\sqrt{a}\,\sqrt{c}\,de(d+ex^2)\sqrt{1-\frac{cx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)-i\sqrt{c}\,d\left(-\sqrt{c}\,d+\sqrt{a}\,e\right)(d+ex^2)\sqrt{1-\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)-3icd^3\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+iade^2\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)-3icd^2ex^2\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+iae^3x^2\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{2\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d^2\left(cd^2-ae^2\right)(d+ex^2)\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e*x^2)^2*Sqrt[a - c*x^4]),x]

[Out] (-(a*Sqrt[-(Sqrt[c]/Sqrt[a])]*d*e^2*x) + Sqrt[-(Sqrt[c]/Sqrt[a])]*c*d*e^2*x^5 + I*Sqrt[a]*Sqrt[c]*d*e*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - I*Sqrt[c]*d*(-(Sqrt[c]*d) + Sqrt[a]*e)*(d + e*x^2)*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (3*I)*c*d^3*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*d*e^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] - (3*I)*c*d^2*e*x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + I*a*e^3*x^2*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(2*Sqrt[-(Sqrt[c]/Sqrt[a])]*d^2*(c*d^2 - a*e^2)*(d + e*x^2)*Sqrt[a - c*x^4])

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(245) = 490$.
time = 0.12, size = 523, normalized size = 1.75

method	result
default	$\frac{e^{2} x \sqrt{-c\,x^{4}+a}}{2\left(a\,e^{2}-c\,d^{2}\right) d \left(e\,x^{2}+d\right)}+\frac{c \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{2\left(a\,e^{2}-c\,d^{2}\right) \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}-\frac{\sqrt{c}\, e \sqrt{a}\, \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}}{2\left(a\,e^{2}-c\,d^{2}\right) d}$
elliptic	$\frac{e^{2} x \sqrt{-c\,x^{4}+a}}{2\left(a\,e^{2}-c\,d^{2}\right) d \left(e\,x^{2}+d\right)}+\frac{c \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}, i\right)}{2\left(a\,e^{2}-c\,d^{2}\right) \sqrt{\frac{\sqrt{c}}{\sqrt{a}}}\, \sqrt{-c\,x^{4}+a}}-\frac{\sqrt{c}\, e \sqrt{a}\, \sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}}{2\left(a\,e^{2}-c\,d^{2}\right) d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/2*c/(a*e^2-c*d^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-1/2*c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2*c^(1/2)*e/(a*e^2-c*d^2)/d*a^(1/2)/(1/a^

```
(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x
^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+1/2/(a*
e^2-c*d^2)/d^2*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2
)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*
c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c
^(1/2))^(1/2))*a-3/2/(a*e^2-c*d^2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
Pi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1
/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(x^2*e + d)^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - cx^4}\,\left(d + ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(-c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + a)*(x^2*e + d)^2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a-c\,x^4}\,{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^2),x)
```

```
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^2), x)
```

3.162 $\int \frac{1}{(d+ex^2)^3 \sqrt{a-cx^4}} \, dx$

Optimal. Leaf size=425

$$-\frac{e^2 x \sqrt{a-cx^4}}{4d\left(cd^2-ae^2\right)\left(d+ex^2\right)^2}-\frac{3e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{8d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)}-\frac{3a^{3/4}\sqrt[4]{c}\,e\left(3cd^2-ae^2\right)\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{8d^2\left(cd^2-ae^2\right)^2\sqrt{a-cx^4}}$$

```
[Out] -1/4*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^2-3/8*e^2*(-a*e^2+3*
c*d^2)*x*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)-3/8*a^(3/4)*c^(1/4
)*e*(-a*e^2+3*c*d^2)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d^2/(
-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)+3/8*a^(1/4)*(a^2*e^4-2*a*c*d^2*e^2+5*c^2*d
^4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1/2)/
c^(1/4)/d^3/(-a*e^2+c*d^2)^2/(-c*x^4+a)^(1/2)-1/8*a^(1/4)*c^(1/4)*EllipticF
(c^(1/4)*x/a^(1/4),I)*(7*c*d^2-3*a*e^2-2*d*e*a^(1/2)*c^(1/2))*(1-c*x^4/a)^(
1/2)/d^2/(-a*e^2+c*d^2)/(e*a^(1/2)+d*c^(1/2))/(-c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1238, 1711, 1731, 1215, 230, 227, 1214, 1213, 435, 1233, 1232}

$$-\frac{3a^{3/4}\sqrt[4]{c}\,e\sqrt{1-\frac{cx^4}{a}}\left(3cd^2-ae^2\right)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}+\frac{3\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(a^2e^4-2acd^2e^2+5c^2d^4\right)\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{c}\,d^3\sqrt{a-cx^4}\left(cd^2-ae^2\right)^2}-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\left(-2\sqrt{a}\sqrt{c}\,de-3ae^2+7cd^2\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8d^2\sqrt{a-cx^4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\left(cd^2-ae^2\right)}-\frac{3e^2x\sqrt{a-cx^4}\left(3cd^2-ae^2\right)}{8d^2\left(d+ex^2\right)\left(cd^2-ae^2\right)^2}-\frac{e^2x\sqrt{a-cx^4}}{4d\left(d+ex^2\right)^2\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

```
[Out] -1/4*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)^2) - (3*e^2*(3*
c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(8*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)) - (
3*a^(3/4)*c^(1/4)*e*(3*c*d^2 - a*e^2)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^4]) - (a^(
1/4)*c^(1/4)*(7*c*d^2 - 2*Sqrt[a]*Sqrt[c]*d*e - 3*a*e^2)*Sqrt[1 - (c*x^4)/a
]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(8*d^2*(Sqrt[c]*d + Sqrt[a]*e
)*(c*d^2 - a*e^2)*Sqrt[a - c*x^4]) + (3*a^(1/4)*(5*c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(8*c^(1/4)*d^3*(c*d^2 - a*e^2)^2*Sqrt[a - c*x^
4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
```

```
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1711

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), I
nt[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2
*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C
*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
&& PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILt
Q[q, -1]
```

Rule 1731

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Di
st[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{4d\,(cd^2-ae^2)\,(d+ex^2)^2} + \frac{\int \frac{4cd^2-3ae^2-4cdex^2+ce^2x^4}{(d+ex^2)^2\sqrt{a-cx^4}}\,dx}{4d\,(cd^2-ae^2)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\,(cd^2-ae^2)\,(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{8d^2\,(cd^2-ae^2)^2\,(d+ex^2)} + \frac{\int \frac{8c^2d^4-5acd^2e^2+3a\cdots}{\cdots}}{\cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\,(cd^2-ae^2)\,(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{8d^2\,(cd^2-ae^2)^2\,(d+ex^2)} - \frac{\int \frac{-3cde^2(3cd^2-ae^2)\cdots}{\cdots}}{8\cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\,(cd^2-ae^2)\,(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{8d^2\,(cd^2-ae^2)^2\,(d+ex^2)} - \frac{\left(\sqrt{c}\left(\sqrt{c}\,d-\sqrt{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\,(cd^2-ae^2)\,(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{8d^2\,(cd^2-ae^2)^2\,(d+ex^2)} + \frac{3\sqrt[4]{a}\,(5c^2d^4-2a\cdots}{\cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\,(cd^2-ae^2)\,(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{8d^2\,(cd^2-ae^2)^2\,(d+ex^2)} - \frac{\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{c}\,d-\cdots\right.}{\cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{4d\,(cd^2-ae^2)\,(d+ex^2)^2} - \frac{3e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{8d^2\,(cd^2-ae^2)^2\,(d+ex^2)} - \frac{3a^{3/4}\sqrt[4]{c}\,e(3cd^2\cdots}{8\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.67, size = 321, normalized size = 0.76

$$\frac{\frac{de^2x(a-cx^4)(ae^2(5d+3ex^2)-cd^2(11d+9ex^2))}{(d+ex^2)^2} - \frac{i\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}\,de(-3cd^2+ae^2)E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+\left(-7c^2d^4+9\sqrt{a}\,c^{3/2}d^3e+acd^2e^2-3a^{3/2}\sqrt{c}\,de^3\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+3(5c^2d^4-2acd^2e^2+a^2e^4)\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}}{8d^3\,(cd^2-ae^2)^2\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^3*Sqrt[a - c*x^4]),x]
```

```
[Out] ((d*e^2*x*(a - c*x^4)*(a*e^2*(5*d + 3*e*x^2) - c*d^2*(11*d + 9*e*x^2)))/(d
+ e*x^2)^2 - (I*Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*Sqrt[c]*d*e*(-3*c*d^2 + a*e^
```

```
2)*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + (-7*c^2*d^4 + 9*S
qrt[a]*c^(3/2)*d^3*e + a*c*d^2*e^2 - 3*a^(3/2)*Sqrt[c]*d*e^3)*EllipticF[I*A
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + 3*(5*c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt
[a])]*x], -1]))/Sqrt[-(Sqrt[c]/Sqrt[a])])/(8*d^3*(c*d^2 - a*e^2)^2*Sqrt[a -
 c*x^4])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 960 vs. $2(363) = 726$.
time = 0.12, size = 961, normalized size = 2.26 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+3/8*e^2*(a*e^2-3*c*d
^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)+1/8*c/d/(a*e^2-c*d^2)^
2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*
a*e^2-7/8*c^2*d/(a*e^2-c*d^2)^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x
*(1/a^(1/2)*c^(1/2))^(1/2),I)-3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^(3/2)/(
1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/
2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+9/8
*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
F(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8*c^(1/2)*e^3/(a*e^2-c*d^2)^2/d^2*a^(3/2
)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-
9/8*c^(3/2)*e/(a*e^2-c*d^2)^2*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+3/8/(a*e^2-c*d^2)^2/d^3*e^4/(1/a^(1/2)*
c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^
(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a^2-3/4/(a*e^2-
c*d^2)^2/d*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1
+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1
/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/
2))^(1/2))*a*c+15/8/(a*e^2-c*d^2)^2*d/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Ellip
ticPi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))
^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2))*c^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(x^2*e + d)^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - cx^4}\,(d + ex^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**3/(-c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^3/(-c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(x^2*e + d)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a - c\,x^4}\,{\left(e\,x^2 + d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^3),x)
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^3), x)
```

3.163 $\int \frac{1}{(d+ex^2)^4 \sqrt{a-cx^4}} \, dx$

Optimal. Leaf size=563

$$-\frac{e^2 x\sqrt{a-cx^4}}{6d\left(cd^2-ae^2\right)\left(d+ex^2\right)^3}-\frac{5e^2\left(3cd^2-ae^2\right)x\sqrt{a-cx^4}}{24d^2\left(cd^2-ae^2\right)^2\left(d+ex^2\right)^2}-\frac{e^2\left(29c^2d^4-14acd^2e^2+5a^2e^4\right)x\sqrt{a-cx^4}}{16d^3\left(cd^2-ae^2\right)^3\left(d+ex^2\right)}-\frac{a^{3/}}{}$$

```
[Out] -1/6*e^2*x*(-c*x^4+a)^(1/2)/d/(-a*e^2+c*d^2)/(e*x^2+d)^3-5/24*e^2*(-a*e^2+3
*c*d^2)*x*(-c*x^4+a)^(1/2)/d^2/(-a*e^2+c*d^2)^2/(e*x^2+d)^2-1/16*e^2*(5*a^2
*e^4-14*a*c*d^2*e^2+29*c^2*d^4)*x*(-c*x^4+a)^(1/2)/d^3/(-a*e^2+c*d^2)^3/(e*
x^2+d)-1/16*a^(3/4)*c^(1/4)*e*(5*a^2*e^4-14*a*c*d^2*e^2+29*c^2*d^4)*Ellipti
cE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/d^3/(-a*e^2+c*d^2)^3/(-c*x^4+a)^(
1/2)+1/16*a^(1/4)*(-5*a^3*e^6+17*a^2*c*d^2*e^4-7*a*c^2*d^4*e^2+35*c^3*d^6)*
EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1/2)/c^(1
/4)/d^4/(-a*e^2+c*d^2)^3/(-c*x^4+a)^(1/2)-1/48*a^(1/4)*c^(1/4)*EllipticF(c^
(1/4)*x/a^(1/4),I)*(57*c^2*d^4-32*a*c*d^2*e^2+15*a^2*e^4-30*c^(3/2)*d^3*e*a
^(1/2)+10*a^(3/2)*d*e^3*c^(1/2))*(1-c*x^4/a)^(1/2)/d^3/(-e*a^(1/2)+d*c^(1/2
))^2/(e*a^(1/2)+d*c^(1/2))^3/(-c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1238, 1711, 1731, 1215, 230, 227, 1214, 1213, 435, 1233, 1232}

$$-\frac{e^2x\sqrt{a-cx^4}\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)}{16d^3\left(d+ex^2\right)\left(cd^2-ae^2\right)^3}-\frac{a^{3/4}\sqrt[4]{c}\,e\sqrt{1-\frac{cx^4}{a}}\left(5a^2e^4-14acd^2e^2+29c^2d^4\right)E\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{16d^3\sqrt{a-cx^4}\left(cd^2-ae^2\right)^3}-\frac{\sqrt[4]{a}\sqrt[4]{c}\sqrt{1-\frac{cx^4}{a}}\left(10a^{3/2}\sqrt{c}\,de^3+15a^2e^4-30\sqrt{a}\,c^{3/2}d^3e-32acd^2e^2+57c^2d^4\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{48d^3\sqrt{a-cx^4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)^3}+\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\left(-5a^3e^6+17a^2cd^2e^4-7ac^2d^4e^2+35c^3d^6\right)\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{16\sqrt[4]{c}\,d^4\sqrt{a-cx^4}\left(cd^2-ae^2\right)^3}-\frac{5e^2x\sqrt{a-cx^4}\left(3cd^2-ae^2\right)}{24d^2\left(d+ex^2\right)^2\left(cd^2-ae^2\right)^2}-\frac{e^2x\sqrt{a-cx^4}}{6d\left(d+ex^2\right)^3\left(cd^2-ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```

```
[Out] -1/6*(e^2*x*Sqrt[a - c*x^4])/(d*(c*d^2 - a*e^2)*(d + e*x^2)^3) - (5*e^2*(3*
c*d^2 - a*e^2)*x*Sqrt[a - c*x^4])/(24*d^2*(c*d^2 - a*e^2)^2*(d + e*x^2)^2)
- (e^2*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*x*Sqrt[a - c*x^4])/(16*d^3
*(c*d^2 - a*e^2)^3*(d + e*x^2)) - (a^(3/4)*c^(1/4)*e*(29*c^2*d^4 - 14*a*c*d
^2*e^2 + 5*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4
)], -1])/(16*d^3*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4]) - (a^(1/4)*c^(1/4)*(57*
c^2*d^4 - 30*Sqrt[a]*c^(3/2)*d^3*e - 32*a*c*d^2*e^2 + 10*a^(3/2)*Sqrt[c]*d*
e^3 + 15*a^2*e^4)*Sqrt[1 - (c*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)]
, -1])/(48*d^3*(Sqrt[c]*d - Sqrt[a]*e)^2*(Sqrt[c]*d + Sqrt[a]*e)^3*Sqrt[a -
 c*x^4]) + (a^(1/4)*(35*c^3*d^6 - 7*a*c^2*d^4*e^2 + 17*a^2*c*d^2*e^4 - 5*a^
3*e^6)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[(c
^(1/4)*x)/a^(1/4)], -1])/(16*c^(1/4)*d^4*(c*d^2 - a*e^2)^3*Sqrt[a - c*x^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
```

```
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp
[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(2*d*(q + 1)*(c*d^2 + a*e^2)
)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqr
t[a + c*x^4])*Simp[a*e^2*(2*q + 3) + 2*c*d^2*(q + 1) - 2*e*c*d*(q + 1)*x^2
+ c*e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, c, d, e}, x] && ILtQ[q, -1]
```

Rule 1711

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol
] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]
}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sqrt[a + c*x^4]/(
2*d*(q + 1)*(c*d^2 + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 + a*e^2)), I
nt[((d + e*x^2)^(q + 1)/Sqrt[a + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2
*q + 3) + 2*c*d^2*(q + 1)) + 2*d*(B*c*d - A*c*e + a*C*e)*(q + 1)*x^2 + c*(C
*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a, c, d, e}, x]
&& PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[c*d^2 + a*e^2, 0] && ILt
Q[q, -1]
```

Rule 1731

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Di
st[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Dist[(C
*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /;
FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^4\sqrt{a-cx^4}}\,dx = -\frac{e^2x\sqrt{a-cx^4}}{6d\,(cd^2-ae^2)\,(d+ex^2)^3} + \frac{\int \frac{6cd^2-5ae^2-6cdex^2+3ce^2x^4}{(d+ex^2)^3\sqrt{a-cx^4}}\,dx}{6d\,(cd^2-ae^2)}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\,(cd^2-ae^2)\,(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\,(cd^2-ae^2)^2\,(d+ex^2)^2} + \frac{\int \frac{24c^2d^4-29acd^2e^2+ \cdots}{\cdots}}{\cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\,(cd^2-ae^2)\,(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\,(cd^2-ae^2)^2\,(d+ex^2)^2} - \frac{e^2(29c^2d^4-14ac \cdots}{16d^3\,(cd^2 \cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\,(cd^2-ae^2)\,(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\,(cd^2-ae^2)^2\,(d+ex^2)^2} - \frac{e^2(29c^2d^4-14ac \cdots}{16d^3\,(cd^2 \cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\,(cd^2-ae^2)\,(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\,(cd^2-ae^2)^2\,(d+ex^2)^2} - \frac{e^2(29c^2d^4-14ac \cdots}{16d^3\,(cd^2 \cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\,(cd^2-ae^2)\,(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\,(cd^2-ae^2)^2\,(d+ex^2)^2} - \frac{e^2(29c^2d^4-14ac \cdots}{16d^3\,(cd^2 \cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\,(cd^2-ae^2)\,(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\,(cd^2-ae^2)^2\,(d+ex^2)^2} - \frac{e^2(29c^2d^4-14ac \cdots}{16d^3\,(cd^2 \cdots}$$

$$= -\frac{e^2x\sqrt{a-cx^4}}{6d\,(cd^2-ae^2)\,(d+ex^2)^3} - \frac{5e^2(3cd^2-ae^2)\,x\sqrt{a-cx^4}}{24d^2\,(cd^2-ae^2)^2\,(d+ex^2)^2} - \frac{e^2(29c^2d^4-14ac \cdots}{16d^3\,(cd^2 \cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.05, size = 458, normalized size = 0.81

$$\frac{-\frac{de^2x(a-cx^4)\left(8(cd^3-ade^2)^2+10d(cd^2-ae^2)(3cd^2-ae^2)(d+ex^2)+3(29c^2d^4-14acd^2e^2+5a^2e^4)(d+ex^2)^2\right)}{(cd^2-ae^2)^3(d+ex^2)^3} - \frac{i\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\sqrt{c}\,de(29c^2d^4-14acd^2e^2+5a^2e^4)E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+\sqrt{c}\,d\left(57c^{5/2}d^5-87\sqrt{a}\,c^2d^4e-2ac^{3/2}d^3e^2+42a^{3/2}cd^2e^3+5a^2\sqrt{c}\,de^4-15a^{5/2}e^5\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+3(-35c^3d^6+7ac^2d^4e^2-17a^2cd^2e^4+5a^3e^6)\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,(-cd^2+ae^2)^3}}{48d^4\sqrt{a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^4*Sqrt[a - c*x^4]),x]
```

```
[Out] (-((d*e^2*x*(a - c*x^4)*(8*(c*d^3 - a*d*e^2)^2 + 10*d*(c*d^2 - a*e^2)*(3*c*
d^2 - a*e^2)*(d + e*x^2) + 3*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*(d +
 e*x^2)^2))/((c*d^2 - a*e^2)^3*(d + e*x^2)^3)) - (I*Sqrt[1 - (c*x^4)/a]*(3*
Sqrt[a]*Sqrt[c]*d*e*(29*c^2*d^4 - 14*a*c*d^2*e^2 + 5*a^2*e^4)*EllipticE[I*A
rcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1] + Sqrt[c]*d*(57*c^(5/2)*d^5 - 87*Sq
rt[a]*c^2*d^4*e - 2*a*c^(3/2)*d^3*e^2 + 42*a^(3/2)*c*d^2*e^3 + 5*a^2*Sqrt[c
]*d*e^4 - 15*a^(5/2)*e^5)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x],
-1] + 3*(-35*c^3*d^6 + 7*a*c^2*d^4*e^2 - 17*a^2*c*d^2*e^4 + 5*a^3*e^6)*Elli
pticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -
1]))/(Sqrt[-(Sqrt[c]/Sqrt[a])]*(-(c*d^2) + a*e^2)^3))/(48*d^4*Sqrt[a - c*x^
4])
```

<u>**Maple**</u> [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1419 vs. $2(489) = 978$.
time = 0.13, size = 1420, normalized size = 2.52

method	result	size
default	Expression too large to display	1420
elliptic	Expression too large to display	1420

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*e^2/(a*e^2-c*d^2)/d*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^3+5/24*e^2*(a*e^2-3*c*
d^2)/(a*e^2-c*d^2)^2/d^2*x*(-c*x^4+a)^(1/2)/(e*x^2+d)^2+1/16*e^2*(5*a^2*e^4
-14*a*c*d^2*e^2+29*c^2*d^4)/(a*e^2-c*d^2)^3/d^3*x*(-c*x^4+a)^(1/2)/(e*x^2+d
)-35/16/(a*e^2-c*d^2)^3*d^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1
/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/
a^(1/2)*c^(1/2))^(1/2))*c^3+19/16*c^3*d^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2
))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-
c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)+5/16/(a*e^2-c*d^2)^
3/d^4*e^6/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^
(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2))^
(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2))^(
1/2))*a^3+7/16/(a*e^2-c*d^2)^3*e^2/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*Elliptic
Pi(x*(1/a^(1/2)*c^(1/2))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1
/2)/(1/a^(1/2)*c^(1/2))^(1/2))*a*c^2-5/16*c^(1/2)*e^5/(a*e^2-c*d^2)^3/d^3*a
^(5/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/
2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2
),I)+7/8*c^(3/2)*e^3/(a*e^2-c*d^2)^3/d*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1
-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1
/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-29/16*c^(5/2)*e/(a*e^2-c*d^2)^
3*d*a^(1/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/
```

```
a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))
^(1/2),I)+5/16*c^(1/2)*e^5/(a*e^2-c*d^2)^3/d^3*a^(5/2)/(1/a^(1/2)*c^(1/2))^
(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x
^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-7/8*c^(3/2)*e^3/(a*e^2
-c*d^2)^3/d*a^(3/2)/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/
2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*
c^(1/2))^(1/2),I)+29/16*c^(5/2)*e/(a*e^2-c*d^2)^3*d*a^(1/2)/(1/a^(1/2)*c^(1
/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/
(-c*x^4+a)^(1/2)*EllipticE(x*(1/a^(1/2)*c^(1/2))^(1/2),I)-17/16/(a*e^2-c*d^
2)^3/d^2*e^4/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1
/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*c^(1/2
))^(1/2),-e*a^(1/2)/d/c^(1/2),(-1/a^(1/2)*c^(1/2))^(1/2)/(1/a^(1/2)*c^(1/2)
)^(1/2))*a^2*c+5/48*c/d^2/(a*e^2-c*d^2)^3/(1/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^
(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*E
llipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*a^2*e^4-1/24*c^2/(a*e^2-c*d^2)^3/(1
/a^(1/2)*c^(1/2))^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+1/a^(1/2)*c^(1/2
)*x^2)^(1/2)/(-c*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*c^(1/2))^(1/2),I)*a*e^
2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(x^2*e + d)^4), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - cx^4}\,(d + ex^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**4/(-c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a - c*x**4)*(d + e*x**2)**4), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^4/(-c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + a)*(x^2*e + d)^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a-c\,x^4}\,{\left(e\,x^2+d\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^4),x)
[Out] int(1/((a - c*x^4)^(1/2)*(d + e*x^2)^4), x)
```

3.164 $\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=126

$$\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{-a+cx^4}}+\frac{a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{-a+cx^4}}$$

```
[Out] a^(3/4)*e*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(3/4)/(c*x^4-a
)^(1/2)+a^(3/4)*EllipticF(c^(1/4)*x/a^(1/4),I)*(-e+d*c^(1/2)/a^(1/2))*(1-c*
x^4/a)^(1/2)/c^(3/4)/(c*x^4-a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1215, 230, 227, 1214, 1213, 435}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)F\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}+\frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (a^(3/4)*e*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/
(c^(3/4)*Sqrt[-a + c*x^4]) + (a^(3/4)*((Sqrt[c]*d)/Sqrt[a] - e)*Sqrt[1 - (c
*x^4)/a]*EllipticF[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(3/4)*Sqrt[-a + c*x
^4])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rule 1215

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Dist[(d*q - e)/q, Int[1/Sqrt[a + c*x^4], x], x] + Dist[e/q,
Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[
c/a] && NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{-a+cx^4}}\,dx = \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{-a+cx^4}}\,dx}{\sqrt{c}} + \left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-a+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}\,e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{c}\,\sqrt{-a+cx^4}} + \frac{\left(\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{cx^4}{a}}}\,d\cdots}{\sqrt{-a+cx^4}}$$

$$= \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,\sqrt{-a+cx^4}} + \frac{\left(\sqrt{a}\,e\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1\cdots}}{\sqrt{1\cdots}}}{\sqrt{c}\,\sqrt{-a+cx^4}}$$

$$= \frac{a^{3/4}e\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{c^{3/4}\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\cdots\right.\right.}{\sqrt[4]{c}\,\sqrt{-a+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 78, normalized size = 0.62

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + e
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4])
```

Maple [A]
time = 0.12, size = 160, normalized size = 1.27

method	result
default	$\frac{e\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}\,\sqrt{c}}+\frac{d\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{}$
elliptic	$\frac{e\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}\,\sqrt{c}}+\frac{d\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a
^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-1/a^(1/2)*
c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))+d/(-1/a^(1/2)*
c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/sqrt(c*x^4 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 1.05, size = 73, normalized size = 0.58

$$-\frac{idx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}-\frac{iex^3\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a)*gamma(5/4)
) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamm
a(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)/sqrt(c*x^4 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{\sqrt{c\,x^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(c*x^4 - a)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(c*x^4 - a)^(1/2), x)
```

3.165 $\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{-a+cx^4}}$$

```
[Out] a^(1/4)*EllipticPi(c^(1/4)*x/a^(1/4),-e*a^(1/2)/d/c^(1/2),I)*(1-c*x^4/a)^(1
/2)/c^(1/4)/d/(c*x^4-a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1233, 1232}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), ArcSin[
(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*Sqrt[-a + c*x^4])
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{-a+cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1}{(d+ex^2)\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}$$
$$= \frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,d\sqrt{-a+cx^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 92, normalized size = 1.26

$$-\frac{i\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,d\sqrt{-a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 - (c*x^4)/a]*EllipticPi[-((Sqrt[a]*e)/(Sqrt[c]*d)), I*ArcSinh[
Sqrt[-(Sqrt[c]/Sqrt[a])]*x], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*d*Sqrt[-a + c*x
^4])
```

Maple [A]
time = 0.13, size = 99, normalized size = 1.36

method	result	size
default	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},\frac{e\sqrt{a}}{d\sqrt{c}},\frac{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}$	99

elliptic	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},\frac{e\sqrt{a}}{d\sqrt{c}},\frac{\sqrt{\frac{\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}-a}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticPi(x*(-1/a^(1/2)*c^(1/2))^(1/2)
,e*a^(1/2)/d/c^(1/2),(1/a^(1/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 - a)*(x^2*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 - a)/(c*d*x^4 - a*d + (c*x^6 - a*x^2)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a+cx^{4}}\,(d+ex^{2})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a + c*x**4)*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 - a)*(x^2*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x^4-a}\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^4 - a)^(1/2)*(d + e*x^2)),x)
```

```
[Out] int(1/((c*x^4 - a)^(1/2)*(d + e*x^2)), x)
```

3.166 $\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=54

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,\sqrt{-a+cx^4}}$$

```
[Out] a^(3/4)*EllipticE(c^(1/4)*x/a^(1/4),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/(c*x^4-a)^
(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1214, 1213, 435}

$$\frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\,E\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (a^(3/4)*Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c
^(1/4)*Sqrt[-a + c*x^4])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{-a+cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{a}+\sqrt{c}\,x^2}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{\left(\sqrt{a}\sqrt{1-\frac{cx^4}{a}}\right)\int \frac{\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}{\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{a^{3/4}\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,\sqrt{-a+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 86, normalized size = 1.59

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(3\sqrt{a}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+\sqrt{c}\,x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a] + Sqrt[c]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*Sqrt[a]*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/
a] + Sqrt[c]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a +
 c*x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(42) = 84.
time = 0.14, size = 158, normalized size = 2.93

method	result
default	$\frac{\sqrt{a}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4-a}}+\frac{\sqrt{a}\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{Ellip}\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}}$

elliptic	$\frac{\left(\sqrt{a}+x^2\sqrt{c}\right)\sqrt{-\left(-c\,x^4+a\right)c}\,\sqrt{-\left(-c\,x^4+a\right)a}\left(\frac{\sqrt{c}\,\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\mathrm{EllipticF}\left(\right.\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c^2x}}\right.}{\sqrt{c\,x^4-a}\left(c\,x^2\sqrt{-\left(-c\,x^4+a\right)c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(
1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1
/2),I)+a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(
1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*(EllipticF(x*(-1/a^(1/2)*c^(
1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 1.01, size = 70, normalized size = 1.30

$$-\frac{ix\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}-\frac{i\sqrt{c}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/2)+x**2*c**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*gamma(5/4)) - I*sqrt
(c)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)*gamma(7/
4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/2)+x^2*c^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((sqrt(c)*x^2 + sqrt(a))/sqrt(c*x^4 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a}\ + \sqrt{c}\ x^2}{\sqrt{c\,x^4 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/2) + c^(1/2)*x^2)/(c*x^4 - a)^(1/2),x)
```

```
[Out] int((a^(1/2) + c^(1/2)*x^2)/(c*x^4 - a)^(1/2), x)
```

3.167 $\int \frac{1+\sqrt{\frac{c}{a}}\,x^2}{\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=52

$$\frac{\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}\,x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\,\sqrt{-a+cx^4}}$$

```
[Out] EllipticE((c/a)^(1/4)*x,I)*(1-c*x^4/a)^(1/2)/(c/a)^(1/4)/(c*x^4-a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1214, 1213, 435}

$$\frac{\sqrt{1-\frac{cx^4}{a}}\,E\left(\text{ArcSin}\left(\sqrt[4]{\frac{c}{a}}\,x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\,\sqrt{cx^4-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*EllipticE[ArcSin[(c/a)^(1/4)*x], -1])/((c/a)^(1/4)*Sqr
t[-a + c*x^4])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1214

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x], x
] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] &&  !GtQ[
```

```
a, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{\frac{c}{a}}\,x^2}{\sqrt{-a+cx^4}}\,dx = \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{1+\sqrt{\frac{c}{a}}\,x^2}{\sqrt{1-\frac{cx^4}{a}}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{\sqrt{1-\frac{cx^4}{a}}\int \frac{\sqrt{1+\sqrt{\frac{c}{a}}\,x^2}}{\sqrt{1-\sqrt{\frac{c}{a}}\,x^2}}\,dx}{\sqrt{-a+cx^4}}$$

$$= \frac{\sqrt{1-\frac{cx^4}{a}}\,E\left(\sin^{-1}\left(\sqrt[4]{\frac{c}{a}}\,x\right)\middle|-1\right)}{\sqrt[4]{\frac{c}{a}}\,\sqrt{-a+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 85, normalized size = 1.63

$$\frac{\sqrt{1-\frac{cx^4}{a}}\left(3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\frac{cx^4}{a}\right)+\sqrt{\frac{c}{a}}\,x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};\frac{cx^4}{a}\right)\right)}{3\sqrt{-a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[c/a]*x^2)/Sqrt[-a + c*x^4],x]
```

```
[Out] (Sqrt[1 - (c*x^4)/a]*(3*x*Hypergeometric2F1[1/4, 1/2, 5/4, (c*x^4)/a] + Sqr
t[c/a]*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, (c*x^4)/a]))/(3*Sqrt[-a + c*x^4
])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(44) = 88$.
time = 0.13, size = 165, normalized size = 3.17

method	result

default	$\frac{\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\text{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}+\frac{\sqrt{\frac{c}{a}}\,\sqrt{a}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\text{Ellipti}\right.}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c:}}$
elliptic	$\frac{\left(1+x^2\sqrt{\frac{c}{a}}\right)\sqrt{-\frac{(-c\,x^4+a)c}{a}}\,a\left(\frac{\sqrt{c}\,\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{-\frac{\sqrt{}}{\sqrt{}}}\right.\right.}{\sqrt{a}\,\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{\frac{c^2x^4}{a}-c}}\right.}{c\,x^2\sqrt{c\,x^4-a}+\sqrt{-\frac{(-c\,x^4+a)c}{a}}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c
^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),I)
+(c/a)^(1/2)*a^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(
1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(
-1/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-1/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(37) = 74.

time = 1.01, size = 76, normalized size = 1.46

$$-\frac{ix^{3}\sqrt{\frac{c}{a}}\,\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{cx^{4}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}-\frac{ix\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^{4}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**2*(c/a)**(1/2))/(c*x**4-a)**(1/2),x)
```

```
[Out] -I*x**3*sqrt(c/a)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4/a)/(4*sqrt(a)
*gamma(7/4)) - I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4/a)/(4*sqrt(a
)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^2*(c/a)^(1/2))/(c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*sqrt(c/a) + 1)/sqrt(c*x^4 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,\sqrt{\frac{c}{a}}+1}{\sqrt{c\,x^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c/a)^(1/2) + 1)/(c*x^4 - a)^(1/2),x)
```

```
[Out] int((x^2*(c/a)^(1/2) + 1)/(c*x^4 - a)^(1/2), x)
```

3.168 $\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx$

Optimal. Leaf size=236

$$-\frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}}+\frac{\sqrt[4]{a}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right.}{}$$

```
[Out] -e*x*(-c*x^4-a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*arctan
(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(si
n(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a
)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-c*x^4-a)^(1/2)+1/2*a^(1/4)*(cos(
2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elli
pticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(
e+d*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-c*
x^4-a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1212, 226, 1210}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{-a-cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{-a-cx^4}}-\frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a - c*x^4],x]
```

```
[Out] -((e*x*Sqrt[-a - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^(1/4)*e*(S
qrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[-a - c*x^4]) + (a^(1/4)
*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/
4)*Sqrt[-a - c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{-a-cx^4}}\,dx = -\frac{\left(\sqrt{a}\,e\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{-a-cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-a-cx^4}}\,dx$$

$$= -\frac{ex\sqrt{-a-cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{-a-cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 80, normalized size = 0.34

$$\frac{\sqrt{1+\frac{cx^4}{a}}\left(3dx\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+ex^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)}{3\sqrt{-a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a - c*x^4],x]
[Out] (Sqrt[1 + (c*x^4)/a]*(3*d*x*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)]
+ e*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)]))/(3*Sqrt[-a - c*x^4
])
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 175, normalized size = 0.74

method	result

default	$-\frac{ie\sqrt{a}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4-a}\,\sqrt{c}}+\frac{d\sqrt{1+\frac{i}{}}}{}$
elliptic	$-\frac{ie\sqrt{a}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4-a}\,\sqrt{c}}+\frac{d\sqrt{1+\frac{i}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*e*a^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)/c^(1/2)*(EllipticF(x*(-I/a^(1
/2)*c^(1/2))^(1/2),I)-EllipticE(x*(-I/a^(1/2)*c^(1/2))^(1/2),I))+d/(-I/a^(1
/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2
)^(1/2)/(-c*x^4-a)^(1/2)*EllipticF(x*(-I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/sqrt(-c*x^4 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.96, size = 83, normalized size = 0.35

$$-\frac{idx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}-\frac{iex^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] -I*d*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sq
rt(a)*gamma(5/4)) - I*e*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*ex
p_polar(I*pi)/a)/(4*sqrt(a)*gamma(7/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)/sqrt(-c*x^4 - a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{\sqrt{-c\,x^4-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(- a - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(- a - c*x^4)^(1/2), x)
```

3.169 $\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx$

Optimal. Leaf size=347

$$\frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{-cd^2-ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\,\sqrt{-cd^2-ae^2}}+\frac{\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{-a-cx^4}}-\frac{a^{3/4}}{\cdots}$$

[Out] 1/2*arctan(x*(-a*e^2-c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(-c*x^4-a)^(1/2))*e^(1/2)
/d^(1/2)/(-a*e^2-c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/a^(1/4)/(-e*a^(1/2)+d*c^(1/2))/(-c*x^4-a)^(1/2)-1/4*a^(3/4)*(c
os(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*E
llipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/
e/a^(1/2)/c^(1/2),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^
2*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(-c*x^
4-a)^(1/2)

Rubi [A]
time = 0.21, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1231, 226, 1721}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,de};2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{c}\,d\sqrt{-a-cx^4}\,\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{x\sqrt{-ae^2-cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\,\sqrt{-ae^2-cd^2}}+\frac{\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt{-a-cx^4}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]

[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a - c*x^4
])])/(2*Sqrt[d]*Sqrt[-(c*d^2) - a*e^2]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a - c*x^4]) - (a^(
3/4)*((Sqrt[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a
]*Sqrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*c^(1/4)*d*(c*d^2 -
a*e^2)*Sqrt[-a - c*x^4])

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{-a-cx^4}}\,dx = \frac{\sqrt{c}\int \frac{1}{\sqrt{-a-cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e} - \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{-a-cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e}$$

$$= \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{-cd^2-ae^2}\,x}{\sqrt{d}\sqrt{e}\sqrt{-a-cx^4}}\right)}{2\sqrt{d}\sqrt{-cd^2-ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right.}}}{2\sqrt[4]{a}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 98, normalized size = 0.28

$$-\frac{i\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{-a-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticPi[((-I)*Sqrt[a]*e)/(Sqrt[c]*d), I*ArcSin
h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*d*Sqrt[-a -
 c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 110, normalized size = 0.32

method	result	size
default	$\frac{\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4-a}}$	110
elliptic	$\frac{\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e}{\sqrt{c}\,d},\frac{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{d\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{-c\,x^4-a}}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(-I/a^(1/2)*c^(1/2))^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-I/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(-c*x^4-a)^(1/2)*EllipticPi(x*(-I/a^(1/2)*c^(1/2))^(1/2
),-I*a^(1/2)/c^(1/2)*e/d,(I/a^(1/2)*c^(1/2))^(1/2)/(-I/a^(1/2)*c^(1/2))^(1/
2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 - a)*(x^2*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c*x^4 - a)/(c*d*x^4 + a*d + (c*x^6 + a*x^2)*e), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a-cx^4}\,(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a - c*x**4)*(d + e*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 - a)*(x^2*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{-c\,x^4-a}\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- a - c*x^4)^(1/2)*(d + e*x^2)),x)
```

```
[Out] int(1/((- a - c*x^4)^(1/2)*(d + e*x^2)), x)
```

3.170 $\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx$

Optimal. Leaf size=40

$$\frac{\Pi\left(-\frac{2b}{\sqrt{5}\,a};\sin^{-1}\left(\frac{\sqrt[4]{5}\,x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\,\sqrt[4]{5}\,a}$$

```
[Out] 1/10*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b/a*5^(1/2),I)*5^(3/4)/a*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1227, 551}

$$\frac{\Pi\left(-\frac{2b}{\sqrt{5}\,a};\text{ArcSin}\left(\frac{\sqrt[4]{5}\,x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\,\sqrt[4]{5}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5^
(1/4)*a)
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1227

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[(-a)*c, 2]}, Dist[Sqrt[-c], Int[1/((d + e*x^2)*Sqrt[q + c*x^2]*Sqrt
[q - c*x^2]), x], x]] /; FreeQ[{a, c, d, e}, x] && GtQ[a, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)\sqrt{4-5x^4}}\,dx = \sqrt{5}\int \frac{1}{\sqrt{2\sqrt{5}-5x^2}\,\sqrt{2\sqrt{5}+5x^2}\,(a+bx^2)}\,dx$$
$$= \frac{\Pi\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

Mathematica [A]
time = 10.10, size = 40, normalized size = 1.00

$$\frac{\Pi\left(-\frac{2b}{\sqrt{5}a};\sin^{-1}\left(\frac{\sqrt[4]{5}x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\sqrt[4]{5}a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 - 5*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(Sqrt[5]*a), ArcSin[(5^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*5^
(1/4)*a)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(32) = 64$.
time = 0.14, size = 79, normalized size = 1.98

method	result	size
default	$\frac{\sqrt{2}\,5^{\frac{3}{4}}\sqrt{1-\frac{x^2\sqrt{5}}{2}}\,\sqrt{1+\frac{x^2\sqrt{5}}{2}}\,\mathrm{EllipticPi}\left(\frac{5^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b\sqrt{5}}{5a},\frac{\sqrt{-\frac{\sqrt{5}}{2}}\,\sqrt{2}\,5^{\frac{3}{4}}}{5}\right)}{5a\sqrt{-5x^4+4}}$	79
elliptic	$\frac{\sqrt{2}\,5^{\frac{3}{4}}\sqrt{1-\frac{x^2\sqrt{5}}{2}}\,\sqrt{1+\frac{x^2\sqrt{5}}{2}}\,\mathrm{EllipticPi}\left(\frac{5^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b\sqrt{5}}{5a},\frac{\sqrt{-\frac{\sqrt{5}}{2}}\,\sqrt{2}\,5^{\frac{3}{4}}}{5}\right)}{5a\sqrt{-5x^4+4}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/a*2^(1/2)*5^(3/4)*(1-1/2*x^2*5^(1/2))^(1/2)*(1+1/2*x^2*5^(1/2))^(1/2)/(
-5*x^4+4)^(1/2)*EllipticPi(1/2*5^(1/4)*x*2^(1/2),-2/5*b/a*5^(1/2),1/5*(-1/2
*5^(1/2))^(1/2)*2^(1/2)*5^(3/4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 - 4*b*x^2 - 4*a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4-5x^4}\,(a+bx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(-5*x**4+4)**(1/2),x)
[Out] Integral(1/(sqrt(4 - 5*x**4)*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-5*x^4+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-5*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\,x^2+a)\,\sqrt{4-5\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(4 - 5*x^4)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(4 - 5*x^4)^(1/2)), x)
```

3.171 $\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx$

Optimal. Leaf size=310

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{5a^2+4b^2}\,x}{\sqrt{a}\sqrt{b}\sqrt{4+5x^4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}}+\frac{\sqrt[4]{5}\left(\sqrt{5}\,a+2b\right)\left(2+\sqrt{5}\,x^2\right)\sqrt{\frac{4+5x^4}{\left(2+\sqrt{5}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{5}\,x}{\sqrt{2}}\right)\right|}{2\sqrt{2}\,(5a^2-4b^2)\sqrt{4+5x^4}}$$

```
[Out] 1/2*arctan(x*(5*a^2+4*b^2)^(1/2)/a^(1/2)/b^(1/2)/(5*x^4+4)^(1/2))*b^(1/2)/a
^(1/2)/(5*a^2+4*b^2)^(1/2)+1/4*5^(1/4)*(cos(2*arctan(1/2*5^(1/4)*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*5^(1/4)*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*
5^(1/4)*x*2^(1/2))),1/2*2^(1/2))*(2*b+a*5^(1/2))*(2+x^2*5^(1/2))*((5*x^4+4)
/(2+x^2*5^(1/2))^2)^(1/2)/(5*a^2-4*b^2)*2^(1/2)/(5*x^4+4)^(1/2)-1/40*(cos(2
*arctan(1/2*5^(1/4)*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*5^(1/4)*x*2^(1/2)
))*EllipticPi(sin(2*arctan(1/2*5^(1/4)*x*2^(1/2))),-1/40*(-2*b+a*5^(1/2))^2
/a/b*5^(1/2),1/2*2^(1/2))*(2*b+a*5^(1/2))^2*(2+x^2*5^(1/2))*((5*x^4+4)/(2+x
^2*5^(1/2))^2)^(1/2)*5^(3/4)/a/(5*a^2-4*b^2)*2^(1/2)/(5*x^4+4)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1231, 226, 1721}

$$\frac{\sqrt{b}\,\mathrm{ArcTan}\left(\frac{x\sqrt{5a^2+4b^2}}{\sqrt{a}\sqrt{b}\sqrt{5x^4+4}}\right)}{2\sqrt{a}\sqrt{5a^2+4b^2}}+\frac{\sqrt[4]{5}\left(\sqrt{5}\,x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}\,x^2+2\right)^2}}\left(\sqrt{5}\,a+2b\right)F\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{5}\,x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{5x^4+4}\,(5a^2-4b^2)}-\frac{\left(\sqrt{5}\,x^2+2\right)\sqrt{\frac{5x^4+4}{\left(\sqrt{5}\,x^2+2\right)^2}}\left(\sqrt{5}\,a+2b\right)^2\Pi\left(-\frac{\left(\sqrt{5}\,a-2b\right)^2}{8\sqrt{5}\,ab};2\mathrm{ArcTan}\left(\frac{\sqrt[4]{5}\,x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{4\sqrt{2}\sqrt[4]{5}\,a\sqrt{5x^4+4}\,(5a^2-4b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[5*a^2 + 4*b^2]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + 5*x^4])])
/(2*Sqrt[a]*Sqrt[5*a^2 + 4*b^2]) + (5^(1/4)*(Sqrt[5]*a + 2*b)*(2 + Sqrt[5]*
x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticF[2*ArcTan[(5^(1/4)*x)/S
qrt[2]], 1/2])/(2*Sqrt[2]*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4]) - ((Sqrt[5]*a +
2*b)^2*(2 + Sqrt[5]*x^2)*Sqrt[(4 + 5*x^4)/(2 + Sqrt[5]*x^2)^2]*EllipticPi[-
1/8*(Sqrt[5]*a - 2*b)^2/(Sqrt[5]*a*b), 2*ArcTan[(5^(1/4)*x)/Sqrt[2]], 1/2])
/(4*Sqrt[2]*5^(1/4)*a*(5*a^2 - 4*b^2)*Sqrt[4 + 5*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)\sqrt{4+5x^4}}\,dx = -\frac{\left(2b\left(\sqrt{5}\,a+2b\right)\right)\int \frac{1+\frac{\sqrt{5}\,x^2}{2}}{(a+bx^2)\sqrt{4+5x^4}}\,dx}{5a^2-4b^2} + \frac{\left(5a+2\sqrt{5}\,b\right)\int \frac{1}{\sqrt{4+5x^4}}\,dx}{5a^2-4b^2}$$

$$= \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{5a^2+4b^2}\,x}{\sqrt{a}\,\sqrt{b}\,\sqrt{4+5x^4}}\right)}{2\sqrt{a}\,\sqrt{5a^2+4b^2}} + \frac{\sqrt[4]{5}\left(\sqrt{5}\,a+2b\right)\left(2+\sqrt{5}\,x^2\right)\sqrt{\frac{4+5\cdots}{\left(2+\sqrt{\cdots}\right.}}}{2\sqrt{2}\,(5a^2-4b^2)\,\sqrt{\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 50, normalized size = 0.16

$$-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\Pi\left(-\frac{2ib}{\sqrt{5}\,a};i\sinh^{-1}\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt[4]{5}\,x\right)\middle|-1\right)}{\sqrt[4]{5}\,a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 + 5*x^4]),x]
```

```
[Out] ((-1/2 - I/2)*EllipticPi[((-2*I)*b)/(Sqrt[5]*a), I*ArcSinh[(1/2 + I/2)*5^(1
/4)*x], -1])/(5^(1/4)*a)
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 86, normalized size = 0.28

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{5}\,x^{2}}{2}}\,\sqrt{1+\frac{i\sqrt{5}\,x^{2}}{2}}\,\text{EllipticPi}\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{5}}\,x}{2},\frac{2i\sqrt{5}\,b}{5a},\frac{\sqrt{-\frac{i\sqrt{5}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{5}}}\right)}{a\sqrt{i\sqrt{5}}\,\sqrt{5x^{4}+4}}$	86
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{i\sqrt{5}\,x^{2}}{2}}\,\sqrt{1+\frac{i\sqrt{5}\,x^{2}}{2}}\,\text{EllipticPi}\left(\frac{\sqrt{2}\,\sqrt{i\sqrt{5}}\,x}{2},\frac{2i\sqrt{5}\,b}{5a},\frac{\sqrt{-\frac{i\sqrt{5}}{2}}\,\sqrt{2}}{\sqrt{i\sqrt{5}}}\right)}{a\sqrt{i\sqrt{5}}\,\sqrt{5x^{4}+4}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(5*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/(1/2*I*5^(1/2))^(1/2)*(1-1/2*I*5^(1/2)*x^2)^(1/2)*(1+1/2*I*5^(1/2)*x^2)
^(1/2)/(5*x^4+4)^(1/2)*EllipticPi((1/2*I*5^(1/2))^(1/2)*x,2/5*I*5^(1/2)*b/a
,(-1/2*I*5^(1/2))^(1/2)/(1/2*I*5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(5*x^4 + 4)/(5*b*x^6 + 5*a*x^4 + 4*b*x^2 + 4*a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)\,\sqrt{5x^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(5*x**4+4)**(1/2),x)
[Out] Integral(1/((a + b*x**2)*sqrt(5*x**4 + 4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(5*x^4+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(5*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)\,\sqrt{5\,x^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(5*x^4 + 4)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(5*x^4 + 4)^(1/2)), x)
```

3.172 $\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx$

Optimal. Leaf size=40

$$\frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\,a\sqrt[4]{d}}$$

```
[Out] 1/2*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d^(1/2),I)/a/d^(1/4)*2^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1232}

$$\frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\text{ArcSin}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\,a\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

```
[Out] EllipticPi[(-2*b)/(a*Sqrt[d]), ArcSin[(d^(1/4)*x)/Sqrt[2]], -1]/(Sqrt[2]*a*
d^(1/4))
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)\sqrt{4-dx^4}}\,dx = \frac{\Pi\left(-\frac{2b}{a\sqrt{d}};\sin^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\,a\sqrt[4]{d}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 59, normalized size = 1.48

$$-\frac{i\Pi\left(-\frac{2b}{a\sqrt{d}};i\sinh^{-1}\left(\frac{\sqrt{-\sqrt{d}}\,x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\,a\sqrt{-\sqrt{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 - d*x^4]),x]
```

```
[Out] ((-I)*EllipticPi[(-2*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[-Sqrt[d]]*x)/Sqrt[2]],
 -1])/(Sqrt[2]*a*Sqrt[-Sqrt[d]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(32) = 64$.
time = 0.12, size = 78, normalized size = 1.95

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{x^{2}\sqrt{d}}{2}}\,\sqrt{1+\frac{x^{2}\sqrt{d}}{2}}\,\mathrm{EllipticPi}\left(\frac{d^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b}{a\sqrt{d}},\frac{\sqrt{-\frac{\sqrt{d}}{2}}\,\sqrt{2}}{d^{\frac{1}{4}}}\right)}{a\,d^{\frac{1}{4}}\sqrt{-d\,x^{4}+4}}$	78
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{x^{2}\sqrt{d}}{2}}\,\sqrt{1+\frac{x^{2}\sqrt{d}}{2}}\,\mathrm{EllipticPi}\left(\frac{d^{\frac{1}{4}}x\sqrt{2}}{2},-\frac{2b}{a\sqrt{d}},\frac{\sqrt{-\frac{\sqrt{d}}{2}}\,\sqrt{2}}{d^{\frac{1}{4}}}\right)}{a\,d^{\frac{1}{4}}\sqrt{-d\,x^{4}+4}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*2^(1/2)/d^(1/4)*(1-1/2*x^2*d^(1/2))^(1/2)*(1+1/2*x^2*d^(1/2))^(1/2)/(-d
*x^4+4)^(1/2)*EllipticPi(1/2*d^(1/4)*x*2^(1/2),-2*b/a/d^(1/2),(-1/2*d^(1/2)
)^(1/2)*2^(1/2)/d^(1/4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-d*x^4 + 4)/(b*d*x^6 + a*d*x^4 - 4*b*x^2 - 4*a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^2)\sqrt{-dx^4 + 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(-d*x**4+4)**(1/2),x)
[Out] Integral(1/((a + b*x**2)*sqrt(-d*x**4 + 4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(-d*x^4+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-d*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\,x^2 + a)\,\sqrt{4 - d\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(4 - d*x^4)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(4 - d*x^4)^(1/2)), x)
```

3.173 $\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx$

Optimal. Leaf size=300

$$\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{4b^2+a^2d}\,x}{\sqrt{a}\,\sqrt{b}\,\sqrt{4+dx^4}}\right)}{2\sqrt{a}\,\sqrt{4b^2+a^2d}}-\frac{\sqrt[4]{d}\left(2+\sqrt{d}\,x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\left(2b-a\sqrt{d}\right)\sqrt{4+dx^4}}+\frac{\left(2b+a\sqrt{\right.}}{}$$

```
[Out] 1/2*arctan(x*(a^2*d+4*b^2)^(1/2)/a^(1/2)/b^(1/2)/(d*x^4+4)^(1/2))*b^(1/2)/a
^(1/2)/(a^2*d+4*b^2)^(1/2)-1/4*d^(1/4)*(cos(2*arctan(1/2*d^(1/4)*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*d^(1/4)*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*
d^(1/4)*x*2^(1/2))),1/2*2^(1/2))*(2+x^2*d^(1/2))*((d*x^4+4)/(2+x^2*d^(1/2))
^2)^(1/2)*2^(1/2)/(2*b-a*d^(1/2))/(d*x^4+4)^(1/2)+1/8*(cos(2*arctan(1/2*d^(
1/4)*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*d^(1/4)*x*2^(1/2)))*EllipticPi(s
in(2*arctan(1/2*d^(1/4)*x*2^(1/2))),-1/8*(2*b-a*d^(1/2))^2/a/b/d^(1/2),1/2*
2^(1/2))*(2*b+a*d^(1/2))*(2+x^2*d^(1/2))*((d*x^4+4)/(2+x^2*d^(1/2))^2)^(1/2
)/a/d^(1/4)*2^(1/2)/(2*b-a*d^(1/2))/(d*x^4+4)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1231, 226, 1721}

$$\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{x\sqrt{a^2d+4b^2}}{\sqrt{a}\,\sqrt{b}\,\sqrt{dx^4+4}}\right)}{2\sqrt{a}\,\sqrt{a^2d+4b^2}}-\frac{\sqrt[4]{d}\left(\sqrt{d}\,x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}\,x^2+2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{2\sqrt{2}\,\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}+\frac{\left(\sqrt{d}\,x^2+2\right)\sqrt{\frac{dx^4+4}{\left(\sqrt{d}\,x^2+2\right)^2}}\left(a\sqrt{d}+2b\right)\Pi\left(-\frac{\left(2b-a\sqrt{d}\right)^2}{8ab\sqrt{d}};2\text{ArcTan}\left(\frac{\sqrt[4]{d}\,x}{\sqrt{2}}\right)|\frac{1}{2}\right)}{4\sqrt{2}\,a\sqrt[4]{d}\,\sqrt{dx^4+4}\left(2b-a\sqrt{d}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[4*b^2 + a^2*d]*x)/(Sqrt[a]*Sqrt[b]*Sqrt[4 + d*x^4])])
/(2*Sqrt[a]*Sqrt[4*b^2 + a^2*d]) - (d^(1/4)*(2 + Sqrt[d]*x^2)*Sqrt[(4 + d*x
^4)/(2 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(2*
Sqrt[2]*(2*b - a*Sqrt[d])*Sqrt[4 + d*x^4]) + ((2*b + a*Sqrt[d])*(2 + Sqrt[d
]*x^2)*Sqrt[(4 + d*x^4)/(2 + Sqrt[d]*x^2)^2]*EllipticPi[-1/8*(2*b - a*Sqrt[
d])^2/(a*b*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/Sqrt[2]], 1/2])/(4*Sqrt[2]*a*(2*b
 - a*Sqrt[d])*d^(1/4)*Sqrt[4 + d*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)\sqrt{4+dx^4}}\,dx = \frac{(2b)\int \frac{1+\frac{\sqrt{d}}{2}x^2}{(a+bx^2)\sqrt{4+dx^4}}\,dx}{2b-a\sqrt{d}} - \frac{\sqrt{d}\int \frac{1}{\sqrt{4+dx^4}}\,dx}{2b-a\sqrt{d}}$$

$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{4b^2+a^2d}\,x}{\sqrt{a}\sqrt{b}\sqrt{4+dx^4}}\right)}{2\sqrt{a}\sqrt{4b^2+a^2d}} - \frac{\sqrt[4]{d}\left(2+\sqrt{d}x^2\right)\sqrt{\frac{4+dx^4}{\left(2+\sqrt{d}x^2\right)^2}}\,F\Big(2 \ldots}{2\sqrt{2}\left(2b-a\sqrt{d}\right)\sqrt{4+ \ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 65, normalized size = 0.22

$$-\frac{i\Pi\left(-\frac{2ib}{a\sqrt{d}};i\sinh^{-1}\left(\frac{\sqrt{i\sqrt{d}}\,x}{\sqrt{2}}\right)\middle|-1\right)}{\sqrt{2}\,a\sqrt{i\sqrt{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)*Sqrt[4 + d*x^4]),x]
```

```
[Out] ((-I)*EllipticPi[((-2*I)*b)/(a*Sqrt[d]), I*ArcSinh[(Sqrt[I*Sqrt[d]]*x)/Sqrt
[2]], -1])/(Sqrt[2]*a*Sqrt[I*Sqrt[d]])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 86, normalized size = 0.29

method	result	size
default	$\frac{\sqrt{2}\, \sqrt{1-\frac{i\sqrt{d}\, x^{2}}{2}}\, \sqrt{1+\frac{i\sqrt{d}\, x^{2}}{2}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{2}\, \sqrt{i\sqrt{d}}\, x}{2}, \frac{2ib}{\sqrt{d}\, a}, \frac{\sqrt{-\frac{i\sqrt{d}}{2}}\, \sqrt{2}}{\sqrt{i\sqrt{d}}}\right)}{a\sqrt{i\sqrt{d}}\, \sqrt{d\,x^{4}+4}}$	86
elliptic	$\frac{\sqrt{2}\, \sqrt{1-\frac{i\sqrt{d}\, x^{2}}{2}}\, \sqrt{1+\frac{i\sqrt{d}\, x^{2}}{2}}\, \mathrm{EllipticPi}\left(\frac{\sqrt{2}\, \sqrt{i\sqrt{d}}\, x}{2}, \frac{2ib}{\sqrt{d}\, a}, \frac{\sqrt{-\frac{i\sqrt{d}}{2}}\, \sqrt{2}}{\sqrt{i\sqrt{d}}}\right)}{a\sqrt{i\sqrt{d}}\, \sqrt{d\,x^{4}+4}}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)/(d*x^4+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/(1/2*I*d^(1/2))^(1/2)*(1-1/2*I*d^(1/2)*x^2)^(1/2)*(1+1/2*I*d^(1/2)*x^2)
^(1/2)/(d*x^4+4)^(1/2)*EllipticPi((1/2*I*d^(1/2))^(1/2)*x,2*I/d^(1/2)*b/a,(
-1/2*I*d^(1/2))^(1/2)/(1/2*I*d^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + 4)/(b*d*x^6 + a*d*x^4 + 4*b*x^2 + 4*a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^2)\sqrt{dx^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)/(d*x**4+4)**(1/2),x)
[Out] Integral(1/((a + b*x**2)*sqrt(d*x**4 + 4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)/(d*x^4+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^4 + 4)*(b*x^2 + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^2+a\right)\,\sqrt{d\,x^4+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^2)*(d*x^4 + 4)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(d*x^4 + 4)^(1/2)), x)
```

3.174 $\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=112

$$\frac{a\sqrt{1-x^2}\,\sqrt{\frac{a(1+x^2)}{a+bx^2}}\,\Pi\left(\frac{b}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}\,x}{\sqrt{a+bx^2}}\right)|-\frac{a-b}{a+b}\right)}{\sqrt{a+b}\,\sqrt{1+x^2}\,\sqrt{\frac{a(1-x^2)}{a+bx^2}}}$$

```
[Out] a*EllipticPi(x*(a+b)^(1/2)/(b*x^2+a)^(1/2),b/(a+b),((-a+b)/(a+b))^(1/2))*(-
x^2+1)^(1/2)*(a*(x^2+1)/(b*x^2+a))^(1/2)/(a+b)^(1/2)/(x^2+1)^(1/2)/(a*(-x^2
+1)/(b*x^2+a))^(1/2)
```

Rubi [F]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
```

```
[Out] Defer[Int][Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

Mathematica [F]
time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4],x]
```

```
[Out] Integrate[Sqrt[a + b*x^2]/Sqrt[1 - x^4], x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^{2}+a}}{\sqrt{-x^{4}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)*sqrt(b*x^2 + a)/(x^4 - 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^{2}}}{\sqrt{-\left(x-1\right)\left(x+1\right)\left(x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(-x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^2+a}}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(1 - x^4)^(1/2),x)
```

```
[Out] int((a + b*x^2)^(1/2)/(1 - x^4)^(1/2), x)
```

3.175 $\int (c+ex^2)^q (a+bx^4)^p \, dx$

Optimal. Leaf size=22

$$\text{Int}\left(\left(c+ex^2\right)^q \left(a+bx^4\right)^p,x\right)$$

```
[Out] Unintegrable((e*x^2+c)^q*(b*x^4+a)^p,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

```
[Out] Defer[Int][(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Rubi steps

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx = \int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(c+ex^2\right)^q \left(a+bx^4\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + e*x^2)^q*(a + b*x^4)^p,x]
```

```
[Out] Integrate[(c + e*x^2)^q*(a + b*x^4)^p, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+c\right)^q \left(b\,x^4+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^q*(b*x^4+a)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^p*(x^2*e + c)^q, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p*(x^2*e + c)^q, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**q*(b*x**4+a)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+a)^p,x, algorithm="giac")
[Out] integrate((b*x^4 + a)^p*(x^2*e + c)^q, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \left(b\,x^4+a\right)^p\,\left(e\,x^2+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p*(c + e*x^2)^q,x)
[Out] int((a + b*x^4)^p*(c + e*x^2)^q, x)
```

3.176 $\int (c+ex^2)^3 (a+bx^4)^p \, dx$

Optimal. Leaf size=204

$$\frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)}+c^3x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)-\frac{e(ae^2-bc^2(7+4p))\,x^3(a+bx^4)^p\,(}{b(7+}$$

```
[Out] e^3*x^3*(b*x^4+a)^(1+p)/b/(7+4*p)+c^3*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/
4],-b*x^4/a)/((1+b*x^4/a)^p)-e*(a*e^2-b*c^2*(7+4*p))*x^3*(b*x^4+a)^p*hyperg
eom([3/4, -p],[7/4],-b*x^4/a)/b/(7+4*p)/((1+b*x^4/a)^p)+3/5*c*e^2*x^5*(b*x^
4+a)^p*hypergeom([5/4, -p],[9/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A]
time = 0.15, antiderivative size = 196, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1221, 1907, 252, 251, 372, 371}

$$c^3x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c^2-\frac{ae^2}{4bp+7b}\right){}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+\frac{3}{5}ce^2x^5(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+\frac{e^3x^3(a+bx^4)^{p+1}}{b(4p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

```
[Out] (e^3*x^3*(a + b*x^4)^(1 + p))/(b*(7 + 4*p)) + (c^3*x*(a + b*x^4)^p*Hypergeo
metric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)/a)^p + (e*(c^2 - (a*e^2
)/(7*b + 4*b*p))*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4
)/a)])/(1 + (b*x^4)/a)^p + (3*c*e^2*x^5*(a + b*x^4)^p*Hypergeometric2F1[5/4
, -p, 9/4, -((b*x^4)/a)])/(5*(1 + (b*x^4)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}\int \left(c+ex^2\right)^3\left(a+bx^4\right)^p\,dx &= \frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)}+\frac{\int (a+bx^4)^p\left(bc^3(7+4p)-3e(ae^2-bc^2(7+4p))\,x^2+3bc\right.}{b(7+4p)}\\ &= \frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)}+\frac{\int \left(bc^3(7+4p)\,(a+bx^4)^p+3e(-ae^2+bc^2(7+4p))\,x^2(a+\right.}{b(7+4p)}\\ &= \frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)}+c^3\int \left(a+bx^4\right)^p\,dx+\left(3ce^2\right)\int x^4\left(a+bx^4\right)^p\,dx+\left(3e\left(c\right.\right.\\ &= \frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)}+\left(c^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int\left(1+\frac{bx^4}{a}\right)^p\,dx+\left(3ce^2\right.\\ &= \frac{e^3x^3(a+bx^4)^{1+p}}{b(7+4p)}+c^3x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+e\left(c\right.\end{aligned}$$

Mathematica [A]
time = 0.59, size = 136, normalized size = 0.67

$$\frac{1}{35}x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(35c^3\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^2\left(35c^2\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+ex^2\left(21c\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)+5ex^2\,{}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-\frac{bx^4}{a}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)^3*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(35*c^3*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*
x^2*(35*c^2*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + e*x^2*(21*c*Hyp
ergeometric2F1[5/4, -p, 9/4, -((b*x^4)/a)] + 5*e*x^2*Hypergeometric2F1[7/4,
 -p, 11/4, -((b*x^4)/a)]))))/(35*(1 + (b*x^4)/a)^p)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+c\right)^{3} \left(b\,x^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^3*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^3*(b*x^4+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + c)^3*(b*x^4 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^6*e^3 + 3*c*x^4*e^2 + 3*c^2*x^2*e + c^3)*(b*x^4 + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 65.26, size = 167, normalized size = 0.82

$$\frac{a^{p}c^{3}x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{3a^{p}c^{2}ex^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},-p\\\frac{7}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{3a^{p}ce^{2}x^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},-p\\\frac{9}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{a^{p}e^{3}x^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{7}{4},-p\\\frac{11}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**3*(b*x**4+a)**p,x)
```

```
[Out] a**p*c**3*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4)) + 3*a**p*c**2*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x*
*4*exp_polar(I*pi)/a)/(4*gamma(7/4)) + 3*a**p*c*e**2*x**5*gamma(5/4)*hyper(
(5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4)) + a**p*e**3*x**
7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(1
1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2*e + c)^3*(b*x^4 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\,x^4+a\right)^p\,\left(e\,x^2+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p*(c + e*x^2)^3,x)
```

```
[Out] int((a + b*x^4)^p*(c + e*x^2)^3, x)
```

3.177 $\int (c+ex^2)^2 (a+bx^4)^p \, dx$

Optimal. Leaf size=150

$$\frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)}-\frac{(ae^2-bc^2(5+4p))\,x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)}{b(5+4p)}+\frac{2}{3}cex^3(a+bx^4)^p\left(1+\right.$$

```
[Out] e^2*x*(b*x^4+a)^(1+p)/b/(5+4*p)-(a*e^2-b*c^2*(5+4*p))*x*(b*x^4+a)^p*hyperge
om([1/4, -p],[5/4],-b*x^4/a)/b/(5+4*p)/((1+b*x^4/a)^p)+2/3*c*e*x^3*(b*x^4+a
)^p*hypergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A]
time = 0.09, antiderivative size = 142, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1221, 1218, 252, 251, 372, 371}

$$x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c^2-\frac{ae^2}{4bp+5b}\right){}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{2}{3}cex^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+\frac{e^2x(a+bx^4)^{p+1}}{b(4p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

```
[Out] (e^2*x*(a + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c^2 - (a*e^2)/(5*b + 4*b*p))*
x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4)
/a)^p + (2*c*e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/
a)])/(3*(1 + (b*x^4)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+ex^2)^2 (a+bx^4)^p \, dx &= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} + \frac{\int (-ae^2+bc^2(5+4p)+2bce(5+4p)x^2)(a+bx^4)^p \, dx}{b(5+4p)} \\
&= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} + \frac{\int \left(-ae^2\left(1-\frac{bc^2(5+4p)}{ae^2}\right)(a+bx^4)^p + 2bce(5+4p)x^2(a+b\right.}{b(5+4p)} \\
&= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} + (2ce)\int x^2(a+bx^4)^p \, dx - \left(-c^2+\frac{ae^2}{5b+4bp}\right)\int (a+bx^4 \\
&= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} + \left(2ce(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int x^2\left(1+\frac{bx^4}{a}\right)^p dx - \Bigg(\Bigg(\\
&= \frac{e^2x(a+bx^4)^{1+p}}{b(5+4p)} + \left(c^2-\frac{ae^2}{5b+4bp}\right)x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};\right.
\end{aligned}$$

Mathematica [A]
time = 0.55, size = 106, normalized size = 0.71

$$\frac{1}{15}x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(15c^2\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^2\left(10c\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)+3ex^2\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{bx^4}{a}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)^2*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(15*c^2*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*
x^2*(10*c*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)] + 3*e*x^2*Hypergeom
etric2F1[5/4, -p, 9/4, -((b*x^4)/a)])))/(15*(1 + (b*x^4)/a)^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+c\right)^{2} \left(b\,x^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)^2*(b*x^4+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + c)^2*(b*x^4 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^4*e^2 + 2*c*x^2*e + c^2)*(b*x^4 + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 36.21, size = 119, normalized size = 0.79

$$\frac{a^{p} c^{2} x \Gamma\left(\frac{1}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4}, -p \\ \frac{5}{4}\end{matrix}\right| \frac{b x^{4} e^{i\pi}}{a}\right)}{4 \Gamma\left(\frac{5}{4}\right)} + \frac{a^{p} c e x^{3} \Gamma\left(\frac{3}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4}, -p \\ \frac{7}{4}\end{matrix}\right| \frac{b x^{4} e^{i\pi}}{a}\right)}{2 \Gamma\left(\frac{7}{4}\right)} + \frac{a^{p} e^{2} x^{5} \Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\left.\begin{matrix}\frac{5}{4}, -p \\ \frac{9}{4}\end{matrix}\right| \frac{b x^{4} e^{i\pi}}{a}\right)}{4 \Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**2*(b*x**4+a)**p,x)
```

```
[Out] a**p*c**2*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4)) + a**p*c*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*ex
p_polar(I*pi)/a)/(2*gamma(7/4)) + a**p*e**2*x**5*gamma(5/4)*hyper((5/4, -p)
, (9/4,), b*x**4*exp_polar(I*pi)/a)/(4*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2*e + c)^2*(b*x^4 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^4+a\right)}^p\,{\left(e\,x^2+c\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p*(c + e*x^2)^2,x)
```

```
[Out] int((a + b*x^4)^p*(c + e*x^2)^2, x)
```

3.178 $\int (c+ex^2)\left(a+bx^4\right)^p\,dx$

Optimal. Leaf size=96

$$cx\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)$$

```
[Out] c*x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)+1/3*e*x
^3*(b*x^4+a)^p*hypergeom([3/4, -p],[7/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A]
time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1218, 252, 251, 372, 371}

$$cx\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+\frac{1}{3}ex^3\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)*(a + b*x^4)^p,x]
```

```
[Out] (c*x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x
^4)/a)^p + (e*x^3*(a + b*x^4)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a
)])/(3*(1 + (b*x^4)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\int (c+ex^2)(a+bx^4)^p\,dx = \int \left(c(a+bx^4)^p + ex^2(a+bx^4)^p\right)dx$$
$$= c\int (a+bx^4)^p\,dx + e\int x^2(a+bx^4)^p\,dx$$
$$= \left(c(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int\left(1+\frac{bx^4}{a}\right)^p dx + \left(e(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right.$$
$$= cx(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right) + \frac{1}{3}ex^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right.$$

Mathematica [A]
time = 0.44, size = 75, normalized size = 0.78

$$\frac{1}{3}x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(3c\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)+ex^2\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{bx^4}{a}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)*(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*(3*c*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)] + e*x^2
*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^4)/a)]))/(3*(1 + (b*x^4)/a)^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (e\,x^2+c)\,(b\,x^4+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)*(b*x^4+a)^p,x)
```

```
[Out] int((e*x^2+c)*(b*x^4+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + c)*(b*x^4 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^2*e + c)*(b*x^4 + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 18.21, size = 75, normalized size = 0.78

$$\frac{a^{p}cx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{a^{p}ex^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},-p\\\frac{7}{4}\end{matrix}\middle|\frac{bx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)*(b*x**4+a)**p,x)
```

```
[Out] a**p*c*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*g
amma(5/4)) + a**p*e*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_pol
ar(I*pi)/a)/(4*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2*e + c)*(b*x^4 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^4+a\right)^p\,\left(e\,x^2+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p*(c + e*x^2),x)
[Out] int((a + b*x^4)^p*(c + e*x^2), x)
```

3.179 $\int (a+bx^4)^p \, dx$

Optimal. Leaf size=44

$$x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

```
[Out] x*(b*x^4+a)^p*hypergeom([1/4, -p],[5/4],-b*x^4/a)/((1+b*x^4/a)^p)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {252, 251}

$$x(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p,x]
```

```
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4
)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+bx^4)^p \, dx &= \left((a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p}\right) \int \left(1+\frac{bx^4}{a}\right)^p \, dx \\
&= x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 44, normalized size = 1.00

$$x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{bx^4}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^p,x]
[Out] (x*(a + b*x^4)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^4)/a)])/(1 + (b*x^4
)/a)^p
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p,x)
[Out] int((b*x^4+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.83, size = 34, normalized size = 0.77

$$\frac{a^p x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p,x)
```

```
[Out] a**p*x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi)/a)/(4*gam
ma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p, x)
```

Mupad [B]
time = 4.36, size = 41, normalized size = 0.93

$$\frac{x\,{\left(b\,x^4+a\right)}^p\,{}_2\mathrm{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -\frac{b\,x^4}{a}\right)}{{\left(\frac{b\,x^4}{a}+1\right)}^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p,x)
```

```
[Out] (x*(a + b*x^4)^p*hypergeom([1/4, -p], 5/4, -(b*x^4)/a))/((b*x^4)/a + 1)^p
```

3.180 $\int \frac{(a+bx^4)^p}{c+ex^2}\,dx$

Optimal. Leaf size=123

$$\frac{x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{3}{4};-p,1;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

```
[Out] x*(b*x^4+a)^p*AppellF1(1/4,1,-p,5/4,e^2*x^4/c^2,-b*x^4/a)/c/((1+b*x^4/a)^p)
-1/3*e*x^3*(b*x^4+a)^p*AppellF1(3/4,1,-p,7/4,e^2*x^4/c^2,-b*x^4/a)/c^2/((1+
b*x^4/a)^p)
```

Rubi [A]
time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$,
Rules used = {1254, 441, 440, 525, 524}

$$\frac{x(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3(a+bx^4)^p\left(\frac{bx^4}{a}+1\right)^{-p}F_1\left(\frac{3}{4};-p,1;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p/(c + e*x^2),x]
```

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 1, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c
*(1 + (b*x^4)/a)^p) - (e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 1, 7/4, -((b*x
^4)/a), (e^2*x^4)/c^2])/(3*c^2*(1 + (b*x^4)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^4)^p}{c+ex^2}\,dx &= \int \left(\frac{c(a+bx^4)^p}{c^2-e^2x^4}+\frac{ex^2(a+bx^4)^p}{-c^2+e^2x^4}\right)dx\\ &= c\int \frac{(a+bx^4)^p}{c^2-e^2x^4}\,dx+e\int \frac{x^2(a+bx^4)^p}{-c^2+e^2x^4}\,dx\\ &= \left(c(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{c^2-e^2x^4}\,dx+\left(e(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{x^2\left(1\right.}{-c^2}\\ &= \frac{x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{1}{4};-p,1;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c}-\frac{ex^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{3}{4};\right.}{3c^2}\end{aligned}$$

Mathematica [F]
time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^4)^p}{c+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x^4)^p/(c + e*x^2),x]
[Out] Integrate[(a + b*x^4)^p/(c + e*x^2), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4+a)^p}{e\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p/(e*x^2+c),x)
[Out] int((b*x^4+a)^p/(e*x^2+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^p/(x^2*e + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p/(x^2*e + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p/(e*x**2+c),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p/(x^2*e + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^4+a\right)^p}{e\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p/(c + e*x^2),x)
```

```
[Out] int((a + b*x^4)^p/(c + e*x^2), x)
```

3.181 $\int \frac{(a+bx^4)^p}{(c+ex^2)^2} \, dx$

Optimal. Leaf size=189

$$\frac{x(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2} - \frac{2ex^3(a+bx^4)^p \left(1+\frac{bx^4}{a}\right)^{-p} F_1\left(\frac{3}{4};-p,2;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3} -$$

```
[Out] x*(b*x^4+a)^p*AppellF1(1/4,2,-p,5/4,e^2*x^4/c^2,-b*x^4/a)/c^2/((1+b*x^4/a)^
p)-2/3*e*x^3*(b*x^4+a)^p*AppellF1(3/4,2,-p,7/4,e^2*x^4/c^2,-b*x^4/a)/c^3/((
1+b*x^4/a)^p)+1/5*e^2*x^5*(b*x^4+a)^p*AppellF1(5/4,2,-p,9/4,e^2*x^4/c^2,-b*
x^4/a)/c^4/((1+b*x^4/a)^p)
```

Rubi [A]
time = 0.12, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1254, 441, 440, 525, 524}

$$\frac{x(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2} + \frac{e^2x^5(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} F_1\left(\frac{5}{4};-p,2;\frac{9}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{5c^4} - \frac{2ex^3(a+bx^4)^p \left(\frac{bx^4}{a}+1\right)^{-p} F_1\left(\frac{3}{4};-p,2;\frac{7}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p/(c + e*x^2)^2,x]
```

```
[Out] (x*(a + b*x^4)^p*AppellF1[1/4, -p, 2, 5/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(c
^2*(1 + (b*x^4)/a)^p) - (2*e*x^3*(a + b*x^4)^p*AppellF1[3/4, -p, 2, 7/4, -(
(b*x^4)/a), (e^2*x^4)/c^2])/(3*c^3*(1 + (b*x^4)/a)^p) + (e^2*x^5*(a + b*x^4
)^p*AppellF1[5/4, -p, 2, 9/4, -((b*x^4)/a), (e^2*x^4)/c^2])/(5*c^4*(1 + (b*
x^4)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx &= \int \left(\frac{c^2(a+bx^4)^p}{(c^2-e^2x^4)^2}-\frac{2cex^2(a+bx^4)^p}{(c^2-e^2x^4)^2}+\frac{e^2x^4(a+bx^4)^p}{(-c^2+e^2x^4)^2}\right)dx\\ &= c^2\int \frac{(a+bx^4)^p}{(c^2-e^2x^4)^2}\,dx-(2ce)\int \frac{x^2(a+bx^4)^p}{(c^2-e^2x^4)^2}\,dx+e^2\int \frac{x^4(a+bx^4)^p}{(-c^2+e^2x^4)^2}\,dx\\ &= \left(c^2(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{(c^2-e^2x^4)^2}\,dx-\left(2ce(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \frac{3}{}\\ &= \frac{x(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{1}{4};-p,2;\frac{5}{4};-\frac{bx^4}{a},\frac{e^2x^4}{c^2}\right)}{c^2}-\frac{2ex^3(a+bx^4)^p\left(1+\frac{bx^4}{a}\right)^{-p}F_1\left(\frac{3}{4}\right.}{3c^3}\end{aligned}$$

Mathematica [F]
time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^4)^p}{(c+ex^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x^4)^p/(c + e*x^2)^2,x]
```

```
[Out] Integrate[(a + b*x^4)^p/(c + e*x^2)^2, x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+a\right)^{p}}{\left(e\,x^{2}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p/(e*x^2+c)^2,x)
[Out] int((b*x^4+a)^p/(e*x^2+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^p/(x^2*e + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="fricas")
[Out] integral((b*x^4 + a)^p/(x^4*e^2 + 2*c*x^2*e + c^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p/(e*x**2+c)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p/(e*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p/(x^2*e + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^4+a)^p}{(e\,x^2+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^p/(c + e*x^2)^2,x)
```

```
[Out] int((a + b*x^4)^p/(c + e*x^2)^2, x)
```

3.182 $\int (1-x^2)^3 (1+bx^4)^p \, dx$

Optimal. Leaf size=108

$$-\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)}+x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)+\frac{(1-b(7+4p))x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)}{b(7+4p)}+\frac{3}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)$$

```
[Out] -x^3*(b*x^4+1)^(1+p)/b/(7+4*p)+x*hypergeom([1/4, -p],[5/4],-b*x^4)+(1-b*(7+
4*p))*x^3*hypergeom([3/4, -p],[7/4],-b*x^4)/b/(7+4*p)+3/5*x^5*hypergeom([5/
4, -p],[9/4],-b*x^4)
```

Rubi [A]
time = 0.07, antiderivative size = 103, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1221, 1907, 251, 371}

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)+\frac{3}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)-x^3\left(1-\frac{1}{4bp+7b}\right){}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)-\frac{x^3(bx^4+1)^{p+1}}{b(4p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

```
[Out] -((x^3*(1 + b*x^4)^(1 + p))/(b*(7 + 4*p))) + x*Hypergeometric2F1[1/4, -p, 5
/4, -(b*x^4)] - (1 - (7*b + 4*b*p)^(-1))*x^3*Hypergeometric2F1[3/4, -p, 7/4
, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
```

```
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int (1-x^2)^3 (1+bx^4)^p \, dx = -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + \frac{\int (1+bx^4)^p (b(7+4p)+3(1-b(7+4p))x^2+3b(7+4p)}{b(7+4p)}$$

$$= -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + \frac{\int (b(7+4p)(1+bx^4)^p + 3(1-b(7+4p))x^2(1+bx^4)^p +}{b(7+4p)}$$

$$= -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + 3\int x^4(1+bx^4)^p \, dx - \left(3\left(1-\frac{1}{7b+4bp}\right)\right)\int x^2(1+bx$$

$$= -\frac{x^3(1+bx^4)^{1+p}}{b(7+4p)} + x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \left(1-\frac{1}{7b+4bp}\right)x^3\,{}_2F_1\left(\frac{3}{4},-p;\right.$$

Mathematica [A]
time = 0.86, size = 86, normalized size = 0.80

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right) + \frac{3}{5}x^5\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right) - \frac{1}{7}x^7\,{}_2F_1\left(\frac{7}{4},-p;\frac{11}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^3*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - x^3*Hypergeometric2F1[3/4, -p
, 7/4, -(b*x^4)] + (3*x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5 - (x
^7*Hypergeometric2F1[7/4, -p, 11/4, -(b*x^4)])/7
```

Maple [A]
time = 0.14, size = 75, normalized size = 0.69

method	result
meijerg	$-\frac{x^7 \operatorname{hypergeom}\left(\left[\frac{7}{4},-p\right],\left[\frac{11}{4}\right],-b\,x^4\right)}{7} + \frac{3x^5 \operatorname{hypergeom}\left(\left[\frac{5}{4},-p\right],\left[\frac{9}{4}\right],-b\,x^4\right)}{5} - x^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},-p\right],\left[\frac{7}{4}\right],-b\,x^4\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^3*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*x^7*hypergeom([7/4,-p],[11/4],-b*x^4)+3/5*x^5*hypergeom([5/4,-p],[9/4]
,-b*x^4)-x^3*hypergeom([3/4,-p],[7/4],-b*x^4)+x*hypergeom([1/4,-p],[5/4],-b
*x^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral(-(x^6 - 3*x^4 + 3*x^2 - 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 55.55, size = 129, normalized size = 1.19

$$-\frac{x^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{7}{4},-p\\\frac{11}{4}\end{matrix}\middle|bx^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{3x^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},-p\\\frac{9}{4}\end{matrix}\middle|bx^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{3x^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{4},-p\\\frac{7}{4}\end{matrix}\middle|bx^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\middle|bx^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**3*(b*x**4+1)**p,x)
```

```
[Out] -x**7*gamma(7/4)*hyper((7/4, -p), (11/4,), b*x**4*exp_polar(I*pi))/(4*gamma
(11/4)) + 3*x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi)
)/(4*gamma(9/4)) - 3*x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_po
lar(I*pi))/(4*gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*ex
p_polar(I*pi))/(4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^3*(b*x^4+1)^p,x, algorithm="giac")
[Out] integrate(-(x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int (x^2-1)^3\,(b\,x^4+1)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)^3*(b*x^4 + 1)^p,x)
[Out] -int((x^2 - 1)^3*(b*x^4 + 1)^p, x)
```

3.183 $\int (1-x^2)^2 (1+bx^4)^p \, dx$

Optimal. Leaf size=86

$$\frac{x(1+bx^4)^{1+p}}{b(5+4p)} - \frac{(1-b(5+4p))x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)}{b(5+4p)} - \frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

```
[Out] x*(b*x^4+1)^(1+p)/b/(5+4*p)-(1-b*(5+4*p))*x*hypergeom([1/4, -p],[5/4],-b*x^
4)/b/(5+4*p)-2/3*x^3*hypergeom([3/4, -p],[7/4],-b*x^4)
```

Rubi [A]
time = 0.05, antiderivative size = 79, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1221, 1218, 251, 371}

$$x\left(1-\frac{1}{4bp+5b}\right){}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \frac{2}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right) + \frac{x(bx^4+1)^{p+1}}{b(4p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

```
[Out] (x*(1 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + (1 - (5*b + 4*b*p)^(-1))*x*Hypergeo
metric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4, -p, 7/4,
-(b*x^4)])/3
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 1221

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[e^q*x^(2*q - 3)*((a + c*x^4)^(p + 1)/(c*(4*p + 2*q + 1))), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int (1-x^2)^2 (1+bx^4)^p \, dx = \frac{x(1+bx^4)^{1+p}}{b(5+4p)} + \frac{\int (-1+b(5+4p)-2b(5+4p)x^2)(1+bx^4)^p \, dx}{b(5+4p)}$$

$$= \frac{x(1+bx^4)^{1+p}}{b(5+4p)} + \frac{\int \left((-1+b(5+4p))(1+bx^4)^p - 2b(5+4p)x^2(1+bx^4)^p\right) dx}{b(5+4p)}$$

$$= \frac{x(1+bx^4)^{1+p}}{b(5+4p)} - 2\int x^2(1+bx^4)^p \, dx + \left(1-\frac{1}{5b+4bp}\right)\int (1+bx^4)^p \, dx$$

$$= \frac{x(1+bx^4)^{1+p}}{b(5+4p)} + \left(1-\frac{1}{5b+4bp}\right) x \, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \frac{2}{3}x^3 \, {}_2F_1\left(\frac{3}{4},-p;\right.$$

Mathematica [A]
time = 0.78, size = 65, normalized size = 0.76

$$x \, {}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \frac{2}{3}x^3 \, {}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right) + \frac{1}{5}x^5 \, {}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^2*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (2*x^3*Hypergeometric2F1[3/4,
 -p, 7/4, -(b*x^4)])/3 + (x^5*Hypergeometric2F1[5/4, -p, 9/4, -(b*x^4)])/5
```

Maple [A]
time = 0.13, size = 56, normalized size = 0.65

method	result	si
meijerg	$\frac{x^5 \operatorname{hypergeom}\left(\left[\frac{5}{4},-p\right],\left[\frac{9}{4}\right],-b\,x^4\right)}{5} - \frac{2x^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},-p\right],\left[\frac{7}{4}\right],-b\,x^4\right)}{3} + x \operatorname{hypergeom}\left(\left[\frac{1}{4},-p\right],\left[\frac{5}{4}\right],-b\,x^4\right)$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^2*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5*hypergeom([5/4,-p],[9/4],-b*x^4)-2/3*x^3*hypergeom([3/4,-p],[7/4],-
b*x^4)+x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral((x^4 - 2*x^2 + 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 30.55, size = 94, normalized size = 1.09

$$\frac{x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{5}{4},-p\\\frac{9}{4}\end{matrix}\right|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},-p\\\frac{7}{4}\end{matrix}\right|bx^4e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\right|bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**2*(b*x**4+1)**p,x)
```

```
[Out] x**5*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**4*exp_polar(I*pi))/(4*gamma(9
/4)) - x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(2*
gamma(7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))
/(4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^2*(b*x^4+1)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2-1\right)^2\,\left(b\,x^4+1\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^2*(b*x^4 + 1)^p,x)
[Out] int((x^2 - 1)^2*(b*x^4 + 1)^p, x)
```

3.184 $\int (1-x^2)(1+bx^4)^p \, dx$

Optimal. Leaf size=42

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

```
[Out] x*hypergeom([1/4, -p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4, -p],[7/4],-b*x^
4)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1218, 251, 371}

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)-\frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)*(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)])/3
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1218

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rubi steps

$$\begin{aligned}\int (1-x^2)\left(1+bx^4\right)^p\,dx &= \int \left(\left(1+bx^4\right)^p - x^2\left(1+bx^4\right)^p\right)\,dx\\ &= \int \left(1+bx^4\right)^p\,dx - \int x^2\left(1+bx^4\right)^p\,dx\\ &= x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)\end{aligned}$$

Mathematica [A]
time = 0.55, size = 42, normalized size = 1.00

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right) - \frac{1}{3}x^3\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)*(1 + b*x^4)^p,x]
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)] - (x^3*Hypergeometric2F1[3/4, -
p, 7/4, -(b*x^4)])/3
```

Maple [A]
time = 0.11, size = 37, normalized size = 0.88

method	result	size
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},-p\right],\left[\frac{5}{4}\right],-b\,x^4\right)-\frac{x^3\,\mathrm{hypergeom}\left(\left[\frac{3}{4},-p\right],\left[\frac{7}{4}\right],-b\,x^4\right)}{3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)*(b*x^4+1)^p,x,method=_RETURNVERBOSE)
[Out] x*hypergeom([1/4,-p],[5/4],-b*x^4)-1/3*x^3*hypergeom([3/4,-p],[7/4],-b*x^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="maxima")
[Out] -integrate((x^2 - 1)*(b*x^4 + 1)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 15.34, size = 61, normalized size = 1.45

$$-\frac{x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle| bx^4e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)*(b*x**4+1)**p,x)
```

```
[Out] -x**3*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**4*exp_polar(I*pi))/(4*gamma(
7/4)) + x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*ga
mma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(b*x^4+1)^p,x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)*(b*x^4 + 1)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \left(x^2-1\right)\,{\left(b\,x^4+1\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)*(b*x^4 + 1)^p,x)
```

```
[Out] -int((x^2 - 1)*(b*x^4 + 1)^p, x)
```

3.185 $\int (1+bx^4)^p \, dx$

Optimal. Leaf size=18

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

```
[Out] x*hypergeom([1/4, -p],[5/4],-b*x^4)
```

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {251}

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (1+bx^4)^p \, dx = x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Mathematica [A]
time = 0.04, size = 18, normalized size = 1.00

$$x\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + b*x^4)^p,x]
```

```
[Out] x*Hypergeometric2F1[1/4, -p, 5/4, -(b*x^4)]
```

Maple [A]
time = 0.12, size = 17, normalized size = 0.94

method	result	size
meijerg	$x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},-p\right],\left[\frac{5}{4}\right],-b\,x^{4}\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p,x,method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([1/4,-p],[5/4],-b*x^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + 1)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + 1)^p, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.23, size = 29, normalized size = 1.61

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},-p\\\frac{5}{4}\end{matrix}\right|bx^{4}e^{i\pi}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p,x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**4*exp_polar(I*pi))/(4*gamma(5/4)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p,x, algorithm="giac")
[Out] integrate((b*x^4 + 1)^p, x)
```

Mupad [B]
time = 0.07, size = 15, normalized size = 0.83

$$x\,{}_2\mathrm{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -b\,x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4 + 1)^p,x)
[Out] x*hypergeom([1/4, -p], 5/4, -b*x^4)
```

3.186 $\int \frac{(1+bx^4)^p}{1-x^2}\,dx$

Optimal. Leaf size=50

$$xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)$$

```
[Out] x*AppellF1(1/4,1,-p,5/4,x^4,-b*x^4)+1/3*x^3*AppellF1(3/4,1,-p,7/4,x^4,-b*x^
4)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1254, 440, 524}

$$xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^4)^p/(1 - x^2),x]
```

```
[Out] x*AppellF1[1/4, 1, -p, 5/4, x^4, -(b*x^4)] + (x^3*AppellF1[3/4, 1, -p, 7/4,
 x^4, -(b*x^4)])/3
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+bx^4)^p}{1-x^2}\,dx &= \int \left(\frac{(1+bx^4)^p}{1-x^4} - \frac{x^2(1+bx^4)^p}{-1+x^4}\right)dx\\ &= \int \frac{(1+bx^4)^p}{1-x^4}\,dx - \int \frac{x^2(1+bx^4)^p}{-1+x^4}\,dx\\ &= xF_1\left(\frac{1}{4};1,-p;\frac{5}{4};x^4,-bx^4\right) + \frac{1}{3}x^3F_1\left(\frac{3}{4};1,-p;\frac{7}{4};x^4,-bx^4\right)\end{aligned}$$

Mathematica [F]
time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{(1+bx^4)^p}{1-x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2),x]
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4+1)^p}{-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p/(-x^2+1),x)
[Out] int((b*x^4+1)^p/(-x^2+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="maxima")
[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p/(-x**2+1),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{(b\,x^4+1)^p}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^4 + 1)^p/(x^2 - 1),x)
```

```
[Out] -int((b*x^4 + 1)^p/(x^2 - 1), x)
```

3.187 $\int \frac{(1+bx^4)^p}{(1-x^2)^2} dx$

Optimal. Leaf size=77

$$xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)+\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-bx^4\right)$$

```
[Out] x*AppellF1(1/4,2,-p,5/4,x^4,-b*x^4)+2/3*x^3*AppellF1(3/4,2,-p,7/4,x^4,-b*x^
4)+1/5*x^5*AppellF1(5/4,2,-p,9/4,x^4,-b*x^4)
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1254, 440, 524}

$$xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^4)^p/(1 - x^2)^2,x]
```

```
[Out] x*AppellF1[1/4, 2, -p, 5/4, x^4, -(b*x^4)] + (2*x^3*AppellF1[3/4, 2, -p, 7/
4, x^4, -(b*x^4)])/3 + (x^5*AppellF1[5/4, 2, -p, 9/4, x^4, -(b*x^4)])/5
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx = \int \left(\frac{(1+bx^4)^p}{(-1+x^4)^2}+\frac{2x^2(1+bx^4)^p}{(-1+x^4)^2}+\frac{x^4(1+bx^4)^p}{(-1+x^4)^2}\right)dx$$
$$= 2\int \frac{x^2(1+bx^4)^p}{(-1+x^4)^2}\,dx+\int \frac{(1+bx^4)^p}{(-1+x^4)^2}\,dx+\int \frac{x^4(1+bx^4)^p}{(-1+x^4)^2}\,dx$$
$$= xF_1\left(\frac{1}{4};2,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{2}{3}x^3F_1\left(\frac{3}{4};2,-p;\frac{7}{4};x^4,-bx^4\right)+\frac{1}{5}x^5F_1\left(\frac{5}{4};2,-p;\frac{9}{4};x^4,-$$

Mathematica [F]
time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(1+bx^4)^p}{(1-x^2)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2)^2,x]
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^2, x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4+1)^p}{(-x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p/(-x^2+1)^2,x)
[Out] int((b*x^4+1)^p/(-x^2+1)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="maxima")
[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + 1)^p/(x^4 - 2*x^2 + 1), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p/(-x**2+1)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^4+1)^p}{(x^2-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4 + 1)^p/(x^2 - 1)^2,x)
```

```
[Out] int((b*x^4 + 1)^p/(x^2 - 1)^2, x)
```

3.188 $\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx$

Optimal. Leaf size=101

$$xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right)+x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right)+\frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right)+\frac{1}{7}x^7F_1\left(\frac{7}{4};3,-p;\right.$$

```
[Out] x*AppellF1(1/4,3,-p,5/4,x^4,-b*x^4)+x^3*AppellF1(3/4,3,-p,7/4,x^4,-b*x^4)+3
/5*x^5*AppellF1(5/4,3,-p,9/4,x^4,-b*x^4)+1/7*x^7*AppellF1(7/4,3,-p,11/4,x^4
,-b*x^4)
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1254, 440, 524}

$$xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right)+\frac{1}{7}x^7F_1\left(\frac{7}{4};3,-p;\frac{11}{4};x^4,-bx^4\right)+\frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-bx^4\right)+x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + b*x^4)^p/(1 - x^2)^3,x]
```

```
[Out] x*AppellF1[1/4, 3, -p, 5/4, x^4, -(b*x^4)] + x^3*AppellF1[3/4, 3, -p, 7/4,
x^4, -(b*x^4)] + (3*x^5*AppellF1[5/4, 3, -p, 9/4, x^4, -(b*x^4)])/5 + (x^7*
AppellF1[7/4, 3, -p, 11/4, x^4, -(b*x^4)])/7
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1254

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - e*(x^2/(d^2 - e^2*x^4)
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
```

```
IntegerQ[p] && ILtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx &= \int \left(-\frac{(1+bx^4)^p}{(-1+x^4)^3}-\frac{3x^2(1+bx^4)^p}{(-1+x^4)^3}-\frac{3x^4(1+bx^4)^p}{(-1+x^4)^3}-\frac{x^6(1+bx^4)^p}{(-1+x^4)^3}\right)dx\\ &= -\left(3\int \frac{x^2(1+bx^4)^p}{(-1+x^4)^3}\,dx\right)-3\int \frac{x^4(1+bx^4)^p}{(-1+x^4)^3}\,dx-\int \frac{(1+bx^4)^p}{(-1+x^4)^3}\,dx-\int \frac{x^6(1+bx^4)^p}{(-1+x^4)^3}\\ &= xF_1\left(\frac{1}{4};3,-p;\frac{5}{4};x^4,-bx^4\right)+x^3F_1\left(\frac{3}{4};3,-p;\frac{7}{4};x^4,-bx^4\right)+\frac{3}{5}x^5F_1\left(\frac{5}{4};3,-p;\frac{9}{4};x^4,-\right.\end{aligned}$$

Mathematica [F]
time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{(1+bx^4)^p}{(1-x^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(1 + b*x^4)^p/(1 - x^2)^3,x]
[Out] Integrate[(1 + b*x^4)^p/(1 - x^2)^3, x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4+1)^p}{(-x^2+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+1)^p/(-x^2+1)^3,x)
[Out] int((b*x^4+1)^p/(-x^2+1)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="maxima")
[Out] -integrate((b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="fricas")
[Out] integral(-(b*x^4 + 1)^p/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+1)**p/(-x**2+1)**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+1)^p/(-x^2+1)^3,x, algorithm="giac")
[Out] integrate(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\left(b\,x^4+1\right)^p}{{\left(x^2-1\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b*x^4 + 1)^p/(x^2 - 1)^3,x)
[Out] int(-(b*x^4 + 1)^p/(x^2 - 1)^3, x)
```

3.189 $\int \frac{(d+ex^2)^4}{d^2-e^2x^4} dx$

Optimal. Leaf size=51

$$-7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}$$

```
[Out] -7*d^2*x-4/3*d*e*x^3-1/5*e^2*x^5+8*d^(5/2)*arctanh(x*e^(1/2)/d^(1/2))/e^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1164, 398, 214}

$$\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-7d^2x-\frac{4}{3}dex^3-\frac{1}{5}e^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
```

```
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqr
t[d]])/Sqrt[e]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex^2)^4}{d^2-e^2x^4}\,dx = \int \frac{(d+ex^2)^3}{d-ex^2}\,dx$$
$$= \int \left(-7d^2-4dex^2-e^2x^4+\frac{8d^3}{d-ex^2}\right)dx$$
$$= -7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\left(8d^3\right)\int \frac{1}{d-ex^2}\,dx$$
$$= -7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.00

$$-7d^2x-\frac{4}{3}dex^3-\frac{e^2x^5}{5}+\frac{8d^{5/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(d^2 - e^2*x^4),x]
[Out] -7*d^2*x - (4*d*e*x^3)/3 - (e^2*x^5)/5 + (8*d^(5/2)*ArcTanh[(Sqrt[e]*x)/Sqr
t[d]])/Sqrt[e]
```

Maple [A]
time = 0.13, size = 42, normalized size = 0.82

method	result	size
default	$-\frac{e^2x^5}{5}-\frac{4x^3de}{3}-7d^2x+\frac{8d^3\,\mathrm{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$	42
risch	$-\frac{e^2x^5}{5}-\frac{4x^3de}{3}-7d^2x-\frac{4\sqrt{de}\,d^2\ln\left(\sqrt{de}\,x-d\right)}{e}+\frac{4\sqrt{de}\,d^2\ln\left(-\sqrt{de}\,x-d\right)}{e}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
[Out] -1/5*e^2*x^5-4/3*x^3*d*e-7*d^2*x+8*d^3/(d*e)^(1/2)*arctanh(e*x/(d*e)^(1/2))
```

Maxima [A]
time = 0.51, size = 56, normalized size = 1.10

$$-\frac{1}{5}\,x^5e^2-\frac{4}{3}\,dx^3e-4\,d^{\frac{5}{2}}e^{\left(-\frac{1}{2}\right)}\log\left(\frac{xe-\sqrt{d}\,e^{\frac{1}{2}}}{xe+\sqrt{d}\,e^{\frac{1}{2}}}\right)-7\,d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -1/5*x^5*e^2 - 4/3*d*x^3*e - 4*d^(5/2)*e^(-1/2)*log((x*e - sqrt(d)*e^(1/2))
/(x*e + sqrt(d)*e^(1/2))) - 7*d^2*x
```

Fricas [A]
time = 0.36, size = 109, normalized size = 2.14

$$\left[-\frac{1}{5}x^5e^2 - \frac{4}{3}dx^3e + 4d^{\frac{5}{2}}e^{\left(-\frac{1}{2}\right)}\log\left(\frac{x^2e + 2\sqrt{d}\,xe^{\frac{1}{2}} + d}{x^2e - d}\right) - 7d^2x, -\frac{1}{5}x^5e^2 - \frac{4}{3}dx^3e - 8\sqrt{-de^{(-1)}}\,d^2\arctan\left(\frac{\sqrt{-de^{(-1)}}\,xe}{d}\right) - 7d^2x\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [-1/5*x^5*e^2 - 4/3*d*x^3*e + 4*d^(5/2)*e^(-1/2)*log((x^2*e + 2*sqrt(d)*x*e
^(1/2) + d)/(x^2*e - d)) - 7*d^2*x, -1/5*x^5*e^2 - 4/3*d*x^3*e - 8*sqrt(-d*
e^(-1))*d^2*arctan(sqrt(-d*e^(-1))*x*e/d) - 7*d^2*x]
```

Sympy [A]
time = 0.09, size = 75, normalized size = 1.47

$$-7d^2x - \frac{4dex^3}{3} - \frac{e^2x^5}{5} - 4\sqrt{\frac{d^5}{e}}\log\left(x - \frac{\sqrt{\frac{d^5}{e}}}{d^2}\right) + 4\sqrt{\frac{d^5}{e}}\log\left(x + \frac{\sqrt{\frac{d^5}{e}}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(-e**2*x**4+d**2),x)
```

```
[Out] -7*d**2*x - 4*d*e*x**3/3 - e**2*x**5/5 - 4*sqrt(d**5/e)*log(x - sqrt(d**5/e
)/d**2) + 4*sqrt(d**5/e)*log(x + sqrt(d**5/e)/d**2)
```

Giac [A]
time = 3.32, size = 53, normalized size = 1.04

$$-\frac{8\,d^3\arctan\left(\frac{xe}{\sqrt{-de}}\right)}{\sqrt{-de}} - \frac{1}{15}\left(3\,x^5e^7 + 20\,dx^3e^6 + 105\,d^2xe^5\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] -8*d^3*arctan(x*e/sqrt(-d*e))/sqrt(-d*e) - 1/15*(3*x^5*e^7 + 20*d*x^3*e^6 +
 105*d^2*x*e^5)*e^(-5)
```

Mupad [B]
time = 0.09, size = 42, normalized size = 0.82

$$-7\,d^2\,x-\frac{e^2\,x^5}{5}-\frac{4\,d\,e\,x^3}{3}-\frac{d^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x\,1\mathrm{i}}{\sqrt{d}}\right)\,8\mathrm{i}}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4/(d^2 - e^2*x^4),x)
```

```
[Out] - 7*d^2*x - (e^2*x^5)/5 - (d^(5/2)*atan((e^(1/2)*x*1i)/d^(1/2))*8i)/e^(1/2)
 - (4*d*e*x^3)/3
```

3.190 $\int \frac{(d+ex^2)^3}{d^2-e^2x^4} dx$

Optimal. Leaf size=38

$$-3dx - \frac{ex^3}{3} + \frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}$$

```
[Out] -3*d*x-1/3*e*x^3+4*d^(3/2)*arctanh(x*e^(1/2)/d^(1/2))/e^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1164, 398, 214}

$$\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - 3dx - \frac{ex^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
```

```
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{d^2-e^2x^4}\,dx = \int \frac{(d+ex^2)^2}{d-ex^2}\,dx$$
$$= \int \left(-3d-ex^2+\frac{4d^2}{d-ex^2}\right)dx$$
$$= -3dx-\frac{ex^3}{3}+\left(4d^2\right)\int \frac{1}{d-ex^2}\,dx$$
$$= -3dx-\frac{ex^3}{3}+\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.00

$$-3dx-\frac{ex^3}{3}+\frac{4d^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(d^2 - e^2*x^4),x]
[Out] -3*d*x - (e*x^3)/3 + (4*d^(3/2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Maple [A]
time = 0.12, size = 31, normalized size = 0.82

method	result	size
default	$-\frac{e\,x^3}{3}-3dx+\frac{4d^2\,\mathrm{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$	31
risch	$-\frac{e\,x^3}{3}-3dx+\frac{2\sqrt{de}\;d\ln\left(\sqrt{de}\;x+d\right)}{e}-\frac{2\sqrt{de}\;d\ln\left(-\sqrt{de}\;x+d\right)}{e}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
[Out] -1/3*e*x^3-3*d*x+4*d^2/(d*e)^(1/2)*arctanh(e*x/(d*e)^(1/2))
```

Maxima [A]
time = 0.52, size = 46, normalized size = 1.21

$$-\frac{1}{3}\,x^3e-2\,d^{\frac{3}{2}}e^{\left(-\frac{1}{2}\right)}\log\left(\frac{xe-\sqrt{d}\,e^{\frac{1}{2}}}{xe+\sqrt{d}\,e^{\frac{1}{2}}}\right)-3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -1/3*x^3*e - 2*d^(3/2)*e^(-1/2)*log((x*e - sqrt(d)*e^(1/2))/(x*e + sqrt(d)*
e^(1/2))) - 3*d*x
```

Fricas [A]
time = 0.37, size = 87, normalized size = 2.29

$$\left[-\frac{1}{3}\,x^{3}e+2\,d^{\frac{3}{2}}e^{\left(-\frac{1}{2}\right)}\log\left(\frac{x^{2}e+2\,\sqrt{d}\,xe^{\frac{1}{2}}+d}{x^{2}e-d}\right)-3\,dx,-\frac{1}{3}\,x^{3}e-4\,\sqrt{-de^{(-1)}}\,d\arctan\left(\frac{\sqrt{-de^{(-1)}}\,xe}{d}\right)-3\,dx\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [-1/3*x^3*e + 2*d^(3/2)*e^(-1/2)*log((x^2*e + 2*sqrt(d)*x*e^(1/2) + d)/(x^2
*e - d)) - 3*d*x, -1/3*x^3*e - 4*sqrt(-d*e^(-1))*d*arctan(sqrt(-d*e^(-1))*x
*e/d) - 3*d*x]
```

Sympy [A]
time = 0.08, size = 58, normalized size = 1.53

$$-3dx-\frac{ex^{3}}{3}-2\sqrt{\frac{d^{3}}{e}}\log\left(x-\frac{\sqrt{\frac{d^{3}}{e}}}{d}\right)+2\sqrt{\frac{d^{3}}{e}}\log\left(x+\frac{\sqrt{\frac{d^{3}}{e}}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(-e**2*x**4+d**2),x)
```

```
[Out] -3*d*x - e*x**3/3 - 2*sqrt(d**3/e)*log(x - sqrt(d**3/e)/d) + 2*sqrt(d**3/e)
*log(x + sqrt(d**3/e)/d)
```

Giac [A]
time = 4.20, size = 42, normalized size = 1.11

$$-\frac{4\,d^{2}\arctan\left(\frac{xe}{\sqrt{-de}}\right)}{\sqrt{-de}}-\frac{1}{3}\,\left(x^{3}e^{4}+9\,dxe^{3}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] -4*d^2*arctan(x*e/sqrt(-d*e))/sqrt(-d*e) - 1/3*(x^3*e^4 + 9*d*x*e^3)*e^(-3)
```

Mupad [B]
time = 0.05, size = 28, normalized size = 0.74

$$\frac{4\,d^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{e\,x^3}{3}-3\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(d^2 - e^2*x^4),x)
[Out] (4*d^(3/2)*atanh((e^(1/2)*x)/d^(1/2)))/e^(1/2) - (e*x^3)/3 - 3*d*x
```

3.191 $\int \frac{(d+ex^2)^2}{d^2-e^2x^4} dx$

Optimal. Leaf size=29

$$-x+\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}$$

[Out] -x+2*arctanh(x*e^(1/2)/d^(1/2))*d^(1/2)/e^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1164, 396, 214}

$$\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}-x$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^2/(d^2 - e^2*x^4),x]

[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex^2)^2}{d^2-e^2x^4}\,dx &= \int \frac{d+ex^2}{d-ex^2}\,dx \\
&= -x+(2d)\int \frac{1}{d-ex^2}\,dx \\
&= -x+\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 1.00

$$
-x+\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(d^2 - e^2*x^4),x]
```

```
[Out] -x + (2*Sqrt[d]*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e]
```

Maple [A]
time = 0.13, size = 22, normalized size = 0.76

method	result	size
default	$-x+\frac{2d\,\mathrm{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$	22
risch	$-x-\frac{\sqrt{de}\,\ln\left(\sqrt{de}\,x-d\right)}{e}+\frac{\sqrt{de}\,\ln\left(-\sqrt{de}\,x-d\right)}{e}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -x+2*d/(d*e)^(1/2)*arctanh(e*x/(d*e)^(1/2))
```

Maxima [A]
time = 0.52, size = 38, normalized size = 1.31

$$
-\sqrt{d}\,e^{\left(-\frac{1}{2}\right)}\log\left(\frac{xe-\sqrt{d}\,e^{\frac{1}{2}}}{xe+\sqrt{d}\,e^{\frac{1}{2}}}\right)-x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
[Out] -sqrt(d)*e^(-1/2)*log((x*e - sqrt(d)*e^(1/2))/(x*e + sqrt(d)*e^(1/2))) - x
```

Fricas [A]
time = 0.35, size = 69, normalized size = 2.38

$$\left[\sqrt{d}\, e^{\left(-\frac{1}{2}\right)} \log\left(\frac{x^2 e + 2\sqrt{d}\, x e^{\frac{1}{2}} + d}{x^2 e - d}\right) - x, -2\sqrt{-d e^{(-1)}} \arctan\left(\frac{\sqrt{-d e^{(-1)}}\, x e}{d}\right) - x\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
[Out] [sqrt(d)*e^(-1/2)*log((x^2*e + 2*sqrt(d)*x*e^(1/2) + d)/(x^2*e - d)) - x, -
2*sqrt(-d*e^(-1))*arctan(sqrt(-d*e^(-1))*x*e/d) - x]
```

Sympy [A]
time = 0.06, size = 34, normalized size = 1.17

$$-x - \sqrt{\frac{d}{e}} \log\left(x - \sqrt{\frac{d}{e}}\right) + \sqrt{\frac{d}{e}} \log\left(x + \sqrt{\frac{d}{e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(-e**2*x**4+d**2),x)
[Out] -x - sqrt(d/e)*log(x - sqrt(d/e)) + sqrt(d/e)*log(x + sqrt(d/e))
```

Giac [A]
time = 4.68, size = 26, normalized size = 0.90

$$-\frac{2\, d \arctan\left(\frac{x e}{\sqrt{-d e}}\right)}{\sqrt{-d e}} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
[Out] -2*d*arctan(x*e/sqrt(-d*e))/sqrt(-d*e) - x
```

Mupad [B]
time = 4.43, size = 21, normalized size = 0.72

$$\frac{2\sqrt{d}\ \mathrm{atanh}\left(\frac{\sqrt{e}\ x}{\sqrt{d}}\right)}{\sqrt{e}} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(d^2 - e^2*x^4),x)
[Out] (2*d^(1/2)*atanh((e^(1/2)*x)/d^(1/2)))/e^(1/2) - x
```

3.192 $\int \frac{d+ex^2}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}$$

```
[Out] arctanh(x*e^(1/2)/d^(1/2))/d^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1164, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(d^2 - e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^2}{d^2-e^2x^4}\,dx &= \int \frac{1}{d-ex^2}\,dx\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(d^2 - e^2*x^4),x]
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(Sqrt[d]*Sqrt[e])
```

Maple [A]
time = 0.12, size = 16, normalized size = 0.67

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$	16
risch	$\frac{\ln\left(ex+\sqrt{de}\right)}{2\sqrt{de}}-\frac{\ln\left(-ex+\sqrt{de}\right)}{2\sqrt{de}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
[Out] 1/(d*e)^(1/2)*arctanh(e*x/(d*e)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(14) = 28.
time = 0.52, size = 34, normalized size = 1.42

$$-\frac{e^{\left(-\frac{1}{2}\right)}\log\left(\frac{xe-\sqrt{d}\,e^{\frac{1}{2}}}{xe+\sqrt{d}\,e^{\frac{1}{2}}}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
[Out] -1/2*e^(-1/2)*log((x*e - sqrt(d)*e^(1/2))/(x*e + sqrt(d)*e^(1/2)))/sqrt(d)
```

Fricas [A]
time = 0.33, size = 65, normalized size = 2.71

$$\left[\frac{e^{\left(-\frac{1}{2}\right)}\log\left(\frac{x^2e+2\sqrt{d}\,xe^{\frac{1}{2}}+d}{x^2e-d}\right)}{2\sqrt{d}}, -\frac{\sqrt{-de}\,\arctan\left(\frac{\sqrt{-de}\,x}{d}\right)e^{(-1)}}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/2*e^(-1/2)*log((x^2*e + 2*sqrt(d)*x*e^(1/2) + d)/(x^2*e - d))/sqrt(d), -
sqrt(-d*e)*arctan(sqrt(-d*e)*x/d)*e^(-1)/d]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.
time = 0.05, size = 46, normalized size = 1.92

$$-\frac{\sqrt{\frac{1}{de}}\log\left(-d\sqrt{\frac{1}{de}}+x\right)}{2}+\frac{\sqrt{\frac{1}{de}}\log\left(d\sqrt{\frac{1}{de}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-e**2*x**4+d**2),x)
```

```
[Out] -sqrt(1/(d*e))*log(-d*sqrt(1/(d*e)) + x)/2 + sqrt(1/(d*e))*log(d*sqrt(1/(d*
e)) + x)/2
```

Giac [A]
time = 4.04, size = 21, normalized size = 0.88

$$-\frac{\arctan\left(\frac{xe}{\sqrt{-de}}\right)}{\sqrt{-de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] -arctan(x*e/sqrt(-d*e))/sqrt(-d*e)
```

Mupad [B]
time = 0.06, size = 16, normalized size = 0.67

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(d^2 - e^2*x^4),x)
```

```
[Out] atanh((e^(1/2)*x)/d^(1/2))/(d^(1/2)*e^(1/2))
```

3.193 $\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx$

Optimal. Leaf size=72

$$\frac{x}{4d^2(d+ex^2)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}$$

```
[Out] 1/4*x/d^2/(e*x^2+d)+1/2*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)+1/4*arcta
nh(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1164, 425, 536, 214, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}+\frac{x}{4d^2(d+ex^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(4*d^2*(d + e*x^2)) + ArcTan[(Sqrt[e]*x)/Sqrt[d]]/(2*d^(5/2)*Sqrt[e]) + A
rcTanh[(Sqrt[e]*x)/Sqrt[d]]/(4*d^(5/2)*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex^2)(d^2-e^2x^4)}\,dx &= \int \frac{1}{(d-ex^2)(d+ex^2)^2}\,dx\\ &= \frac{x}{4d^2(d+ex^2)} - \frac{\int \frac{-3de+e^2x^2}{(d-ex^2)(d+ex^2)}\,dx}{4d^2e}\\ &= \frac{x}{4d^2(d+ex^2)} + \frac{\int \frac{1}{d-ex^2}\,dx}{4d^2} + \frac{\int \frac{1}{d+ex^2}\,dx}{2d^2}\\ &= \frac{x}{4d^2(d+ex^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{4d^{5/2}\sqrt{e}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 0.90

$$\frac{\frac{\sqrt{d}\,x}{d+ex^2} + \frac{2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}}{4d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(d^2 - e^2*x^4)),x]
```

```
[Out] ((Sqrt[d]*x)/(d + e*x^2) + (2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] + ArcTan
h[(Sqrt[e]*x)/Sqrt[d]]/Sqrt[e])/(4*d^(5/2))
```

Maple [A]
time = 0.16, size = 54, normalized size = 0.75

method	result	size
default	$\frac{\frac{x}{e\,x^{2}+d}+\frac{2\arctan\left(\frac{e x}{\sqrt{d e}}\right)}{\sqrt{d e}}}{4 d^{2}}+\frac{\operatorname{arctanh}\left(\frac{e x}{\sqrt{d e}}\right)}{4 d^{2} \sqrt{d e}}$	54
risch	$\frac{x}{4 d^{2}\left(e\,x^{2}+d\right)}-\frac{\ln\left(-e x-\sqrt{-d e}\right)}{4 \sqrt{-d e}\, d^{2}}+\frac{\ln\left(e x-\sqrt{-d e}\right)}{4 \sqrt{-d e}\, d^{2}}+\frac{\ln\left(e x+\sqrt{d e}\right)}{8 \sqrt{d e}\, d^{2}}-\frac{\ln\left(-e x+\sqrt{d e}\right)}{8 \sqrt{d e}\, d^{2}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d^2*(x/(e*x^2+d)+2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))+1/4/d^2/(d*e)^(
1/2)*arctanh(e*x/(d*e)^(1/2))
```

Maxima [A]
time = 0.53, size = 68, normalized size = 0.94

$$\frac{x}{4\left(d^{2} x^{2} e+d^{3}\right)}+\frac{\arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)}}{2 d^{\frac{5}{2}}}-\frac{e^{\left(-\frac{1}{2}\right)} \log\left(\frac{x e-\sqrt{d}\, e^{\frac{1}{2}}}{x e+\sqrt{d}\, e^{\frac{1}{2}}}\right)}{8 d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] 1/4*x/(d^2*x^2*e + d^3) + 1/2*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(5/2) -
1/8*e^(-1/2)*log((x*e - sqrt(d)*e^(1/2))/(x*e + sqrt(d)*e^(1/2)))/d^(5/2)
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(47) = 94.
time = 0.36, size = 200, normalized size = 2.78

$$\left[\frac{4\left(x^{2} e+d\right) \sqrt{d} \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\frac{1}{2}}+\left(x^{2} e+d\right) \sqrt{d}\, e^{\frac{1}{2}} \log\left(\frac{x^{2} e+2 \sqrt{d}\, x e^{\frac{1}{2}}+d}{x^{2} e-d}\right)+2\, d x e}{8\left(d^{3} x^{2} e^{2}+d^{4} e\right)}, \frac{d x e-\left(x^{2} e+d\right) \sqrt{-d e} \arctan\left(\frac{\sqrt{-d e}\, x}{d}\right)-\left(x^{2} e+d\right) \sqrt{-d e} \log\left(\frac{x^{2} e-2 \sqrt{-d e}\, x-d}{x^{2} e+d}\right)}{4\left(d^{3} x^{2} e^{2}+d^{4} e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(x^2*e + d)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) + (x^2*e + d)
*sqrt(d)*e^(1/2)*log((x^2*e + 2*sqrt(d)*x*e^(1/2) + d)/(x^2*e - d)) + 2*d*x
*e)/(d^3*x^2*e^2 + d^4*e), 1/4*(d*x*e - (x^2*e + d)*sqrt(-d*e)*arctan(sqrt(
-d*e)*x/d) - (x^2*e + d)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e
 + d)))/(d^3*x^2*e^2 + d^4*e)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(63) = 126$.
time = 0.20, size = 226, normalized size = 3.14

$$\frac{x}{4d^3+4d^2ex^2} - \frac{\sqrt{\frac{1}{d^5e}}\log\left(-\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} - \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} + \frac{\sqrt{\frac{1}{d^5e}}\log\left(\frac{d^8e\left(\frac{1}{d^5e}\right)^{\frac{3}{2}}}{10} + \frac{9d^3\sqrt{\frac{1}{d^5e}}}{10} + x\right)}{8} - \frac{\sqrt{-\frac{1}{d^5e}}\log\left(-\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5} - \frac{9d^3\sqrt{-\frac{1}{d^5e}}}{5} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^5e}}\log\left(\frac{4d^8e\left(-\frac{1}{d^5e}\right)^{\frac{3}{2}}}{5} + \frac{9d^3\sqrt{-\frac{1}{d^5e}}}{5} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-e**2*x**4+d**2),x)
```

```
[Out] x/(4*d**3 + 4*d**2*e*x**2) - sqrt(1/(d**5*e))*log(-d**8*e*(1/(d**5*e))**(3/
2)/10 - 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 + sqrt(1/(d**5*e))*log(d**8*e*(1/
(d**5*e))**(3/2)/10 + 9*d**3*sqrt(1/(d**5*e))/10 + x)/8 - sqrt(-1/(d**5*e))
*log(-4*d**8*e*(-1/(d**5*e))**(3/2)/5 - 9*d**3*sqrt(-1/(d**5*e))/5 + x)/4 +
 sqrt(-1/(d**5*e))*log(4*d**8*e*(-1/(d**5*e))**(3/2)/5 + 9*d**3*sqrt(-1/(d*
*5*e))/5 + x)/4
```

Giac [A]
time = 3.03, size = 56, normalized size = 0.78

$$\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{5}{2}}} - \frac{\arctan\left(\frac{xe}{\sqrt{-de}}\right)}{4\,\sqrt{-de}\,d^2} + \frac{x}{4\,(x^2e+d)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(5/2) - 1/4*arctan(x*e/sqrt(-d*e))
/(sqrt(-d*e)*d^2) + 1/4*x/((x^2*e + d)*d^2)
```

Mupad [B]
time = 0.16, size = 74, normalized size = 1.03

$$\frac{x}{4\,d^2\,\left(e\,x^2+d\right)} + \frac{\mathrm{atanh}\left(\frac{x\,\sqrt{d^5\,e}}{d^3}\right)\,\sqrt{d^5\,e}}{4\,d^5\,e} - \frac{\mathrm{atanh}\left(\frac{x\,\sqrt{-d^5\,e}}{d^3}\right)\,\sqrt{-d^5\,e}}{2\,d^5\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)),x)
```

```
[Out] x/(4*d^2*(d + e*x^2)) + (atanh((x*(d^5*e)^(1/2))/d^3)*(d^5*e)^(1/2))/(4*d^5
*e) - (atanh((x*(-d^5*e)^(1/2))/d^3)*(-d^5*e)^(1/2))/(2*d^5*e)
```

3.194 $\int \frac{1}{(d+ex^2)^2 (d^2-e^2x^4)} \, dx$

Optimal. Leaf size=89

$$\frac{x}{8d^2(d+ex^2)^2}+\frac{5x}{16d^3(d+ex^2)}+\frac{7\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}$$

```
[Out] 1/8*x/d^2/(e*x^2+d)^2+5/16*x/d^3/(e*x^2+d)+7/16*arctan(x*e^(1/2)/d^(1/2))/d
^(7/2)/e^(1/2)+1/8*arctanh(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1164, 425, 541, 536, 214, 211}

$$\frac{7\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}+\frac{5x}{16d^3(d+ex^2)}+\frac{x}{8d^2(d+ex^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(8*d^2*(d + e*x^2)^2) + (5*x)/(16*d^3*(d + e*x^2)) + (7*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(16*d^(7/2)*Sqrt[e]) + ArcTanh[(Sqrt[e]*x)/Sqrt[d]]/(8*d^(7/2)*
Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^2(d^2-e^2x^4)}\,dx &= \int \frac{1}{(d-ex^2)(d+ex^2)^3}\,dx\\
&= \frac{x}{8d^2(d+ex^2)^2} - \frac{\int \frac{-7de+3e^2x^2}{(d-ex^2)(d+ex^2)^2}\,dx}{8d^2e}\\
&= \frac{x}{8d^2(d+ex^2)^2} + \frac{5x}{16d^3(d+ex^2)} + \frac{\int \frac{18d^2e^2-10de^3x^2}{(d-ex^2)(d+ex^2)}\,dx}{32d^4e^2}\\
&= \frac{x}{8d^2(d+ex^2)^2} + \frac{5x}{16d^3(d+ex^2)} + \frac{\int \frac{1}{d-ex^2}\,dx}{8d^3} + \frac{7\int \frac{1}{d+ex^2}\,dx}{16d^3}\\
&= \frac{x}{8d^2(d+ex^2)^2} + \frac{5x}{16d^3(d+ex^2)} + \frac{7\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}\sqrt{e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{7/2}\sqrt{e}}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 76, normalized size = 0.85

$$\frac{\frac{\sqrt{d}\,x(7d+5ex^2)}{(d+ex^2)^2}+\frac{7\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}}}{16d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(d^2 - e^2*x^4)),x]
```

```
[Out] ((Sqrt[d]*x*(7*d + 5*e*x^2))/(d + e*x^2)^2 + (7*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/Sqrt[e] + (2*ArcTanh[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[e])/(16*d^(7/2))
```

Maple [A]
time = 0.15, size = 64, normalized size = 0.72

method	result	size
default	$\frac{\frac{\frac{5}{2}e\,x^3+\frac{7}{2}dx}{(e\,x^2+d)^2}+\frac{7\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}}}{8d^3}+\frac{\operatorname{arctanh}\left(\frac{ex}{\sqrt{de}}\right)}{8d^3\sqrt{de}}$	64
risch	$\frac{\frac{5e\,x^3}{16d^3}+\frac{7x}{16d^2}}{(e\,x^2+d)^2}-\frac{7\ln\left(-ex-\sqrt{-de}\right)}{32\sqrt{-de}\,d^3}+\frac{7\ln\left(ex-\sqrt{-de}\right)}{32\sqrt{-de}\,d^3}+\frac{\ln\left(ex+\sqrt{de}\right)}{16\sqrt{de}\,d^3}-\frac{\ln\left(-ex+\sqrt{de}\right)}{16\sqrt{de}\,d^3}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d^3*((5/2*e*x^3+7/2*d*x)/(e*x^2+d)^2+7/2/(d*e)^(1/2)*arctan(e*x/(d*e)^(
1/2)))+1/8/d^3/(d*e)^(1/2)*arctanh(e*x/(d*e)^(1/2))
```

Maxima [A]
time = 0.52, size = 89, normalized size = 1.00

$$\frac{5\,x^3e+7\,dx}{16\,(d^3x^4e^2+2\,d^4x^2e+d^5)}+\frac{7\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{16\,d^{\frac{7}{2}}}-\frac{e^{\left(-\frac{1}{2}\right)}\log\left(\frac{xe-\sqrt{d}\,e^{\frac{1}{2}}}{xe+\sqrt{d}\,e^{\frac{1}{2}}}\right)}{16\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] 1/16*(5*x^3*e + 7*d*x)/(d^3*x^4*e^2 + 2*d^4*x^2*e + d^5) + 7/16*arctan(x*e^
(1/2)/sqrt(d))*e^(-1/2)/d^(7/2) - 1/16*e^(-1/2)*log((x*e - sqrt(d)*e^(1/2))
/(x*e + sqrt(d)*e^(1/2)))/d^(7/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(63) = 126$.
time = 0.38, size = 281, normalized size = 3.16

$$\left[\frac{5\,dx^3e^2 + 7\,d^2xe + 7\,(x^4e^2 + 2\,dx^2e + d^2)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}} + (x^4e^2 + 2\,dx^2e + d^2)\sqrt{d}\,e^{\frac{1}{2}}\log\left(\frac{x^2e + 2\,\sqrt{d}\,xe^{\frac{1}{2}} + d}{x^2e - d}\right)}{16\,(d^4x^4e^3 + 2\,d^5x^2e^2 + d^6e)}, \frac{10\,dx^3e^2 + 14\,d^2xe - 4\,(x^4e^2 + 2\,dx^2e + d^2)\sqrt{-de}\,\arctan\left(\frac{\sqrt{-de}\,x}{d}\right) - 7\,(x^4e^2 + 2\,dx^2e + d^2)\sqrt{-de}\,\log\left(\frac{x^2e - 2\,\sqrt{-de}\,x - d}{x^2e + d}\right)}{32\,(d^4x^4e^3 + 2\,d^5x^2e^2 + d^6e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] [1/16*(5*d*x^3*e^2 + 7*d^2*x*e + 7*(x^4*e^2 + 2*d*x^2*e + d^2)*sqrt(d)*arct
an(x*e^(1/2)/sqrt(d))*e^(1/2) + (x^4*e^2 + 2*d*x^2*e + d^2)*sqrt(d)*e^(1/2)
*log((x^2*e + 2*sqrt(d)*x*e^(1/2) + d)/(x^2*e - d)))/(d^4*x^4*e^3 + 2*d^5*x
^2*e^2 + d^6*e), 1/32*(10*d*x^3*e^2 + 14*d^2*x*e - 4*(x^4*e^2 + 2*d*x^2*e +
 d^2)*sqrt(-d*e)*arctan(sqrt(-d*e)*x/d) - 7*(x^4*e^2 + 2*d*x^2*e + d^2)*sqr
t(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)))/(d^4*x^4*e^3 + 2*d^5
*x^2*e^2 + d^6*e)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(82) = 164$.
time = 0.27, size = 257, normalized size = 2.89

$$-\frac{\sqrt{\frac{1}{d^7e}}\log\left(-\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371} - \frac{351d^4\sqrt{\frac{1}{d^7e}}}{371} + x\right)}{16} + \frac{\sqrt{\frac{1}{d^7e}}\log\left(\frac{20d^{11}e\left(\frac{1}{d^7e}\right)^{\frac{3}{2}}}{371} + \frac{351d^4\sqrt{\frac{1}{d^7e}}}{371} + x\right)}{16} - \frac{7\sqrt{-\frac{1}{d^7e}}\log\left(-\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106} - \frac{351d^4\sqrt{-\frac{1}{d^7e}}}{106} + x\right)}{32} + \frac{7\sqrt{-\frac{1}{d^7e}}\log\left(\frac{245d^{11}e\left(-\frac{1}{d^7e}\right)^{\frac{3}{2}}}{106} + \frac{351d^4\sqrt{-\frac{1}{d^7e}}}{106} + x\right)}{32} - \frac{-7dx - 5ex^3}{16d^5 + 32d^4ex^2 + 16d^3e^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(-e**2*x**4+d**2),x)
```

```
[Out] -sqrt(1/(d**7*e))*log(-20*d**11*e*(1/(d**7*e))**(3/2)/371 - 351*d**4*sqrt(1
/(d**7*e))/371 + x)/16 + sqrt(1/(d**7*e))*log(20*d**11*e*(1/(d**7*e))**(3/2
)/371 + 351*d**4*sqrt(1/(d**7*e))/371 + x)/16 - 7*sqrt(-1/(d**7*e))*log(-24
5*d**11*e*(-1/(d**7*e))**(3/2)/106 - 351*d**4*sqrt(-1/(d**7*e))/106 + x)/32
 + 7*sqrt(-1/(d**7*e))*log(245*d**11*e*(-1/(d**7*e))**(3/2)/106 + 351*d**4*
sqrt(-1/(d**7*e))/106 + x)/32 - (-7*d*x - 5*e*x**3)/(16*d**5 + 32*d**4*e*x*
*2 + 16*d**3*e**2*x**4)
```

Giac [A]
time = 2.63, size = 67, normalized size = 0.75

$$\frac{7\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{16\,d^{\frac{7}{2}}} - \frac{\arctan\left(\frac{xe}{\sqrt{-de}}\right)}{8\,\sqrt{-de}\,d^3} + \frac{5\,x^3e + 7\,dx}{16\,(x^2e + d)^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 7/16*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(7/2) - 1/8*arctan(x*e/sqrt(-d*e)
)/(sqrt(-d*e)*d^3) + 1/16*(5*x^3*e + 7*d*x)/((x^2*e + d)^2*d^3)
```

Mupad [B]
time = 0.16, size = 96, normalized size = 1.08

$$\frac{\frac{7\,x}{16\,d^2}+\frac{5\,e\,x^3}{16\,d^3}}{d^2+2\,d\,e\,x^2+e^2\,x^4}+\frac{\mathrm{atanh}\left(\frac{x\,\sqrt{d^7\,e}}{d^4}\right)\,\sqrt{d^7\,e}}{8\,d^7\,e}-\frac{7\,\mathrm{atanh}\left(\frac{x\,\sqrt{-d^7\,e}}{d^4}\right)\,\sqrt{-d^7\,e}}{16\,d^7\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^2),x)
```

```
[Out] ((7*x)/(16*d^2) + (5*e*x^3)/(16*d^3))/(d^2 + e^2*x^4 + 2*d*e*x^2) + (atanh(
(x*(d^7*e)^(1/2))/d^4)*(d^7*e)^(1/2))/(8*d^7*e) - (7*atanh((x*(-d^7*e)^(1/2
))/d^4)*(-d^7*e)^(1/2))/(16*d^7*e)
```

3.195 $\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=62

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}+\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

```
[Out] -arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(1/2)+arctanh(x*2^(1/2)*e^(1/2)/(e*x^
2+d)^(1/2))*2^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1164, 399, 223, 212, 385, 214}

$$\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/Sqrt[e]) + (Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^{3/2}}{d^2-e^2x^4}\,dx &= \int \frac{\sqrt{d+ex^2}}{d-ex^2}\,dx\\ &= (2d)\int \frac{1}{(d-ex^2)\sqrt{d+ex^2}}\,dx - \int \frac{1}{\sqrt{d+ex^2}}\,dx\\ &= (2d)\text{Subst}\left(\int \frac{1}{d-2dex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right) - \text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 70, normalized size = 1.13

$$\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{d-ex^2+\sqrt{e}\,x\sqrt{d+ex^2}}{\sqrt{2}\,d}\right) + \log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x]
```

```
[Out] (Sqrt[2]*ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2])/(Sqrt[2]*d)] + Log
[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/Sqrt[e]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1355 vs. 2(46) = 92.
time = 0.24, size = 1356, normalized size = 21.87

method	result	size
default	Expression too large to display	1356

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e/(-d*e)^(1/2)/(-(d*e)^(1/2)+(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))*(
1/3*(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(3/2)-(-
d*e)^(1/2)*(1/4*(2*e*(x+1/e*(-d*e)^(1/2))-2*(-d*e)^(1/2))/e*(e*(x+1/e*(-d*e
)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2)+1/2*d/e^(1/2)*ln((-(-
d*e)^(1/2)+e*(x+1/e*(-d*e)^(1/2)))/e^(1/2)+(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*
e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2))))+1/2*e/(-(d*e)^(1/2)+(-d*e)^(1/2))/(
(d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)*(1/3*(e*(x-1/e*(d*e)^(1/2))^2+2*(d*e)
^(1/2)*(x-1/e*(d*e)^(1/2))+2*d)^(3/2)+(d*e)^(1/2)*(1/4*(2*e*(x-1/e*(d*e)^(1
/2))+2*(d*e)^(1/2))/e*(e*(x-1/e*(d*e)^(1/2))^2+2*(d*e)^(1/2)*(x-1/e*(d*e)^(
1/2))+2*d)^(1/2)+1/2*d/e^(1/2)*ln(((d*e)^(1/2)+e*(x-1/e*(d*e)^(1/2)))/e^(1/
2)+(e*(x-1/e*(d*e)^(1/2))^2+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))+2*d)^(1/2)))+
2*d*((e*(x-1/e*(d*e)^(1/2))^2+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))+2*d)^(1/2)+
(d*e)^(1/2)*ln(((d*e)^(1/2)+e*(x-1/e*(d*e)^(1/2)))/e^(1/2)+(e*(x-1/e*(d*e)^
(1/2))^2+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))+2*d)^(1/2))/e^(1/2)-d^(1/2)*2^(1
/2)*ln((4*d+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))+2*2^(1/2)*d^(1/2)*(e*(x-1/e*(
d*e)^(1/2))^2+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))+2*d)^(1/2))/(x-1/e*(d*e)^(1
/2)))))-1/2*e/(-(d*e)^(1/2)+(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^
(1/2)*(1/3*(e*(x+1/e*(d*e)^(1/2))^2-2*(d*e)^(1/2)*(x+1/e*(d*e)^(1/2))+2*d)^
(3/2)-(d*e)^(1/2)*(1/4*(2*e*(x+1/e*(d*e)^(1/2))-2*(d*e)^(1/2))/e*(e*(x+1/e*
(d*e)^(1/2))^2-2*(d*e)^(1/2)*(x+1/e*(d*e)^(1/2))+2*d)^(1/2)+1/2*d/e^(1/2)*l
n((-(d*e)^(1/2)+e*(x+1/e*(d*e)^(1/2)))/e^(1/2)+(e*(x+1/e*(d*e)^(1/2))^2-2*(
d*e)^(1/2)*(x+1/e*(d*e)^(1/2))+2*d)^(1/2)))+2*d*((e*(x+1/e*(d*e)^(1/2))^2-2
*(d*e)^(1/2)*(x+1/e*(d*e)^(1/2))+2*d)^(1/2)-(d*e)^(1/2)*ln((-(d*e)^(1/2)+e*
(x+1/e*(d*e)^(1/2)))/e^(1/2)+(e*(x+1/e*(d*e)^(1/2))^2-2*(d*e)^(1/2)*(x+1/e*
(d*e)^(1/2))+2*d)^(1/2))/e^(1/2)-d^(1/2)*2^(1/2)*ln((4*d-2*(d*e)^(1/2)*(x+1
/e*(d*e)^(1/2))+2*2^(1/2)*d^(1/2)*(e*(x+1/e*(d*e)^(1/2))^2-2*(d*e)^(1/2)*(x
+1/e*(d*e)^(1/2))+2*d)^(1/2))/(x+1/e*(d*e)^(1/2)))))-1/2*e/(-d*e)^(1/2)/(-(
d*e)^(1/2)+(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))*(1/3*(e*(x-1/e*(-d*e)^(
1/2))^2+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(3/2)+(-d*e)^(1/2)*(1/4*(2*e*(
x-1/e*(-d*e)^(1/2))+2*(-d*e)^(1/2))/e*(e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e)^(1
/2)*(x-1/e*(-d*e)^(1/2)))^(1/2)+1/2*d/e^(1/2)*ln(((-d*e)^(1/2)+e*(x-1/e*(-d
*e)^(1/2)))/e^(1/2)+(e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^
(1/2)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2*e + d)^(3/2)/(x^4*e^2 - d^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(44) = 88$.
time = 0.36, size = 113, normalized size = 1.82

$$\frac{1}{4}\left(\sqrt{2}\,e^{\frac{1}{2}}\log\left(\frac{17\,x^4e^2+14\,dx^2e+4\,\sqrt{2}\,(3\,x^3e^2+dxe)\sqrt{x^2e+d}\,e^{\left(-\frac{1}{2}\right)}+d^2}{x^4e^2-2\,dx^2e+d^2}\right)+2\,e^{\frac{1}{2}}\log\left(-2\,x^2e+2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*e^(1/2)*log((17*x^4*e^2 + 14*d*x^2*e + 4*sqrt(2)*(3*x^3*e^2 +
d*x*e)*sqrt(x^2*e + d)*e^(-1/2) + d^2)/(x^4*e^2 - 2*d*x^2*e + d^2)) + 2*e^(
1/2)*log(-2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) - d))*e^(-1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{d+ex^2}}{-d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(sqrt(d + e*x**2)/(-d + e*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(44) = 88$.
time = 2.84, size = 107, normalized size = 1.73

$$\frac{\sqrt{2}\,de^{\left(-\frac{1}{2}\right)}\log\left(\frac{\left|2\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2-4\,\sqrt{2}\,|d|-6\,d\right|}{\left|2\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2+4\,\sqrt{2}\,|d|-6\,d\right|}\right)}{2\,|d|}+\frac{1}{2}\,e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*d*e^(-1/2)*log(abs(2*(x*e^(1/2) - sqrt(x^2*e + d))^2 - 4*sqrt(2
)*abs(d) - 6*d)/abs(2*(x*e^(1/2) - sqrt(x^2*e + d))^2 + 4*sqrt(2)*abs(d) -
6*d))/abs(d) + 1/2*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(e\,x^2+d)^{3/2}}{d^2-e^2\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4),x)
[Out] int((d + e*x^2)^(3/2)/(d^2 - e^2*x^4), x)
```

3.196 $\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}\,d\sqrt{e}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d*2^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1164, 385, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}\,d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(Sqrt[2]*d*Sqrt[e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{d^2-e^2x^4}\,dx = \int \frac{1}{(d-ex^2)\sqrt{d+ex^2}}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{d-2dex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{2}\,d\sqrt{e}}$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 1.32

$$\frac{\tanh^{-1}\left(\frac{d-ex^2+\sqrt{e}\,x\sqrt{d+ex^2}}{\sqrt{2}\,d}\right)}{\sqrt{2}\,d\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(d^2 - e^2*x^4),x]
```

```
[Out] ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2])/(Sqrt[2]*d)]/(Sqrt[2]*d*Sqr
t[e])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(29) = 58$.
time = 0.25, size = 818, normalized size = 21.53

method	result
default	$-\frac{e\left(\sqrt{e\left(x+\frac{\sqrt{-de}}{e}\right)^2-2\sqrt{-de}\,\left(x+\frac{\sqrt{-de}}{e}\right)}-\frac{\sqrt{-de}\,\ln\left(\frac{-\sqrt{-de}+e\left(x+\frac{\sqrt{-de}}{e}\right)}{\sqrt{e}}+\sqrt{e\left(x+\right.}\right.}{\sqrt{e}}\right.}{2\sqrt{-de}\,\left(\sqrt{de}-\sqrt{-de}\right)\left(\sqrt{de}+\sqrt{-de}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e/(-d*e)^(1/2)/((d*e)^(1/2)-(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))*(
(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2)-(-d*e)
```

```
^(1/2)*ln((-(-d*e)^(1/2)+e*(x+1/e*(-d*e)^(1/2)))/e^(1/2)+(e*(x+1/e*(-d*e)^(
1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2))/e^(1/2))-1/2*e/((d*e)^(
1/2)-(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)*((e*(x-1/e*(d*e)^
(1/2))^2+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))+2*d)^(1/2)+(d*e)^(1/2)*ln(((d*e)
^(1/2)+e*(x-1/e*(d*e)^(1/2)))/e^(1/2)+(e*(x-1/e*(d*e)^(1/2))^2+2*(d*e)^(1/2
)*(x-1/e*(d*e)^(1/2))+2*d)^(1/2))/e^(1/2)-d^(1/2)*2^(1/2)*ln((4*d+2*(d*e)^(
1/2)*(x-1/e*(d*e)^(1/2))+2*2^(1/2)*d^(1/2)*(e*(x-1/e*(d*e)^(1/2))^2+2*(d*e)
^(1/2)*(x-1/e*(d*e)^(1/2))+2*d)^(1/2))/(x-1/e*(d*e)^(1/2))))+1/2*e/((d*e)^(
1/2)-(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)*((e*(x+1/e*(d*e)^
(1/2))^2-2*(d*e)^(1/2)*(x+1/e*(d*e)^(1/2))+2*d)^(1/2)-(d*e)^(1/2)*ln((-(d*e
)^(1/2)+e*(x+1/e*(d*e)^(1/2)))/e^(1/2)+(e*(x+1/e*(d*e)^(1/2))^2-2*(d*e)^(1/
2)*(x+1/e*(d*e)^(1/2))+2*d)^(1/2))/e^(1/2)-d^(1/2)*2^(1/2)*ln((4*d-2*(d*e)^
(1/2)*(x+1/e*(d*e)^(1/2))+2*2^(1/2)*d^(1/2)*(e*(x+1/e*(d*e)^(1/2))^2-2*(d*e
)^(1/2)*(x+1/e*(d*e)^(1/2))+2*d)^(1/2))/(x+1/e*(d*e)^(1/2))))+1/2*e/(-d*e)^
(1/2)/((d*e)^(1/2)-(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))*((e*(x-1/e*(-d*
e)^(1/2))^2+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2)+(-d*e)^(1/2)*ln(((-d
*e)^(1/2)+e*(x-1/e*(-d*e)^(1/2)))/e^(1/2)+(e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e
)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2))/e^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x^2*e + d)/(x^4*e^2 - d^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(28) = 56.
time = 0.33, size = 79, normalized size = 2.08

$$\frac{\sqrt{2}\,e^{\left(-\frac{1}{2}\right)}\log\left(\frac{17\,x^4e^2+14\,dx^2e+4\,\sqrt{2}\,(3\,x^3e+dx)\,\sqrt{x^2e+d}\,e^{\frac{1}{2}}+d^2}{x^4e^2-2\,dx^2e+d^2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*e^(-1/2)*log((17*x^4*e^2 + 14*d*x^2*e + 4*sqrt(2)*(3*x^3*e + d*
x)*sqrt(x^2*e + d)*e^(1/2) + d^2)/(x^4*e^2 - 2*d*x^2*e + d^2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{-d\sqrt{d+ex^2}+ex^2\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d*sqrt(d + e*x**2) + e*x**2*sqrt(d + e*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(28) = 56$.
time = 3.52, size = 81, normalized size = 2.13

$$\frac{\sqrt{2}\,e^{\left(-\frac{1}{2}\right)}\log\left(\frac{\left|2\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2-4\sqrt{2}\,|d|-6\,d\right|}{\left|2\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2+4\sqrt{2}\,|d|-6\,d\right|}\right)}{4\,|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*e^(-1/2)*log(abs(2*(x*e^(1/2) - sqrt(x^2*e + d))^2 - 4*sqrt(2)*
abs(d) - 6*d)/abs(2*(x*e^(1/2) - sqrt(x^2*e + d))^2 + 4*sqrt(2)*abs(d) - 6*
d))/abs(d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{e\,x^2+d}}{d^2-e^2\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4),x)
```

```
[Out] int((d + e*x^2)^(1/2)/(d^2 - e^2*x^4), x)
```

3.197 $\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx$

Optimal. Leaf size=61

$$\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}\,d^2\sqrt{e}}$$

```
[Out] 1/4*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d^2*2^(1/2)/e^(1/2)+1/2*x/d^
2/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1164, 390, 385, 214}

$$\frac{x}{2d^2\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}\,d^2\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(2*d^2*Sqrt[d + e*x^2]) + ArcTanh[(Sqrt[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(2
*Sqrt[2]*d^2*Sqrt[e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{d+ex^2}\,(d^2-e^2x^4)}\,dx &= \int \frac{1}{(d-ex^2)\,(d+ex^2)^{3/2}}\,dx \\
&= \frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\int \frac{1}{(d-ex^2)\sqrt{d+ex^2}}\,dx}{2d} \\
&= \frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\text{Subst}\left(\int \frac{1}{d-2dex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2d} \\
&= \frac{x}{2d^2\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2\sqrt{2}\,d^2\sqrt{e}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 69, normalized size = 1.13

$$\frac{\frac{2x}{\sqrt{d+ex^2}} + \frac{\sqrt{2}\tanh^{-1}\left(\frac{d-ex^2+\sqrt{e}\,x\sqrt{d+ex^2}}{\sqrt{2}\,d}\right)}{\sqrt{e}}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(d^2 - e^2*x^4)),x]
```

```
[Out] ((2*x)/Sqrt[d + e*x^2] + (Sqrt[2]*ArcTanh[(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e
*x^2])/(Sqrt[2]*d)])/Sqrt[e])/(4*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(45) = 90$.
time = 0.24, size = 441, normalized size = 7.23

method	result

default	$\frac{\sqrt{e\left(x+\frac{\sqrt{-de}}{e}\right)^{2}-2\sqrt{-de}\,\left(x+\frac{\sqrt{-de}}{e}\right)}}{2d\left(\sqrt{de}-\sqrt{-de}\right)\left(\sqrt{de}+\sqrt{-de}\right)\left(x+\frac{\sqrt{-de}}{e}\right)}+\frac{e\sqrt{2}\,\ln\left(\frac{4d+2\sqrt{de}\,\left(x-\frac{\sqrt{de}}{e}\right)+2\sqrt{2}\,\sqrt{d}\,\sqrt{e\left(\right.}}{}\right)}{4\left(\sqrt{de}-\sqrt{-de}\right)\left(\sqrt{d}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/((d*e)^(1/2)-(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))/(x+1/e*(-d*e)^(
1/2))*(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2)+
1/4*e/((d*e)^(1/2)-(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))/(d*e)^(1/2)*2^(
1/2)/d^(1/2)*ln((4*d+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))+2*2^(1/2)*d^(1/2)*(e
*(x-1/e*(d*e)^(1/2))^2+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))+2*d)^(1/2))/(x-1/e
*(d*e)^(1/2)))-1/4*e/((d*e)^(1/2)-(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))/
(d*e)^(1/2)*2^(1/2)/d^(1/2)*ln((4*d-2*(d*e)^(1/2)*(x+1/e*(d*e)^(1/2))+2*2^(
1/2)*d^(1/2)*(e*(x+1/e*(d*e)^(1/2))^2-2*(d*e)^(1/2)*(x+1/e*(d*e)^(1/2))+2*d
)^(1/2))/(x+1/e*(d*e)^(1/2)))+1/2/d/((d*e)^(1/2)-(-d*e)^(1/2))/((d*e)^(1/2)
+(-d*e)^(1/2))/(x-1/e*(-d*e)^(1/2))*(e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e)^(1/2
)*(x-1/e*(-d*e)^(1/2)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^4*e^2 - d^2)*sqrt(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(45) = 90$.
time = 0.34, size = 119, normalized size = 1.95

$$\frac{\sqrt{2}\,(x^2e+d)e^{\frac{1}{2}}\log\left(\frac{17\,x^4e^2+14\,dx^2e+4\,\sqrt{2}\,(3\,x^3e+dx)\sqrt{x^2e+d}\,e^{\frac{1}{2}}+d^2}{x^4e^2-2\,dx^2e+d^2}\right)+8\,\sqrt{x^2e+d}\,xe}{16\,(d^2x^2e^2+d^3e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*(x^2*e + d)*e^(1/2)*log((17*x^4*e^2 + 14*d*x^2*e + 4*sqrt(2)*
(3*x^3*e + d*x)*sqrt(x^2*e + d)*e^(1/2) + d^2)/(x^4*e^2 - 2*d*x^2*e + d^2))
 + 8*sqrt(x^2*e + d)*x*e)/(d^2*x^2*e^2 + d^3*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-d^2\sqrt{d+ex^2}+e^2x^4\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(1/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d**2*sqrt(d + e*x**2) + e**2*x**4*sqrt(d + e*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(45) = 90.
time = 3.21, size = 101, normalized size = 1.66

$$\frac{\sqrt{2}\,e^{\left(-\frac{1}{2}\right)}\log\left(\frac{\left|2\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2-4\sqrt{2}\,|d|-6\,d\right|}{\left|2\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2+4\sqrt{2}\,|d|-6\,d\right|}\right)}{8\,d|d|}+\frac{x}{2\sqrt{x^2e+d}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*e^(-1/2)*log(abs(2*(x*e^(1/2) - sqrt(x^2*e + d))^2 - 4*sqrt(2)*
abs(d) - 6*d)/abs(2*(x*e^(1/2) - sqrt(x^2*e + d))^2 + 4*sqrt(2)*abs(d) - 6*
d))/(d*abs(d)) + 1/2*x/(sqrt(x^2*e + d)*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(d^2-e^2\,x^4\right)\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(1/2)),x)
```

```
[Out] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(1/2)), x)
```

3.198 $\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx$

Optimal. Leaf size=80

$$\frac{x}{6d^2(d+ex^2)^{3/2}}+\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}\,d^3\sqrt{e}}$$

```
[Out] 1/6*x/d^2/(e*x^2+d)^(3/2)+1/8*arctanh(x*2^(1/2)*e^(1/2)/(e*x^2+d)^(1/2))/d^
3*2^(1/2)/e^(1/2)+7/12*x/d^3/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1164, 425, 541, 12, 385, 214}

$$\frac{7x}{12d^3\sqrt{d+ex^2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}\,d^3\sqrt{e}}+\frac{x}{6d^2(d+ex^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

```
[Out] x/(6*d^2*(d + e*x^2)^(3/2)) + (7*x)/(12*d^3*Sqrt[d + e*x^2]) + ArcTanh[(Sqr
t[2]*Sqrt[e]*x)/Sqrt[d + e*x^2]]/(4*Sqrt[2]*d^3*Sqrt[e])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
```

```
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a, c, d, e, q}, x]
 && EqQ[c*d^2 + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex^2)^{3/2}(d^2-e^2x^4)}\,dx &= \int \frac{1}{(d-ex^2)(d+ex^2)^{5/2}}\,dx \\
&= \frac{x}{6d^2(d+ex^2)^{3/2}} - \frac{\int \frac{-5de+2e^2x^2}{(d-ex^2)(d+ex^2)^{3/2}}\,dx}{6d^2e} \\
&= \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\int \frac{3d^2e^2}{(d-ex^2)\sqrt{d+ex^2}}\,dx}{12d^4e^2} \\
&= \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\int \frac{1}{(d-ex^2)\sqrt{d+ex^2}}\,dx}{4d^2} \\
&= \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\text{Subst}\left(\int \frac{1}{d-2dex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{4d^2} \\
&= \frac{x}{6d^2(d+ex^2)^{3/2}} + \frac{7x}{12d^3\sqrt{d+ex^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{4\sqrt{2}\,d^3\sqrt{e}}
\end{aligned}$$

Mathematica [A]

time = 0.15, size = 80, normalized size = 1.00

$$\frac{\frac{2(9dx+7ex^3)}{(d+ex^2)^{3/2}}+\frac{3\sqrt{2}\tanh^{-1}\left(\frac{d-ex^2+\sqrt{e}\,x\sqrt{d+ex^2}}{\sqrt{2}\,d}\right)}{\sqrt{e}}}{24d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(d^2 - e^2*x^4)),x]
```

```
[Out] ((2*(9*d*x + 7*e*x^3))/(d + e*x^2)^(3/2) + (3*Sqrt[2]*ArcTanh[(d - e*x^2 +
Sqrt[e]*x*Sqrt[d + e*x^2]])/(Sqrt[2]*d)])/Sqrt[e])/(24*d^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(60) = 120$.
time = 0.21, size = 865, normalized size = 10.81

method	result
default	$-\frac{e\left(\frac{1}{3\sqrt{-de}\left(x+\frac{\sqrt{-de}}{e}\right)\sqrt{e\left(x+\frac{\sqrt{-de}}{e}\right)^2-2\sqrt{-de}\left(x+\frac{\sqrt{-de}}{e}\right)}}+\frac{2e\left(x \cdots\right.}{3\sqrt{-de}\,d\sqrt{e\left(x+\sqrt{\cdots}\right.}}\right.}{2\sqrt{-de}\left(\sqrt{de}-\sqrt{-de}\right)\left(\sqrt{de}+\sqrt{-de}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e/(-d*e)^(1/2)/((d*e)^(1/2)-(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))*(
1/3/(-d*e)^(1/2)/(x+1/e*(-d*e)^(1/2))/(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1
/2)*(x+1/e*(-d*e)^(1/2)))^(1/2)+1/3/(-d*e)^(1/2)*(2*e*(x+1/e*(-d*e)^(1/2))-
2*(-d*e)^(1/2))/d/(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1
/2)))^(1/2))-1/2*e/((d*e)^(1/2)-(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2))/(d
*e)^(1/2)*(1/2/d/(e*(x-1/e*(d*e)^(1/2))^2+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))
+2*d)^(1/2)-1/4*(d*e)^(1/2)/d^2*(2*e*(x-1/e*(d*e)^(1/2))+2*(d*e)^(1/2))/e/(
e*(x-1/e*(d*e)^(1/2))^2+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))+2*d)^(1/2)-1/4/d^
(3/2)*2^(1/2)*ln((4*d+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))+2*2^(1/2)*d^(1/2)*(
e*(x-1/e*(d*e)^(1/2))^2+2*(d*e)^(1/2)*(x-1/e*(d*e)^(1/2))+2*d)^(1/2))/(x-1/
e*(d*e)^(1/2))))+1/2*e/((d*e)^(1/2)-(-d*e)^(1/2))/((d*e)^(1/2)+(-d*e)^(1/2)
```

```
)/(d*e)^(1/2)*(1/2/d/(e*(x+1/e*(d*e)^(1/2))^2-2*(d*e)^(1/2)*(x+1/e*(d*e)^(1
/2))+2*d)^(1/2)+1/4*(d*e)^(1/2)/d^2*(2*e*(x+1/e*(d*e)^(1/2))-2*(d*e)^(1/2))
/e/(e*(x+1/e*(d*e)^(1/2))^2-2*(d*e)^(1/2)*(x+1/e*(d*e)^(1/2))+2*d)^(1/2)-1/
4/d^(3/2)*2^(1/2)*ln((4*d-2*(d*e)^(1/2)*(x+1/e*(d*e)^(1/2))+2*2^(1/2)*d^(1/
2)*(e*(x+1/e*(d*e)^(1/2))^2-2*(d*e)^(1/2)*(x+1/e*(d*e)^(1/2))+2*d)^(1/2))/(
x+1/e*(d*e)^(1/2))))+1/2*e/(-d*e)^(1/2)/((d*e)^(1/2)-(-d*e)^(1/2))/((d*e)^(
1/2)+(-d*e)^(1/2))*(-1/3/(-d*e)^(1/2)/(x-1/e*(-d*e)^(1/2))/(e*(x-1/e*(-d*e)
^(1/2))^2+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2)-1/3/(-d*e)^(1/2)*(2*e*
(x-1/e*(-d*e)^(1/2))+2*(-d*e)^(1/2))/d/(e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e)^(
1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="maxima")
[Out] -integrate(1/((x^4*e^2 - d^2)*(x^2*e + d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(61) = 122.
time = 0.35, size = 151, normalized size = 1.89

$$\frac{3\sqrt{2}\,(x^4e^2+2\,dx^2e+d^2)e^{\frac{1}{2}}\log\left(\frac{17\,x^4e^2+14\,dx^2e+4\,\sqrt{2}\,(3\,x^3e+dx)\sqrt{x^2e+d}\,e^{\frac{1}{2}}+d^2}{x^4e^2-2\,dx^2e+d^2}\right)+8\,(7\,x^3e^2+9\,dxe)\sqrt{x^2e+d}}{96\,(d^3x^4e^3+2\,d^4x^2e^2+d^5e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="fricas")
[Out] 1/96*(3*sqrt(2)*(x^4*e^2 + 2*d*x^2*e + d^2)*e^(1/2)*log((17*x^4*e^2 + 14*d*
x^2*e + 4*sqrt(2)*(3*x^3*e + d*x)*sqrt(x^2*e + d)*e^(1/2) + d^2)/(x^4*e^2 -
 2*d*x^2*e + d^2)) + 8*(7*x^3*e^2 + 9*d*x*e)*sqrt(x^2*e + d))/(d^3*x^4*e^3
+ 2*d^4*x^2*e^2 + d^5*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{-d^3\sqrt{d+ex^2}-d^2ex^2\sqrt{d+ex^2}+de^2x^4\sqrt{d+ex^2}+e^3x^6\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(-e**2*x**4+d**2),x)
```

```
[Out] -Integral(1/(-d**3*sqrt(d + e*x**2) - d**2*e*x**2*sqrt(d + e*x**2) + d*e**2
*x**4*sqrt(d + e*x**2) + e**3*x**6*sqrt(d + e*x**2)), x)
```

Giac [A]
time = 3.88, size = 114, normalized size = 1.42

$$\frac{x\left(\frac{7\,x^2e}{d^3}+\frac{9}{d^2}\right)}{12\,(x^2e+d)^{\frac{3}{2}}}+\frac{\sqrt{2}\,e^{\left(-\frac{1}{2}\right)}\log\left(\frac{\left|2\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2-4\sqrt{2}\,|d|-6\,d\right|}{\left|2\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2+4\sqrt{2}\,|d|-6\,d\right|}\right)}{16\,d^2|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(-e^2*x^4+d^2),x, algorithm="giac")
```

```
[Out] 1/12*x*(7*x^2*e/d^3 + 9/d^2)/(x^2*e + d)^(3/2) + 1/16*sqrt(2)*e^(-1/2)*log(
abs(2*(x*e^(1/2) - sqrt(x^2*e + d))^2 - 4*sqrt(2)*abs(d) - 6*d)/abs(2*(x*e^
(1/2) - sqrt(x^2*e + d))^2 + 4*sqrt(2)*abs(d) - 6*d))/(d^2*abs(d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d^2-e^2\,x^4\right)\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(3/2)),x)
```

```
[Out] int(1/((d^2 - e^2*x^4)*(d + e*x^2)^(3/2)), x)
```

3.199 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=153

$$-\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}}-\frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] -1/4*x*(-b*x^2+a)*(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)-9/8*a*x*(-b*x^2+a)*(
b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/8*a^2*arctan(x*b^(1/2)/(-b*x^2+a)^(1
/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1166, 427, 396, 223, 209}

$$\frac{19a^2\sqrt{a-bx^2}\sqrt{a+bx^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{8\sqrt{b}\sqrt{a^2-b^2x^4}}-\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}}-\frac{x(a-bx^2)(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-9*a*x*(a - b*x^2)*Sqrt[a + b*x^2])/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^
2)*(a + b*x^2)^(3/2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqr
t[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^2
*x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{(a+bx^2)^2}{\sqrt{a-bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= -\frac{x(a-bx^2)\,(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} - \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{-5a^2b-9ab^2x^2}{\sqrt{a-bx^2}}\,dx}{4b\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)\,(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int}{8\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)\,(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\mathrm{S}}{8\sqrt{a^2}}$$

$$= -\frac{9ax(a-bx^2)\sqrt{a+bx^2}}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)\,(a+bx^2)^{3/2}}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan}{8\sqrt{b}\,\sqrt{a^2-b^2x}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.70, size = 98, normalized size = 0.64

$$-\frac{(11ax+2bx^3)\sqrt{a^2-b^2x^4}}{8\sqrt{a+bx^2}} + \frac{19ia^2\log\left(-2i\sqrt{b}\,x+\frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -1/8*((11*a*x + 2*b*x^3)*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2] + (((19*I)/8)
*a^2*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[
b]
```

Maple [**A**]
time = 0.14, size = 96, normalized size = 0.63

method	result
default	$\frac{\sqrt{-b^{2} x^{4}+a^{2}}\,\left(-2 b^{\frac{3}{2}} x^{3} \sqrt{-b\,x^{2}+a}-11 a x \sqrt{-b\,x^{2}+a}\,\sqrt{b}+19 \arctan\left(\frac{x \sqrt{b}}{\sqrt{-b\,x^{2}+a}}\right) a^{2}\right)}{8 \sqrt{b\,x^{2}+a}\,\sqrt{-b\,x^{2}+a}\,\sqrt{b}}$
risch	$-\frac{x \left(2 b\,x^{2}+11 a\right) \sqrt{-b\,x^{2}+a}\,\sqrt{\frac{-b^{2} x^{4}+a^{2}}{b\,x^{2}+a}}\,\sqrt{b\,x^{2}+a}}{8 \sqrt{-b^{2} x^{4}+a^{2}}}+\frac{19 a^{2} \arctan\left(\frac{x \sqrt{b}}{\sqrt{-b\,x^{2}+a}}\right) \sqrt{\frac{-b^{2} x^{4}+a^{2}}{b\,x^{2}+a}}\,\sqrt{b\,x^{2}+a}}{8 \sqrt{b}\,\sqrt{-b^{2} x^{4}+a^{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-b^2*x^4+a^2)^(1/2)*(-2*b^(3/2)*x^3*(-b*x^2+a)^(1/2)-11*a*x*(-b*x^2+a)
^(1/2)*b^(1/2)+19*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*a^2)/(b*x^2+a)^(1/2)/(
-b*x^2+a)^(1/2)/b^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [**A**]
time = 0.35, size = 251, normalized size = 1.64

$$\left[-\frac{19\,(a^{2}bx^{2}+a^{3})\sqrt{-b}\,\log\left(-\frac{2\,b^{2}x^{4}+abx^{2}-2\,\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{bx^{2}+a}\,\sqrt{-b}\,x-a^{2}}{bx^{2}+a}\right)+2\,\sqrt{-b^{2}x^{4}+a^{2}}\,(2\,b^{2}x^{3}+11\,abx)\sqrt{bx^{2}+a}}{16\,(b^{2}x^{2}+ab)},\,-\frac{19\,(a^{2}bx^{2}+a^{3})\sqrt{b}\,\arctan\left(\frac{\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{bx^{2}+a}\,\sqrt{b}}{b^{2}x^{3}+abx}\right)+\sqrt{-b^{2}x^{4}+a^{2}}\,(2\,b^{2}x^{3}+11\,abx)\sqrt{bx^{2}+a}}{8\,(b^{2}x^{2}+ab)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(19*(a^2*b*x^2 + a^3)*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b
^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a)) + 2*sqrt(-b^2*
```

```
x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*sqrt(b*x^2 + a))/(b^2*x^2 + a*b), -1/8*(1
9*(a^2*b*x^2 + a^3)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqr
t(b)/(b^2*x^3 + a*b*x)) + sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 + 11*a*b*x)*sqrt(
b*x^2 + a))/(b^2*x^2 + a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2)^{\frac{5}{2}}}{\sqrt{-(-a + bx^2)(a + bx^2)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral((a + b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^2 + a)^{5/2}}{\sqrt{a^2 - b^2\,x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a + b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.200 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=110

$$-\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}+\frac{3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] -1/2*x*(-b*x^2+a)*(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/2*a*arctan(x*b^(1/
2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2
)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1166, 396, 223, 209}

$$\frac{3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\,\sqrt{a^2-b^2x^4}}-\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -1/2*(x*(a - b*x^2)*Sqrt[a + b*x^2])/Sqrt[a^2 - b^2*x^4] + (3*a*Sqrt[a - b*
x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(2*Sqrt[b]*Sqrt[a
^2 - b^2*x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{a+bx^2}{\sqrt{a-bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{\left(3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}}\,dx}{2\sqrt{a^2-b^2x^4}}$$

$$= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{\left(3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right.}{2\sqrt{a^2-b^2x^4}}$$

$$= -\frac{x(a-bx^2)\sqrt{a+bx^2}}{2\sqrt{a^2-b^2x^4}} + \frac{3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{2\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.26, size = 86, normalized size = 0.78

$$-\frac{x\sqrt{a^2-b^2x^4}}{2\sqrt{a+bx^2}} + \frac{3ia\log\left(-2i\sqrt{b}\,x + \frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -1/2*(x*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2] + (((3*I)/2)*a*Log[(-2*I)*Sqrt
[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A]
time = 0.13, size = 75, normalized size = 0.68

method	result	size

default	$\frac{\sqrt{-b^2x^4+a^2}\left(-x\sqrt{-b\,x^2+a}\,\sqrt{b}+3\arctan\left(\frac{x\sqrt{b}}{\sqrt{-b\,x^2+a}}\right)a\right)}{2\sqrt{b\,x^2+a}\,\sqrt{-b\,x^2+a}\,\sqrt{b}}$	75
risch	$-\frac{x\sqrt{-b\,x^2+a}\,\sqrt{\frac{-b^2x^4+a^2}{b\,x^2+a}}\,\sqrt{b\,x^2+a}}{2\sqrt{-b^2x^4+a^2}}+\frac{3a\arctan\left(\frac{x\sqrt{b}}{\sqrt{-b\,x^2+a}}\right)\sqrt{\frac{-b^2x^4+a^2}{b\,x^2+a}}\,\sqrt{b\,x^2+a}}{2\sqrt{b}\,\sqrt{-b^2x^4+a^2}}$	131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b^2*x^4+a^2)^(1/2)*(-x*(-b*x^2+a)^(1/2)*b^(1/2)+3*arctan(x*b^(1/2)/(-
b*x^2+a)^(1/2))*a)/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [A]
time = 0.34, size = 223, normalized size = 2.03

$$\left[-\frac{2\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,bx+3\left(abx^2+a^2\right)\sqrt{-b}\,\log\left(-\frac{2\,b^2x^4+abx^2-2\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{-b}\,x-a^2}{bx^2+a}\right)}{4\left(b^2x^2+ab\right)},-\frac{\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,bx+3\left(abx^2+a^2\right)\sqrt{b}\,\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{b}}{b^2x^3+abx}\right)}{2\left(b^2x^2+ab\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt(
-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt
(-b)*x - a^2)/(b*x^2 + a)))/(b^2*x^2 + a*b), -1/2*(sqrt(-b^2*x^4 + a^2)*sqr
t(b*x^2 + a)*b*x + 3*(a*b*x^2 + a^2)*sqrt(b)*arctan(sqrt(-b^2*x^4 + a^2)*sq
rt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(b^2*x^2 + a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+bx^2\right)^{\frac{3}{2}}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral((a + b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a + b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.201 $\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}} \, dx$

Optimal. Leaf size=65

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)
/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1166, 223, 209}

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt
[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$
$$= \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}}$$
$$= \frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.96, size = 50, normalized size = 0.77

$$\frac{i\log\left(-2i\sqrt{b}\,x + \frac{2\sqrt{a^2-b^2x^4}}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[a^2 - b^2*x^4],x]
[Out] (I*Log[(-2*I)*Sqrt[b]*x + (2*Sqrt[a^2 - b^2*x^4])/Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A]
time = 0.13, size = 54, normalized size = 0.83

method	result	size
default	$\frac{\sqrt{-b^2x^4+a^2}\,\arctan\left(\frac{x\sqrt{b}}{\sqrt{-b\,x^2+a}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{-b\,x^2+a}\,\sqrt{b}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)/(-b*x^2+a)^(1/2)/b^(1/2)*arctan(x*b^
(1/2)/(-b*x^2+a)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [A]
time = 0.37, size = 121, normalized size = 1.86

$$\left[-\frac{\sqrt{-b}\log\left(-\frac{2\,b^2x^4+abx^2-2\,\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{-b}\,x-a^2}{bx^2+a}\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{b}}{b^2x^3+abx}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-b)*log(-(2*b^2*x^4 + a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^
2 + a)*sqrt(-b)*x - a^2)/(b*x^2 + a))/b, -arctan(sqrt(-b^2*x^4 + a^2)*sqrt(
b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x))/sqrt(b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a+bx^2}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{b\,x^2+a}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.202 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/2*arctan(x*2^(1/2)*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(
1/2)/a*2^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1166, 385, 211}

$$\frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]
])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}\,(a+bx^2)}\,dx}{\sqrt{a^2-b^2x^4}}$$
$$= \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{a+2abx^2}\,dx, x, \frac{x}{\sqrt{a-bx^2}}\right)}{\sqrt{a^2-b^2x^4}}$$
$$= \frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Mathematica [A]
time = 1.37, size = 78, normalized size = 1.00

$$\frac{\sqrt{a^2-b^2x^4}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\,\sqrt{a-bx^2}\,\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(Sqrt[2]*
a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(63) = 126$.
time = 0.27, size = 249, normalized size = 3.19

method	result
default	$\frac{\sqrt{-b^2x^4+a^2}\,\sqrt{b}\,\left(\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^2+a}\,-\sqrt{-ab}\,x+a\right)}{bx-\sqrt{-ab}}\right)\sqrt{b}-\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\right.}{}\right.\right.}{2\sqrt{b\,x^2+a}\,\sqrt{-b\,x^2+a}\,\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(
-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*b^(1/2)-a^(1/2)*2^(1/
2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(
```

```
1/2)))*b^(1/2)+2*(-a*b)^(1/2)*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))-2*(-a*b)^(
1/2)*arctan(b^(1/2)*x/(1/b*(b*x+(a*b)^(1/2))*(-b*x+(a*b)^(1/2)))^(1/2)))/(b
*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/((-a*b)^(1/2)-(a*b)^(1/2))/((-a*b)^(1/2)+(a*
b)^(1/2))/(-a*b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)
```

Fricas [A]
time = 0.37, size = 152, normalized size = 1.95

$$\left[-\frac{\sqrt{2}\,\sqrt{-b}\,\log\left(-\frac{3\,b^2x^4+2\,abx^2-2\,\sqrt{2}\,\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{-b}\,x-a^2}{b^2x^4+2\,abx^2+a^2}\right)}{4\,ab},\,-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{b}}{2\,(b^2x^3+abx)}\right)}{2\,a\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
[Out] [-1/4*sqrt(2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^
4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*
b), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sq
rt(b)/(b^2*x^3 + a*b*x))/(a*sqrt(b))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\,\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(1/2)),x)
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(1/2)), x)
```

3.203 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=125

$$\frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/4*x*(-b*x^2+a)/a^2/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/8*arctan(x*2^(1
/2)*b^(1/2)/(-b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^2*2^(1/2)/
b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1166, 390, 385, 211}

$$\frac{3\sqrt{a+bx^2}\,\sqrt{a-bx^2}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\,\sqrt{a^2-b^2x^4}}+\frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a - b*x^2))/(4*a^2*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b
*x^2]*Sqrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(4*Sqrt[
2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
```

```
x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}\,(a+bx^2)^2}\,dx}{\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}\,(a+bx^2)}}{4a\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{\left(3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{a+2abx^2}\,dx\right.}{4a\sqrt{a^2-b^2x^4}}\\
&= \frac{x(a-bx^2)}{4a^2\sqrt{a+bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\,\sqrt{a^2-b^2x^4}}
\end{aligned}$$

Mathematica [A]
time = 2.34, size = 111, normalized size = 0.89

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}\,x\sqrt{a-bx^2}+3\sqrt{2}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)\right)}{8a^2\sqrt{b}\,\sqrt{a-bx^2}\,(a+bx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2] + 3*Sqrt[2]*(a + b*x^2)*A
rcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(8*a^2*Sqrt[b]*Sqrt[a - b*x^2]
*(a + b*x^2)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(101) = 202$.
time = 0.27, size = 488, normalized size = 3.90

method	result
default	$\frac{\sqrt{-b^2x^4+a^2}\, b^{\frac{5}{2}} \left(3\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x^2+a}+\sqrt{-ab}\,x+a\right)}{bx+\sqrt{-ab}}\right)\sqrt{2}\, b^{\frac{3}{2}} x^2 \sqrt{a} - 3\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{-b\,x}\right.}{bx-\sqrt{}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2
)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*2^(1/2)*b^(3/2)*x^2*a^(1/2)-3*ln(2*
b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2
^(1/2)*b^(3/2)*x^2*a^(1/2)+3*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b
)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*2^(1/2)*a^(3/2)*b^(1/2)-3*ln(2*b*(2^(1/2)*
a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*2^(1/2)*a^(3
/2)*b^(1/2)-4*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*b*x^2*(-a*b)^(1/2)+4*arcta
n(b^(1/2)*x/(1/b*(b*x+(a*b)^(1/2))*(-b*x+(a*b)^(1/2)))^(1/2))*b*x^2*(-a*b)^
(1/2)+4*b^(1/2)*(-a*b)^(1/2)*(-b*x^2+a)^(1/2)*x-4*arctan(x*b^(1/2)/(-b*x^2+
a)^(1/2))*a*(-a*b)^(1/2)+4*arctan(b^(1/2)*x/(1/b*(b*x+(a*b)^(1/2))*(-b*x+(a
*b)^(1/2)))^(1/2))*a*(-a*b)^(1/2))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/((-a*b)
^(1/2)-(a*b)^(1/2))^2/((-a*b)^(1/2)+(a*b)^(1/2))^2/(-a*b)^(1/2)/(b*x+(-a*b)
^(1/2))/(b*x-(-a*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

Fricas [A]
time = 0.34, size = 297, normalized size = 2.38

$$\left[\frac{4\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,bx - 3\sqrt{2}\,(b^2x^4+2\,abx^2+a^2)\sqrt{-b}\,\log\left(-\frac{3\,b^2x^4+2\,abx^2-2\,\sqrt{2}\,\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{-b}\,x-a^2}{b^2x^4+2\,abx^2+a^2}\right)}{16\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)},\ \frac{2\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,bx - 3\sqrt{2}\,(b^2x^4+2\,abx^2+a^2)\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-b^2x^4+a^2}\,\sqrt{bx^2+a}\,\sqrt{b}}{2\,(b^2x^3+abx)}\right)}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2*
a*b*x^2 + a^2)*sqrt(-b)*log(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x
^4 + a^2)*sqrt(b*x^2 + a)*sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(
a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^
2 + a)*b*x - 3*sqrt(2)*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b)*arctan(1/2*sqrt(
2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)/(b^2*x^3 + a*b*x)))/(a^2*b^
3*x^4 + 2*a^3*b^2*x^2 + a^4*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a+bx^{2}\right)\left(a+bx^{2}\right)}\,\left(a+bx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(3/2)),x)
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(3/2)), x)
```

3.204 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=168

$$\frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}\,a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/8*x*(-b*x^2+a)/a^2/(b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)+9/32*x*(-b*x^2+a)
/a^3/(b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/64*arctan(x*2^(1/2)*b^(1/2)/(-
b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^3*2^(1/2)/b^(1/2)/(-b^2*
x^4+a^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1166, 425, 541, 12, 385, 211}

$$\frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a+bx^2}\sqrt{a-bx^2}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)}{32\sqrt{2}\,a^3\sqrt{b}\sqrt{a^2-b^2x^4}}+\frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a - b*x^2))/(8*a^2*(a + b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a - b
*x^2))/(32*a^3*Sqrt[a + b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*S
qrt[a + b*x^2]*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]])/(32*Sqrt[2]*a^3
*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a-bx^2}\,(a+bx^2)^3}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} - \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{-7ab+2b^2x^2}{\sqrt{a-bx^2}\,(a+ba}}{8a^2b\sqrt{a^2-b^2x^4}}$$

$$= \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx\right.}{}$$

$$= \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-\right.}{}$$

$$= \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-\right.}{}$$

$$= \frac{x(a-bx^2)}{8a^2(a+bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a-bx^2)}{32a^3\sqrt{a+bx^2}\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-bx}}{3}$$

Mathematica [A]
time = 2.65, size = 123, normalized size = 0.73

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}\,x\sqrt{a-bx^2}\,(13a+9bx^2)+19\sqrt{2}\,(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a-bx^2}}\right)\right)}{64a^3\sqrt{b}\sqrt{a-bx^2}\,(a+bx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a - b*x^2]*(13*a + 9*b*x^2) + 19*Sqr
t[2]*(a + b*x^2)^2*ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a - b*x^2]]))/(64*a^3*Sq
rt[b]*Sqrt[a - b*x^2]*(a + b*x^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(138) = 276.
time = 0.28, size = 711, normalized size = 4.23

method	result

default	$\frac{\sqrt{-b^{2}x^{4}+a^{2}}\, b^{\frac{9}{2}} \left(19\sqrt{2}\, \ln\left(\frac{2b\left(\sqrt{2}\, \sqrt{a}\, \sqrt{-b\,x^{2}+a}\, +\sqrt{-ab}\, x+a\right)}{bx+\sqrt{-ab}}\right) b^{\frac{5}{2}} x^{4} \sqrt{a}\, -19\sqrt{2}\, \ln\left(\frac{2b\left(\sqrt{2}\, \sqrt{a}\, \sqrt{\right.}\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(-b^2*x^4+a^2)^(1/2)*b^(9/2)*(19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x
^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*b^(5/2)*x^4*a^(1/2)-19*2^
(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b
)^(1/2)))*b^(5/2)*x^4*a^(1/2)+38*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)
^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b)^(1/2)))*a^(3/2)*b^(3/2)*x^2-38*2^(1/2)
*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/
2)))*a^(3/2)*b^(3/2)*x^2-16*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*b^2*x^4*(-a*
b)^(1/2)+16*arctan(b^(1/2)*x/(1/b*(b*x+(a*b)^(1/2))*(-b*x+(a*b)^(1/2)))^(1/
2))*b^2*x^4*(-a*b)^(1/2)+36*b^(3/2)*(-a*b)^(1/2)*(-b*x^2+a)^(1/2)*x^3+19*2^
(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/2)+(-a*b)^(1/2)*x+a)/(b*x+(-a*b
)^(1/2)))*a^(5/2)*b^(1/2)-19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(-b*x^2+a)^(1/
2)-(-a*b)^(1/2)*x+a)/(b*x-(-a*b)^(1/2)))*a^(5/2)*b^(1/2)-32*arctan(x*b^(1/2
)/(-b*x^2+a)^(1/2))*a*b*x^2*(-a*b)^(1/2)+32*arctan(b^(1/2)*x/(1/b*(b*x+(a*b
)^(1/2))*(-b*x+(a*b)^(1/2)))^(1/2))*a*b*x^2*(-a*b)^(1/2)+52*b^(1/2)*a*(-a*b
)^(1/2)*(-b*x^2+a)^(1/2)*x-16*arctan(x*b^(1/2)/(-b*x^2+a)^(1/2))*a^2*(-a*b)
^(1/2)+16*arctan(b^(1/2)*x/(1/b*(b*x+(a*b)^(1/2))*(-b*x+(a*b)^(1/2)))^(1/2)
)*a^2*(-a*b)^(1/2))/(b*x^2+a)^(1/2)/(-b*x^2+a)^(1/2)/(-a*b)^(1/2)/(-(-a*b)^
(1/2)+(a*b)^(1/2))^3/((-a*b)^(1/2)+(a*b)^(1/2))^3/(b*x-(-a*b)^(1/2))^2/(b*x
+(-a*b)^(1/2))^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

Fricas [A]
time = 0.35, size = 365, normalized size = 2.17

$$\left[-\frac{19\,\sqrt{2}\,(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3})\sqrt{-b}\,\log\left(-\frac{3\,b^{2}x^{4}+2\,abx^{2}-2\,\sqrt{2}\,\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{bx^{2}+a}\,\sqrt{-b}\,x-a^{2}}{b^{2}x^{4}+2\,abx^{2}+a^{2}}\right)-4\,\sqrt{-b^{2}x^{4}+a^{2}}\,(9\,b^{2}x^{3}+13\,abx)\sqrt{bx^{2}+a}}{128\,(a^{3}b^{4}x^{6}+3\,a^{4}b^{3}x^{4}+3\,a^{5}b^{2}x^{2}+a^{6}b)}, -\frac{19\,\sqrt{2}\,(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3})\sqrt{b}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{bx^{2}+a}\,\sqrt{b}}{2\,(b^{2}x^{3}+abx)}\right)-2\,\sqrt{-b^{2}x^{4}+a^{2}}\,(9\,b^{2}x^{3}+13\,abx)\sqrt{bx^{2}+a}}{64\,(a^{3}b^{4}x^{6}+3\,a^{4}b^{3}x^{4}+3\,a^{5}b^{2}x^{2}+a^{6}b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(19*sqrt(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-b)*lo
g(-(3*b^2*x^4 + 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*
sqrt(-b)*x - a^2)/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*sqrt(-b^2*x^4 + a^2)*(9*
b^2*x^3 + 13*a*b*x)*sqrt(b*x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b
^2*x^2 + a^6*b), -1/64*(19*sqrt(2)*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a
^3)*sqrt(b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(b*x^2 + a)*sqrt(b)
/(b^2*x^3 + a*b*x)) - 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 + 13*a*b*x)*sqrt(b*
x^2 + a))/(a^3*b^4*x^6 + 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 + a^6*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a+bx^{2}\right)\left(a+bx^{2}\right)}\,\left(a+bx^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a + b*x**2)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(b*x^2 + a)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(5/2)),x)
```

```
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a + b*x^2)^(5/2)), x)
```

3.205 $\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=152

$$-\frac{9ax\sqrt{a-bx^2}\,(a+bx^2)}{8\sqrt{a^2-b^2x^4}}-\frac{x(a-bx^2)^{3/2}\,(a+bx^2)}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] -1/4*x*(-b*x^2+a)^(3/2)*(b*x^2+a)/(-b^2*x^4+a^2)^(1/2)-9/8*a*x*(b*x^2+a)*(-
b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1166, 427, 396, 223, 212}

$$-\frac{9ax\sqrt{a-bx^2}\,(a+bx^2)}{8\sqrt{a^2-b^2x^4}}-\frac{x(a-bx^2)^{3/2}\,(a+bx^2)}{4\sqrt{a^2-b^2x^4}}+\frac{19a^2\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-9*a*x*Sqrt[a - b*x^2]*(a + b*x^2))/(8*Sqrt[a^2 - b^2*x^4]) - (x*(a - b*x^
2)^(3/2)*(a + b*x^2))/(4*Sqrt[a^2 - b^2*x^4]) + (19*a^2*Sqrt[a - b*x^2]*Sqr
t[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b]*Sqrt[a^2 - b^
2*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(a-bx^2)^{5/2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{(a-bx^2)^2}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= -\frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{5a^2b-9ab^2x^2}{\sqrt{a+bx^2}}\,dx}{4b\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax\sqrt{a-bx^2}\,(a+bx^2)}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int}{8\sqrt{a^2-b^2x^4}}$$

$$= -\frac{9ax\sqrt{a-bx^2}\,(a+bx^2)}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{\left(19a^2\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\mathrm{S}}{8\sqrt{a^2}}$$

$$= -\frac{9ax\sqrt{a-bx^2}\,(a+bx^2)}{8\sqrt{a^2-b^2x^4}} - \frac{x(a-bx^2)^{3/2}(a+bx^2)}{4\sqrt{a^2-b^2x^4}} + \frac{19a^2\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\mathrm{tanh}}{8\sqrt{b}\,\sqrt{a^2-b^2x}}$$

Mathematica [A]
time = 2.75, size = 123, normalized size = 0.81

$$\frac{1}{8}\left(\frac{x(-11a+2bx^2)\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}} - \frac{19a^2\log(-a+bx^2)}{\sqrt{b}} + \frac{19a^2\log\left(abx-b^2x^3+\sqrt{b}\,\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(5/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] ((x*(-11*a + 2*b*x^2)*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2] - (19*a^2*Log[-a
 + b*x^2])/Sqrt[b] + (19*a^2*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*
Sqrt[a^2 - b^2*x^4]])/Sqrt[b])/8
```

Maple [A]
time = 0.13, size = 94, normalized size = 0.62

method	result
default	$\frac{\sqrt{-b^{2} x^{4}+a^{2}}\, \left(2 b^{\frac{3}{2}} x^{3} \sqrt{b\,x^{2}+a}-11 a x \sqrt{b\,x^{2}+a}\, \sqrt{b}+19 \ln \left(\sqrt{b}\, x+\sqrt{b\,x^{2}+a}\right) a^{2}\right)}{8 \sqrt{-b\,x^{2}+a}\, \sqrt{b\,x^{2}+a}\, \sqrt{b}}$
risch	$\frac{x \left(-2 b\,x^{2}+11 a \right) \sqrt{b\,x^{2}+a}\, \sqrt{\frac{\left(-b\,x^{2}+a \right) \left(-b^{2} x^{4}+a^{2}\right)}{\left(b\,x^{2}-a \right)^{2}}}\, \left(b\,x^{2}-a \right)}{8 \sqrt{-b\,x^{2}+a}\, \sqrt{-b^{2} x^{4}+a^{2}}}-\frac{19 a^{2} \ln \left(\sqrt{b}\, x+\sqrt{b\,x^{2}+a}\right) \sqrt{\frac{\left(-b\,x^{2}+a \right) \left(-b^{2} x^{4}+a^{2}\right)}{\left(b\,x^{2}-a \right)^{2}}}}{8 \sqrt{b}\, \sqrt{-b\,x^{2}+a}\, \sqrt{-b^{2} x^{4}+a^{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-b^2*x^4+a^2)^(1/2)*(2*b^(3/2)*x^3*(b*x^2+a)^(1/2)-11*a*x*(b*x^2+a)^(1
/2)*b^(1/2)+19*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*a^2)/(-b*x^2+a)^(1/2)/(b*x^2+a
)^(1/2)/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [A]
time = 0.34, size = 265, normalized size = 1.74

$$\left[\frac{19 \, {\left(a^{2} b x^{2} - a^{3}\right)} \sqrt{b} \log\left(\frac{2 \, b^{2} x^{4} - a b x^{2} - 2 \, \sqrt{-b^{2} x^{4} + a^{2}} \sqrt{-b x^{2} + a} \sqrt{b} x - a^{2}}{b x^{2} - a}\right) - 2 \, \sqrt{-b^{2} x^{4} + a^{2}} {\left(2 \, b^{2} x^{3} - 11 \, a b x\right)} \sqrt{-b x^{2} + a}}{16 \, {\left(b^{2} x^{2} - a b\right)}}, \frac{19 \, {\left(a^{2} b x^{2} - a^{3}\right)} \sqrt{-b} \arctan\left(\frac{\sqrt{-b^{2} x^{4} + a^{2}} \sqrt{-b x^{2} + a} \sqrt{-b}}{b^{2} x^{3} - a b x}\right) - \sqrt{-b^{2} x^{4} + a^{2}} {\left(2 \, b^{2} x^{3} - 11 \, a b x\right)} \sqrt{-b x^{2} + a}}{8 \, {\left(b^{2} x^{2} - a b\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(19*(a^2*b*x^2 - a^3)*sqrt(b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*
x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b*x^2 - a)) - 2*sqrt(-b^2*x^4
```

```
 + a^2)*(2*b^2*x^3 - 11*a*b*x)*sqrt(-b*x^2 + a))/(b^2*x^2 - a*b), 1/8*(19*(
a^2*b*x^2 - a^3)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt
(-b)/(b^2*x^3 - a*b*x)) - sqrt(-b^2*x^4 + a^2)*(2*b^2*x^3 - 11*a*b*x)*sqrt(
-b*x^2 + a))/(b^2*x^2 - a*b)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a-bx^2)^{\frac{5}{2}}}{\sqrt{-(-a+bx^2)(a+bx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral((a - b*x**2)**(5/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(5/2)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x^2)^{5/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a - b*x^2)^(5/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.206 $\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=109

$$-\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}}+\frac{3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] -1/2*x*(b*x^2+a)*(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/2*a*arctanh(x*b^(1
/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(-b^2*x^4+a^2
)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1166, 396, 223, 212}

$$\frac{3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\,\sqrt{a^2-b^2x^4}}-\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] -1/2*(x*Sqrt[a - b*x^2]*(a + b*x^2))/Sqrt[a^2 - b^2*x^4] + (3*a*Sqrt[a - b*
x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]*Sqrt[
a^2 - b^2*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^2)^{3/2}}{\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{a-bx^2}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\ &= -\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}}+\frac{\left(3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2\sqrt{a^2-b^2x^4}}\\ &= -\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}}+\frac{\left(3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2\sqrt{a^2-b^2x^4}}\\ &= -\frac{x\sqrt{a-bx^2}\,(a+bx^2)}{2\sqrt{a^2-b^2x^4}}+\frac{3a\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}\,\sqrt{a^2-b^2x^4}}\end{aligned}$$

Mathematica [A]
time = 2.27, size = 110, normalized size = 1.01

$$\frac{1}{2}\left(-\frac{x\sqrt{a^2-b^2x^4}}{\sqrt{a-bx^2}}-\frac{3a\log\left(-a+bx^2\right)}{\sqrt{b}}+\frac{3a\log\left(abx-b^2x^3+\sqrt{b}\,\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^(3/2)/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-((x*Sqrt[a^2 - b^2*x^4])/Sqrt[a - b*x^2]) - (3*a*Log[-a + b*x^2])/Sqrt[b]
 + (3*a*Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]])
/Sqrt[b])/2
```

Maple [A]
time = 0.13, size = 74, normalized size = 0.68

method	result	s
default	$\frac{\sqrt{-b^{2}x^{4}+a^{2}}\,\left(-x\sqrt{b}\,\sqrt{b\,x^{2}+a}+3\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)a\right)}{2\sqrt{-b\,x^{2}+a}\,\sqrt{b\,x^{2}+a}\,\sqrt{b}}$	7
risch	$\frac{x\sqrt{b\,x^{2}+a}\,\sqrt{\frac{(-b\,x^{2}+a)(-b^{2}x^{4}+a^{2})}{(b\,x^{2}-a)^{2}}}\,(b\,x^{2}-a)}{2\sqrt{-b\,x^{2}+a}\,\sqrt{-b^{2}x^{4}+a^{2}}}-\frac{3a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)\sqrt{\frac{(-b\,x^{2}+a)(-b^{2}x^{4}+a^{2})}{(b\,x^{2}-a)^{2}}}\,(b\,x^{2}-a)}{2\sqrt{b}\,\sqrt{-b\,x^{2}+a}\,\sqrt{-b^{2}x^{4}+a^{2}}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b^2*x^4+a^2)^(1/2)*(-x*b^(1/2)*(b*x^2+a)^(1/2)+3*ln(b^(1/2)*x+(b*x^2+
a)^(1/2))*a)/(-b*x^2+a)^(1/2)/(b*x^2+a)^(1/2)/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [A]
time = 0.34, size = 236, normalized size = 2.17

$$\left[\frac{2\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{-bx^{2}+a}\,bx+3\,(abx^{2}-a^{2})\sqrt{b}\,\log\left(\frac{2\,b^{2}x^{4}-abx^{2}-2\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{-bx^{2}+a}\,\sqrt{b}\,x-a^{2}}{bx^{2}-a}\right)}{4\,(b^{2}x^{2}-ab)},\ \frac{\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{-bx^{2}+a}\,bx+3\,(abx^{2}-a^{2})\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{-bx^{2}+a}\,\sqrt{-b}}{b^{2}x^{3}-abx}\right)}{2\,(b^{2}x^{2}-ab)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt(
b)*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(
b)*x - a^2)/(b*x^2 - a)))/(b^2*x^2 - a*b), 1/2*(sqrt(-b^2*x^4 + a^2)*sqrt(-
b*x^2 + a)*b*x + 3*(a*b*x^2 - a^2)*sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sqr
t(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/(b^2*x^2 - a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a-bx^{2})^{\frac{3}{2}}}{\sqrt{-(-a+bx^{2})\,(a+bx^{2})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral((a - b*x**2)**(3/2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-b*x^2 + a)^(3/2)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a-b\,x^2\right)}^{3/2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2),x)
```

```
[Out] int((a - b*x^2)^(3/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.207 $\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=64

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)
/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1166, 223, 212}

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqr
t[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$
$$= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}}$$
$$= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [A]
time = 0.96, size = 67, normalized size = 1.05

$$\frac{-\log\left(-a+bx^2\right)+\log\left(abx-b^2x^3+\sqrt{b}\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[a^2 - b^2*x^4],x]
```

```
[Out] (-Log[-a + b*x^2] + Log[a*b*x - b^2*x^3 + Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a^2
- b^2*x^4]])/Sqrt[b]
```

Maple [A]
time = 0.16, size = 54, normalized size = 0.84

method	result	size
default	$\frac{\sqrt{-b^2x^4+a^2}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{-b\,x^2+a}\,\sqrt{b\,x^2+a}\,\sqrt{b}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-b*x^2+a)^(1/2)*(-b^2*x^4+a^2)^(1/2)/(b*x^2+a)^(1/2)*ln(b^(1/2)*x+(b*x^2
+a)^(1/2))/b^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

Fricas [A]
time = 0.35, size = 125, normalized size = 1.95

$$\left[\frac{\log\left(\frac{2\,b^2x^4-abx^2-2\,\sqrt{-b^2x^4+a^2}\,\sqrt{-bx^2+a}\,\sqrt{b}\,x-a^2}{bx^2-a}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b^2x^4+a^2}\,\sqrt{-bx^2+a}\,\sqrt{-b}}{b^2x^3-abx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*log((2*b^2*x^4 - a*b*x^2 - 2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqr
t(b)*x - a^2)/(b*x^2 - a))/sqrt(b), sqrt(-b)*arctan(sqrt(-b^2*x^4 + a^2)*sq
rt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
[Out] Integral(sqrt(a - b*x**2)/sqrt(-(-a + b*x**2)*(a + b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-b^2*x^4 + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a-b\,x^2}}{\sqrt{a^2-b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2),x)
[Out] int((a - b*x^2)^(1/2)/(a^2 - b^2*x^4)^(1/2), x)
```

3.208 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(
1/2)/a*2^(1/2)/b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1166, 385, 214}

$$\frac{\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\,\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(Sqrt[2]*a*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\int \frac{1}{(a-bx^2)\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$
$$= \frac{\left(\sqrt{a-bx^2}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{a-2abx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{\sqrt{a^2-b^2x^4}}$$
$$= \frac{\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Mathematica [**A**]
time = 1.36, size = 77, normalized size = 1.00

$$\frac{\sqrt{a^2-b^2x^4}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{2}\,a\sqrt{b}\sqrt{a-bx^2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[2]
*a*Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[a + b*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(62) = 124$.
time = 0.25, size = 256, normalized size = 3.32

method	result
default	$\frac{\sqrt{-b^2x^4+a^2}\,\sqrt{b}\left(\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b\,x^2+a}-\sqrt{ab}\,x+a\right)}{bx+\sqrt{ab}}\right)\sqrt{b}-\sqrt{a}\,\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{\cdots}\right.}{bx\cdots}\right.\right.}{2\sqrt{-b\,x^2+a}\,\sqrt{b\,x^2+a}\,(\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b^2*x^4+a^2)^(1/2)*b^(1/2)*(a^(1/2)*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(
b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*b^(1/2)-a^(1/2)*2^(1/2)*
ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))
```

```
*b^(1/2)-2*(a*b)^(1/2)*ln((b^(1/2)*(-1/b*(b*x+(-a*b)^(1/2))*(-b*x+(-a*b)^(1
/2)))^(1/2)+b*x)/b^(1/2))+2*(a*b)^(1/2)*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^
(1/2)))/(-b*x^2+a)^(1/2)/(b*x^2+a)^(1/2)/((-a*b)^(1/2)-(a*b)^(1/2))/((-a*b)
^(1/2)+(a*b)^(1/2))/(a*b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)
```

Fricas [A]
time = 0.36, size = 155, normalized size = 2.01

$$\left[\frac{\sqrt{2}\,\log\left(-\frac{3\,b^{2}x^{4}-2\,abx^{2}-2\,\sqrt{2}\,\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{-bx^{2}+a}\,\sqrt{b}\,x-a^{2}}{b^{2}x^{4}-2\,abx^{2}+a^{2}}\right)}{4\,a\sqrt{b}}, \frac{\sqrt{2}\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{-bx^{2}+a}\,\sqrt{-b}}{2\,(b^{2}x^{3}-abx)}\right)}{2\,ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*s
qrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2))/(a*sqrt(b)),
1/2*sqrt(2)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 +
a)*sqrt(-b)/(b^2*x^3 - a*b*x))/(a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a+bx^{2}\right)\left(a+bx^{2}\right)}\,\sqrt{a-bx^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(1/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*sqrt(a - b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,\sqrt{a-b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(1/2)),x)
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(1/2)), x)
```

3.209 $\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=124

$$\frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/4*x*(b*x^2+a)/a^2/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+3/8*arctanh(x*2^(
1/2)*b^(1/2)/(b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^2*2^(1/2)/
b^(1/2)/(-b^2*x^4+a^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1166, 390, 385, 214}

$$\frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2))/(4*a^2*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (3*Sqrt[a - b
*x^2]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(4*Sqrt
[2]*a^2*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
```

```
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}\,dx &= \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{(a-bx^2)^2\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}\\ &= \frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}}+\frac{\left(3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{(a-bx^2)\sqrt{a+bx^2}}}{4a\sqrt{a^2-b^2x^4}}\\ &= \frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}}+\frac{\left(3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{a-2abx^2}\,dx\right.}{4a\sqrt{a^2-b^2x^4}}\\ &= \frac{x(a+bx^2)}{4a^2\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}}+\frac{3\sqrt{a-bx^2}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{4\sqrt{2}\,a^2\sqrt{b}\,\sqrt{a^2-b^2x^4}}\end{aligned}$$

Mathematica [**A**]
time = 2.38, size = 110, normalized size = 0.89

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}\,x\sqrt{a+bx^2}+3\sqrt{2}\,(a-bx^2)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\right)}{8a^2\sqrt{b}\,(a-bx^2)^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*Sqrt[a + b*x^2] + 3*Sqrt[2]*(a - b*x^2)*A
rcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(8*a^2*Sqrt[b]*(a - b*x^2)^(3
/2)*Sqrt[a + b*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(100) = 200$.
time = 0.26, size = 499, normalized size = 4.02

method	result
default	$-\frac{\sqrt{-b^{2}x^{4}+a^{2}}\, b^{\frac{5}{2}}\left(-3\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b\,x^{2}+a}+\sqrt{ab}\,x+a\right)}{bx-\sqrt{ab}}\right)b^{\frac{3}{2}}x^{2}\sqrt{a}+3\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b}\right.}{bx}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-b^2*x^4+a^2)^(1/2)*b^(5/2)*(-3*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^
2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*b^(3/2)*x^2*a^(1/2)+3*2^(1/2
)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)
))*b^(3/2)*x^2*a^(1/2)+3*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a
*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*a^(3/2)*b^(1/2)-3*2^(1/2)*ln(2*b*(2^(1/2)
*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*a^(3/2)*b^(1/2
)+4*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*b*x^2*(a*b)^(1/2)-4*ln((b^(1/
2)*(-1/b*(b*x+(-a*b)^(1/2))*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*b*x^2*
(a*b)^(1/2)+4*b^(1/2)*(a*b)^(1/2)*(b*x^2+a)^(1/2)*x-4*ln((b^(1/2)*(b*x^2+a)
^(1/2)+b*x)/b^(1/2))*a*(a*b)^(1/2)+4*ln((b^(1/2)*(-1/b*(b*x+(-a*b)^(1/2))*(
-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a*(a*b)^(1/2))/(-b*x^2+a)^(1/2)/(b*
x^2+a)^(1/2)/(-(-a*b)^(1/2)+(a*b)^(1/2))^2/((-a*b)^(1/2)+(a*b)^(1/2))^2/(b*
x-(a*b)^(1/2))/(b*x+(a*b)^(1/2))/(a*b)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

Fricas [A]
time = 0.35, size = 302, normalized size = 2.44

$$\left[\frac{4\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{-bx^{2}+a}\,bx+3\sqrt{2}\,(b^{2}x^{4}-2\,abx^{2}+a^{2})\sqrt{b}\,\log\left(-\frac{3\,b^{2}x^{4}-2\,abx^{2}-2\sqrt{2}\,\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{-bx^{2}+a}\,\sqrt{b}\,x-a^{2}}{b^{2}x^{4}-2\,abx^{2}+a^{2}}\right)}{16\,(a^{2}b^{3}x^{4}-2\,a^{3}b^{2}x^{2}+a^{4}b)},\ \frac{2\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{-bx^{2}+a}\,bx+3\sqrt{2}\,(b^{2}x^{4}-2\,abx^{2}+a^{2})\sqrt{-b}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-b^{2}x^{4}+a^{2}}\,\sqrt{-bx^{2}+a}\,\sqrt{-b}}{2\,(b^{2}x^{3}-abx)}\right)}{8\,(a^{2}b^{3}x^{4}-2\,a^{3}b^{2}x^{2}+a^{4}b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 - 2
*a*b*x^2 + a^2)*sqrt(b)*log(-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x
^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2)))/(
a^2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b), 1/8*(2*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x
^2 + a)*b*x + 3*sqrt(2)*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(-b)*arctan(1/2*sqr
t(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b)/(b^2*x^3 - a*b*x)))/(a^
2*b^3*x^4 - 2*a^3*b^2*x^2 + a^4*b)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\,\left(a-bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(3/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(3/2)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(3/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,{\left(a-b\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(3/2)),x)
```

```
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(3/2)), x)
```

3.210 $\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx$

Optimal. Leaf size=167

$$\frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}\,a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

```
[Out] 1/8*x*(b*x^2+a)/a^2/(-b*x^2+a)^(3/2)/(-b^2*x^4+a^2)^(1/2)+9/32*x*(b*x^2+a)/
a^3/(-b*x^2+a)^(1/2)/(-b^2*x^4+a^2)^(1/2)+19/64*arctanh(x*2^(1/2)*b^(1/2)/(
b*x^2+a)^(1/2))*(-b*x^2+a)^(1/2)*(b*x^2+a)^(1/2)/a^3*2^(1/2)/b^(1/2)/(-b^2*
x^4+a^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1166, 425, 541, 12, 385, 214}

$$\frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}}+\frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\sqrt{a^2-b^2x^4}}+\frac{19\sqrt{a-bx^2}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{32\sqrt{2}\,a^3\sqrt{b}\sqrt{a^2-b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2))/(8*a^2*(a - b*x^2)^(3/2)*Sqrt[a^2 - b^2*x^4]) + (9*x*(a + b
*x^2))/(32*a^3*Sqrt[a - b*x^2]*Sqrt[a^2 - b^2*x^4]) + (19*Sqrt[a - b*x^2]*S
qrt[a + b*x^2]*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]])/(32*Sqrt[2]*a^
3*Sqrt[b]*Sqrt[a^2 - b^2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a-bx^2)^{5/2}\sqrt{a^2-b^2x^4}}\,dx = \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{1}{(a-bx^2)^3\sqrt{a+bx^2}}\,dx}{\sqrt{a^2-b^2x^4}}$$

$$= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx^2}\,\sqrt{a+bx^2}\right)\int \frac{7ab+2b^2x^2}{(a-bx^2)^2\sqrt{a+b}}$$

$$8a^2b\sqrt{a^2-b^2x^4}$$

$$= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{\left(\sqrt{a-bx\right.}{}$$

$$= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-\right.}{}$$

$$= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{\left(19\sqrt{a-\right.}{}$$

$$= \frac{x(a+bx^2)}{8a^2(a-bx^2)^{3/2}\sqrt{a^2-b^2x^4}} + \frac{9x(a+bx^2)}{32a^3\sqrt{a-bx^2}\,\sqrt{a^2-b^2x^4}} + \frac{19\sqrt{a-b}}{}$$

Mathematica [A]
time = 2.65, size = 122, normalized size = 0.73

$$\frac{\sqrt{a^2-b^2x^4}\left(2\sqrt{b}\,x(13a-9bx^2)\sqrt{a+bx^2}+19\sqrt{2}\,(a-bx^2)^2\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\right)}{64a^3\sqrt{b}\,(a-bx^2)^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - b*x^2)^(5/2)*Sqrt[a^2 - b^2*x^4]),x]
```

```
[Out] (Sqrt[a^2 - b^2*x^4]*(2*Sqrt[b]*x*(13*a - 9*b*x^2)*Sqrt[a + b*x^2] + 19*Sqr
t[2]*(a - b*x^2)^2*ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[a + b*x^2]]))/(64*a^3*S
qrt[b]*(a - b*x^2)^(5/2)*Sqrt[a + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 727 vs. 2(137) = 274.
time = 0.28, size = 728, normalized size = 4.36

method	result

default	$-\frac{\sqrt{-b^2x^4+a^2}\, b^{\frac{9}{2}} \left(19\sqrt{2}\, \ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b\,x^2+a}+\sqrt{ab}\,x+a\right)}{bx-\sqrt{ab}}\right) b^{\frac{5}{2}}x^4\sqrt{a}-19\sqrt{2}\,\ln\left(\frac{2b\left(\sqrt{2}\,\sqrt{a}\,\sqrt{b}\right.}{bx+}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/16*(-b^2*x^4+a^2)^(1/2)*b^(9/2)*(19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*b^(5/2)*x^4*a^(1/2)-19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*b^(5/2)*x^4*a^(1/2)-38*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*a^(3/2)*b^(3/2)*x^2+38*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*a^(3/2)*b^(3/2)*x^2-16*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*b^2*x^4*(a*b)^(1/2)+16*ln((b^(1/2)*(-1/b*(b*x+(-a*b)^(1/2))*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*b^2*x^4*(a*b)^(1/2)-36*(a*b)^(1/2)*b^(3/2)*(b*x^2+a)^(1/2)*x^3+19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)+(a*b)^(1/2)*x+a)/(b*x-(a*b)^(1/2)))*a^(5/2)*b^(1/2)-19*2^(1/2)*ln(2*b*(2^(1/2)*a^(1/2)*(b*x^2+a)^(1/2)-(a*b)^(1/2)*x+a)/(b*x+(a*b)^(1/2)))*a^(5/2)*b^(1/2)+32*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*a*b*x^2*(a*b)^(1/2)-32*ln((b^(1/2)*(-1/b*(b*x+(-a*b)^(1/2))*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a*b*x^2*(a*b)^(1/2)+52*a*(a*b)^(1/2)*(b*x^2+a)^(1/2)*b^(1/2)*x-16*ln((b^(1/2)*(b*x^2+a)^(1/2)+b*x)/b^(1/2))*a^2*(a*b)^(1/2)+16*ln((b^(1/2)*(-1/b*(b*x+(-a*b)^(1/2))*(-b*x+(-a*b)^(1/2)))^(1/2)+b*x)/b^(1/2))*a^2*(a*b)^(1/2))/(-b*x^2+a)^(1/2)/(b*x^2+a)^(1/2)/((-a*b)^(1/2)-(a*b)^(1/2))^3/((-a*b)^(1/2)+(a*b)^(1/2))^3/(b*x+(a*b)^(1/2))^2/(a*b)^(1/2)/(b*x-(a*b)^(1/2))^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)

Fricas [A]
time = 0.34, size = 376, normalized size = 2.25

$$\left[\frac{19\sqrt{2}\left(b^3x^6-3ab^2x^4+3a^2bx^2-a^3\right)\sqrt{b}\log\left(-\frac{3b^2x^4-2abx^2-2\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{b}x-a^2}{b^2x^4-2abx^2+a^2}\right)+4\sqrt{-b^2x^4+a^2}\left(9b^2x^3-13abx\right)\sqrt{-bx^2+a}}{128\left(a^3b^4x^6-3a^4b^3x^4+3a^5b^2x^2-a^6b\right)}, \frac{19\sqrt{2}\left(b^3x^6-3ab^2x^4+3a^2bx^2-a^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{2}\sqrt{-b^2x^4+a^2}\sqrt{-bx^2+a}\sqrt{-b}}{2\left(b^2x^3-abx\right)}\right)+2\sqrt{-b^2x^4+a^2}\left(9b^2x^3-13abx\right)\sqrt{-bx^2+a}}{64\left(a^3b^4x^6-3a^4b^3x^4+3a^5b^2x^2-a^6b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/128*(19*sqrt(2)*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3)*sqrt(b)*log(
-(3*b^2*x^4 - 2*a*b*x^2 - 2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*s
qrt(b)*x - a^2)/(b^2*x^4 - 2*a*b*x^2 + a^2)) + 4*sqrt(-b^2*x^4 + a^2)*(9*b^
2*x^3 - 13*a*b*x)*sqrt(-b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^5*b^
2*x^2 - a^6*b), 1/64*(19*sqrt(2)*(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3
)*sqrt(-b)*arctan(1/2*sqrt(2)*sqrt(-b^2*x^4 + a^2)*sqrt(-b*x^2 + a)*sqrt(-b
)/(b^2*x^3 - a*b*x)) + 2*sqrt(-b^2*x^4 + a^2)*(9*b^2*x^3 - 13*a*b*x)*sqrt(-
b*x^2 + a))/(a^3*b^4*x^6 - 3*a^4*b^3*x^4 + 3*a^5*b^2*x^2 - a^6*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(-a+bx^2\right)\left(a+bx^2\right)}\left(a-bx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**2+a)**(5/2)/(-b**2*x**4+a**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(-a + b*x**2)*(a + b*x**2))*(a - b*x**2)**(5/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^2+a)^(5/2)/(-b^2*x^4+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b^2*x^4 + a^2)*(-b*x^2 + a)^(5/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2-b^2\,x^4}\,{\left(a-b\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(5/2)),x)
```

```
[Out] int(1/((a^2 - b^2*x^4)^(1/2)*(a - b*x^2)^(5/2)), x)
```

3.211 $\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=30

$$\frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,\sinh^{-1}(x)}{\sqrt{-1+x^4}}$$

[Out] arcsinh(x)*(x^2-1)^(1/2)*(x^2+1)^(1/2)/(x^4-1)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1166, 221}

$$\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]

[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx &= \frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{\sqrt{-1+x^4}}\\ &= \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,\sinh^{-1}(x)}{\sqrt{-1+x^4}}\end{aligned}$$

Mathematica [A]
time = 0.48, size = 38, normalized size = 1.27

$$-\log\left(1-x^2\right)+\log\left(-x+x^3+\sqrt{-1+x^2}\,\sqrt{-1+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^2]/Sqrt[-1 + x^4],x]
```

```
[Out] -Log[1 - x^2] + Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [A]
time = 0.14, size = 25, normalized size = 0.83

method	result	size
default	$\frac{\sqrt{x^4-1}\,\operatorname{arcsinh}(x)}{\sqrt{x^2-1}\,\sqrt{x^2+1}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(1/2)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(24) = 48.
time = 0.35, size = 73, normalized size = 2.43

$$\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\,\sqrt{x^2-1}-x}{x^3-x}\right)-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\,\sqrt{x^2-1}-x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 - 1) - x)/(x^3 - x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)}}{\sqrt{(x-1)(x+1)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(1/2)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)/sqrt(x^4 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2),x)
```

```
[Out] int((x^2 - 1)^(1/2)/(x^4 - 1)^(1/2), x)
```

3.212 $\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=24

$$-\frac{\sqrt{-1+x^4}\,\sin^{-1}(x)}{\sqrt{1-x^4}}$$

```
[Out] -arcsin(x)*(x^4-1)^(1/2)/(-x^4+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.67, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1166, 223, 212}

$$\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx = \frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^4}}$$
$$= \frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}$$
$$= \frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)}{\sqrt{-1+x^4}}$$

Mathematica [A]
time = 0.48, size = 34, normalized size = 1.42

$$-\log\left(1+x^2\right)+\log\left(x+x^3+\sqrt{1+x^2}\,\sqrt{-1+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[-1 + x^4],x]
[Out] -Log[1 + x^2] + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [A]
time = 0.14, size = 33, normalized size = 1.38

method	result	size
default	$\frac{\sqrt{x^4-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}\,\sqrt{x^2-1}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(x^2+1)^(1/2)*(x^4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(20) = 40$.
time = 0.35, size = 65, normalized size = 2.71

$$\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\sqrt{x^2+1}+x}{x^3+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2+1}}{\sqrt{(x-1)(x+1)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/sqrt((x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(x^4 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)^(1/2)/(x^4 - 1)^(1/2), x)
```

3.213 $\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=73

$$-\frac{\sqrt{-1+x^4}\,\sin^{-1}(x)}{\sqrt{1-x^2}\,\sqrt{1+x^2}}+\frac{\sqrt{-1+x^2}\,\sqrt{-1+x^4}\,\sinh^{-1}(x)}{(1-x^2)\,\sqrt{1+x^2}}$$

```
[Out] -arcsin(x)*(x^4-1)^(1/2)/(-x^2+1)^(1/2)/(x^2+1)^(1/2)+arcsinh(x)*(x^2-1)^(1
/2)*(x^4-1)^(1/2)/(-x^2+1)/(x^2+1)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {6874, 1166, 221, 223, 212}

$$\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^4-1}}-\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,\sinh^{-1}(x)}{\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

```
[Out] -((Sqrt[-1 + x^2]*Sqrt[1 + x^2]*ArcSinh[x])/Sqrt[-1 + x^4]) + (Sqrt[-1 + x^
2]*Sqrt[1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^4]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dis
t[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d + c*(x^2/e))^FracPa
```

```
rt[p]), Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, p, q}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-\sqrt{-1+x^2}+\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx &= \int \left(-\frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}+\frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\right)dx \\
&= -\int \frac{\sqrt{-1+x^2}}{\sqrt{-1+x^4}}\,dx+\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^4}}\,dx \\
&= \frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx}{\sqrt{-1+x^4}}-\frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\int \frac{}{\sqrt{}}}{\sqrt{-1+x^4}} \\
&= -\frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,\sinh^{-1}(x)}{\sqrt{-1+x^4}}+\frac{\left(\sqrt{-1+x^2}\,\sqrt{1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-}\right.}{\sqrt{-1+x^4}} \\
&= -\frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,\sinh^{-1}(x)}{\sqrt{-1+x^4}}+\frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1}}\right.}{\sqrt{-1+x^4}}
\end{aligned}
$$

Mathematica [**A**]
time = 3.54, size = 71, normalized size = 0.97

$$\log\left(1-x^2\right)-\log\left(1+x^2\right)-\log\left(-x+x^3+\sqrt{-1+x^2}\,\sqrt{-1+x^4}\right)+\log\left(x+x^3+\sqrt{1+x^2}\,\sqrt{-1+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-1 + x^2] + Sqrt[1 + x^2])/Sqrt[-1 + x^4],x]
```

```
[Out] Log[1 - x^2] - Log[1 + x^2] - Log[-x + x^3 + Sqrt[-1 + x^2]*Sqrt[-1 + x^4]]
 + Log[x + x^3 + Sqrt[1 + x^2]*Sqrt[-1 + x^4]]
```

Maple [**A**]
time = 0.14, size = 59, normalized size = 0.81

method	result	size

default	$-\frac{\sqrt{x^4-1}\,\operatorname{arcsinh}(x)}{\sqrt{x^2-1}\,\sqrt{x^2+1}}+\frac{\sqrt{x^4-1}\,\ln\left(x+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}\,\sqrt{x^2-1}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(x^2-1)^(1/2)*(x^4-1)^(1/2)/(x^2+1)^(1/2)*arcsinh(x)+1/(x^2+1)^(1/2)*(x^
4-1)^(1/2)/(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(53) = 106$.
time = 0.37, size = 137, normalized size = 1.88

$$\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\,\sqrt{x^2+1}+x}{x^3+x}\right)-\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\,\sqrt{x^2+1}+x}{x^3+x}\right)-\frac{1}{2}\log\left(\frac{x^3+\sqrt{x^4-1}\,\sqrt{x^2-1}-x}{x^3-x}\right)+\frac{1}{2}\log\left(-\frac{x^3-\sqrt{x^4-1}\,\sqrt{x^2-1}-x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/2*log((x^3 + sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log(-(x^3
- sqrt(x^4 - 1)*sqrt(x^2 + 1) + x)/(x^3 + x)) - 1/2*log((x^3 + sqrt(x^4 - 1
)*sqrt(x^2 - 1) - x)/(x^3 - x)) + 1/2*log(-(x^3 - sqrt(x^4 - 1)*sqrt(x^2 -
1) - x)/(x^3 - x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{-\sqrt{x^2-1}+\sqrt{x^2+1}}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x**2-1)**(1/2)+(x**2+1)**(1/2))/(x**4-1)**(1/2),x)
```

```
[Out] Integral((-sqrt(x**2 - 1) + sqrt(x**2 + 1))/sqrt((x - 1)*(x + 1)*(x**2 + 1)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(x^2-1)^(1/2)+(x^2+1)^(1/2))/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((sqrt(x^2 + 1) - sqrt(x^2 - 1))/sqrt(x^4 - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{x^2-1}-\sqrt{x^2+1}}{\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2),x)
```

```
[Out] int(-((x^2 - 1)^(1/2) - (x^2 + 1)^(1/2))/(x^4 - 1)^(1/2), x)
```

3.214 $\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=121

$$\frac{(7c^2d^2-5bcde+b^2e^2)\,x}{c^3}+\frac{e(4cd-be)x^3}{3c^2}+\frac{e^2x^5}{5c}-\frac{(2cd-be)^3\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\,\sqrt{cd-be}}$$

```
[Out] (b^2*e^2-5*b*c*d*e+7*c^2*d^2)*x/c^3+1/3*e*(-b*e+4*c*d)*x^3/c^2+1/5*e^2*x^5/
c-(-b*e+2*c*d)^3*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(7/2)/e^(1/2
)/(-b*e+c*d)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1163, 398, 214}

$$\frac{x(b^2e^2-5bcde+7c^2d^2)}{c^3}-\frac{(2cd-be)^3\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{c^{7/2}\sqrt{e}\,\sqrt{cd-be}}+\frac{ex^3(4cd-be)}{3c^2}+\frac{e^2x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ((7*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*x)/c^3 + (e*(4*c*d - b*e)*x^3)/(3*c^2) +
 (e^2*x^5)/(5*c) - ((2*c*d - b*e)^3*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d -
b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
```

```
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex^2)^4}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(d+ex^2)^3}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$
$$= \int \left(\frac{7c^2d^2-5bcde+b^2e^2}{c^3}+\frac{e(4cd-be)x^2}{c^2}+\frac{e^2x^4}{c}+\frac{8c^3d^3-12bc^2d^2e}{c^3(-cd+}\right.$$
$$= \frac{(7c^2d^2-5bcde+b^2e^2)\,x}{c^3}+\frac{e(4cd-be)x^3}{3c^2}+\frac{e^2x^5}{5c}+\frac{(2cd-be)^3\int\frac{}{-cd}}{c^3}$$
$$= \frac{(7c^2d^2-5bcde+b^2e^2)\,x}{c^3}+\frac{e(4cd-be)x^3}{3c^2}+\frac{e^2x^5}{5c}-\frac{(2cd-be)^3\tanh^{-}}{c^{7/2}\sqrt{e}\,\sqrt{}}$$

Mathematica [A]
time = 0.05, size = 121, normalized size = 1.00

$$-\frac{(-7c^2d^2+5bcde-b^2e^2)\,x}{c^3}-\frac{e(-4cd+be)x^3}{3c^2}+\frac{e^2x^5}{5c}-\frac{(-2cd+be)^3\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{-cd+be}}\right)}{c^{7/2}\sqrt{e}\,\sqrt{-cd+be}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((-7*c^2*d^2 + 5*b*c*d*e - b^2*e^2)*x)/c^3) - (e*(-4*c*d + b*e)*x^3)/(3*c
^2) + (e^2*x^5)/(5*c) - ((-2*c*d + b*e)^3*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-
(c*d) + b*e]])/(c^(7/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

Maple [A]
time = 0.13, size = 134, normalized size = 1.11

method	result
default	$\frac{\frac{1}{5}x^5e^2c^2-\frac{1}{3}bc\,e^2x^3+\frac{4}{3}c^2de\,x^3+e^2b^2x-5bcdex+7c^2d^2x}{c^3}+\frac{(-b^3e^3+6b^2cd\,e^2-12b\,c^2d^2e+8c^3d^3)\arctan\left(\frac{cex}{\sqrt{(eb-cd)\,ce}}\right)}{c^3\sqrt{(eb-cd)\,ce}}$
risch	$\frac{e^2x^5}{5c}-\frac{b\,e^2x^3}{3c^2}+\frac{4de\,x^3}{3c}+\frac{e^2b^2x}{c^3}-\frac{5bdex}{c^2}+\frac{7d^2x}{c}-\frac{\ln\left(cex-\sqrt{-(eb-cd)\,ce}\right)b^3e^3}{2c^3\sqrt{-(eb-cd)\,ce}}+\frac{3\ln\left(cex-\sqrt{-(eb-c}\right.}{c^2\sqrt{-(eb-c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/5*x^5*e^2*c^2-1/3*b*c*e^2*x^3+4/3*c^2*d*e*x^3+e^2*b^2*x-5*b*c*d*e*
x+7*c^2*d^2*x)+(-b^3*e^3+6*b^2*c*d*e^2-12*b*c^2*d^2*e+8*c^3*d^3)/c^3/((b*e-
c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e*b>0)', see `assume?` for mor
e detai
```

Fricas [**A**]
time = 0.34, size = 440, normalized size = 3.64

$$\left[\frac{210\,c^{4}d^{3}xe-15\,(8\,c^{3}d^{3}-12\,bc^{2}d^{2}e+6\,b^{2}cde^{2}-b^{3}e^{3})\sqrt{c^{2}de-bce^{2}}\,\log\left(-\frac{cd+(cx^{2}-b)e+2\,\sqrt{c^{2}de-bce^{2}}\,x}{cd-(cx^{2}+b)e}\right)-2\,(3\,bc^{3}x^{5}-5\,b^{2}c^{2}x^{3}+15\,b^{3}cx)e^{4}+2\,(3\,c^{4}dx^{5}-25\,bc^{3}dx^{3}+90\,b^{2}c^{2}dx)e^{3}+40\,(c^{4}d^{2}x^{3}-9\,bc^{3}d^{2}x)e^{2}}{30\,(c^{5}de-bc^{4}e^{2})},\ \frac{105\,c^{4}d^{3}xe-15\,(8\,c^{3}d^{3}-12\,bc^{2}d^{2}e+6\,b^{2}cde^{2}-b^{3}e^{3})\sqrt{-c^{2}de+bce^{2}}\,\arctan\left(-\frac{\sqrt{-c^{2}de+bce^{2}}\,x}{cd-be}\right)-(3\,bc^{3}x^{5}-5\,b^{2}c^{2}x^{3}+15\,b^{3}cx)e^{4}+(3\,c^{4}dx^{5}-25\,bc^{3}dx^{3}+90\,b^{2}c^{2}dx)e^{3}+20\,(c^{4}d^{2}x^{3}-9\,bc^{3}d^{2}x)e^{2}}{15\,(c^{5}de-bc^{4}e^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/30*(210*c^4*d^3*x*e - 15*(8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b
^3*e^3)*sqrt(c^2*d*e - b*c*e^2)*log(-(c*d + (c*x^2 - b)*e + 2*sqrt(c^2*d*e
- b*c*e^2)*x)/(c*d - (c*x^2 + b)*e)) - 2*(3*b*c^3*x^5 - 5*b^2*c^2*x^3 + 15*
b^3*c*x)*e^4 + 2*(3*c^4*d*x^5 - 25*b*c^3*d*x^3 + 90*b^2*c^2*d*x)*e^3 + 40*(
c^4*d^2*x^3 - 9*b*c^3*d^2*x)*e^2)/(c^5*d*e - b*c^4*e^2), 1/15*(105*c^4*d^3*
x*e - 15*(8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*sqrt(-c^2*d
*e + b*c*e^2)*arctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)) - (3*b*c^3*x^
5 - 5*b^2*c^2*x^3 + 15*b^3*c*x)*e^4 + (3*c^4*d*x^5 - 25*b*c^3*d*x^3 + 90*b^
2*c^2*d*x)*e^3 + 20*(c^4*d^2*x^3 - 9*b*c^3*d^2*x)*e^2)/(c^5*d*e - b*c^4*e^2
)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(110) = 220$.
time = 0.50, size = 345, normalized size = 2.85

$$x^{3}\left(-\frac{be^{2}}{3c^{2}}+\frac{4de}{3c}\right)+x\left(\frac{b^{2}e^{2}}{c^{3}}-\frac{5bde}{c^{2}}+\frac{7d^{2}}{c}\right)+\frac{\sqrt{-\frac{1}{c^{7}e\,(be-cd)}}\,(be-2cd)^{3}\log\left(x+\frac{-bc^{3}e\sqrt{-\frac{1}{c^{7}e\,(be-cd)}}\,(be-2cd)^{3}+c^{4}d\sqrt{-\frac{1}{c^{7}e\,(be-cd)}}\,(be-2cd)^{3}}{b^{3}e^{3}-6b^{2}cde^{2}+12bc^{2}d^{2}e-8c^{3}d^{3}}\right)}{2}-\frac{\sqrt{-\frac{1}{c^{7}e\,(be-cd)}}\,(be-2cd)^{3}\log\left(x+\frac{bc^{3}e\sqrt{-\frac{1}{c^{7}e\,(be-cd)}}\,(be-2cd)^{3}-c^{4}d\sqrt{-\frac{1}{c^{7}e\,(be-cd)}}\,(be-2cd)^{3}}{b^{3}e^{3}-6b^{2}cde^{2}+12bc^{2}d^{2}e-8c^{3}d^{3}}\right)}{2}+\frac{e^{2}x^{5}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] x**3*(-b*e**2/(3*c**2) + 4*d*e/(3*c)) + x*(b**2*e**2/c**3 - 5*b*d*e/c**2 +
7*d**2/c) + sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log(x + (-b*c**3
*e*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 + c**4*d*sqrt(-1/(c**7*e*
(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 + 12*b*c**2*d*
*2*e - 8*c**3*d**3))/2 - sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3*log
(x + (b*c**3*e*sqrt(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3 - c**4*d*sqrt
(-1/(c**7*e*(b*e - c*d)))*(b*e - 2*c*d)**3)/(b**3*e**3 - 6*b**2*c*d*e**2 +
12*b*c**2*d**2*e - 8*c**3*d**3))/2 + e**2*x**5/(5*c)
```

Giac [A]
time = 3.59, size = 151, normalized size = 1.25

$$\frac{\left(8\,c^3d^3 - 12\,bc^2d^2e + 6\,b^2cde^2 - b^3e^3\right)\arctan\left(\frac{cxe}{\sqrt{-c^2de + bce^2}}\right)}{\sqrt{-c^2de + bce^2}\,c^3} + \frac{\left(3\,c^4x^5e^7 + 20\,c^4dx^3e^6 - 5\,bc^3x^3e^7 + 105\,c^4d^2xe^5 - 75\,bc^3dxe^6 + 15\,b^2c^2xe^7\right)e^{(-5)}}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] (8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*e^3)*arctan(c*x*e/sqrt(-c
^2*d*e + b*c*e^2))/(sqrt(-c^2*d*e + b*c*e^2)*c^3) + 1/15*(3*c^4*x^5*e^7 + 2
0*c^4*d*x^3*e^6 - 5*b*c^3*x^3*e^7 + 105*c^4*d^2*x*e^5 - 75*b*c^3*d*x*e^6 +
15*b^2*c^2*x*e^7)*e^(-5)/c^5
```

Mupad [B]
time = 4.53, size = 182, normalized size = 1.50

$$x\,\left(\frac{3\,d^2}{c}+\frac{\left(\frac{e\,\left(b\,e-c\,d\right)}{c^2}-\frac{3\,d\,e}{c}\right)\,\left(b\,e-c\,d\right)}{c\,e}\right)-x^3\,\left(\frac{e\,\left(b\,e-c\,d\right)}{3\,c^2}-\frac{d\,e}{c}\right)+\frac{e^2\,x^5}{5\,c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,e\,x\,{\left(b\,e-2\,c\,d\right)}^3}{\sqrt{b\,e^2-c\,d\,e}\,\left(b^3\,e^3-6\,b^2\,c\,d\,e^2+12\,b\,c^2\,d^2\,e-8\,c^3\,d^3\right)}\right)\,{\left(b\,e-2\,c\,d\right)}^3}{c^{7/2}\,\sqrt{b\,e^2-c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] x*((3*d^2)/c + (((e*(b*e - c*d))/c^2 - (3*d*e)/c)*(b*e - c*d))/(c*e)) - x^3
*((e*(b*e - c*d))/(3*c^2) - (d*e)/c) + (e^2*x^5)/(5*c) - (atan((c^(1/2)*e*x
*(b*e - 2*c*d)^3)/((b*e^2 - c*d*e)^(1/2)*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^
2*e - 6*b^2*c*d*e^2)))*(b*e - 2*c*d)^3)/(c^(7/2)*(b*e^2 - c*d*e)^(1/2))
```

3.215 $\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=86

$$\frac{(3cd-be)x}{c^2}+\frac{ex^3}{3c}-\frac{(2cd-be)^2\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\,\sqrt{cd-be}}$$

```
[Out] (-b*e+3*c*d)*x/c^2+1/3*e*x^3/c-(-b*e+2*c*d)^2*arctanh(x*c^(1/2)*e^(1/2)/(-b
*e+c*d)^(1/2))/c^(5/2)/e^(1/2)/(-b*e+c*d)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1163, 398, 214}

$$-\frac{(2cd-be)^2\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\,\sqrt{cd-be}}+\frac{x(3cd-be)}{c^2}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ((3*c*d - b*e)*x)/c^2 + (e*x^3)/(3*c) - ((2*c*d - b*e)^2*ArcTanh[(Sqrt[c]*S
qrt[e]*x)/Sqrt[c*d - b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(d+ex^2)^2}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$
$$= \int \left(\frac{3cd-be}{c^2}+\frac{ex^2}{c}+\frac{4c^2d^2-4bcde+b^2e^2}{c^2\left(-cd+be+cex^2\right)}\right)dx$$
$$= \frac{(3cd-be)x}{c^2}+\frac{ex^3}{3c}+\frac{(2cd-be)^2\int \frac{1}{-cd+be+cex^2}\,dx}{c^2}$$
$$= \frac{(3cd-be)x}{c^2}+\frac{ex^3}{3c}-\frac{(2cd-be)^2\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{c^{5/2}\sqrt{e}\,\sqrt{cd-be}}$$

Mathematica [A]
time = 0.03, size = 84, normalized size = 0.98

$$-\frac{(-3cd+be)x}{c^2}+\frac{ex^3}{3c}+\frac{(-2cd+be)^2\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{-cd+be}}\right)}{c^{5/2}\sqrt{e}\,\sqrt{-cd+be}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(((-3*c*d + b*e)*x)/c^2) + (e*x^3)/(3*c) + ((-2*c*d + b*e)^2*ArcTan[(Sqrt[
c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(5/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

Maple [A]
time = 0.15, size = 81, normalized size = 0.94

method	result
default	$-\frac{-\frac{1}{3}ce\,x^3+ebx-3cdx}{c^2}+\frac{\left(e^2b^2-4bcde+4c^2d^2\right)\arctan\left(\frac{cex}{\sqrt{(eb-cd)\,ce}}\right)}{c^2\sqrt{(eb-cd)\,ce}}$
risch	$\frac{e\,x^3}{3c}-\frac{ebx}{c^2}+\frac{3dx}{c}-\frac{\ln\left(cex+\sqrt{-(eb-cd)\,ce}\right)e^2b^2}{2c^2\sqrt{-(eb-cd)\,ce}}+\frac{2\ln\left(cex+\sqrt{-(eb-cd)\,ce}\right)bde}{c\sqrt{-(eb-cd)\,ce}}-\frac{2\ln\left(cex+\sqrt{-(e\right.}{\sqrt{-(eb}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(-1/3*c*e*x^3+e*b*x-3*c*d*x)+(b^2*e^2-4*b*c*d*e+4*c^2*d^2)/c^2/((b*e
-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e*b>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.35, size = 312, normalized size = 3.63

$$\left[\frac{18\,c^{3}d^{2}xe+3\,(4\,c^{2}d^{2}-4\,bcde+b^{2}e^{2})\sqrt{c^{2}de-bce^{2}}\log\left(-\frac{cd+(cx^{2}-b)e-2\,\sqrt{c^{2}de-bce^{2}}\,x}{cd-(cx^{2}+b)e}\right)-2\,(bc^{2}x^{3}-3\,b^{2}cx)e^{3}+2\,(c^{3}dx^{3}-12\,bc^{2}dx)e^{2}}{6\,(c^{4}de-bc^{3}e^{2})},\ \frac{9\,c^{3}d^{2}xe-3\,(4\,c^{2}d^{2}-4\,bcde+b^{2}e^{2})\sqrt{-c^{2}de+bce^{2}}\arctan\left(-\frac{\sqrt{-c^{2}de+bce^{2}}\,x}{cd-be}\right)-(bc^{2}x^{3}-3\,b^{2}cx)e^{3}+(c^{3}dx^{3}-12\,bc^{2}dx)e^{2}}{3\,(c^{4}de-bc^{3}e^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/6*(18*c^3*d^2*x*e + 3*(4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*sqrt(c^2*d*e - b
*c*e^2)*log(-(c*d + (c*x^2 - b)*e - 2*sqrt(c^2*d*e - b*c*e^2)*x)/(c*d - (c*
x^2 + b)*e)) - 2*(b*c^2*x^3 - 3*b^2*c*x)*e^3 + 2*(c^3*d*x^3 - 12*b*c^2*d*x)
*e^2)/(c^4*d*e - b*c^3*e^2), 1/3*(9*c^3*d^2*x*e - 3*(4*c^2*d^2 - 4*b*c*d*e
+ b^2*e^2)*sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d
 - b*e)) - (b*c^2*x^3 - 3*b^2*c*x)*e^3 + (c^3*d*x^3 - 12*b*c^2*d*x)*e^2)/(c
^4*d*e - b*c^3*e^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(75) = 150$.
time = 0.37, size = 275, normalized size = 3.20

$$x\left(-\frac{be}{c^{2}}+\frac{3d}{c}\right)-\frac{\sqrt{-\frac{1}{c^{5}e\,(be-cd)}}\,(be-2cd)^{2}\log\left(x+\frac{-bc^{2}e\sqrt{-\frac{1}{c^{5}e\,(be-cd)}}\,(be-2cd)^{2}+c^{3}d\sqrt{-\frac{1}{c^{5}e\,(be-cd)}}\,(be-2cd)^{2}}{b^{2}e^{2}-4bcde+4c^{2}d^{2}}\right)}{2}+\frac{\sqrt{-\frac{1}{c^{5}e\,(be-cd)}}\,(be-2cd)^{2}\log\left(x+\frac{bc^{2}e\sqrt{-\frac{1}{c^{5}e\,(be-cd)}}\,(be-2cd)^{2}-c^{3}d\sqrt{-\frac{1}{c^{5}e\,(be-cd)}}\,(be-2cd)^{2}}{b^{2}e^{2}-4bcde+4c^{2}d^{2}}\right)}{2}+\frac{ex^{3}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] x*(-b*e/c**2 + 3*d/c) - sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(
x + (-b*c**2*e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 + c**3*d*sqrt
(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2
*d**2))/2 + sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2*log(x + (b*c**2*
```

```
e*sqrt(-1/(c**5*e*(b*e - c*d)))*(b*e - 2*c*d)**2 - c**3*d*sqrt(-1/(c**5*e*(
b*e - c*d)))*(b*e - 2*c*d)**2)/(b**2*e**2 - 4*b*c*d*e + 4*c**2*d**2))/2 + e
*x**3/(3*c)
```

Giac [A]
time = 2.72, size = 98, normalized size = 1.14

$$\frac{(4\,c^2d^2-4\,bcde+b^2e^2)\arctan\left(\frac{cxe}{\sqrt{-c^2de+bce^2}}\right)}{\sqrt{-c^2de+bce^2}\,c^2}+\frac{(c^2x^3e^4+9\,c^2dxe^3-3\,bcxe^4)e^{(-3)}}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] (4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*arctan(c*x*e/sqrt(-c^2*d*e + b*c*e^2))/(s
qrt(-c^2*d*e + b*c*e^2)*c^2) + 1/3*(c^2*x^3*e^4 + 9*c^2*d*x*e^3 - 3*b*c*x*e
^4)*e^(-3)/c^3
```

Mupad [B]
time = 4.52, size = 113, normalized size = 1.31

$$x\left(\frac{2\,d}{c}-\frac{b\,e-c\,d}{c^2}\right)+\frac{e\,x^3}{3\,c}+\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,e\,x\,(b\,e-2\,c\,d)^2}{\sqrt{b\,e^2-c\,d\,e}\,(b^2\,e^2-4\,b\,c\,d\,e+4\,c^2\,d^2)}\right)\,(b\,e-2\,c\,d)^2}{c^{5/2}\,\sqrt{b\,e^2-c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] x*((2*d)/c - (b*e - c*d)/c^2) + (e*x^3)/(3*c) + (atan((c^(1/2)*e*x*(b*e - 2
*c*d)^2)/((b*e^2 - c*d*e)^(1/2)*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)))*(b*e -
2*c*d)^2)/(c^(5/2)*(b*e^2 - c*d*e)^(1/2))
```

3.216 $\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=64

$$\frac{x}{c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\,\sqrt{cd-be}}$$

```
[Out] x/c-(-b*e+2*c*d)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(3/2)/e^(1/2
)/(-b*e+c*d)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1163, 396, 214}

$$\frac{x}{c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\,\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] x/c - ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(c^(3/2)
*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{d+ex^2}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$
$$= \frac{x}{c} - \frac{\left(-cde+\frac{e(-cd^2+bde)}{d}\right)\int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx}{ce}$$
$$= \frac{x}{c} - \frac{(2cd-be)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{c^{3/2}\sqrt{e}\,\sqrt{cd-be}}$$

Mathematica [A]
time = 0.04, size = 63, normalized size = 0.98

$$\frac{x}{c} - \frac{(-2cd+be)\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{-cd+be}}\right)}{c^{3/2}\sqrt{e}\,\sqrt{-cd+be}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
[Out] x/c - ((-2*c*d + b*e)*ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(c^(3
/2)*Sqrt[e]*Sqrt[-(c*d) + b*e])
```

Maple [A]
time = 0.13, size = 51, normalized size = 0.80

method	result
default	$\frac{x}{c}+\frac{(-eb+2cd)\arctan\left(\frac{cex}{\sqrt{(eb-cd)ce}}\right)}{c\sqrt{(eb-cd)ce}}$
risch	$\frac{x}{c}-\frac{\ln\left(cex-\sqrt{-(eb-cd)ce}\right)eb}{2c\sqrt{-(eb-cd)ce}}+\frac{\ln\left(cex-\sqrt{-(eb-cd)ce}\right)d}{\sqrt{-(eb-cd)ce}}+\frac{\ln\left(-cex-\sqrt{-(eb-cd)ce}\right)eb}{2c\sqrt{-(eb-cd)ce}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
[Out] x/c+(-b*e+2*c*d)/c/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2)
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e*b>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.38, size = 215, normalized size = 3.36

$$\left[\frac{2\,c^2dxe-2\,bcxe^2-\sqrt{c^2de-bce^2}\,(2\,cd-be)\log\left(-\frac{cd+(cx^2-b)e+2\,\sqrt{c^2de-bce^2}\,x}{cd-(cx^2+b)e}\right)}{2\,(c^3de-bc^2e^2)},\ \frac{c^2dxe-bcxe^2-\sqrt{-c^2de+bce^2}\,(2\,cd-be)\arctan\left(-\frac{\sqrt{-c^2de+bce^2}\,x}{cd-be}\right)}{c^3de-bc^2e^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [1/2*(2*c^2*d*x*e - 2*b*c*x*e^2 - sqrt(c^2*d*e - b*c*e^2)*(2*c*d - b*e)*log
(-(c*d + (c*x^2 - b)*e + 2*sqrt(c^2*d*e - b*c*e^2)*x)/(c*d - (c*x^2 + b)*e)
))/(c^3*d*e - b*c^2*e^2), (c^2*d*x*e - b*c*x*e^2 - sqrt(-c^2*d*e + b*c*e^2)
*(2*c*d - b*e)*arctan(-sqrt(-c^2*d*e + b*c*e^2)*x/(c*d - b*e)))/(c^3*d*e -
b*c^2*e^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(54) = 108$.
time = 0.25, size = 212, normalized size = 3.31

$$\frac{\sqrt{-\frac{1}{c^3e\,(be-cd)}}\,(be-2cd)\log\left(x+\frac{-bce\sqrt{-\frac{1}{c^3e\,(be-cd)}}\,(be-2cd)+c^2d\sqrt{-\frac{1}{c^3e\,(be-cd)}}\,(be-2cd)}{be-2cd}\right)}{2}-\frac{\sqrt{-\frac{1}{c^3e\,(be-cd)}}\,(be-2cd)\log\left(x+\frac{bce\sqrt{-\frac{1}{c^3e\,(be-cd)}}\,(be-2cd)-c^2d\sqrt{-\frac{1}{c^3e\,(be-cd)}}\,(be-2cd)}{be-2cd}\right)}{2}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(x + (-b*c*e*sqrt(-1/(c**3*e
*(b*e - c*d)))*(b*e - 2*c*d) + c**2*d*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e -
2*c*d))/(b*e - 2*c*d))/2 - sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d)*log(
x + (b*c*e*sqrt(-1/(c**3*e*(b*e - c*d)))*(b*e - 2*c*d) - c**2*d*sqrt(-1/(c*
*3*e*(b*e - c*d)))*(b*e - 2*c*d))/(b*e - 2*c*d))/2 + x/c
```

Giac [A]
time = 3.95, size = 58, normalized size = 0.91

$$\frac{(2\,cd-be)\arctan\left(\frac{cxe}{\sqrt{-c^2de+bce^2}}\right)}{\sqrt{-c^2de+bce^2}\,c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] (2*c*d - b*e)*arctan(c*x*e/sqrt(-c^2*d*e + b*c*e^2))/(sqrt(-c^2*d*e + b*c*e
^2)*c) + x/c
```

Mupad [B]
time = 0.07, size = 52, normalized size = 0.81

$$\frac{x}{c}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,e\,x}{\sqrt{b\,e^2-c\,d\,e}}\right)\,\left(b\,e-2\,c\,d\right)}{c^{3/2}\,\sqrt{b\,e^2-c\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] x/c - (atan((c^(1/2)*e*x)/(b*e^2 - c*d*e)^(1/2))*(b*e - 2*c*d))/(c^(3/2)*(b
*e^2 - c*d*e)^(1/2))
```

3.217 $\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{\sqrt{c}\,\sqrt{e}\,\sqrt{cd-be}}$$

```
[Out] -arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/c^(1/2)/e^(1/2)/(-b*e+c*d)^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {1163, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{\sqrt{c}\,\sqrt{e}\,\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[c*d -
b*e]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^2}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx &= \int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{\sqrt{c}\,\sqrt{e}\,\sqrt{cd-be}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 0.98

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\sqrt{e}\,x}{\sqrt{-cd+be}}\right)}{\sqrt{c}\sqrt{e}\sqrt{-cd+be}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]]/(Sqrt[c]*Sqrt[e]*Sqrt[-(c*d)
 + b*e])
```

Maple [A]
time = 0.03, size = 33, normalized size = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{cex}{\sqrt{(eb-cd)ce}}\right)}{\sqrt{(eb-cd)ce}}$	33
risch	$-\frac{\ln\left(cex+\sqrt{-(eb-cd)ce}\right)}{2\sqrt{-(eb-cd)ce}}+\frac{\ln\left(-cex+\sqrt{-(eb-cd)ce}\right)}{2\sqrt{-(eb-cd)ce}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((b*e-c*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e*b>0)', see `assume?` for mor
e detai
```

Fricas [A]
time = 0.34, size = 139, normalized size = 2.84

$$\left[\frac{\log\left(-\frac{cd+(cx^2-b)e-2\sqrt{c^2de-bce^2}\,x}{cd-(cx^2+b)e}\right)}{2\sqrt{c^2de-bce^2}}, -\frac{\sqrt{-c^2de+bce^2}\,\arctan\left(-\frac{\sqrt{-c^2de+bce^2}\,x}{cd-be}\right)}{c^2de-bce^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fricas"
)
```

```
[Out] [1/2*log(-(c*d + (c*x^2 - b)*e - 2*sqrt(c^2*d*e - b*c*e^2)*x)/(c*d - (c*x^2
 + b)*e))/sqrt(c^2*d*e - b*c*e^2), -sqrt(-c^2*d*e + b*c*e^2)*arctan(-sqrt(-
c^2*d*e + b*c*e^2)*x/(c*d - b*e))/(c^2*d*e - b*c*e^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(44) = 88$.
time = 0.10, size = 124, normalized size = 2.53

$$-\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(-be\sqrt{-\frac{1}{ce(be-cd)}}+cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ce(be-cd)}}\log\left(be\sqrt{-\frac{1}{ce(be-cd)}}-cd\sqrt{-\frac{1}{ce(be-cd)}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] -sqrt(-1/(c*e*(b*e - c*d)))*log(-b*e*sqrt(-1/(c*e*(b*e - c*d))) + c*d*sqrt(
-1/(c*e*(b*e - c*d))) + x)/2 + sqrt(-1/(c*e*(b*e - c*d)))*log(b*e*sqrt(-1/(
c*e*(b*e - c*d))) - c*d*sqrt(-1/(c*e*(b*e - c*d))) + x)/2
```

Giac [A]
time = 3.21, size = 39, normalized size = 0.80

$$\frac{\arctan\left(\frac{cxe}{\sqrt{-c^2de+bce^2}}\right)}{\sqrt{-c^2de+bce^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac")
```

```
[Out] arctan(c*x*e/sqrt(-c^2*d*e + b*c*e^2))/sqrt(-c^2*d*e + b*c*e^2)
```

Mupad [B]
time = 4.49, size = 38, normalized size = 0.78

$$\frac{\mathrm{atan}\left(\frac{c\,e\,x}{\sqrt{b\,c\,e^2-c^2\,d\,e}}\right)}{\sqrt{b\,c\,e^2-c^2\,d\,e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
[Out] atan((c*e*x)/(b*c*e^2 - c^2*d*e)^(1/2))/(b*c*e^2 - c^2*d*e)^(1/2)
```

3.218 $\int \frac{1}{(d+ex^2)\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$

Optimal. Leaf size=136

$$-\frac{x}{2d(2cd-be)\left(d+ex^2\right)}-\frac{(4cd-be)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}\,(2cd-be)^2}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^2}$$

```
[Out] -1/2*x/d/(-b*e+2*c*d)/(e*x^2+d)-1/2*(-b*e+4*c*d)*arctan(x*e^(1/2)/d^(1/2))/
d^(3/2)/(-b*e+2*c*d)^2/e^(1/2)-c^(3/2)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)
^(1/2))/(-b*e+2*c*d)^2/e^(1/2)/(-b*e+c*d)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {1163, 425, 536, 211, 214}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(4cd-be)}{2d^{3/2}\sqrt{e}\,(2cd-be)^2}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^2}-\frac{x}{2d\left(d+ex^2\right)(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -1/2*x/(d*(2*c*d - b*e)*(d + e*x^2)) - ((4*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sq
rt[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqr
t[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx &= \int \frac{1}{(d+ex^2)^2\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx\\
&= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)}+\frac{\int \frac{e(3cd-be)-ce^2x^2}{(d+ex^2)\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2de(2cd-be)}\\
&= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)}+\frac{c^2\int \frac{1}{\frac{-cd^2+bde}{d}+cex^2}\,dx}{(2cd-be)^2}-\frac{(4cd-}{2d(2}\\
&= -\frac{x}{2d(2cd-be)\left(d+ex^2\right)}-\frac{(4cd-be)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}\,(2cd-be)^2}-\frac{c^{}}{\sqrt{}}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 133, normalized size = 0.98

$$
-\frac{x}{2d(2cd-be)\left(d+ex^2\right)}+\frac{(-4cd+be)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}\,(2cd-be)^2}+\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{-cd+be}}\right)}{\sqrt{e}\,(-2cd+be)^2\sqrt{-cd+be}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -1/2*x/(d*(2*c*d - b*e)*(d + e*x^2)) + ((-4*c*d + b*e)*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(2*d^(3/2)*Sqrt[e]*(2*c*d - b*e)^2) + (c^(3/2)*ArcTan[(Sqrt[c]*Sqr
t[e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*(-2*c*d + b*e)^2*Sqrt[-(c*d) + b*e])
```

Maple [A]

time = 0.18, size = 109, normalized size = 0.80

method	result
default	$\frac{\frac{(eb-2cd)x}{2d(e\,x^2+d)}+\frac{(eb-4cd)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}}{(eb-2cd)^2}+\frac{c^2\arctan\left(\frac{cex}{\sqrt{(eb-cd)\,ce}}\right)}{(eb-2cd)^2\sqrt{(eb-cd)\,ce}}$
risch	$\frac{x}{2d(eb-2cd)(e\,x^2+d)}-\frac{\ln\left(d\,e^2x-(-de)^{\frac{3}{2}}\right)eb}{4\sqrt{-de}\,(eb-2cd)^2d}+\frac{\ln\left(d\,e^2x-(-de)^{\frac{3}{2}}\right)c}{\sqrt{-de}\,(eb-2cd)^2}+\frac{\ln\left(-d\,e^2x-(-de)^{\frac{3}{2}}\right)eb}{4\sqrt{-de}\,(eb-2cd)^2d}-\frac{\ln\left(-d\,e^2x-(-de)^{\frac{3}{2}}\right)c}{\sqrt{-de}\,(eb-2cd)^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*e-2*c*d)^2*(1/2*(b*e-2*c*d)/d*x/(e*x^2+d)+1/2*(b*e-4*c*d)/d/(d*e)^(1/2
)*arctan(e*x/(d*e)^(1/2)))+c^2/(b*e-2*c*d)^2/((b*e-c*d)*c*e)^(1/2)*arctan(c
*e*x/((b*e-c*d)*c*e)^(1/2))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="maxim
a")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e*b>0)', see `assume?` for mor
e detai
```

Fricas [A]

time = 0.46, size = 919, normalized size = 6.76

$$\left[-\frac{4\,cd^2xe-2\,bdxe^2+(bx^2e^2-4\,cd^2-(4\,cdx^2-bd)e)\sqrt{-de}\,\log\left(\frac{x^2e-2\sqrt{-de}\,x-d}{x^2e+d}\right)-2\,(cd^2x^2e^2+cd^3e)\sqrt{\frac{c}{cde-be^2}}\,\log\left(-\frac{cd+(cx^2-b)e-2\,(cdxe-bxe^2)\sqrt{\frac{c}{cde-be^2}}}{cd-(cx^2+b)e}\right)}{4\,(4\,c^2d^5e+b^2d^2x^2e^4-(4\,bcd^3x^2-b^2d^3)e^3+4\,(c^2d^4x^2-bcd^4)e^2)},\ -\frac{2\,cd^2xe-bdxe^2-(bx^2e^2-4\,cd^2-(4\,cdx^2-bd)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-(cd^2x^2e^2+cd^3e)\sqrt{\frac{c}{cde-be^2}}\,\log\left(-\frac{cd+(cx^2-b)e-2\,(cdxe-bxe^2)\sqrt{\frac{c}{cde-be^2}}}{cd-(cx^2+b)e}\right)}{2\,(4\,c^2d^5e+b^2d^2x^2e^4-(4\,bcd^3x^2-b^2d^3)e^3+4\,(c^2d^4x^2-bcd^4)e^2)},\ -\frac{4\,cd^2xe-2\,bdxe^2-4\,(cd^2x^2e^2+cd^3e)\sqrt{-\frac{c}{cde-be^2}}\,\arctan\left(x\sqrt{-\frac{c}{cde-be^2}}\,e\right)+(bx^2e^2-4\,cd^2-(4\,cdx^2-bd)e)\sqrt{-de}\,\log\left(\frac{x^2e-2\sqrt{-de}\,x-d}{x^2e+d}\right)}{4\,(4\,c^2d^5e+b^2d^2x^2e^4-(4\,bcd^3x^2-b^2d^3)e^3+4\,(c^2d^4x^2-bcd^4)e^2)},\ -\frac{2\,cd^2xe-bdxe^2-(bx^2e^2-4\,cd^2-(4\,cdx^2-bd)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-2\,(cd^2x^2e^2+cd^3e)\sqrt{-\frac{c}{cde-be^2}}\,\arctan\left(x\sqrt{-\frac{c}{cde-be^2}}\,e\right)}{2\,(4\,c^2d^5e+b^2d^2x^2e^4-(4\,bcd^3x^2-b^2d^3)e^3+4\,(c^2d^4x^2-bcd^4)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="frica
s")
```

```
[Out] [-1/4*(4*c*d^2*x*e - 2*b*d*x*e^2 + (b*x^2*e^2 - 4*c*d^2 - (4*c*d*x^2 - b*d)
*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) - 2*(c*d^2*x^2
*e^2 + c*d^3*e)*sqrt(c/(c*d*e - b*e^2))*log(-(c*d + (c*x^2 - b)*e - 2*(c*d*
```

```
x*e - b*x*e^2)*sqrt(c/(c*d*e - b*e^2)))/(c*d - (c*x^2 + b)*e)))/(4*c^2*d^5*
e + b^2*d^2*x^2*e^4 - (4*b*c*d^3*x^2 - b^2*d^3)*e^3 + 4*(c^2*d^4*x^2 - b*c*
d^4)*e^2), -1/2*(2*c*d^2*x*e - b*d*x*e^2 - (b*x^2*e^2 - 4*c*d^2 - (4*c*d*x^
2 - b*d)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - (c*d^2*x^2*e^2 + c*
d^3*e)*sqrt(c/(c*d*e - b*e^2))*log(-(c*d + (c*x^2 - b)*e - 2*(c*d*x*e - b*x
*e^2)*sqrt(c/(c*d*e - b*e^2)))/(c*d - (c*x^2 + b)*e)))/(4*c^2*d^5*e + b^2*d
^2*x^2*e^4 - (4*b*c*d^3*x^2 - b^2*d^3)*e^3 + 4*(c^2*d^4*x^2 - b*c*d^4)*e^2)
, -1/4*(4*c*d^2*x*e - 2*b*d*x*e^2 - 4*(c*d^2*x^2*e^2 + c*d^3*e)*sqrt(-c/(c*
d*e - b*e^2))*arctan(x*sqrt(-c/(c*d*e - b*e^2))*e) + (b*x^2*e^2 - 4*c*d^2 -
 (4*c*d*x^2 - b*d)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e +
d)))/(4*c^2*d^5*e + b^2*d^2*x^2*e^4 - (4*b*c*d^3*x^2 - b^2*d^3)*e^3 + 4*(c^
2*d^4*x^2 - b*c*d^4)*e^2), -1/2*(2*c*d^2*x*e - b*d*x*e^2 - (b*x^2*e^2 - 4*c
*d^2 - (4*c*d*x^2 - b*d)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - 2*(
c*d^2*x^2*e^2 + c*d^3*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(x*sqrt(-c/(c*d*e -
 b*e^2))*e))/(4*c^2*d^5*e + b^2*d^2*x^2*e^4 - (4*b*c*d^3*x^2 - b^2*d^3)*e^3
 + 4*(c^2*d^4*x^2 - b*c*d^4)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.20, size = 147, normalized size = 1.08

$$\frac{c^2\arctan\left(\frac{cxe}{\sqrt{-c^2de+bce^2}}\right)}{(4\,c^2d^2-4\,bcde+b^2e^2)\sqrt{-c^2de+bce^2}}-\frac{(4\,cd-be)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,(4\,c^2d^3-4\,bcd^2e+b^2de^2)\sqrt{d}}-\frac{x}{2\,(2\,cd^2-bde)(x^2e+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="giac"
)
```

```
[Out] c^2*arctan(c*x*e/sqrt(-c^2*d*e + b*c*e^2))/((4*c^2*d^2 - 4*b*c*d*e + b^2*e^
2)*sqrt(-c^2*d*e + b*c*e^2)) - 1/2*(4*c*d - b*e)*arctan(x*e^(1/2)/sqrt(d))*
e^(-1/2)/((4*c^2*d^3 - 4*b*c*d^2*e + b^2*d*e^2)*sqrt(d)) - 1/2*x/((2*c*d^2
- b*d*e)*(x^2*e + d))
```

Mupad [**B**]
time = 5.40, size = 2500, normalized size = 18.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

```
[Out] - x/(2*(d + e*x^2)*(2*c*d^2 - b*d*e)) - (atan((((((96*c^7*d^6*e^6 - 224*b*
c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 +
 22*b^4*c^3*d^2*e^10)/(2*(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*
c^2*d^4*e)) - (x*(-c^3*e*(b*e - c*d))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b^2*c^
5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^
12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e + 8*
b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e^4 -
4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)) - (x*(b^2*c^3*e^8 + 20*c^5*
d^2*e^6 - 8*b*c^4*d*e^7))/(4*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))*(-c^
3*e*(b*e - c*d))^(1/2)*1i)/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2
*c*d*e^3) - (((((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 20
8*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(2*(8*c^3*d^5
 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e)) + (x*(-c^3*e*(b*e - c*d
))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 -
128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 -
4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-
c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b
^2*c*d*e^3)) + (x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(4*(4*c^2
*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))*(-c^3*e*(b*e - c*d))^(1/2)*1i)/(b^3*e^4
 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3))/((((((96*c^7*d^6*e^6 - 2
24*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*
e^9 + 22*b^4*c^3*d^2*e^10)/(2*(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 -
12*b*c^2*d^4*e)) - (x*(-c^3*e*(b*e - c*d))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b
^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d
^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e
 + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e
^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)) - (x*(b^2*c^3*e^8 + 20
*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(4*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))
*(-c^3*e*(b*e - c*d))^(1/2))/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b
^2*c*d*e^3) - ((b*c^4*e^6)/2 - 2*c^5*d*e^5)/(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^
2*c*d^3*e^2 - 12*b*c^2*d^4*e) + (((((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2
*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2
*e^10)/(2*(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e)) + (
x*(-c^3*e*(b*e - c*d))^(1/2)*(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384
*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*
d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 -
 5*b^2*c*d*e^3)))*(-c^3*e*(b*e - c*d))^(1/2))/(2*(b^3*e^4 - 4*c^3*d^3*e + 8
*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)) + (x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c
^4*d*e^7))/(4*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)))*(-c^3*e*(b*e - c*d)
)^(1/2))/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c*d*e^3)))*(-c^3*
e*(b*e - c*d))^(1/2)*1i)/(b^3*e^4 - 4*c^3*d^3*e + 8*b*c^2*d^2*e^2 - 5*b^2*c
```

```
*d*e^3) - (atan(((((x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(2*(4
*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)) - ((-d^3*e)^(1/2)*((96*c^7*d^6*e^6 -
 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^4*d^
3*e^9 + 22*b^4*c^3*d^2*e^10)/(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2 - 1
2*b*c^2*d^4*e) - (x*(-d^3*e)^(1/2)*(b*e - 4*c*d)*(256*b*c^6*d^6*e^8 - 512*b
^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c^2*d
^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(4*c^2*d^5*e + b^2*d^3
*e^3 - 4*b*c*d^4*e^2)))*(b*e - 4*c*d))/(4*(4*c^2*d^5*e + b^2*d^3*e^3 - 4*b*
c*d^4*e^2)))*(-d^3*e)^(1/2)*(b*e - 4*c*d)*1i)/(4*(4*c^2*d^5*e + b^2*d^3*e^3
 - 4*b*c*d^4*e^2)) + (((x*(b^2*c^3*e^8 + 20*c^5*d^2*e^6 - 8*b*c^4*d*e^7))/(
2*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)) + ((-d^3*e)^(1/2)*((96*c^7*d^6*e
^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c^5*d^4*e^8 - 96*b^3*c^
4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(8*c^3*d^5 - b^3*d^2*e^3 + 6*b^2*c*d^3*e^2
 - 12*b*c^2*d^4*e) + (x*(-d^3*e)^(1/2)*(b*e - 4*c*d)*(256*b*c^6*d^6*e^8 - 5
12*b^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c^3*d^3*e^11 + 16*b^5*c
^2*d^2*e^12))/(8*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)*(4*c^2*d^5*e + b^2
*d^3*e^3 - 4*b*c*d^4*e^2)))*(b*e - 4*c*d))/(4*(4*c^2*d^5*e + b^2*d^3*e^3 -
4*b*c*d^4*e^2)))*(-d^3*e)^(1/2)*(b*e - 4*c*d)*1i)/(4*(4*c^2*d^5*e + b^2*d^3
*e^3 - 4*b*c*d^4*e^2)))/(((b*c^4*e^6)/2 - 2*c^5*d*e^5)/(8*c^3*d^5 - b^3*d^2
*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e) + (((x*(b^2*c^3*e^8 + 20*c^5*d^2*e
^6 - 8*b*c^4*d*e^7))/(2*(4*c^2*d^4 + b^2*d^2*e^2 - 4*b*c*d^3*e)) - ((-d^3*e
)^(1/2)*((96*c^7*d^6*e^6 - 224*b*c^6*d^5*e^7 - 2*b^5*c^2*d*e^11 + 208*b^2*c
^5*d^4*e^8 - 96*b^3*c^4*d^3*e^9 + 22*b^4*c^3*d^2*e^10)/(8*c^3*d^5 - b^3*d^2
*e^3 + 6*b^2*c*d^3*e^2 - 12*b*c^2*d^4*e) - (x*(-d^3*e)^(1/2)*(b*e - 4*c*d)*
(256*b*c^6*d^6*e^8 - 512*b^2*c^5*d^5*e^9 + 384*b^3*c^4*d^4*e^10 - 128*b^4*c
^3*d^3*e^11 + 16*b^5*c^2*d^2*e^12))/(8*(4*c^2*d...
```

3.219 $\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

Optimal. Leaf size=187

$$-\frac{x}{4d(2cd-be)\left(d+ex^2\right)^2}-\frac{(10cd-3be)x}{8d^2(2cd-be)^2\left(d+ex^2\right)}-\frac{\left(28c^2d^2-16bcde+3b^2e^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}\,(2cd-be)^3}-\frac{c^{5/2}\tanh^{-}}{\sqrt{e}\,\sqrt{cd-}}$$

[Out] -1/4*x/d/(-b*e+2*c*d)/(e*x^2+d)^2-1/8*(-3*b*e+10*c*d)*x/d^2/(-b*e+2*c*d)^2/(e*x^2+d)-1/8*(3*b^2*e^2-16*b*c*d*e+28*c^2*d^2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/(-b*e+2*c*d)^3/e^(1/2)-c^(5/2)*arctanh(x*c^(1/2)*e^(1/2)/(-b*e+c*d)^(1/2))/(-b*e+2*c*d)^3/e^(1/2)/(-b*e+c*d)^(1/2)

Rubi [A]
time = 0.18, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1163, 425, 541, 536, 211, 214}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(3b^2e^2-16bcde+28c^2d^2\right)}{8d^{5/2}\sqrt{e}\,(2cd-be)^3}-\frac{c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{cd-be}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^3}-\frac{x(10cd-3be)}{8d^2\left(d+ex^2\right)(2cd-be)^2}-\frac{x}{4d\left(d+ex^2\right)^2(2cd-be)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]

[Out] -1/4*x/(d*(2*c*d - b*e)*(d + e*x^2)^2) - ((10*c*d - 3*b*e)*x)/(8*d^2*(2*c*d - b*e)^2*(d + e*x^2)) - ((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]*(2*c*d - b*e)^3) - (c^(5/2)*ArcTanh[(Sqrt[c]*Sqrt[e]*x)/Sqrt[c*d - b*e]])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^3)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c - a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
```

```
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \int \frac{1}{(d+ex^2)^3\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx$$

$$= -\frac{x}{4d(2cd-be)(d+ex^2)^2} + \frac{\int \frac{e(7cd-3be)-3ce^2x^2}{(d+ex^2)^2\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{4de(2cd-be)}$$

$$= -\frac{x}{4d(2cd-be)(d+ex^2)^2} - \frac{(10cd-3be)x}{8d^2(2cd-be)^2(d+ex^2)} + \ldots$$

$$= -\frac{x}{4d(2cd-be)(d+ex^2)^2} - \frac{(10cd-3be)x}{8d^2(2cd-be)^2(d+ex^2)} + \frac{c^3}{\ldots}$$

$$= -\frac{x}{4d(2cd-be)(d+ex^2)^2} - \frac{(10cd-3be)x}{8d^2(2cd-be)^2(d+ex^2)} - \frac{(2\ldots}{\ldots}$$

Mathematica [A]
time = 0.26, size = 177, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{(28c^2d^2-16bcde+3b^2e^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}\,(2cd-be)^3} - \frac{\frac{(-2cd+be)x(-be(5d+3ex^2)+2cd(7d+5ex^2))}{d^2(d+ex^2)^2} + \frac{8c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{e}\,x}{\sqrt{-cd+be}}\right)}{\sqrt{e}\,\sqrt{-cd+be}}}{(-2cd+be)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] (-(((28*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(
5/2)*Sqrt[e]*(2*c*d - b*e)^3)) - (((-2*c*d + b*e)*x*(-(b*e*(5*d + 3*e*x^2))
 + 2*c*d*(7*d + 5*e*x^2)))/(d^2*(d + e*x^2)^2) + (8*c^(5/2)*ArcTan[(Sqrt[c]
*Sqrt[e]*x)/Sqrt[-(c*d) + b*e]])/(Sqrt[e]*Sqrt[-(c*d) + b*e]))/(-2*c*d + b*
e)^3)/8
```

Maple [A]
time = 0.20, size = 174, normalized size = 0.93

method	result
default	$\frac{\frac{\frac{e(3e^2b^2-16bcde+20c^2d^2)x^3}{8d^2}+\frac{(5e^2b^2-24bcde+28c^2d^2)x}{8d}}{(e\,x^2+d)^2}+\frac{(3e^2b^2-16bcde+28c^2d^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8d^2\sqrt{de}}}{(eb-2cd)^3} - \frac{c^3\arctan\left(\frac{cex}{\sqrt{(eb-cd)ce}}\right)}{(eb-2cd)^3\sqrt{(eb-cd)ce}}$

risch	$\frac{\frac{(3eb-10cd)e\,x^3}{8d^2\left(e^2b^2-4bcde+4c^2d^2\right)}+\frac{(5eb-14cd)x}{8d\left(e^2b^2-4bcde+4c^2d^2\right)}}{\left(e\,x^2+d\right)^2}-\frac{3\ln\left(d\,e^2x-(-de)^{\frac{3}{2}}\right)e^2b^2}{16\sqrt{-de}\,(eb-2cd)^3d^2}+\frac{\ln\left(d\,e^2x-(-de)^{\frac{3}{2}}\right)bce}{\sqrt{-de}\,(eb-2cd)^3d}-\frac{7\ln\left(d\,e^2x-(-de)^{\frac{3}{2}}\right)}{4\sqrt{-de}\,(eb-2cd)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBOSE
)
```

```
[Out] 1/(b*e-2*c*d)^3*((1/8*e*(3*b^2*e^2-16*b*c*d*e+20*c^2*d^2)/d^2*x^3+1/8*(5*b^
2*e^2-24*b*c*d*e+28*c^2*d^2)/d*x)/(e*x^2+d)^2+1/8*(3*b^2*e^2-16*b*c*d*e+28*
c^2*d^2)/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-c^3/(b*e-2*c*d)^3/((b*e-c
*d)*c*e)^(1/2)*arctan(c*e*x/((b*e-c*d)*c*e)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="max
ima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d-%e*b>0)', see `assume?` for mor
e detai
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(166) = 332.
time = 0.93, size = 1799, normalized size = 9.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="fri
cas")
```

```
[Out] [-1/16*(56*c^2*d^4*x*e + 6*b^2*d*x^3*e^4 - (3*b^2*x^4*e^4 + 28*c^2*d^4 - 2*
(8*b*c*d*x^4 - 3*b^2*d*x^2)*e^3 + (28*c^2*d^2*x^4 - 32*b*c*d^2*x^2 + 3*b^2*
d^2)*e^2 + 8*(7*c^2*d^3*x^2 - 2*b*c*d^3)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(
-d*e)*x - d)/(x^2*e + d)) + 8*(c^2*d^3*x^4*e^3 + 2*c^2*d^4*x^2*e^2 + c^2*d^
5*e)*sqrt(c/(c*d*e - b*e^2))*log(-(c*d + (c*x^2 - b)*e + 2*(c*d*x*e - b*x*e
^2)*sqrt(c/(c*d*e - b*e^2)))/(c*d - (c*x^2 + b)*e)) - 2*(16*b*c*d^2*x^3 - 5
*b^2*d^2*x)*e^3 + 8*(5*c^2*d^3*x^3 - 6*b*c*d^3*x)*e^2)/(8*c^3*d^8*e - b^3*d
^3*x^4*e^6 + 2*(3*b^2*c*d^4*x^4 - b^3*d^4*x^2)*e^5 - (12*b*c^2*d^5*x^4 - 12
```

```
*b^2*c*d^5*x^2 + b^3*d^5)*e^4 + 2*(4*c^3*d^6*x^4 - 12*b*c^2*d^6*x^2 + 3*b^2
*c*d^6)*e^3 + 4*(4*c^3*d^7*x^2 - 3*b*c^2*d^7)*e^2), -1/8*(28*c^2*d^4*x*e +
3*b^2*d*x^3*e^4 + (3*b^2*x^4*e^4 + 28*c^2*d^4 - 2*(8*b*c*d*x^4 - 3*b^2*d*x^
2)*e^3 + (28*c^2*d^2*x^4 - 32*b*c*d^2*x^2 + 3*b^2*d^2)*e^2 + 8*(7*c^2*d^3*x
^2 - 2*b*c*d^3)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) + 4*(c^2*d^3*x
^4*e^3 + 2*c^2*d^4*x^2*e^2 + c^2*d^5*e)*sqrt(c/(c*d*e - b*e^2))*log(-(c*d +
 (c*x^2 - b)*e + 2*(c*d*x*e - b*x*e^2)*sqrt(c/(c*d*e - b*e^2)))/(c*d - (c*x
^2 + b)*e)) - (16*b*c*d^2*x^3 - 5*b^2*d^2*x)*e^3 + 4*(5*c^2*d^3*x^3 - 6*b*c
*d^3*x)*e^2)/(8*c^3*d^8*e - b^3*d^3*x^4*e^6 + 2*(3*b^2*c*d^4*x^4 - b^3*d^4*
x^2)*e^5 - (12*b*c^2*d^5*x^4 - 12*b^2*c*d^5*x^2 + b^3*d^5)*e^4 + 2*(4*c^3*d
^6*x^4 - 12*b*c^2*d^6*x^2 + 3*b^2*c*d^6)*e^3 + 4*(4*c^3*d^7*x^2 - 3*b*c^2*d
^7)*e^2), -1/16*(56*c^2*d^4*x*e + 6*b^2*d*x^3*e^4 - 16*(c^2*d^3*x^4*e^3 + 2
*c^2*d^4*x^2*e^2 + c^2*d^5*e)*sqrt(-c/(c*d*e - b*e^2))*arctan(x*sqrt(-c/(c*
d*e - b*e^2))*e) - (3*b^2*x^4*e^4 + 28*c^2*d^4 - 2*(8*b*c*d*x^4 - 3*b^2*d*x
^2)*e^3 + (28*c^2*d^2*x^4 - 32*b*c*d^2*x^2 + 3*b^2*d^2)*e^2 + 8*(7*c^2*d^3*
x^2 - 2*b*c*d^3)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)
) - 2*(16*b*c*d^2*x^3 - 5*b^2*d^2*x)*e^3 + 8*(5*c^2*d^3*x^3 - 6*b*c*d^3*x)*
e^2)/(8*c^3*d^8*e - b^3*d^3*x^4*e^6 + 2*(3*b^2*c*d^4*x^4 - b^3*d^4*x^2)*e^5
 - (12*b*c^2*d^5*x^4 - 12*b^2*c*d^5*x^2 + b^3*d^5)*e^4 + 2*(4*c^3*d^6*x^4 -
 12*b*c^2*d^6*x^2 + 3*b^2*c*d^6)*e^3 + 4*(4*c^3*d^7*x^2 - 3*b*c^2*d^7)*e^2)
, -1/8*(28*c^2*d^4*x*e + 3*b^2*d*x^3*e^4 + (3*b^2*x^4*e^4 + 28*c^2*d^4 - 2*
(8*b*c*d*x^4 - 3*b^2*d*x^2)*e^3 + (28*c^2*d^2*x^4 - 32*b*c*d^2*x^2 + 3*b^2*
d^2)*e^2 + 8*(7*c^2*d^3*x^2 - 2*b*c*d^3)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d
))*e^(1/2) - 8*(c^2*d^3*x^4*e^3 + 2*c^2*d^4*x^2*e^2 + c^2*d^5*e)*sqrt(-c/(c
*d*e - b*e^2))*arctan(x*sqrt(-c/(c*d*e - b*e^2))*e) - (16*b*c*d^2*x^3 - 5*b
^2*d^2*x)*e^3 + 4*(5*c^2*d^3*x^3 - 6*b*c*d^3*x)*e^2)/(8*c^3*d^8*e - b^3*d^3
*x^4*e^6 + 2*(3*b^2*c*d^4*x^4 - b^3*d^4*x^2)*e^5 - (12*b*c^2*d^5*x^4 - 12*b
^2*c*d^5*x^2 + b^3*d^5)*e^4 + 2*(4*c^3*d^6*x^4 - 12*b*c^2*d^6*x^2 + 3*b^2*c
*d^6)*e^3 + 4*(4*c^3*d^7*x^2 - 3*b*c^2*d^7)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.36, size = 235, normalized size = 1.26

$$\frac{c^3 \arctan\left(\frac{cxe}{\sqrt{-c^2de+bce^2}}\right)}{(8\,c^3d^3-12\,bc^2d^2e+6\,b^2cde^2-b^3e^3)\sqrt{-c^2de+bce^2}}-\frac{(28\,c^2d^2-16\,bcde+3\,b^2e^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,(8\,c^3d^5-12\,bc^2d^4e+6\,b^2cd^3e^2-b^3d^2e^3)\sqrt{d}}-\frac{10\,cdx^3e-3\,bx^3e^2+14\,cd^2x-5\,bdxe}{8\,(4\,c^2d^4-4\,bcd^3e+b^2d^2e^2)(x^2e+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="gia
c")
```

```
[Out] c^3*arctan(c*x*e/sqrt(-c^2*d*e + b*c*e^2))/((8*c^3*d^3 - 12*b*c^2*d^2*e + 6
*b^2*c*d*e^2 - b^3*e^3)*sqrt(-c^2*d*e + b*c*e^2)) - 1/8*(28*c^2*d^2 - 16*b*
c*d*e + 3*b^2*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((8*c^3*d^5 - 12*b*c^
2*d^4*e + 6*b^2*c*d^3*e^2 - b^3*d^2*e^3)*sqrt(d)) - 1/8*(10*c*d*x^3*e - 3*b
*x^3*e^2 + 14*c*d^2*x - 5*b*d*x*e)/((4*c^2*d^4 - 4*b*c*d^3*e + b^2*d^2*e^2)
*(x^2*e + d)^2)
```

Mupad [B]
time = 6.45, size = 2500, normalized size = 13.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^2*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

```
[Out] ((x*(5*b*e - 14*c*d))/(8*d*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)) + (e*x^3*(3*b
*e - 10*c*d))/(8*d^2*(b^2*e^2 + 4*c^2*d^2 - 4*b*c*d*e)))/(d^2 + e^2*x^4 + 2
*d*e*x^2) - (atan(((((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e
^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8 + b^4*d^4*e^4
 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - (((576*c^10*d^
10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 +
 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*
b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^6*d^10 + b^6*d^4*e^6
 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^
2*d^6*e^4 - 192*b*c^5*d^9*e)) - (x*(-c^5*e*(b*e - c*d))^(1/2)*(16384*b*c^8*
d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*b^4*c^5*d
^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7*c^2*d^4*
e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^
2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*
d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(2*(b^4*e^5 + 8*c^4*
d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b
*e - c*d))^(1/2)*1i)/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2
*d^2*e^3 - 7*b^3*c*d*e^4) + (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*
c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8 + b^
4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) + (((57
6*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*
d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^
12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^6*d^10 + b^
6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 +
60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) + (x*(-c^5*e*(b*e - c*d))^(1/2)*(163
84*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*
b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7
*c^2*d^4*e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c
```

^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(2*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2)*1i)/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4))/(((9*b^3*c^5*e^8)/32 - (35*c^8*d^3*e^5)/4 + (61*b*c^7*d^2*e^6)/8 - (39*b^2*c^6*d*e^7)/16)/(64*c^6*d^10 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e) + (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) - (((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^6*d^10 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) - (x*(-c^5*e*(b*e - c*d))^(1/2)*(16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7*c^2*d^4*e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(2*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4) - (((x*(9*b^4*c^3*e^10 + 848*c^7*d^4*e^6 - 896*b*c^6*d^3*e^7 - 96*b^3*c^4*d*e^9 + 424*b^2*c^5*d^2*e^8))/(64*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)) + (((576*c^10*d^10*e^6 - 2144*b*c^9*d^9*e^7 + 3504*b^2*c^8*d^8*e^8 - 3288*b^3*c^7*d^7*e^9 + 1940*b^4*c^6*d^6*e^10 - 738*b^5*c^5*d^5*e^11 + 177*b^6*c^4*d^4*e^12 - (49*b^7*c^3*d^3*e^13)/2 + (3*b^8*c^2*d^2*e^14)/2)/(2*(64*c^6*d^10 + b^6*d^4*e^6 - 12*b^5*c*d^5*e^5 + 240*b^2*c^4*d^8*e^2 - 160*b^3*c^3*d^7*e^3 + 60*b^4*c^2*d^6*e^4 - 192*b*c^5*d^9*e)) + (x*(-c^5*e*(b*e - c*d))^(1/2)*(16384*b*c^8*d^10*e^8 - 49152*b^2*c^7*d^9*e^9 + 61440*b^3*c^6*d^8*e^10 - 40960*b^4*c^5*d^7*e^11 + 15360*b^5*c^4*d^6*e^12 - 3072*b^6*c^3*d^5*e^13 + 256*b^7*c^2*d^4*e^14))/(128*(16*c^4*d^8 + b^4*d^4*e^4 - 8*b^3*c*d^5*e^3 + 24*b^2*c^2*d^6*e^2 - 32*b*c^3*d^7*e)*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(2*(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2))/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18*b^2*c^2*d^2*e^3 - 7*b^3*c*d*e^4)))*(-c^5*e*(b*e - c*d))^(1/2)*1i)/(b^4*e^5 + 8*c^4*d^4*e - 20*b*c^3*d^3*e^2 + 18...

3.220 $\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=139

$$\frac{x\sqrt{d+ex^2}}{2c}+\frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}-\frac{(2cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{2cd-be}\,x}{\sqrt{cd-be}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}\,\sqrt{cd-be}}$$

```
[Out] 1/2*(-2*b*e+5*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^2/e^(1/2)-(-b*e+2*c
*d)^(3/2)*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(
1/2))/c^2/e^(1/2)/(-b*e+c*d)^(1/2)+1/2*x*(e*x^2+d)^(1/2)/c
```

Rubi [A]
time = 0.21, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1163, 427, 537, 223, 212, 385, 214}

$$-\frac{(2cd-be)^{3/2}\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\,\sqrt{cd-be}}\right)}{c^2\sqrt{e}\,\sqrt{cd-be}}+\frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) + ((5*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e
*x^2]])/(2*c^2*Sqrt[e]) - ((2*c*d - b*e)^(3/2)*ArcTanh[(Sqrt[e]*Sqrt[2*c*d
- b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c^2*Sqrt[e]*Sqrt[c*d - b*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex^2)^{5/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{(d+ex^2)^{3/2}}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{\int \frac{de(3cd-be)+e^2(5cd-2be)x^2}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2ce}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c^2} + \frac{(2cd-be)^2 \int \frac{}{\sqrt{d+ex^2}}}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} + \frac{(2cd-}{}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(5cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{(2cd-be)^{3/2}\tanh}{c^2\sqrt{}}$$

Mathematica [A]
time = 0.38, size = 172, normalized size = 1.24

$$-\frac{-cx\sqrt{d+ex^2} + \frac{2(2cd-be)\sqrt{2c^2d^2-3bcde+b^2e^2}\tanh^{-1}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}\,x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{\sqrt{e}\,(cd-be)} + \frac{(5cd-2be)\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{\sqrt{e}}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(5/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -1/2*(-(c*x*Sqrt[d + e*x^2]) + (2*(2*c*d - b*e)*Sqrt[2*c^2*d^2 - 3*b*c*d*e
+ b^2*e^2]*ArcTanh[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2]))/Sqr
t[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]])/(Sqrt[e]*(c*d - b*e)) + ((5*c*d - 2*b*
e)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/Sqrt[e])/c^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3791 vs. $2(113) = 226$.
time = 0.35, size = 3792, normalized size = 27.28

method	result

risch	$\frac{x\sqrt{e\,x^2+d}}{2c}-\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)\sqrt{e}\,b}{c^2}+\frac{5\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)d}{2c\sqrt{e}}-\frac{\ln\left(\frac{-\frac{2(eb-2cd)}{c}+\frac{2\sqrt{-(eb-c}}{}}{}\right)}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBO
SE)
```

```
[Out] -1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c
*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*(1/5*(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/
e)^2+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/
c)^(5/2)+(-(b*e-c*d)*c*e)^(1/2)/c*(1/8*(2*e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+
2*(-(b*e-c*d)*c*e)^(1/2)/c)/e*(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-
c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(3/2)+3/16*
(-4*e*(b*e-2*c*d)/c+4*(b*e-c*d)/c*e)/e*(1/4*(2*e*(x-(-(b*e-c*d)*c*e)^(1/2)/
c/e)+2*(-(b*e-c*d)*c*e)^(1/2)/c)/e*(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-
(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)+
1/8*(-4*e*(b*e-2*c*d)/c+4*(b*e-c*d)/c*e)/e^(3/2)*ln(((-(b*e-c*d)*c*e)^(1/2)
/c+e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))/e^(1/2)+(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c
/e)^2+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)
/c)^(1/2))))-(b*e-2*c*d)/c*(1/3*(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*
e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(3/2)+(-(
b*e-c*d)*c*e)^(1/2)/c*(1/4*(2*e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-c*d
)*c*e)^(1/2)/c)/e*(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-c*d)*c*e)^(1
/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)+1/8*(-4*e*(b*e-2*
c*d)/c+4*(b*e-c*d)/c*e)/e^(3/2)*ln(((-(b*e-c*d)*c*e)^(1/2)/c+e*(x-(-(b*e-c*
d)*c*e)^(1/2)/c/e))/e^(1/2)+(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-c*
d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)))-(b*e-
2*c*d)/c*((e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x
-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)+(-(b*e-c*d)*c*e)^(1/2)/c*
ln(((-(b*e-c*d)*c*e)^(1/2)/c+e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))/e^(1/2)+(e*(
x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c
*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/e^(1/2)+(b*e-2*c*d)/c/(-(b*e-2*c*d)/c)
^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^
(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(
-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)
```

```
)/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)))))-1/2*e*c/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(
-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*(1/5*(e*(x+1
/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(5/2)-(-d*e)^(1/2)*
(1/8*(2*e*(x+1/e*(-d*e)^(1/2))-2*(-d*e)^(1/2))/e*(e*(x+1/e*(-d*e)^(1/2))^2-
2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(3/2)+3/4*d*(1/4*(2*e*(x+1/e*(-d*e)^(1
/2))-2*(-d*e)^(1/2))/e*(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*
e)^(1/2)))^(1/2)+1/2*d/e^(1/2)*ln((-(-d*e)^(1/2)+e*(x+1/e*(-d*e)^(1/2)))/e^
(1/2)+(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2))
)))+1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*
e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*(1/5*(e*(x+(-(b*e-c*d)*c*e)^(1/2)
/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*
d)/c)^(5/2)-(-(b*e-c*d)*c*e)^(1/2)/c*(1/8*(2*e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/
e)-2*(-(b*e-c*d)*c*e)^(1/2)/c)/e*(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b
*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(3/2)+3/
16*(-4*e*(b*e-2*c*d)/c+4*(b*e-c*d)/c*e)/e*(1/4*(2*e*(x+(-(b*e-c*d)*c*e)^(1/
2)/c/e)-2*(-(b*e-c*d)*c*e)^(1/2)/c)/e*(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2
*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/
2)+1/8*(-4*e*(b*e-2*c*d)/c+4*(b*e-c*d)/c*e)/e^(3/2)*ln((-(-(b*e-c*d)*c*e)^(
1/2)/c+e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))/e^(1/2)+(e*(x+(-(b*e-c*d)*c*e)^(1/
2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*
c*d)/c)^(1/2))))-(b*e-2*c*d)/c*(1/3*(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(
-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(3/2)
-(-(b*e-c*d)*c*e)^(1/2)/c*(1/4*(2*e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-2*(-(b*e
-c*d)*c*e)^(1/2)/c)/e*(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e
)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)+1/8*(-4*e*(b*
e-2*c*d)/c+4*(b*e-c*d)/c*e)/e^(3/2)*ln((-(-(b*e-c*d)*c*e)^(1/2)/c+e*(x+(-(b
*e-c*d)*c*e)^(1/2)/c/e))/e^(1/2)+(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b
*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)))-
(b*e-2*c*d)/c*((e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)
/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)-(-(b*e-c*d)*c*e)^(1/
2)/c*ln((-(-(b*e-c*d)*c*e)^(1/2)/c+e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))/e^(1/2
)+(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-
c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/e^(1/2)+(b*e-2*c*d)/c/(-(b*e-2*c
*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)
*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)
^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)
^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)))))+1/2*e*c/(-d*e)^(1/2)/((-d*e)^(1/
2)*c+(-(b*e-c*d)*c*e)^(1/2))/((-d*e)^(1/2)*c-(-(b*e-c*d)*c*e)^(1/2))*(1/5*(
e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e)^(1/2)*(x-1/e*...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate((x^2*e + d)^(5/2)/(c*x^4*e^2 + b*x^2*e^2 - c*d^2 + b*d*e), x)
```

Fricas [A]
time = 0.66, size = 564, normalized size = 4.06

$$\left[\frac{\left(2\sqrt{x^2e+d}\,cxe-(5\,cd-2\,be)e^{\frac{1}{2}}\log\left(-2\,x^2e+2\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)-(2\,cde-be^2)\sqrt{\frac{2\,cd-be}{cde-be^2}}\log\left(\frac{8\,b^2x^4e^4+c^2d^4+4\,(c^2d^3xe+2\,b^2x^3e^4-(5\,bcdx^3-b^2dx)e^3+(3\,c^2d^2x^3-2\,bcd^2x)e^2)\sqrt{x^2e+d}\sqrt{\frac{2\,cd-be}{cde-be^2}}-8\,(3\,bcdx^4-b^2dx^2)e^3+(17\,c^2d^2x^4-22\,bcd^2x^2+b^2d^2)e^2+2\,(7\,c^2d^3x^2-bcd^3)e}{c^2d^2+(c^2x^4+2\,bcx^2+b^2)e^2-2\,(c^2dx^2+bcd)e}\right)\right)e^{(-1)}}{4\,c^2}, \frac{\left(2\sqrt{x^2e+d}\,cxe-(5\,cd-2\,be)e^{\frac{1}{2}}\log\left(-2\,x^2e+2\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)+2\,(2\,cde-be^2)\sqrt{-\frac{2\,cd-be}{cde-be^2}}\arctan\left(\frac{(2\,bx^2e^2-cd^2-(3\,cdx^2-bd)e)\sqrt{x^2e+d}\sqrt{-\frac{2\,cd-be}{cde-be^2}}}{2\,(bx^3e^2-2\,cd^2x-(2\,cdx^3-bdx)e)}\right)\right)e^{(-1)}}{4\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*(2*sqrt(x^2*e + d)*c*x*e - (5*c*d - 2*b*e)*e^(1/2)*log(-2*x^2*e + 2*sq
rt(x^2*e + d)*x*e^(1/2) - d) - (2*c*d*e - b*e^2)*sqrt((2*c*d - b*e)/(c*d*e
- b*e^2))*log((8*b^2*x^4*e^4 + c^2*d^4 + 4*(c^2*d^3*x*e + 2*b^2*x^3*e^4 - (
5*b*c*d*x^3 - b^2*d*x)*e^3 + (3*c^2*d^2*x^3 - 2*b*c*d^2*x)*e^2)*sqrt(x^2*e
+ d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2)) - 8*(3*b*c*d*x^4 - b^2*d*x^2)*e^3
+ (17*c^2*d^2*x^4 - 22*b*c*d^2*x^2 + b^2*d^2)*e^2 + 2*(7*c^2*d^3*x^2 - b*c*
d^3)*e)/(c^2*d^2 + (c^2*x^4 + 2*b*c*x^2 + b^2)*e^2 - 2*(c^2*d*x^2 + b*c*d)*
e)))*e^(-1)/c^2, 1/4*(2*sqrt(x^2*e + d)*c*x*e - (5*c*d - 2*b*e)*e^(1/2)*log
(-2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(1/2) - d) + 2*(2*c*d*e - b*e^2)*sqrt(-(2
*c*d - b*e)/(c*d*e - b*e^2))*arctan(1/2*(2*b*x^2*e^2 - c*d^2 - (3*c*d*x^2 -
 b*d)*e)*sqrt(x^2*e + d)*sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))/(b*x^3*e^2 -
2*c*d^2*x - (2*c*d*x^3 - b*d*x)*e)))*e^(-1)/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{\frac{3}{2}}}{be-cd+cex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(b*e - c*d + c*e*x**2), x)
```

Giac [A]
time = 3.62, size = 167, normalized size = 1.20

$$-\frac{(5\,cd-2\,be)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{4\,c^2}+\frac{\sqrt{x^2e+d}\,x}{2\,c}-\frac{(4\,c^2d^2-4\,bcde+b^2e^2)\arctan\left(\frac{\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2c-3\,cd+2\,be}{2\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}\right)e^{\left(-\frac{1}{2}\right)}}{\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] -1/4*(5*c*d - 2*b*e)*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^2 + 1/
2*sqrt(x^2*e + d)*x/c - (4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*arctan(1/2*((x*e^
(1/2) - sqrt(x^2*e + d))^2*c - 3*c*d + 2*b*e)/sqrt(-2*c^2*d^2 + 3*b*c*d*e -
 b^2*e^2))*e^(-1/2)/(sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2)*c^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^{5/2}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(5/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] int((d + e*x^2)^(5/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

3.221 $\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=108

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{2cd-be}\,x}{\sqrt{cd-be}\,\sqrt{d+ex^2}}\right)}{c\sqrt{e}\,\sqrt{cd-be}}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(1/2)-arctanh(x*e^(1/2)*(-b*e+2*c*d)
^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))*(-b*e+2*c*d)^(1/2)/c/e^(1/2)/(-b*e
+c*d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {1163, 399, 223, 212, 385, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}-\frac{\sqrt{2cd-be}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\,\sqrt{cd-be}}\right)}{c\sqrt{e}\,\sqrt{cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]/(c*Sqrt[e]) - (Sqrt[2*c*d - b*e]*ArcTa
nh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(c*Sqr
t[e]*Sqrt[c*d - b*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{\sqrt{d+ex^2}}{\frac{-cd^2+bde}{d}+cex^2}\,dx$$

$$= \frac{\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c} - \frac{\left(-cde+\frac{e(-cd^2+bde)}{d}\right)\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{ce}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c} - \frac{\left(-cde+\frac{e(-cd^2+bde)}{d}\right)\text{Subst}\left(\int \ldots\right)}{\ldots}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\sqrt{2cd-be}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{2cd-be}\,x}{\sqrt{cd-be}\,\sqrt{d+ex^2}}\right)}{c\sqrt{e}\,\sqrt{cd-be}}$$

Mathematica [A]
time = 0.19, size = 139, normalized size = 1.29

$$-\frac{\sqrt{2c^2d^2-3bcde+b^2e^2}\,\tanh^{-1}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}\,x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)+(cd-be)\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{c\sqrt{e}\,(cd-be)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -((Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]*ArcTanh[(-(b*e) + c*(d - e*x^2 + S
qrt[e]*x*Sqrt[d + e*x^2]))/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]] + (c*d -
b*e)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(c*Sqrt[e]*(c*d - b*e)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2435 vs. $2(88) = 176$.
time = 0.32, size = 2436, normalized size = 22.56

method	result	size
default	Expression too large to display	2436

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c
*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*(1/3*(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/
e)^2+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/
c)^(3/2)+(-(b*e-c*d)*c*e)^(1/2)/c*(1/4*(2*e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+
2*(-(b*e-c*d)*c*e)^(1/2)/c)/e*(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-
c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)+1/8*(
-4*e*(b*e-2*c*d)/c+4*(b*e-c*d)/c*e)/e^(3/2)*ln(((-(b*e-c*d)*c*e)^(1/2)/c+e*
(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))/e^(1/2)+(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2
+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(
1/2)))-(b*e-2*c*d)/c*((e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-c*d)*c*e
)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)+(-(b*e-c*d)*c
*e)^(1/2)/c*ln(((-(b*e-c*d)*c*e)^(1/2)/c+e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))/
e^(1/2)+(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(
-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/e^(1/2)+(b*e-2*c*d)/c/(-(b
*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*
e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*(e*(x-(-(b*e-c*d)*c*e)^(1/2
)/c/e)^2+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c
*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e))))+1/2*e*c/(-d*e)^(1/2)/((-d*e
)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*
(1/3*(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(3/2)-(
-d*e)^(1/2)*(1/4*(2*e*(x+1/e*(-d*e)^(1/2))-2*(-d*e)^(1/2))/e*(e*(x+1/e*(-d*
e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2)+1/2*d/e^(1/2)*ln((-(
-d*e)^(1/2)+e*(x+1/e*(-d*e)^(1/2)))/e^(1/2)+(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d
*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2))))-1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c
*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(
1/2)*(1/3*(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x
+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(3/2)-(-(b*e-c*d)*c*e)^(1/2)/c*
(1/4*(2*e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-2*(-(b*e-c*d)*c*e)^(1/2)/c)/e*(e*(
x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c
```

```
*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)+1/8*(-4*e*(b*e-2*c*d)/c+4*(b*e-c*d)/c*e
)/e^(3/2)*ln((-(-(b*e-c*d)*c*e)^(1/2)/c+e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))/e
^(1/2)+(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-
(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)))-(b*e-2*c*d)/c*((e*(x+(-(b*
e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/
2)/c/e)-(b*e-2*c*d)/c)^(1/2)-(-(b*e-c*d)*c*e)^(1/2)/c*ln((-(-(b*e-c*d)*c*e)
^(1/2)/c+e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))/e^(1/2)+(e*(x+(-(b*e-c*d)*c*e)^(
1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-
2*c*d)/c)^(1/2))/e^(1/2)+(b*e-2*c*d)/c/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2
*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-
2*c*d)/c)^(1/2)*(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2
)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e
)^(1/2)/c/e))))-1/2*e*c/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2)
)/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*(1/3*(e*(x-1/e*(-d*e)^(1/2))^2+2
*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(3/2)+(-d*e)^(1/2)*(1/4*(2*e*(x-1/e*(-d
*e)^(1/2))+2*(-d*e)^(1/2))/e*(e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e)^(1/2)*(x-1/
e*(-d*e)^(1/2)))^(1/2)+1/2*d/e^(1/2)*ln(((-d*e)^(1/2)+e*(x-1/e*(-d*e)^(1/2)
))/e^(1/2)+(e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(
1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate((x^2*e + d)^(3/2)/(c*x^4*e^2 + b*x^2*e^2 - c*d^2 + b*d*e), x)
```

Fricas [A]
time = 0.41, size = 489, normalized size = 4.53

$$\left[\frac{\left(\sqrt{\frac{2\,cd-be}{cde-be^2}}\,e\log\left(\frac{8\,b^2x^4e^4+c^2d^4-4\,(c^2d^3xe+2\,b^2x^3e^4-(5\,bcdx^3-b^2dx)e^3+(3\,c^2d^2x^3-2\,bcd^2x)e^2)\sqrt{x^2e+d}\sqrt{\frac{2\,cd-be}{cde-be^2}}-8\,(3\,bcdx^4-b^2dx^2)e^3+(17\,c^2d^2x^4-22\,bcd^2x^2+b^2d^2)e^2+2\,(7\,c^2d^3x^2-bcd^3)e}{c^2d^2+(c^2x^4+2\,bcx^2+b^2)e^2-2\,(c^2dx^2+bcd)e}\right)+2\,e^{\frac{1}{2}}\log\left(-2\,x^2e-2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)\right)e^{(-1)}}{4\,c}, \frac{\left(\sqrt{-\frac{2\,cd-be}{cde-be^2}}\arctan\left(\frac{(2\,bx^2e^2-cd^2-(3\,cdx^2-bd)e)\sqrt{x^2e+d}\sqrt{-\frac{2\,cd-be}{cde-be^2}}}{2\,(bx^3e^2-2\,cd^2x-(2\,cdx^3-bdx)e)}\right)e+e^{\frac{1}{2}}\log\left(-2\,x^2e-2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)\right)e^{(-1)}}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*(sqrt((2*c*d - b*e)/(c*d*e - b*e^2))*e*log((8*b^2*x^4*e^4 + c^2*d^4 -
4*(c^2*d^3*x*e + 2*b^2*x^3*e^4 - (5*b*c*d*x^3 - b^2*d*x)*e^3 + (3*c^2*d^2*x
^3 - 2*b*c*d^2*x)*e^2)*sqrt(x^2*e + d)*sqrt((2*c*d - b*e)/(c*d*e - b*e^2))
- 8*(3*b*c*d*x^4 - b^2*d*x^2)*e^3 + (17*c^2*d^2*x^4 - 22*b*c*d^2*x^2 + b^2*
```

```
d^2)*e^2 + 2*(7*c^2*d^3*x^2 - b*c*d^3)*e)/(c^2*d^2 + (c^2*x^4 + 2*b*c*x^2 +
 b^2)*e^2 - 2*(c^2*d*x^2 + b*c*d)*e)) + 2*e^(1/2)*log(-2*x^2*e - 2*sqrt(x^2
*e + d)*x*e^(1/2) - d))*e^(-1)/c, 1/2*(sqrt(-(2*c*d - b*e)/(c*d*e - b*e^2))
*arctan(1/2*(2*b*x^2*e^2 - c*d^2 - (3*c*d*x^2 - b*d)*e)*sqrt(x^2*e + d)*sqr
t(-(2*c*d - b*e)/(c*d*e - b*e^2))/(b*x^3*e^2 - 2*c*d^2*x - (2*c*d*x^3 - b*d
*x)*e))*e + e^(1/2)*log(-2*x^2*e - 2*sqrt(x^2*e + d)*x*e^(1/2) - d))*e^(-1)
/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{be-cd+cex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(b*e - c*d + c*e*x**2), x)
```

Giac [A]
time = 3.53, size = 129, normalized size = 1.19

$$-\frac{(2\,cd-be)\arctan\left(\frac{\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2c-3\,cd+2\,be}{2\,\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}\right)e^{\left(-\frac{1}{2}\right)}}{\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}\,c}-\frac{e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] -(2*c*d - b*e)*arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c - 3*c*d + 2*b*
e)/sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2))*e^(-1/2)/(sqrt(-2*c^2*d^2 + 3*b*
c*d*e - b^2*e^2)*c) - 1/2*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^{3/2}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] int((d + e*x^2)^(3/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

3.222 $\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx$

Optimal. Leaf size=76

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-be}\,x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

```
[Out] -arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/e^(
1/2)/(-b*e+c*d)^(1/2)/(-b*e+2*c*d)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.073$, Rules used = {1163, 385, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\sqrt{2cd-be}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])]/
(Sqrt[e]*Sqrt[c*d - b*e]*Sqrt[2*c*d - b*e]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{-cd^2+bde+be^2x^2+ce^2x^4}\,dx = \int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e(-cd^2+bde)}{d}\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{2cd-be}\,x}{\sqrt{cd-be}\,\sqrt{d+ex^2}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,\sqrt{2cd-be}}$$

Mathematica [A]
time = 0.11, size = 94, normalized size = 1.24

$$-\frac{\tanh^{-1}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}\,x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{\sqrt{e}\,\sqrt{2c^2d^2-3bcde+b^2e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4),x]
```

```
[Out] -(ArcTanh[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2]))/Sqrt[2*c^2*d
^2 - 3*b*c*d*e + b^2*e^2]]/(Sqrt[e]*Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1424 vs. $2(62) = 124$.
time = 0.35, size = 1425, normalized size = 18.75

method	result	size
default	Expression too large to display	1425

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVERBO
SE)
```

```
[Out] 1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c
*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2
+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(
1/2)+(-(b*e-c*d)*c*e)^(1/2)/c*ln(((-(b*e-c*d)*c*e)^(1/2)/c+e*(x-(-(b*e-c*d)
*c*e)^(1/2)/c/e))/e^(1/2)+(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-c*d)
*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/e^(1/2)+
(b*e-2*c*d)/c/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e
```

```
)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*(e*(x-(-(
b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(
1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)))+1/2*e*c/(-
d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-
c*d)*c*e)^(1/2))*((e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1
/2)))^(1/2)-(-d*e)^(1/2)*ln((-(-d*e)^(1/2)+e*(x+1/e*(-d*e)^(1/2)))/e^(1/2)+
(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2))/e^(1/
2))-1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b
*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*((e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/
e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/
c)^(1/2)-(-(b*e-c*d)*c*e)^(1/2)/c*ln((-(-(b*e-c*d)*c*e)^(1/2)/c+e*(x+(-(b*e
-c*d)*c*e)^(1/2)/c/e))/e^(1/2)+(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e
-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/e^(
1/2)+(b*e-2*c*d)/c/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d
)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*(e*(
x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c
*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)))-1/2*e
*c/(-d*e)^(1/2)/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-
(b*e-c*d)*c*e)^(1/2))*((e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e)^(1/2)*(x-1/e*(-d*
e)^(1/2)))^(1/2)+(-d*e)^(1/2)*ln(((-d*e)^(1/2)+e*(x-1/e*(-d*e)^(1/2)))/e^(1
/2)+(e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2))/e
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(x^2*e + d)/(c*x^4*e^2 + b*x^2*e^2 - c*d^2 + b*d*e), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(65) = 130$.
time = 0.38, size = 437, normalized size = 5.75

$$\left[\frac{\log\left(\frac{8\,b^2x^4e^4+c^2d^4+4\,(2\,bx^3e^2-cd^2x-(3\,cdx^3-bdx)e)\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}\,\sqrt{x^2e+d}-8\,(3\,bcdx^4-b^2dx^2)e^3+(17\,c^2d^2x^4-22\,bcd^2x^2+b^2d^2)e^2+2\,(7\,c^2d^3x^2-bcd^3)e}{c^2d^2+(c^2x^4+2\,bcx^2+b^2)e^2-2\,(c^2dx^2+bcd)e}\right)}{4\,\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}}, -\frac{\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\,\arctan\left(\frac{\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\,(2\,bx^2e^2-cd^2-(3\,cdx^2-bd)e)\sqrt{x^2e+d}}{2\,(2\,c^2d^3xe+b^2x^3e^4-(3\,bcdx^3-b^2dx)e^3+(2\,c^2d^2x^3-3\,bcd^2x)e^2)}\right)}{2\,(2\,c^2d^2e-3\,bcde^2+b^2e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="f
ricas")
```

```
[Out] [1/4*log((8*b^2*x^4*e^4 + c^2*d^4 + 4*(2*b*x^3*e^2 - c*d^2*x - (3*c*d*x^3 -
 b*d*x)*e)*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(x^2*e + d) - 8*(3
```

```
*b*c*d*x^4 - b^2*d*x^2)*e^3 + (17*c^2*d^2*x^4 - 22*b*c*d^2*x^2 + b^2*d^2)*e
^2 + 2*(7*c^2*d^3*x^2 - b*c*d^3)*e)/(c^2*d^2 + (c^2*x^4 + 2*b*c*x^2 + b^2)*
e^2 - 2*(c^2*d*x^2 + b*c*d)*e))/sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3),
-1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*arctan(1/2*sqrt(-2*c^2*d^2*
e + 3*b*c*d*e^2 - b^2*e^3)*(2*b*x^2*e^2 - c*d^2 - (3*c*d*x^2 - b*d)*e)*sqrt
(x^2*e + d)/(2*c^2*d^3*x*e + b^2*x^3*e^4 - (3*b*c*d*x^3 - b^2*d*x)*e^3 + (2
*c^2*d^2*x^3 - 3*b*c*d^2*x)*e^2))/(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\,(be-cd+cex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x**2)*(b*e - c*d + c*e*x**2)), x)
```

Giac [A]
time = 4.41, size = 88, normalized size = 1.16

$$-\frac{\arctan\left(\frac{\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2c-3\,cd+2\,be}{2\,\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}\right)e^{\left(-\frac{1}{2}\right)}}{\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm="g
iac")
```

```
[Out] -arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c - 3*c*d + 2*b*e)/sqrt(-2*c^2
*d^2 + 3*b*c*d*e - b^2*e^2))*e^(-1/2)/sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,x^2+d}}{-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e),x)
```

```
[Out] int((d + e*x^2)^(1/2)/(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e), x)
```

3.223 $\int \frac{1}{\sqrt{d+ex^2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx$

Optimal. Leaf size=106

$$-\frac{x}{d(2cd-be)\sqrt{d+ex^2}}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{2cd-be}\,x}{\sqrt{cd-be}\,\sqrt{d+ex^2}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^{3/2}}$$

```
[Out] -c*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/(
-b*e+2*c*d)^(3/2)/e^(1/2)/(-b*e+c*d)^(1/2)-x/d/(-b*e+2*c*d)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {1163, 390, 385, 214}

$$-\frac{x}{d\sqrt{d+ex^2}\,(2cd-be)}-\frac{c\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\,\sqrt{cd-be}}\right)}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -(x/(d*(2*c*d - b*e)*Sqrt[d + e*x^2])) - (c*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b
*e]*x)/(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d
- b*e)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
```

```
tQ[q, -1]) && NeQ[p, -1]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\,(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx = \int \frac{1}{(d+ex^2)^{3/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx$$

$$= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} + \frac{c\int \frac{1}{\sqrt{d+ex^2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx}{2cd-be}$$

$$= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} + \frac{c\text{Subst}\left(\int \frac{1}{\frac{-cd^2+bde}{d}-\left(-cde+\frac{e\left(-cd^2\right.}{d}\right.}\right.}{2cd-}$$

$$= -\frac{x}{d(2cd-be)\sqrt{d+ex^2}} - \frac{c\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{2cd-be}\,x}{\sqrt{cd-be}\,\sqrt{d+ex^2}}\right.}{\sqrt{e}\,\sqrt{cd-be}\,(2cd-be)^{3/2}}$$

Mathematica [A]
time = 0.26, size = 172, normalized size = 1.62

$$-\frac{\sqrt{e}\,(2c^2d^2-3bcde+b^2e^2)\,x+cd\sqrt{2c^2d^2-3bcde+b^2e^2}\,\sqrt{d+ex^2}\,\tanh^{-1}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}\,x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{d\sqrt{e}\,(cd-be)(-2cd+be)^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -((Sqrt[e]*(2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*x + c*d*Sqrt[2*c^2*d^2 - 3*b*c
*d*e + b^2*e^2]*Sqrt[d + e*x^2]*ArcTanh[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*x*
Sqrt[d + e*x^2]))/Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]])/(d*Sqrt[e]*(c*d -
 b*e)*(-2*c*d + b*e)^2*Sqrt[d + e*x^2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 770 vs. 2(90) = 180.
time = 0.33, size = 771, normalized size = 7.27

method	result
default	$-\frac{e\,c^2\ln\left(\frac{-\frac{2(eb-2cd)}{c}+\frac{2\sqrt{-(eb-cd)\,ce}\left(x-\frac{\sqrt{-(eb-cd)\,ce}}{ce}\right)}{c}+2\sqrt{-\frac{eb-2cd}{c}}\sqrt{e\left(x-\frac{\sqrt{-(eb-cd)}}{ce}\right.}}{x-\frac{\sqrt{-(eb-cd)\,ce}}{ce}}\right.}{2\left(\sqrt{-de}\,c+\sqrt{-(eb-cd)\,ce}\right)\left(-\sqrt{-de}\,c+\sqrt{-(eb-cd)\,ce}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVER
BOSE)
```

```
[Out] -1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-
c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-
2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e
-2*c*d)/c)^(1/2)*(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-c*d)*c*e)^(1/
2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2))/(x-(-(b*e-c*d)*c*
e)^(1/2)/c/e))-1/2*c/d/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/
2)*c+(-(b*e-c*d)*c*e)^(1/2))/(x+1/e*(-d*e)^(1/2))*(e*(x+1/e*(-d*e)^(1/2))^2
-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))^(1/2)+1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*
e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e
)^(1/2)/(-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2
)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*(e*(x+(-(b*e-c*
d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c
/e)-(b*e-2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e))-1/2*c/d/((-d*e)^(
1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(x-
1/e*(-d*e)^(1/2))*(e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e)^(1/2)*(x-1/e*(-d*e)^(1
/2)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((c*x^4*e^2 + b*x^2*e^2 - c*d^2 + b*d*e)*sqrt(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(95) = 190.

time = 0.46, size = 717, normalized size = 6.76

$$\left[-\frac{\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}\,(cdx^2e+cd^2)\log\left(\frac{8\,b^2x^4e^4+c^2d^4-4\,(2\,bx^3e^2-cd^2x-(3\,cdx^3-bdx)e)\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}\,\sqrt{x^2e+d}-8\,(3\,bcdx^4-b^2dx^2)e^3+(17\,c^2d^2x^4-22\,bcd^2x^2+b^2d^2)e^2+2\,(7\,c^2d^3x^2-bcd^3)e}{c^2d^2+(c^2x^4+2\,bcx^2+b^2)e^2-2\,(c^2dx^2+bcd)e}\right)+4\,(2\,c^2d^2xe-3\,bcdxe^2+b^2xe^3)\sqrt{x^2e+d}}{4\,(4\,c^3d^5e-b^3dx^2e^5+(5\,b^2cd^2x^2-b^3d^2)e^4-(8\,bc^2d^3x^2-5\,b^2cd^3)e^3+4\,(c^3d^4x^2-2\,bc^2d^4)e^2)},-\frac{\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\,(cdx^2e+cd^2)\arctan\left(\frac{\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\,(2\,bx^2e^2-cd^2-(3\,cdx^2-bd)e)\sqrt{x^2e+d}}{2\,(2\,c^2d^3xe+b^2x^3e^4-(3\,bcdx^3-b^2dx)e^3+(2\,c^2d^2x^3-3\,bcd^2x)e^2)}\right)+2\,(2\,c^2d^2xe-3\,bcdxe^2+b^2xe^3)\sqrt{x^2e+d}}{2\,(4\,c^3d^5e-b^3dx^2e^5+(5\,b^2cd^2x^2-b^3d^2)e^4-(8\,bc^2d^3x^2-5\,b^2cd^3)e^3+4\,(c^3d^4x^2-2\,bc^2d^4)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"fricas")
```

```
[Out] [-1/4*(sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*(c*d*x^2*e + c*d^2)*log((8
*b^2*x^4*e^4 + c^2*d^4 - 4*(2*b*x^3*e^2 - c*d^2*x - (3*c*d*x^3 - b*d*x)*e)*
sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(x^2*e + d) - 8*(3*b*c*d*x^4
- b^2*d*x^2)*e^3 + (17*c^2*d^2*x^4 - 22*b*c*d^2*x^2 + b^2*d^2)*e^2 + 2*(7*c
^2*d^3*x^2 - b*c*d^3)*e)/(c^2*d^2 + (c^2*x^4 + 2*b*c*x^2 + b^2)*e^2 - 2*(c^
2*d*x^2 + b*c*d)*e)) + 4*(2*c^2*d^2*x*e - 3*b*c*d*x*e^2 + b^2*x*e^3)*sqrt(x
^2*e + d))/(4*c^3*d^5*e - b^3*d*x^2*e^5 + (5*b^2*c*d^2*x^2 - b^3*d^2)*e^4 -
 (8*b*c^2*d^3*x^2 - 5*b^2*c*d^3)*e^3 + 4*(c^3*d^4*x^2 - 2*b*c^2*d^4)*e^2),
-1/2*(sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(c*d*x^2*e + c*d^2)*arctan
(1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(2*b*x^2*e^2 - c*d^2 - (3*c
*d*x^2 - b*d)*e)*sqrt(x^2*e + d)/(2*c^2*d^3*x*e + b^2*x^3*e^4 - (3*b*c*d*x^
3 - b^2*d*x)*e^3 + (2*c^2*d^2*x^3 - 3*b*c*d^2*x)*e^2)) + 2*(2*c^2*d^2*x*e -
 3*b*c*d*x*e^2 + b^2*x*e^3)*sqrt(x^2*e + d))/(4*c^3*d^5*e - b^3*d*x^2*e^5 +
 (5*b^2*c*d^2*x^2 - b^3*d^2)*e^4 - (8*b*c^2*d^3*x^2 - 5*b^2*c*d^3)*e^3 + 4*
(c^3*d^4*x^2 - 2*b*c^2*d^4)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^{\frac{3}{2}}\,(be-cd+cex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(1/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/((d + e*x**2)**(3/2)*(b*e - c*d + c*e*x**2)), x)
```

Giac [A]
time = 3.85, size = 132, normalized size = 1.25

$$-\frac{c\arctan\left(\frac{\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2c-3\,cd+2\,be}{2\,\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}\right)e^{\frac{1}{2}}}{\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}\,(2\,cde-be^2)}-\frac{x}{(2\,cd^2-bde)\sqrt{x^2e+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(1/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"giac")
```

```
[Out] -c*arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c - 3*c*d + 2*b*e)/sqrt(-2*c
^2*d^2 + 3*b*c*d*e - b^2*e^2))*e^(1/2)/(sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e
^2)*(2*c*d*e - b*e^2)) - x/((2*c*d^2 - b*d*e)*sqrt(x^2*e + d))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{e\,x^2+d}\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^(1/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
[Out] int(1/((d + e*x^2)^(1/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)), x)
```

3.224 $\int \frac{1}{(d+ex^2)^{3/2}(-cd^2+bde+be^2x^2+ce^2x^4)}\,dx$

Optimal. Leaf size=149

$$-\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}}-\frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{2cd-be}\,x}{\sqrt{cd-be}\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{cd-be}\,(2cd-be)^{5/2}}$$

```
[Out] -1/3*x/d/(-b*e+2*c*d)/(e*x^2+d)^(3/2)-c^2*arctanh(x*e^(1/2)*(-b*e+2*c*d)^(1
/2)/(-b*e+c*d)^(1/2)/(e*x^2+d)^(1/2))/(-b*e+2*c*d)^(5/2)/e^(1/2)/(-b*e+c*d)
^(1/2)-1/3*(-2*b*e+7*c*d)*x/d^2/(-b*e+2*c*d)^2/(e*x^2+d)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {1163, 425, 541, 12, 385, 214}

$$-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{e}\,x\sqrt{2cd-be}}{\sqrt{d+ex^2}\sqrt{cd-be}}\right)}{\sqrt{e}\sqrt{cd-be}\,(2cd-be)^{5/2}}-\frac{x(7cd-2be)}{3d^2\sqrt{d+ex^2}\,(2cd-be)^2}-\frac{x}{3d\left(d+ex^2\right)^{3/2}(2cd-be)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),x]
```

```
[Out] -1/3*x/(d*(2*c*d - b*e)*(d + e*x^2)^(3/2)) - ((7*c*d - 2*b*e)*x)/(3*d^2*(2*
c*d - b*e)^2*Sqrt[d + e*x^2]) - (c^2*ArcTanh[(Sqrt[e]*Sqrt[2*c*d - b*e]*x)/
(Sqrt[c*d - b*e]*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[c*d - b*e]*(2*c*d - b*e)^
(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 1163

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[(d + e*x^2)^(p + q)*(a/d + (c/e)*x^2)^p, x] /; FreeQ[{a,
b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^{3/2}\left(-cd^2+bde+be^2x^2+ce^2x^4\right)}\,dx = \int \frac{1}{(d+ex^2)^{5/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}\,dx$$

$$= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}} + \frac{\int \frac{e(5cd-2be)-2ce^2x^2}{(d+ex^2)^{3/2}\left(\frac{-cd^2+bde}{d}+cex^2\right)}dx}{3de(2cd-be)}$$

$$= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}}$$

$$= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}}$$

$$= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}}$$

$$= -\frac{x}{3d(2cd-be)\left(d+ex^2\right)^{3/2}} - \frac{(7cd-2be)x}{3d^2(2cd-be)^2\sqrt{d+ex^2}}$$

Mathematica [A]
time = 0.43, size = 174, normalized size = 1.17

$$-\frac{\frac{(-2cd+be)x\left(-be\left(3d+2ex^2\right)+cd\left(9d+7ex^2\right)\right)}{d^2(d+ex^2)^{3/2}} + \frac{3c^2\sqrt{2c^2d^2-3bcde+b^2e^2}\tanh^{-1}\left(\frac{-be+c\left(d-ex^2+\sqrt{e}\,x\sqrt{d+ex^2}\right)}{\sqrt{2c^2d^2-3bcde+b^2e^2}}\right)}{\sqrt{e}\,(-cd+be)}}{3(-2cd+be)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(-(c*d^2) + b*d*e + b*e^2*x^2 + c*e^2*x^4)),
x]
```

```
[Out] -1/3*(((-2*c*d + b*e)*x*(-(b*e*(3*d + 2*e*x^2)) + c*d*(9*d + 7*e*x^2)))/(d^
2*(d + e*x^2)^(3/2)) + (3*c^2*Sqrt[2*c^2*d^2 - 3*b*c*d*e + b^2*e^2]*ArcTanh
[(-(b*e) + c*(d - e*x^2 + Sqrt[e]*x*Sqrt[d + e*x^2]))/Sqrt[2*c^2*d^2 - 3*b*
c*d*e + b^2*e^2]])/(Sqrt[e]*(-(c*d) + b*e)))/(-2*c*d + b*e)^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1550 vs. $2(127) = 254$.
time = 0.29, size = 1551, normalized size = 10.41

method	result	size
default	Expression too large to display	1551

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c
*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*(-1/(b*e-2*c*d)*c/(e*(x-(-(b*e-c*d)*
c*e)^(1/2)/c/e)^2+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)
-(b*e-2*c*d)/c)^(1/2)+2*(-(b*e-c*d)*c*e)^(1/2)/(b*e-2*c*d)*(2*e*(x-(-(b*e-c
*d)*c*e)^(1/2)/c/e)+2*(-(b*e-c*d)*c*e)^(1/2)/c)/(-4*e*(b*e-2*c*d)/c+4*(b*e-
c*d)/c*e)/(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)^2+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x
-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)+1/(b*e-2*c*d)*c/(-(b*e-2*
c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d
)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*(e*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e
)^2+2*(-(b*e-c*d)*c*e)^(1/2)/c*(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c
)^(1/2))/(x-(-(b*e-c*d)*c*e)^(1/2)/c/e)))+1/2*e*c/(-d*e)^(1/2)/((-d*e)^(1/2
)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))*(1/3/(
-d*e)^(1/2)/(x+1/e*(-d*e)^(1/2))/(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(
x+1/e*(-d*e)^(1/2)))^(1/2)+1/3/(-d*e)^(1/2)*(2*e*(x+1/e*(-d*e)^(1/2))-2*(-d
*e)^(1/2))/d/(e*(x+1/e*(-d*e)^(1/2))^2-2*(-d*e)^(1/2)*(x+1/e*(-d*e)^(1/2)))
^(1/2))-1/2*e*c^2/((-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+
(-(b*e-c*d)*c*e)^(1/2))/(-(b*e-c*d)*c*e)^(1/2)*(-1/(b*e-2*c*d)*c/(e*(x+(-(b
*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1
/2)/c/e)-(b*e-2*c*d)/c)^(1/2)-2*(-(b*e-c*d)*c*e)^(1/2)/(b*e-2*c*d)*(2*e*(x+
(-(b*e-c*d)*c*e)^(1/2)/c/e)-2*(-(b*e-c*d)*c*e)^(1/2)/c)/(-4*e*(b*e-2*c*d)/c
+4*(b*e-c*d)/c*e)/(e*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1
/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-2*c*d)/c)^(1/2)+1/(b*e-2*c*d)*c/(
-(b*e-2*c*d)/c)^(1/2)*ln((-2*(b*e-2*c*d)/c-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-
(b*e-c*d)*c*e)^(1/2)/c/e)+2*(-(b*e-2*c*d)/c)^(1/2)*(e*(x+(-(b*e-c*d)*c*e)^(
1/2)/c/e)^2-2*(-(b*e-c*d)*c*e)^(1/2)/c*(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)-(b*e-
2*c*d)/c)^(1/2))/(x+(-(b*e-c*d)*c*e)^(1/2)/c/e)))-1/2*e*c/(-d*e)^(1/2)/((-d
*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2))/(-(-d*e)^(1/2)*c+(-(b*e-c*d)*c*e)^(1/2)
)*(-1/3/(-d*e)^(1/2)/(x-1/e*(-d*e)^(1/2))/(e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e
)^(1/2)*(x-1/e*(-d*e)^(1/2)))^(1/2)-1/3/(-d*e)^(1/2)*(2*e*(x-1/e*(-d*e)^(1/
2))+2*(-d*e)^(1/2))/d/(e*(x-1/e*(-d*e)^(1/2))^2+2*(-d*e)^(1/2)*(x-1/e*(-d*e
)^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/((c*x^4*e^2 + b*x^2*e^2 - c*d^2 + b*d*e)*(x^2*e + d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(135) = 270$.
time = 0.63, size = 1085, normalized size = 7.28

$$\left[\frac{3\,(c^2d^2x^4e^2+2\,c^2d^3x^2e+c^2d^4)\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}\,\log\left(\frac{8\,b^2x^4e^4+c^2d^4+4\,(2\,bx^3e^2-cd^2x-(3\,cdx^3-bdx)e)\sqrt{2\,c^2d^2e-3\,bcde^2+b^2e^3}\,\sqrt{x^2e+d}-8\,(3\,bcdx^4-b^2dx^2)e^3+(17\,c^2d^2x^4-22\,bcd^2x^2+b^2d^2)e^2+2\,(7\,c^2d^3x^2-bcd^3)e}{c^2d^2+(c^2x^4+2\,bcx^2+b^2)e^2-2\,(c^2dx^2+bcd)e}\right)-4\,(18\,c^3d^4xe-2\,b^3x^3e^5+(13\,b^2cdx^3-3\,b^3dx)e^4-(25\,bc^2d^2x^3-18\,b^2cd^2x)e^3+(14\,c^3d^3x^3-33\,bc^2d^3x)e^2)\sqrt{x^2e+d}}{12\,(8\,c^4d^8e+b^4d^2x^4e^7-(7\,b^3cd^3x^4-2\,b^4d^3x^2)e^6+(18\,b^2c^2d^4x^4-14\,b^3cd^4x^2+b^4d^4)e^5-(20\,bc^3d^5x^4-36\,b^2c^2d^5x^2+7\,b^3cd^5)e^4+2\,(4\,c^4d^6x^4-20\,bc^3d^6x^2+9\,b^2c^2d^6)e^3+4\,(4\,c^4d^7x^2-5\,bc^3d^7)e^2)}, -\frac{3\,(c^2d^2x^4e^2+2\,c^2d^3x^2e+c^2d^4)\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\,\arctan\left(\frac{\sqrt{-2\,c^2d^2e+3\,bcde^2-b^2e^3}\,(2\,bx^2e^2-cd^2-(3\,cdx^2-bd)e)\sqrt{x^2e+d}}{2\,(2\,c^2d^3xe+b^2x^3e^4-(3\,bcdx^3-b^2dx)e^3+(2\,c^2d^2x^3-3\,bcd^2x)e^2)}\right)+2\,(18\,c^3d^4xe-2\,b^3x^3e^5+(13\,b^2cdx^3-3\,b^3dx)e^4-(25\,bc^2d^2x^3-18\,b^2cd^2x)e^3+(14\,c^3d^3x^3-33\,bc^2d^3x)e^2)\sqrt{x^2e+d}}{6\,(8\,c^4d^8e+b^4d^2x^4e^7-(7\,b^3cd^3x^4-2\,b^4d^3x^2)e^6+(18\,b^2c^2d^4x^4-14\,b^3cd^4x^2+b^4d^4)e^5-(20\,bc^3d^5x^4-36\,b^2c^2d^5x^2+7\,b^3cd^5)e^4+2\,(4\,c^4d^6x^4-20\,bc^3d^6x^2+9\,b^2c^2d^6)e^3+4\,(4\,c^4d^7x^2-5\,bc^3d^7)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"fricas")
```

```
[Out] [1/12*(3*(c^2*d^2*x^4*e^2 + 2*c^2*d^3*x^2*e + c^2*d^4)*sqrt(2*c^2*d^2*e - 3
*b*c*d*e^2 + b^2*e^3)*log((8*b^2*x^4*e^4 + c^2*d^4 + 4*(2*b*x^3*e^2 - c*d^2
*x - (3*c*d*x^3 - b*d*x)*e)*sqrt(2*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*sqrt(
x^2*e + d) - 8*(3*b*c*d*x^4 - b^2*d*x^2)*e^3 + (17*c^2*d^2*x^4 - 22*b*c*d^2
*x^2 + b^2*d^2)*e^2 + 2*(7*c^2*d^3*x^2 - b*c*d^3)*e)/(c^2*d^2 + (c^2*x^4 +
2*b*c*x^2 + b^2)*e^2 - 2*(c^2*d*x^2 + b*c*d)*e)) - 4*(18*c^3*d^4*x*e - 2*b^
3*x^3*e^5 + (13*b^2*c*d*x^3 - 3*b^3*d*x)*e^4 - (25*b*c^2*d^2*x^3 - 18*b^2*c
*d^2*x)*e^3 + (14*c^3*d^3*x^3 - 33*b*c^2*d^3*x)*e^2)*sqrt(x^2*e + d))/(8*c^
4*d^8*e + b^4*d^2*x^4*e^7 - (7*b^3*c*d^3*x^4 - 2*b^4*d^3*x^2)*e^6 + (18*b^2
*c^2*d^4*x^4 - 14*b^3*c*d^4*x^2 + b^4*d^4)*e^5 - (20*b*c^3*d^5*x^4 - 36*b^2
*c^2*d^5*x^2 + 7*b^3*c*d^5)*e^4 + 2*(4*c^4*d^6*x^4 - 20*b*c^3*d^6*x^2 + 9*b
^2*c^2*d^6)*e^3 + 4*(4*c^4*d^7*x^2 - 5*b*c^3*d^7)*e^2), -1/6*(3*(c^2*d^2*x^
4*e^2 + 2*c^2*d^3*x^2*e + c^2*d^4)*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^
3)*arctan(1/2*sqrt(-2*c^2*d^2*e + 3*b*c*d*e^2 - b^2*e^3)*(2*b*x^2*e^2 - c*d
^2 - (3*c*d*x^2 - b*d)*e)*sqrt(x^2*e + d)/(2*c^2*d^3*x*e + b^2*x^3*e^4 - (3
*b*c*d*x^3 - b^2*d*x)*e^3 + (2*c^2*d^2*x^3 - 3*b*c*d^2*x)*e^2)) + 2*(18*c^3
*d^4*x*e - 2*b^3*x^3*e^5 + (13*b^2*c*d*x^3 - 3*b^3*d*x)*e^4 - (25*b*c^2*d^2
*x^3 - 18*b^2*c*d^2*x)*e^3 + (14*c^3*d^3*x^3 - 33*b*c^2*d^3*x)*e^2)*sqrt(x^
2*e + d))/(8*c^4*d^8*e + b^4*d^2*x^4*e^7 - (7*b^3*c*d^3*x^4 - 2*b^4*d^3*x^2
)*e^6 + (18*b^2*c^2*d^4*x^4 - 14*b^3*c*d^4*x^2 + b^4*d^4)*e^5 - (20*b*c^3*d
^5*x^4 - 36*b^2*c^2*d^5*x^2 + 7*b^3*c*d^5)*e^4 + 2*(4*c^4*d^6*x^4 - 20*b*c^
3*d^6*x^2 + 9*b^2*c^2*d^6)*e^3 + 4*(4*c^4*d^7*x^2 - 5*b*c^3*d^7)*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^{\frac{5}{2}}\,(be-cd+cex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(c*e**2*x**4+b*e**2*x**2+b*d*e-c*d**2),x)
```

```
[Out] Integral(1/((d + e*x**2)**(5/2)*(b*e - c*d + c*e*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(135) = 270.
time = 3.81, size = 331, normalized size = 2.22

$$-\frac{c^2 \arctan\left(\frac{\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2c-3\,cd+2\,be}{2\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}\right)e^{\frac{1}{2}}}{(4\,c^2d^2e-4\,bcde^2+b^2e^3)\sqrt{-2\,c^2d^2+3\,bcde-b^2e^2}}-\frac{\left(\frac{(28\,c^3d^3e^2-36\,bc^2d^2e^3+15\,b^2cde^4-2\,b^3e^5)x^2}{16\,c^4d^6e-32\,bc^3d^5e^2+24\,b^2c^2d^4e^3-8\,b^3cd^3e^4+b^4d^2e^5}+\frac{3\,(12\,c^3d^4e-16\,bc^2d^3e^2+7\,b^2cd^2e^3-b^3de^4)}{16\,c^4d^6e-32\,bc^3d^5e^2+24\,b^2c^2d^4e^3-8\,b^3cd^3e^4+b^4d^2e^5}\right)x}{3\,(x^2e+d)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*e^2*x^4+b*e^2*x^2+b*d*e-c*d^2),x, algorithm=
"giac")
```

```
[Out] -c^2*arctan(1/2*((x*e^(1/2) - sqrt(x^2*e + d))^2*c - 3*c*d + 2*b*e)/sqrt(-2
*c^2*d^2 + 3*b*c*d*e - b^2*e^2))*e^(1/2)/((4*c^2*d^2*e - 4*b*c*d*e^2 + b^2*
e^3)*sqrt(-2*c^2*d^2 + 3*b*c*d*e - b^2*e^2)) - 1/3*((28*c^3*d^3*e^2 - 36*b*
c^2*d^2*e^3 + 15*b^2*c*d*e^4 - 2*b^3*e^5)*x^2/(16*c^4*d^6*e - 32*b*c^3*d^5*
e^2 + 24*b^2*c^2*d^4*e^3 - 8*b^3*c*d^3*e^4 + b^4*d^2*e^5) + 3*(12*c^3*d^4*e
 - 16*b*c^2*d^3*e^2 + 7*b^2*c*d^2*e^3 - b^3*d*e^4)/(16*c^4*d^6*e - 32*b*c^3
*d^5*e^2 + 24*b^2*c^2*d^4*e^3 - 8*b^3*c*d^3*e^4 + b^4*d^2*e^5))*x/(x^2*e +
d)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(e\,x^2+d\right)}^{3/2}\,\left(-c\,d^2+b\,d\,e+c\,e^2\,x^4+b\,e^2\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^(3/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)),x)
```

```
[Out] int(1/((d + e*x^2)^(3/2)*(b*e^2*x^2 - c*d^2 + c*e^2*x^4 + b*d*e)), x)
```

3.225 $\int (1+x^2)^3 \sqrt{1+x^2+x^4}\, dx$

Optimal. Leaf size=183

$$\frac{26x\sqrt{1+x^2+x^4}}{45\,(1+x^2)}+\frac{2}{45}x(7+6x^2)\sqrt{1+x^2+x^4}+\frac{1}{3}x(1+x^2+x^4)^{3/2}+\frac{1}{9}x^3(1+x^2+x^4)^{3/2}-\frac{26(1+x^2)\sqrt{}}{4}$$

```
[Out] 1/3*x*(x^4+x^2+1)^(3/2)+1/9*x^3*(x^4+x^2+1)^(3/2)+26/45*x*(x^4+x^2+1)^(1/2)
/(x^2+1)+2/45*x*(6*x^2+7)*(x^4+x^2+1)^(1/2)-26/45*(x^2+1)*(cos(2*arctan(x))
^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^
2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+7/15*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos
(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)
/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1220, 1693, 1190, 1211, 1117, 1209}

$$\frac{7(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}}-\frac{26(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{45\sqrt{x^4+x^2+1}}+\frac{1}{3}(x^4+x^2+1)^{3/2}x+\frac{2}{45}(6x^2+7)\sqrt{x^4+x^2+1}\,x+\frac{26\sqrt{x^4+x^2+1}\,x}{45\,(x^2+1)}+\frac{1}{9}(x^4+x^2+1)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (26*x*Sqrt[1 + x^2 + x^4])/(45*(1 + x^2)) + (2*x*(7 + 6*x^2)*Sqrt[1 + x^2 +
 x^4])/45 + (x*(1 + x^2 + x^4)^(3/2))/3 + (x^3*(1 + x^2 + x^4)^(3/2))/9 - (
26*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])
/(45*Sqrt[1 + x^2 + x^4]) + (7*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*
EllipticF[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
```

```
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(1+x^2\right)^3 \sqrt{1+x^2+x^4}\,dx = \frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2} + \frac{1}{9}\int \sqrt{1+x^2+x^4}\left(9+24x^2+21x^4\right)\,dx$$

$$= \frac{1}{3}x\left(1+x^2+x^4\right)^{3/2} + \frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2} + \frac{1}{63}\int \left(42+84x^2\right)\sqrt{1+x^2+$$

$$= \frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4} + \frac{1}{3}x\left(1+x^2+x^4\right)^{3/2} + \frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}$$

$$= \frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4} + \frac{1}{3}x\left(1+x^2+x^4\right)^{3/2} + \frac{1}{9}x^3\left(1+x^2+x^4\right)^{3/2}$$

$$= \frac{26x\sqrt{1+x^2+x^4}}{45\left(1+x^2\right)} + \frac{2}{45}x\left(7+6x^2\right)\sqrt{1+x^2+x^4} + \frac{1}{3}x\left(1+x^2+x^4\right)^{3/2} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.95, size = 169, normalized size = 0.92

$$\frac{x(29+61x^2+81x^4+57x^6+25x^8+5x^{10})+26\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+2(-1)^{5/6}\left(9i+4\sqrt{3}\right)\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)}{45\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^3*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(29 + 61*x^2 + 81*x^4 + 57*x^6 + 25*x^8 + 5*x^10) + 26*(-1)^(1/3)*Sqrt[1
 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*
x], (-1)^(2/3)] + 2*(-1)^(5/6)*(9*I + 4*Sqrt[3])*Sqrt[1 + (-1)^(1/3)*x^2]*S
qrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(45
*Sqrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 263, normalized size = 1.44

method	result
risch	$\frac{x(5x^6+20x^4+32x^2+29)\sqrt{x^4+x^2+1}}{45} - \frac{104\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\left(\text{EllipticF}\right.}{45\sqrt{-2+2}}$
default	$\frac{x^7\sqrt{x^4+x^2+1}}{9} + \frac{4x^5\sqrt{x^4+x^2+1}}{9} + \frac{32x^3\sqrt{x^4+x^2+1}}{45} + \frac{29x\sqrt{x^4+x^2+1}}{45} + \frac{32\sqrt{1-\left(-\frac{1}{2}+\right.}}{}$

elliptic	$\frac{x^7\sqrt{x^4+x^2+1}}{9}+\frac{4x^5\sqrt{x^4+x^2+1}}{9}+\frac{32x^3\sqrt{x^4+x^2+1}}{45}+\frac{29x\sqrt{x^4+x^2+1}}{45}+\frac{32\sqrt{1-\left(-\frac{1}{2}\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^3*(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*x^7*(x^4+x^2+1)^(1/2)+4/9*x^5*(x^4+x^2+1)^(1/2)+32/45*x^3*(x^4+x^2+1)^(
1/2)+29/45*x*(x^4+x^2+1)^(1/2)+32/45/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*
3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*El
lipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-104/45/(-2
+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/
2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(
1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**3*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x^2+1\right)}^3\,\sqrt{x^4+x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2), x)
```

3.226 $\int (1+x^2)^2 \sqrt{1+x^2+x^4}\, dx$

Optimal. Leaf size=164

$$\frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)}+\frac{2}{21}x(4+3x^2)\sqrt{1+x^2+x^4}+\frac{1}{7}x(1+x^2+x^4)^{3/2}-\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}\right.}{3\sqrt{1+x^2+x^4}}$$

```
[Out] 1/7*x*(x^4+x^2+1)^(3/2)+2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+2/21*x*(3*x^2+4)*(x
^4+x^2+1)^(1/2)-2/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Ell
ipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2
)+4/7*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*a
rctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1190, 1211, 1117, 1209}

$$\frac{4(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,\middle|\,\frac{1}{4}\right)}{7\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,\middle|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}+\frac{1}{7}x(x^4+x^2+1)^{3/2}+\frac{2}{21}x(3x^2+4)\sqrt{x^4+x^2+1}+\frac{2x\sqrt{x^4+x^2+1}}{3(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (2*x*(4 + 3*x^2)*Sqrt[1 + x^2 + x
^4])/21 + (x*(1 + x^2 + x^4)^(3/2))/7 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(
1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (4*(1 +
x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(7*Sqrt
[1 + x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int (1+x^2)^2\sqrt{1+x^2+x^4}\,dx = \frac{1}{7}x(1+x^2+x^4)^{3/2}+\frac{1}{7}\int (6+10x^2)\sqrt{1+x^2+x^4}\,dx$$

$$= \frac{2}{21}x(4+3x^2)\sqrt{1+x^2+x^4}+\frac{1}{7}x(1+x^2+x^4)^{3/2}+\frac{1}{105}\int \frac{50+70x^2}{\sqrt{1+x^2+x^4}}$$

$$= \frac{2}{21}x(4+3x^2)\sqrt{1+x^2+x^4}+\frac{1}{7}x(1+x^2+x^4)^{3/2}-\frac{2}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,d$$

$$= \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)}+\frac{2}{21}x(4+3x^2)\sqrt{1+x^2+x^4}+\frac{1}{7}x(1+x^2+x^4)^{3/2}-$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.52, size = 162, normalized size = 0.99

$$\frac{x(11+20x^2+23x^4+12x^6+3x^8)+14\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+2\sqrt[3]{-1}\left(-7+5\sqrt[3]{-1}\right)\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)}{21\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(11 + 20*x^2 + 23*x^4 + 12*x^6 + 3*x^8) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(1
/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(
2/3)] + 2*(-1)^(1/3)*(-7 + 5*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
(-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(21*Sqrt[1
+ x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 248, normalized size = 1.51

method	result
risch	$\frac{x(3x^4+9x^2+11)\sqrt{x^4+x^2+1}}{21}-\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-}}{}\right.\right.}{3\sqrt{-2+2i\sqrt{3}}}$
default	$\frac{x^5\sqrt{x^4+x^2+1}}{7}+\frac{3x^3\sqrt{x^4+x^2+1}}{7}+\frac{11x\sqrt{x^4+x^2+1}}{21}+\frac{20\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-(}}{21\sqrt{}}$
elliptic	$\frac{x^5\sqrt{x^4+x^2+1}}{7}+\frac{3x^3\sqrt{x^4+x^2+1}}{7}+\frac{11x\sqrt{x^4+x^2+1}}{21}+\frac{20\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-(}}{21\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2*(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x^5*(x^4+x^2+1)^(1/2)+3/7*x^3*(x^4+x^2+1)^(1/2)+11/21*x*(x^4+x^2+1)^(1/
2)+20/21/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2
-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2
))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/
2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2
)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2)
)^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(x^2 - x + 1)(x^2 + x + 1)} (x^2 + 1)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2*(x**4+x**2+1)**(1/2),x)
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*(x^4+x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (x^2 + 1)^2 \sqrt{x^4 + x^2 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2),x)
[Out] int((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2), x)
```

3.227 $\int (1+x^2)\sqrt{1+x^2+x^4}\,dx$

Optimal. Leaf size=145

$$\frac{3x\sqrt{1+x^2+x^4}}{5\left(1+x^2\right)}+\frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4}-\frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{5\sqrt{1+x^2+x^4}}+\frac{3(1+x^2)\sqrt{\frac{1-}{(}}}{5}$$

```
[Out] 3/5*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/5*x*(x^2+2)*(x^4+x^2+1)^(1/2)-3/5*(x^2+1)
*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2
)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+3/5*(x^2+1)*(cos(2*arctan
(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)
/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1190, 1211, 1117, 1209}

$$\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}+\frac{1}{5}(x^2+2)\sqrt{x^4+x^2+1}\,x+\frac{3\sqrt{x^4+x^2+1}\,x}{5\,(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (3*x*Sqrt[1 + x^2 + x^4])/(5*(1 + x^2)) + (x*(2 + x^2)*Sqrt[1 + x^2 + x^4])
/5 - (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x],
1/4])/(5*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\int \left(1+x^2\right)\sqrt{1+x^2+x^4}\,dx &= \frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4}+\frac{1}{15}\int \frac{9+9x^2}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4}-\frac{3}{5}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+\frac{6}{5}\int \frac{1}{\sqrt{1+x^2+x^4}}\,d\\ &= \frac{3x\sqrt{1+x^2+x^4}}{5\left(1+x^2\right)}+\frac{1}{5}x\left(2+x^2\right)\sqrt{1+x^2+x^4}-\frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E}{5\sqrt{1+x^2+}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.24, size = 168, normalized size = 1.16

$$\frac{2x+3x^3+3x^5+x^7+3\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\frac{3}{2}\sqrt{2+\left(1-i\sqrt{3}\right)x^2}\sqrt{2+\left(1+i\sqrt{3}\right)x^2}\,F\left(\sin^{-1}\left(\frac{1}{2}\left(x+i\sqrt{3}x\right)\right)|\frac{1}{2}i\left(i+\sqrt{3}\right)\right)}{5\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)*Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (2*x + 3*x^3 + 3*x^5 + x^7 + 3*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
 (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (3*Sqrt[2
 + (1 - I*Sqrt[3])*x^2]*Sqrt[2 + (1 + I*Sqrt[3])*x^2]*EllipticF[ArcSin[(x +
 I*Sqrt[3]*x)/2], (I/2)*(I + Sqrt[3])])/2)/(5*Sqrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 233, normalized size = 1.61

method	result
risch	$\frac{x(x^2+2)\sqrt{x^4+x^2+1}}{5} - \frac{12\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-2+\cdots}}{2}\cdots\right.\right.}{5\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4\cdots}}$
default	$\frac{x^3\sqrt{x^4+x^2+1}}{5} + \frac{2x\sqrt{x^4+x^2+1}}{5} + \frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\text{Ellipti}\cdots}{5\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x\cdots}}$
elliptic	$\frac{x^3\sqrt{x^4+x^2+1}}{5} + \frac{2x\sqrt{x^4+x^2+1}}{5} + \frac{6\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\text{Ellipti}\cdots}{5\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^3*(x^4+x^2+1)^(1/2)+2/5*x*(x^4+x^2+1)^(1/2)+6/5/(-2+2*I*3^(1/2))^(1/2
)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^
4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^
(1/2))-12/5/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-
1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/
2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+
2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(x^2 - x + 1)(x^2 + x + 1)} \left(x^2 + 1\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)*(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2 + 1\right) \sqrt{x^4 + x^2 + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)*(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)*(x^2 + x^4 + 1)^(1/2), x)
```

3.228 $\int \frac{\sqrt{1+x^2+x^4}}{1+x^2} dx$

Optimal. Leaf size=137

$$\frac{x\sqrt{1+x^2+x^4}}{1+x^2}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)-\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}+\frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+\cdots}}}{4\sqrt{1\cdots}}$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))+x*(x^4+x^2+1)^(1/2)/(x^2+1)-(x^2+1)*(cos(2*
arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+
x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+3/4*(x^2+1)*(cos(2*arctan(x))^2)^
(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)
^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1222, 1153, 1117, 1209, 1224, 1712, 209}

$$\frac{1}{2}\text{ArcTan}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}\,x}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - ((1 +
 x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1
 + x^2 + x^4] + (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*
ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1224

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^4}}{1+x^2}\,dx = \int \frac{x^2}{\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\sqrt{1+x^2+x^4}}\,dx -$$

$$= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{3(1+x^2)\sqrt{\frac{1+x^2}{(1+x}}}{4\sqrt{1+}}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}\right.}{\sqrt{1+x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.09, size = 117, normalized size = 0.85

$$\frac{\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)-F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+\sqrt[3]{-1}\,\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)\right)}{\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2),x]
```

```
[Out] ((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1
)^(2/3)] + (-1)^(1/3)*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^
(2/3)]))/Sqrt[1 + x^2 + x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 293, normalized size = 2.14

method	result
default	$-\frac{4\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(1+i\sqrt{3}\right)}+\frac{4\sqrt{1+}}{}$
elliptic	$-\frac{4\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right)}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}\left(1+i\sqrt{3}\right)}+\frac{4\sqrt{1+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -4/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/
2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*x*(-2+
2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+4/(-2+2*I*3^(1/2))^(1/2)*(1+
1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x
^2+1)^(1/2)/(1+I*3^(1/2))*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*
I*3^(1/2))^(1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2)
)^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-
1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/
2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}}{x^2 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4+x^2+1}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1),x)
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1), x)
```

3.229 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2} \, dx$

Optimal. Leaf size=49

$$\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}$$

```
[Out] 1/2*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arc
tan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1239}

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

```
[Out] ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(
2*Sqrt[1 + x^2 + x^4])
```

Rule 1239

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> With[{q = Rt[e/d, 2]}, Simp[c*(d + e*x^2)*(Sqrt[(e^2*(a + b*x^2 + c
*x^4))/(c*(d + e*x^2)^2)]/(2*d*e^2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*
ArcTan[q*x], (2*c*d - b*e)/(4*c*d)], x]] /; FreeQ[{a, b, c, d, e}, x] && Ne
Q[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
 && PosQ[e/d]
```

Rubi steps

$$\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^2} \, dx = \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.21, size = 164, normalized size = 3.35

$$\frac{\frac{x+x^3+x^5}{1+x^2}+(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}\,x^2}\,\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\sqrt[3]{-1}\,\sqrt{1+\sqrt[3]{-1}\,x^2}\,\sqrt{1-(-1)^{2/3}x^2}\,\left(-E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)}{2\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^2,x]
```

```
[Out] ((x + x^3 + x^5)/(1 + x^2) + (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*
Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1
)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/
(2*Sqrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 224, normalized size = 4.57

method	result
default	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}+\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$
risch	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}+\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}+\frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(
1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*Ellip
ticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*3^(
1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)
^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1
/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*
(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x^2-x+1)(x^2+x+1)}}{(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**2,x)
```

```
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x^4+x^2+1}}{(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^2,x)
```

```
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^2, x)
```

3.230 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^3} \, dx$

Optimal. Leaf size=93

$$\frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

```
[Out] 1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/4*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 13, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {1242, 1237, 1710, 1607, 1726, 1209, 1714, 1117, 1712, 209, 12, 1331, 1224}

$$\frac{1}{4}\text{ArcTan}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,\middle|\,\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}\,x}{4(x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
 ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/
(4*Sqrt[1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1224

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1331

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1714

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x^2+x^4}}{\left(1+x^2\right)^3}\,dx &= \int \left(\frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}} - \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}} + \frac{1}{\left(1+x^2\right)\sqrt{1+x^2+x^4}}\right) \\
&= \int \frac{1}{\left(1+x^2\right)^3\sqrt{1+x^2+x^4}}\,dx - \int \frac{1}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{\left(1+x^2\right)\sqrt{1+ } \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} - \frac{x\sqrt{1+x^2+x^4}}{2\left(1+x^2\right)} - \frac{1}{4}\int \frac{-3+2x^2-x^4}{\left(1+x^2\right)^2\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{}{\sqrt{1+ } \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{8} \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{\left(1+x^2\right)\sqrt{\frac{1+}{(1}}}{2\sqrt{}} \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \frac{\left(1+x^2\right)\sqrt{\frac{1+}{(1}}}{2\sqrt{}} \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(\right.}{4\sqrt{1+x^2+x^4}} \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx\right. \\
&= \frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^2+x^4}{\left(1+x^2\right)^2}}\,E\left(2\tan^{-1}(\right.}{4\sqrt{1+x^2+x^4}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 176, normalized size = 1.89

$$
\frac{\frac{x(2+x^2)(1+x^2+x^4)}{(1+x^2)^2} + \sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\left(-E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right) + F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)\right) + 2(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)}{4\sqrt{1+x^2+x^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^3,x]
```

```
[Out] ((x*(2 + x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 + (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)
*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2
```

```
/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) + 2*(-1)^(2/3)*Sqrt[
1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSi
nh[(-1)^(5/6)*x], (-1)^(2/3)])/(4*Sqrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 333, normalized size = 3.58

method	result
risch	$\frac{\sqrt{x^4+x^2+1}\, x(x^2+2)}{4(x^2+1)^2} + \frac{\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-2+2}}{2}\right.\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4}}$
default	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2} + \frac{x\sqrt{x^4+x^2+1}}{4x^2+4} + \frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\text{EllipticF}\left(\frac{x\sqrt{-2+2}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,(1+i\sqrt{}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2} + \frac{x\sqrt{x^4+x^2+1}}{4x^2+4} + \frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\text{EllipticF}\left(\frac{x\sqrt{-2+2}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,(1+i\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/(-2+2*I
*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^
(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*x*(-2+2*I*3^(1/2
))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-1/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/
2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2
)/(1+I*3^(1/2))*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))
^(1/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*
(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*
I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2
+1/2*I*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}}{(x^2 + 1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**3,x)
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^3,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{(x^2 + 1)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^3,x)
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^3, x)
```

3.231 $\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4} \, dx$

Optimal. Leaf size=166

$$\frac{x\sqrt{1+x^2+x^4}}{6\,(1+x^2)^3}+\frac{x\sqrt{1+x^2+x^4}}{6\,(1+x^2)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\big(2\tan^{-1}(x)|\frac{1}{4}\big)}{3\sqrt{1+x^2+x^4}}-$$

```
[Out] 1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^3+1/6*x*(x^
4+x^2+1)^(1/2)/(x^2+1)^2+1/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arcta
n(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^
2+1)^(1/2)-1/8*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Elliptic
F(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {1242, 1237, 1710, 1600, 1211, 1117, 1209, 1607, 1726, 1714, 1712, 209, 12, 1331}

$$\frac{1}{4}\text{ArcTan}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{8\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}\,x}{6\,(x^2+1)^2}+\frac{\sqrt{x^4+x^2+1}\,x}{6\,(x^2+1)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x^2)^3) + (x*Sqrt[1 + x^2 + x^4])/(6*(1 + x
^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/
(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) - ((1 + x
^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(8*Sqrt[
1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1331

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
```

```
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1714

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^2+x^4}}{(1+x^2)^4}\,dx &= \int \left(\frac{1}{(1+x^2)^4\sqrt{1+x^2+x^4}} - \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}} + \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\right.\\
&= \int \frac{1}{(1+x^2)^4\sqrt{1+x^2+x^4}}\,dx - \int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx + \int \frac{1}{(1+x^2)^2\sqrt{1+\cdots}}\\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} - \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{x\sqrt{1+x^2+x^4}}{2(1+x^2)} - \frac{1}{6}\int \frac{-5+2x^2-3x^4}{(1+x^2)^3\sqrt{1+x^2+\cdots}}\\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{24}\int \frac{10-8x^2+10x^4}{(1+x^2)^2\sqrt{1+x^2+\cdots}}\\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E(\cdots}{2\sqrt{1+x^2+x\cdots}}\\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E(\cdots}{2\sqrt{1+x^2+x\cdots}}\\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} + \frac{7x\sqrt{1+x^2+x^4}}{12(1+x^2)} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\cdots}{4\sqrt{1+x^2+\cdots}}\\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+\cdots}{(1+\cdots}}}{3\sqrt{\cdots}}\\
&= \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^3} + \frac{x\sqrt{1+x^2+x^4}}{6(1+x^2)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+\cdots}{(1+\cdots}}}{3\sqrt{\cdots}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.20, size = 240, normalized size = 1.45

$$\frac{\frac{x(1+x^2+x^4)(4+5x^2+2x^4)}{(1+x^2)^3} - 2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right) - F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right) - (-1)^{2/3}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right) + 3(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)}{6\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2 + x^4]/(1 + x^2)^4,x]
```

```
[Out] ((x*(1 + x^2 + x^4)*(4 + 5*x^2 + 2*x^4))/(1 + x^2)^3 - 2*(-1)^(1/3)*Sqrt[1
+ (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*
x], (-1)^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - (-1)^(2
/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(
-1)^(5/6)*x], (-1)^(2/3)] + 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
(-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]
)/(6*Sqrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 438, normalized size = 2.64

method	result
risch	$\frac{\sqrt{x^4+x^2+1}\,x\left(2x^4+5x^2+4\right)}{6\left(x^2+1\right)^3} + \frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-\ldots}}{\ldots}\right.\right.\ldots}{3\sqrt{-2+2i\sqrt{3}}\ldots}$
default	$\frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^3} + \frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^2} + \frac{x\sqrt{x^4+x^2+1}}{3x^2+3} - \frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\ldots}{3\sqrt{-2+2i\sqrt{\ldots}}\ldots}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^3} + \frac{x\sqrt{x^4+x^2+1}}{6\left(x^2+1\right)^2} + \frac{x\sqrt{x^4+x^2+1}}{3x^2+3} - \frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\ldots}{3\sqrt{-2+2i\sqrt{\ldots}}\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)^(1/2)/(x^2+1)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^3+1/6*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+1/3*x*(
x^4+x^2+1)^(1/2)/(x^2+1)-1/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^
(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*Elliptic
F(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+4/3/(-2+2*I*3^(1
/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2)
)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1
```

```
/2),1/2*(-2+2*I*3^(1/2))^(1/2))-4/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I
*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(
1+I*3^(1/2))*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1
/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+
1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3
^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/
2*I*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}}{(x^2 + 1)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)**(1/2)/(x**2+1)**4,x)
[Out] Integral(sqrt((x**2 - x + 1)*(x**2 + x + 1))/(x**2 + 1)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)^(1/2)/(x^2+1)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + x^2 + 1)/(x^2 + 1)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4 + x^2 + 1}}{\left(x^2 + 1\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^4,x)
```

```
[Out] int((x^2 + x^4 + 1)^(1/2)/(x^2 + 1)^4, x)
```

3.232 $\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=159

$$\frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{14x\sqrt{1+x^2+x^4}}{15\,(1+x^2)}-\frac{14(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{15\sqrt{1+x^2+x^4}}+ 3($$

```
[Out] 11/15*x*(x^4+x^2+1)^(1/2)+1/5*x^3*(x^4+x^2+1)^(1/2)+14/15*x*(x^4+x^2+1)^(1/
2)/(x^2+1)-14/15*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*Ellipt
icE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+3
/5*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arct
an(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1693, 1211, 1117, 1209}

$$\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{5\sqrt{x^4+x^2+1}}-\frac{14(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{15\sqrt{x^4+x^2+1}}+\frac{14\sqrt{x^4+x^2+1}\,x}{15\,(x^2+1)}+\frac{11}{15}\sqrt{x^4+x^2+1}\,x+\frac{1}{5}\sqrt{x^4+x^2+1}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (11*x*Sqrt[1 + x^2 + x^4])/15 + (x^3*Sqrt[1 + x^2 + x^4])/5 + (14*x*Sqrt[1
+ x^2 + x^4])/(15*(1 + x^2)) - (14*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticE[2*ArcTan[x], 1/4])/(15*Sqrt[1 + x^2 + x^4]) + (3*(1 + x^2)*Sq
rt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(5*Sqrt[1 + x^
2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
```

```
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^2)^3}{\sqrt{1+x^2+x^4}}\,dx &= \frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{1}{5}\int \frac{5+12x^2+11x^4}{\sqrt{1+x^2+x^4}}\,dx\\
&= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{1}{15}\int \frac{4+14x^2}{\sqrt{1+x^2+x^4}}\,dx\\
&= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}-\frac{14}{15}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+\frac{6}{5}\int \frac{1}{\sqrt{1+x^2}\ldots}\\
&= \frac{11}{15}x\sqrt{1+x^2+x^4}+\frac{1}{5}x^3\sqrt{1+x^2+x^4}+\frac{14x\sqrt{1+x^2+x^4}}{15\,(1+x^2)}-\frac{14(1+x^2)\sqrt{\frac{1+\ldots}{(1-\ldots}}}{15\sqrt{1\ldots}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 157, normalized size = 0.99

$$\frac{x(11+14x^2+14x^4+3x^6)+14\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+2\sqrt[3]{-1}\left(-7+2\sqrt[3]{-1}\right)\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)}{15\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^3/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*(11 + 14*x^2 + 14*x^4 + 3*x^6) + 14*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*
Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2
*(-1)^(1/3)*(-7 + 2*(-1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3
)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(15*Sqrt[1 + x^2 + x
^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 233, normalized size = 1.47

method	result
risch	$\frac{x(3x^2+11)\sqrt{x^4+x^2+1}}{15}-\frac{56\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-2+}}{2}\right.\right.}{15\sqrt{-2+2i\sqrt{3}}\sqrt{x}}$
default	$\frac{x^3\sqrt{x^4+x^2+1}}{5}+\frac{11x\sqrt{x^4+x^2+1}}{15}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\text{Elliptic}}{15\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x}}$
elliptic	$\frac{x^3\sqrt{x^4+x^2+1}}{5}+\frac{11x\sqrt{x^4+x^2+1}}{15}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\text{Elliptic}}{15\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^3/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^3*(x^4+x^2+1)^(1/2)+11/15*x*(x^4+x^2+1)^(1/2)+8/15/(-2+2*I*3^(1/2))^(
1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/
(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2
))^(1/2))-56/15/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(
1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(Elliptic
F(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*
(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+1)^3}{\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**3/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)**3/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^3/sqrt(x^4 + x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2+1)^3}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(1/2), x)
```

3.233 $\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=137

$$\frac{1}{3}x\sqrt{1+x^2+x^4}+\frac{4x\sqrt{1+x^2+x^4}}{3(1+x^2)}-\frac{4(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,F}{\sqrt{1+x^2+x}}$$

```
[Out] 1/3*x*(x^4+x^2+1)^(1/2)+4/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)-4/3*(x^2+1)*(cos(2*
arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+
x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+(x^2+1)*(cos(2*arctan(x))^2)^(1/2
)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^
(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1220, 1211, 1117, 1209}

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{4(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}+\frac{4\sqrt{x^4+x^2+1}\,x}{3(x^2+1)}+\frac{1}{3}\sqrt{x^4+x^2+1}\,x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/3 + (4*x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) - (4*(1
 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*S
qrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*Elliptic
F[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x^2)^2}{\sqrt{1+x^2+x^4}}\,dx &= \frac{1}{3}x\sqrt{1+x^2+x^4}+\frac{1}{3}\int \frac{2+4x^2}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{1+x^2+x^4}-\frac{4}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+2\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{1+x^2+x^4}+\frac{4x\sqrt{1+x^2+x^4}}{3(1+x^2)}-\frac{4(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 143, normalized size = 1.04

$$\frac{x+x^3+x^5+4\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+2\sqrt[3]{-1}\left(-2+\sqrt[3]{-1}\right)\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)}{3\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x + x^3 + x^5 + 4*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*
x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(1/3)*(-2 + (-
```

```
1)^(1/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*Arc
Sinh[(-1)^(5/6)*x], (-1)^(2/3)])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 218, normalized size = 1.59

method	result
default	$\frac{x\sqrt{x^4+x^2+1}}{3}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{}}{}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$
risch	$\frac{x\sqrt{x^4+x^2+1}}{3}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{}}{}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{3}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{}}{}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(x^4+x^2+1)^(1/2)+4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*
x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1
/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-16/3/(-2+2*I*3^(1/2
))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1
/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2)
,1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2
+2*I*3^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+1)^2}{\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)**2/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/sqrt(x^4 + x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2+1)^2}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(1/2), x)
```

3.234 $\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=115

$$\frac{x\sqrt{1+x^2+x^4}}{1+x^2}-\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}}$$

```
[Out] x*(x^4+x^2+1)^(1/2)/(x^2+1)-(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan
(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2
+1)^(1/2)+(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin
(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1211, 1117, 1209}

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}\,x}{x^2+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(1 + x^2) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^
2)^2]*EllipticE[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4] + ((1 + x^2)*Sqrt[(1
 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{1+x^2}{\sqrt{1+x^2+x^4}}\,dx = 2\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx - \int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{1+x^2} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+}{(1+x^2}}}{\sqrt{1+x}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 94, normalized size = 0.82

$$\frac{\sqrt[3]{-1}\,\sqrt{1+\sqrt[3]{-1}\,x^2}\,\sqrt{1-(-1)^{2/3}x^2}\,\left(E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\left(-1+\sqrt[3]{-1}\right)F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)}{\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] ((-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*
ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1 + (-1)^(1/3))*EllipticF[I*ArcSinh[
(-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 205, normalized size = 1.78

method	result
default	$-\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right.\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}$
elliptic	$-\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i\sqrt{3}}}{2}\right.\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}\,\left(1+i\sqrt{3}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I
*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2
*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/
2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))+2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/
2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2
)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2+1}{\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/sqrt(x^4 + x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^2 + x^4 + 1)^(1/2),x)
```

```
[Out] int((x^2 + 1)/(x^2 + x^4 + 1)^(1/2), x)
```

3.235 $\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))+1/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(
2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/
(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1224, 1117, 1712, 209}

$$\frac{1}{2}\text{ArcTan}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1224

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
```

```
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{}{\sqrt{1+}}\right.$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 72, normalized size = 1.04

$$\frac{(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}\,x^2}\,\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{-1}\,;i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)}{\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[(-
1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/Sqrt[1 + x^2 + x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 104, normalized size = 1.51

method	result	s

default	$\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\text{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}$	10
elliptic	$\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\text{EllipticPi}\left(\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,x,-\frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}},\frac{\sqrt{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}\,\sqrt{x^4+x^2+1}}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2
+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))
^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(
1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Fricas [A]
time = 0.11, size = 54, normalized size = 0.78

$$-\frac{1}{8}\sqrt{2}\left(\sqrt{-3}+1\right)\sqrt{\sqrt{-3}-1}\,\text{ellipticF}\left(\frac{1}{2}\sqrt{2}\,x\sqrt{\sqrt{-3}-1},\frac{1}{2}\sqrt{-3}-\frac{1}{2}\right)+\frac{1}{2}\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*(sqrt(-3) + 1)*sqrt(sqrt(-3) - 1)*ellipticF(1/2*sqrt(2)*x*sqrt
(sqrt(-3) - 1), 1/2*sqrt(-3) - 1/2) + 1/2*arctan(x/sqrt(x^4 + x^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2+1\right)\,\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(1/2)), x)
```

3.236 $\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=118

$$\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\big(2\tan^{-1}(x)|\frac{1}{4}\big)}{2\sqrt{1+x^2+x^4}}-\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,F\big(2\tan^{-1}(x$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))+1/2*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(
2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/
(x^4+x^2+1)^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*E
llipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1
/2)
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1237, 1726, 1209, 12, 1331, 1117, 1712, 209}

$$\frac{1}{2}\text{ArcTan}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\big(2\text{ArcTan}(x)\,|\,\frac{1}{4}\big)}{4\sqrt{x^4+x^2+1}}+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\big(2\text{ArcTan}(x)\,|\,\frac{1}{4}\big)}{2\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)
^2]*EllipticE[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[
(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 +
 x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1331

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
```

```
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx &= \frac{x\sqrt{1+x^2+x^4}}{2(1+x^2)} - \frac{1}{2}\int \frac{-1+2x^2+x^4}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{x\sqrt{1+x^2+x^4}}{2(1+x^2)} + \frac{1}{2}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx - \frac{1}{2}\int \frac{2x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,d \\
&= \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \int \frac{x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx \\
&= \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \\
&= \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,F(2\tan}{4\sqrt{1+x^2+x^4}} \\
&= \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{2\sqrt{1+x^2+x^4}} -
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.22, size = 226, normalized size = 1.92

$$\frac{\frac{x+x^3+x^5}{1+x^2} - (-1)^{2/3}\sqrt{1+\sqrt[3]{-1}\,x^2}\,\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right) + \sqrt[3]{-1}\,\sqrt{1+\sqrt[3]{-1}\,x^2}\,\sqrt{1-(-1)^{2/3}x^2}\left(-E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right) + F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)\right) + 2(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}\,x^2}\,\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)}{2\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^2*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((x + x^3 + x^5)/(1 + x^2) - (-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (
-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*
Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(-EllipticE[I*ArcSinh[(-1
)^(5/6)*x], (-1)^(2/3)] + EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) +
 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticPi[
(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(2*Sqrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 397, normalized size = 3.36

method	result
risch	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}\right.\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2}}$
default	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i}}{2}\right.}{\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x
^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*Ell
ipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-2+2*I*3
^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1
/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*x*(-2+2*I*3^(1/2))
^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-2/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*
I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/
(1+I*3^(1/2))*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(
1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1
/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^
(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2
*I*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2-x+1)(x^2+x+1)}\,(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^2+1)^2\,\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(1/2)), x)
```

3.237 $\int \frac{1}{(1+x^2)^3 \sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=142

$$\frac{x\sqrt{1+x^2+x^4}}{4\left(1+x^2\right)^2}+\frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}}-\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+}}}{2\sqrt{}}$$

```
[Out] 1/4*arctan(x/(x^4+x^2+1)^(1/2))+1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+3/4*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)-1/2*(x^2+1)*(cos(2*arct
an(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+
1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {1237, 1710, 1607, 1726, 1209, 1714, 1117, 1712, 209}

$$\frac{1}{4}\text{ArcTan}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\middle|\frac{1}{4}\right)}{2\sqrt{x^4+x^2+1}}+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}\,x}{4\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^2)^2) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/4 +
 (3*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4]
)/(4*Sqrt[1 + x^2 + x^4]) - ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*El
lipticF[2*ArcTan[x], 1/4])/(2*Sqrt[1 + x^2 + x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1714

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx = \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} - \frac{1}{4}\int \frac{-3+2x^2-x^4}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{8}\int \frac{-10x^2-6x^4}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{8}\int \frac{x^2(-10-6x^2)}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3x\sqrt{1+x^2+x^4}}{4(1+x^2)} + \frac{1}{8}\int \frac{-6-10x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx +$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{4}\int \frac{}{(1+}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} - \frac{(1+x^2)}{}$$

$$= \frac{x\sqrt{1+x^2+x^4}}{4(1+x^2)^2} + \frac{1}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{3(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}}{4\sqrt{1+x^2+}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.17, size = 235, normalized size = 1.65

$$\frac{\frac{x(4+3x^2)(1+x^2+x^4)}{(1+x^2)^2}-3\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\left(E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)-2(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+2(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)}{4\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^3*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((x*(4 + 3*x^2)*(1 + x^2 + x^4))/(1 + x^2)^2 - 3*(-1)^(1/3)*Sqrt[1 + (-1)^(
1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)
^(2/3)] - EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]) - 2*(-1)^(2/3)*Sq
rt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5
/6)*x], (-1)^(2/3)] + 2*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(
2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(4*S
qrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 418, normalized size = 2.94

method	result
risch	$\frac{\sqrt{x^4+x^2+1}\,x(3x^2+4)}{4(x^2+1)^2}+\frac{3\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2 \ldots}}{2}\ldots\right.\right.}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4 \ldots}}$
default	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2}+\frac{3x\sqrt{x^4+x^2+1}}{4(x^2+1)}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2 \ldots}}{2}\ldots\right.}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2}+\frac{3x\sqrt{x^4+x^2+1}}{4(x^2+1)}-\frac{\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2 \ldots}}{2}\ldots\right.}{\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+3/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)-1/(-2+2*I
*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^
(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(
-2+2*I*3^(1/2))^(1/2))+3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2
))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2)
)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-3/(-2+
```

```
2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2
*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticE(1/2*x*(-2+2*I*3^(
1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+1/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1
/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^
2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),
(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2 - x + 1)(x^2 + x + 1)}\,(x^2 + 1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3/(x**4+x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt((x**2 - x + 1)*(x**2 + x + 1))*(x**2 + 1)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^2+1)^3 \sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(1/2)), x)
```

3.238 $\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=144

$$-\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)}-\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}}{\sqrt{1+x^2+}}$$

```
[Out] -1/3*x*(-x^2+1)/(x^4+x^2+1)^(1/2)+2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)-2/3*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)+(x^2+1)*(cos(2*arctan(x
))^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(
x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1219, 1211, 1117, 1209}

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}+\frac{2\sqrt{x^4+x^2+1}\,x}{3(x^2+1)}-\frac{(1-x^2)\,x}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] -1/3*(x*(1 - x^2))/Sqrt[1 + x^2 + x^4] + (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 +
x^2)) - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x
], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2
)^2]*EllipticF[2*ArcTan[x], 1/4])/Sqrt[1 + x^2 + x^4]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
```

```
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^2)^3}{(1+x^2+x^4)^{3/2}}\,dx &= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}}+\frac{1}{3}\int \frac{4+2x^2}{\sqrt{1+x^2+x^4}}\,dx\\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}}-\frac{2}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx+2\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx\\
&= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3\,(1+x^2)}-\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.11, size = 136, normalized size = 0.94

$$\frac{-x+x^3+2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}\,x^2}\,\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+2(-1)^{5/6}\sqrt{3+3\sqrt[3]{-1}\,x^2}\,\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)}{3\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^3/(1 + x^2 + x^4)^(3/2),x]
```

[Out] (-x + x^3 + 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*(-1)^(5/6)*Sqrt[3 + 3*(-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)])/(3*Sqrt[1 + x^2 + x^4])

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 268, normalized size = 1.86

method	result
risch	$\frac{(x^2-1)x}{3\sqrt{x^4+x^2+1}} - \frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}\right.\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2}}$
elliptic	$-\frac{2\left(\frac{1}{6}x-\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}} - \frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}\right.\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2}}$
default	$-\frac{4\left(-\frac{1}{6}x+\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}} + \frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)^3/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)

[Out] -4*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^(1/2)+8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))-6*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)-6*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="maxima")

[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2+1)^3}{((x^2-x+1)(x^2+x+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**3/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)**3/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^3/(x^4 + x^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x^2+1)^3}{(x^4+x^2+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(3/2),x)
```

```
[Out] int((x^2 + 1)^3/(x^2 + x^4 + 1)^(3/2), x)
```

3.239 $\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}}-\frac{2x\sqrt{1+x^2+x^4}}{3\,(1+x^2)}+\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

```
[Out] 1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)-2/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+2/3*(x^2+
1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1
/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1219, 1209}

$$\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,|\,\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}-\frac{2\sqrt{x^4+x^2+1}\,x}{3\,(x^2+1)}+\frac{(2x^2+1)\,x}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(1 + 2*x^2))/(3*Sqrt[1 + x^2 + x^4]) - (2*x*Sqrt[1 + x^2 + x^4])/(3*(1 +
 x^2)) + (2*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[
x], 1/4])/(3*Sqrt[1 + x^2 + x^4])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
```

```
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(1+x^2)^2}{(1+x^2+x^4)^{3/2}}\,dx = \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}}+\frac{1}{3}\int \frac{2-2x^2}{\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}}-\frac{2x\sqrt{1+x^2+x^4}}{3\,(1+x^2)}+\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 158, normalized size = 1.61

$$\frac{x+2x^3-2\sqrt[3]{-1}\,\sqrt{1+\sqrt[3]{-1}\,x^2}\,\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)-i\sqrt{2+\left(1+i\sqrt{3}\right)x^2}\,\sqrt{6+\left(3-3i\sqrt{3}\right)x^2}\,F\left(\sin^{-1}\left(\frac{1}{2}\left(x+i\sqrt{3}\,x\right)\right)\middle|\frac{1}{2}i\left(i+\sqrt{3}\right)\right)}{3\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^2/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (x + 2*x^3 - 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]
*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - I*Sqrt[2 + (1 + I*Sqrt[3]
)*x^2]*Sqrt[6 + (3 - (3*I)*Sqrt[3])*x^2]*EllipticF[ArcSin[(x + I*Sqrt[3]*x)
/2], (I/2)*(I + Sqrt[3])])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 268, normalized size = 2.73

method	result
risch	$\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\right.\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+}}$
elliptic	$-\frac{2\left(-\frac{1}{3}x^3-\frac{1}{6}x\right)}{\sqrt{x^4+x^2+1}}+\frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}\right.\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2}}$

default	$-\frac{2\left(\frac{1}{6}x^3+\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}}+\frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)+4/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/
2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2
)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+8/3/(-
2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1
/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3^
(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^(
1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))-4*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)-2*(-
1/6*x+1/6*x^3)/(x^4+x^2+1)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^2+1\right)^2}{\left(\left(x^2-x+1\right)\left(x^2+x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)**2/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2/(x^4 + x^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(x^2+1\right)^2}{\left(x^4+x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(3/2),x)
```

```
[Out] int((x^2 + 1)^2/(x^2 + x^4 + 1)^(3/2), x)
```

3.240 $\int \frac{1+x^2}{(1+x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=96

$$\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} - \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

```
[Out] 1/3*x*(x^2+2)/(x^4+x^2+1)^(1/2)-1/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+1/3*(x^2+1)
*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2
)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1192, 1209}

$$\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}} - \frac{\sqrt{x^4+x^2+1}\,x}{3(x^2+1)} + \frac{(x^2+2)\,x}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(2 + x^2))/(3*Sqrt[1 + x^2 + x^4]) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2
)) + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/
4])/(3*Sqrt[1 + x^2 + x^4])
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
```

```
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1+x^2}{\left(1+x^2+x^4\right)^{3/2}}\,dx = \frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3}\int \frac{1-x^2}{\sqrt{1+x^2+x^4}}\,dx$$

$$= \frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} - \frac{x\sqrt{1+x^2+x^4}}{3\,(1+x^2)} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{3\sqrt{1+x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 160, normalized size = 1.67

$$\frac{2x+x^3-\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}\,x^2}\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)-\frac{1}{2}i\sqrt{2+\left(1+i\sqrt{3}\right)x^2}\sqrt{6+\left(3-3i\sqrt{3}\right)x^2}\,F\left(\sin^{-1}\left(\frac{1}{2}\left(x+i\sqrt{3}\,x\right)\right)\middle|\frac{1}{2}i\left(i+\sqrt{3}\right)\right)}{3\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(1 + x^2 + x^4)^(3/2),x]
```

```
[Out] (2*x + x^3 - (-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*E
llipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - (I/2)*Sqrt[2 + (1 + I*Sqrt[
3])*x^2]*Sqrt[6 + (3 - (3*I)*Sqrt[3])*x^2]*EllipticF[ArcSin[(x + I*Sqrt[3]*
x)/2], (I/2)*(I + Sqrt[3])])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 247, normalized size = 2.57

method	result
risch	$\frac{x(x^2+2)}{3\sqrt{x^4+x^2+1}} + \frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\right.\right.}{3\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2+}}$
elliptic	$-\frac{2\left(-\frac{1}{6}x^3-\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}} + \frac{4\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}\right.\right.}{3\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x^2}}$

default	$-\frac{2\left(-\frac{1}{3}x^{3}-\frac{1}{6}x\right)}{\sqrt{x^{4}+x^{2}+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^{2}}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^{2}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/3*x^3-1/6*x)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1
/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/
2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+4/3/(
-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(
1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*(EllipticF(1/2*x*(-2+2*I*3
^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-EllipticE(1/2*x*(-2+2*I*3^(1/2))^
(1/2),1/2*(-2+2*I*3^(1/2))^(1/2)))-2*(-1/6*x+1/6*x^3)/(x^4+x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}+1}{\left(\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral((x**2 + 1)/((x**2 - x + 1)*(x**2 + x + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate((x^2 + 1)/(x^4 + x^2 + 1)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2+1}{\left(x^4+x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)/(x^2 + x^4 + 1)^(3/2),x)
[Out] int((x^2 + 1)/(x^2 + x^4 + 1)^(3/2), x)
```

3.241 $\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}}+\frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)}+\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)-\frac{2(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\right|}{3\sqrt{1+x^2+x^4}}$$

```
[Out] 1/2*arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(2*x^2+1)/(x^4+x^2+1)^(1/2)+2/3*x*(x^
4+x^2+1)^(1/2)/(x^2+1)-2/3*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(
x))*EllipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+
1)^(1/2)+3/4*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticF(
sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1235, 1133, 1211, 1117, 1209, 1224, 1712, 209}

$$\frac{1}{2}\text{ArcTan}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{3(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}-\frac{2(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\middle|\frac{1}{4}\right)}{3\sqrt{x^4+x^2+1}}+\frac{2\sqrt{x^4+x^2+1}\,x}{3(x^2+1)}-\frac{(2x^2+1)x}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/3*(x*(1 + 2*x^2))/Sqrt[1 + x^2 + x^4] + (2*x*Sqrt[1 + x^2 + x^4])/(3*(1
+ x^2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]]/2 - (2*(1 + x^2)*Sqrt[(1 + x^2 + x^
4)/(1 + x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(3*Sqrt[1 + x^2 + x^4]) + (3*(
1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*
Sqrt[1 + x^2 + x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1224

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
```

```
0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{1}{(1+x^2)(1+x^2+x^4)^{3/2}}\,dx = -\int \frac{x^2}{(1+x^2+x^4)^{3/2}}\,dx + \int \frac{1}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$$

$$= -\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{1}{3}\int \frac{1+2x^2}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \frac{1}{\sqrt{1+x^2+x^4}}\,dx + \ldots$$

$$= -\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{4\sqrt{1+x^2+x^4}} + \frac{1}{2}\text{Subst}\Big(\ldots$$

$$= -\frac{x(1+2x^2)}{3\sqrt{1+x^2+x^4}} + \frac{2x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) - \ldots$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.12, size = 204, normalized size = 1.23

$$\frac{-x-2x^3+2\sqrt[3]{-1}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+\sqrt[3]{-1}\left(-2+\sqrt[3]{-1}\right)\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)+3(-1)^{2/3}\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\,\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)\middle|(-1)^{2/3}\right)}{3\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (-x - 2*x^3 + 2*(-1)^(1/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2
]*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-1)^(1/3)*(-2 + (-1)^(1
/3))*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[
(-1)^(5/6)*x], (-1)^(2/3)] + 3*(-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 -
 (-1)^(2/3)*x^2]*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)
])/(3*Sqrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 398, normalized size = 2.40

method	result
risch	$-\frac{x(2x^2+1)}{3\sqrt{x^4+x^2+1}} - \frac{8\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2}\ldots\right.\right.}{3\sqrt{-2+2i\sqrt{3}}\sqrt{x^4+x\ldots}}$

default	$-\frac{2\left(\frac{1}{3}x^{3}+\frac{1}{6}x\right)}{\sqrt{x^{4}+x^{2}+1}}+\frac{2\sqrt{1+\frac{x^{2}}{2}-\frac{i x^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{i x^{2}\sqrt{3}}{2}}\,\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i}}{2}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}$
elliptic	$-\frac{2\left(\frac{1}{3}x^{3}+\frac{1}{6}x\right)}{\sqrt{x^{4}+x^{2}+1}}+\frac{2\sqrt{1+\frac{x^{2}}{2}-\frac{i x^{2}\sqrt{3}}{2}}\,\sqrt{1+\frac{x^{2}}{2}+\frac{i x^{2}\sqrt{3}}{2}}\,\text{EllipticF}\left(\frac{x\sqrt{-2+2i\sqrt{3}}}{2},\frac{\sqrt{-2+2i}}{2}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^{4}+x^{2}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1/3*x^3+1/6*x)/(x^4+x^2+1)^(1/2)+2/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-
1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1
/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-8/3/
(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I
*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*x*(-2+2*I
*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+8/3/(-2+2*I*3^(1/2))^(1/2)*(1+1
/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^
2+1)^(1/2)/(1+I*3^(1/2))*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I
*3^(1/2))^(1/2))+1/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))
^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1
/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2
)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)\right)^{\frac{3}{2}}\left(x^{2}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**4+x**2+1)**(3/2),x)
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^4+x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^{2}+1\right)\,\left(x^{4}+x^{2}+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(3/2)),x)
[Out] int(1/((x^2 + 1)*(x^2 + x^4 + 1)^(3/2)), x)
```

3.242 $\int \frac{1}{\left(1+x^2\right)^2\left(1+x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$-\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{3\left(1+x^2\right)}+\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E\left(2\tan^{-1}(x)\middle|\frac{1}{4}\right)}{6\sqrt{1+x^2+x^4}}$$

```
[Out] arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(x^2+2)/(x^4+x^2+1)^(1/2)+1/3*x*(x^4+x^2+
1)^(1/2)/(x^2+1)+1/6*(x^2+1)*(cos(2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*El
lipticE(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/
2)
```

Rubi [A]
time = 0.18, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {1242, 1192, 1209, 1237, 1726, 12, 1331, 1117, 1712, 209, 1224}

$$\text{ArcTan}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)+\frac{(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}\,E\left(2\text{ArcTan}(x)\,\middle|\,\frac{1}{4}\right)}{6\sqrt{x^4+x^2+1}}+\frac{\sqrt{x^4+x^2+1}\,x}{3\left(x^2+1\right)}-\frac{\left(x^2+2\right)x}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/3*(x*(2 + x^2))/Sqrt[1 + x^2 + x^4] + (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^
2)) + ArcTan[x/Sqrt[1 + x^2 + x^4]] + ((1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 +
x^2)^2]*EllipticE[2*ArcTan[x], 1/4])/(6*Sqrt[1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1224

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> Dist[1/(2*d), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[1/(2*d
), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
&& EqQ[c*d^2 - a*e^2, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1331

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(1+x^2)^2 (1+x^2+x^4)^{3/2}}\,dx = \int \left(\frac{-1-x^2}{(1+x^2+x^4)^{3/2}} + \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}} + \frac{1}{(1+x^2)\sqrt{1+x^2+\cdots}} \right.$$

$$= \int \frac{-1-x^2}{(1+x^2+x^4)^{3/2}}\,dx + \int \frac{1}{(1+x^2)^2\sqrt{1+x^2+x^4}}\,dx + \int \frac{\cdots}{(1+x^2)\sqrt{\cdots}}$$

$$= -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{2(1+x^2)} + \frac{1}{3}\int \frac{-1+x^2}{\sqrt{1+x^2+x^4}}\,dx + \frac{1}{2}\int \cdots$$

$$= -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{5x\sqrt{1+x^2+x^4}}{6(1+x^2)} - \frac{(1+x^2)\sqrt{\frac{1+x^2+x^4}{(1+x^2)^2}}\,E(2\tan\cdots}{3\sqrt{1+x^2+x^4}}$$

$$= -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1\cdots}{\cdots}$$

$$= -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1\cdots}{\cdots}$$

$$= -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1\cdots}{\cdots}$$

$$= -\frac{x(2+x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{3(1+x^2)} + \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right) + \frac{(1+\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.22, size = 168, normalized size = 1.51

$$\frac{-2x(1+x^2)(2+x^2)+3x(1+x^2+x^4)-\sqrt[3]{-1}\,(1+x^2)\sqrt{1+\sqrt[3]{-1}\,x^2}\,\sqrt{1-(-1)^{2/3}x^2}\,\left(E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\left(-1+5\sqrt[3]{-1}\right)F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-12\sqrt[3]{-1}\,\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)}{6(1+x^2)\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^2*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (-2*x*(1 + x^2)*(2 + x^2) + 3*x*(1 + x^2 + x^4) - (-1)^(1/3)*(1 + x^2)*Sqrt
[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticE[I*ArcSinh[(-1)^(5/
6)*x], (-1)^(2/3)] + (-1 + 5*(-1)^(1/3))*EllipticF[I*ArcSinh[(-1)^(5/6)*x],
 (-1)^(2/3)] - 12*(-1)^(1/3)*EllipticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x]
, (-1)^(2/3)]))/(6*(1 + x^2)*Sqrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 419, normalized size = 3.77

method	result
risch	$\frac{x(x^4-3x^2-1)}{6(x^2+1)\sqrt{x^4+x^2+1}}+\frac{2\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\,\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-2+2i}}{2}\right.\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+}}$
default	$-\frac{2\left(\frac{1}{6}x^3+\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}}+\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{5\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\text{EllipticF}\left(\frac{x\sqrt{-2+}}{2}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$
elliptic	$-\frac{2\left(\frac{1}{6}x^3+\frac{1}{3}x\right)}{\sqrt{x^4+x^2+1}}+\frac{x\sqrt{x^4+x^2+1}}{2x^2+2}-\frac{5\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\,\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}\,\text{EllipticF}\left(\frac{x\sqrt{-2+}}{2}\right.}{3\sqrt{-2+2i\sqrt{3}}\,\sqrt{x^4+x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1/6*x^3+1/3*x)/(x^4+x^2+1)^(1/2)+1/2*x*(x^4+x^2+1)^(1/2)/(x^2+1)-5/3/(-
2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x
^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),
1/2*(-2+2*I*3^(1/2))^(1/2))+2/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2
*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*
3^(1/2))*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))
-2/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+
1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticE(1/2*x*(-
2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-1/2+1/2*I*3^(1/2))^(1/
2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/
(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^
(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)\right)^{\frac{3}{2}}\left(x^{2}+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2/(x**4+x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^2+1\right)}^2\,{\left(x^4+x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(3/2)),x)
```

```
[Out] int(1/((x^2 + 1)^2*(x^2 + x^4 + 1)^(3/2)), x)
```

3.243 $\int \frac{1}{(1+x^2)^3 (1+x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=190

$$-\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}}+\frac{x\sqrt{1+x^2+x^4}}{4\,(1+x^2)^2}-\frac{x\sqrt{1+x^2+x^4}}{3\,(1+x^2)}+\frac{3}{4}\tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)+\frac{19(1+x^2)\sqrt{\frac{1+x^2+}{(1+x^2)}}}{12\sqrt{1+x^2}}$$

```
[Out] 3/4*arctan(x/(x^4+x^2+1)^(1/2))-1/3*x*(-x^2+1)/(x^4+x^2+1)^(1/2)+1/4*x*(x^4
+x^2+1)^(1/2)/(x^2+1)^2-1/3*x*(x^4+x^2+1)^(1/2)/(x^2+1)+19/12*(x^2+1)*(cos(
2*arctan(x))^2)^(1/2)/cos(2*arctan(x))*EllipticE(sin(2*arctan(x)),1/2)*((x^
4+x^2+1)/(x^2+1)^2)^(1/2)/(x^4+x^2+1)^(1/2)-5/4*(x^2+1)*(cos(2*arctan(x))^2
)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2)*((x^4+x^2+1)/(x^2+
1)^2)^(1/2)/(x^4+x^2+1)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {1242, 1106, 1211, 1117, 1209, 1237, 1710, 1607, 1726, 1714, 1712, 209, 12, 1331}

$$\frac{3}{4}\text{ArcTan}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)-\frac{5(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}F\left(2\text{ArcTan}(x)\middle|\frac{1}{4}\right)}{4\sqrt{x^4+x^2+1}}+\frac{19(x^2+1)\sqrt{\frac{x^4+x^2+1}{(x^2+1)^2}}E\left(2\text{ArcTan}(x)\middle|\frac{1}{4}\right)}{12\sqrt{x^4+x^2+1}}-\frac{\sqrt{x^4+x^2+1}\,x}{3\,(x^2+1)}+\frac{\sqrt{x^4+x^2+1}\,x}{4\,(x^2+1)^2}-\frac{(1-x^2)\,x}{3\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/3*(x*(1 - x^2))/Sqrt[1 + x^2 + x^4] + (x*Sqrt[1 + x^2 + x^4])/(4*(1 + x^
2)^2) - (x*Sqrt[1 + x^2 + x^4])/(3*(1 + x^2)) + (3*ArcTan[x/Sqrt[1 + x^2 +
x^4]])/4 + (19*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(1 + x^2)^2]*EllipticE[2*ArcT
an[x], 1/4])/(12*Sqrt[1 + x^2 + x^4]) - (5*(1 + x^2)*Sqrt[(1 + x^2 + x^4)/(
1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/4])/(4*Sqrt[1 + x^2 + x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1331

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> Dist[d/(2*d*e), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist
[d/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x] /
; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1712

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 1714

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[(B*d + A*e)/(2*d*e), Int[1/Sqrt[a + b*x^
2 + c*x^4], x], x] - Dist[(B*d - A*e)/(2*d*e), Int[(d - e*x^2)/((d + e*x^2)
*Sqrt[a + b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
&& NeQ[B*d + A*e, 0]
```

Rule 1726

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-C/e^2, Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] +
 Dist[1/e^2, Int[(C*d^2 + A*e^2 + B*e^2*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 +
c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(1+x^2)^3\,(1+x^2+x^4)^{3/2}}\,dx = \int \left(-\frac{1}{(1+x^2+x^4)^{3/2}} + \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}} + \frac{1}{(1+x^2)^2\sqrt{1+x^2}}\right.$$

$$= -\int \frac{1}{(1+x^2+x^4)^{3/2}}\,dx + \int \frac{1}{(1+x^2)^3\sqrt{1+x^2+x^4}}\,dx + \int \frac{}{(1+x^2)}$$

$$= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\,(1+x^2)^2} + \frac{x\sqrt{1+x^2+x^4}}{2\,(1+x^2)} - \frac{1}{4}\int \frac{-3}{(1+x^2)}$$

$$= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\,(1+x^2)^2} + \frac{5x\sqrt{1+x^2+x^4}}{4\,(1+x^2)} + \frac{1}{8}\int \frac{-}{(1+x}$$

$$= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\,(1+x^2)^2} + \frac{5x\sqrt{1+x^2+x^4}}{12\,(1+x^2)} + \frac{5(1+x^2)}{}$$

$$= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\,(1+x^2)^2} + \frac{5x\sqrt{1+x^2+x^4}}{12\,(1+x^2)} + \frac{5(1+x^2)}{}$$

$$= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\,(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\,(1+x^2)} + \frac{19(1+x^2)}{}$$

$$= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\,(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\,(1+x^2)} + \frac{1}{2}\tan^{-1}\left(\frac{}{\sqrt{}}\right.$$

$$= -\frac{x(1-x^2)}{3\sqrt{1+x^2+x^4}} + \frac{x\sqrt{1+x^2+x^4}}{4\,(1+x^2)^2} - \frac{x\sqrt{1+x^2+x^4}}{3\,(1+x^2)} + \frac{3}{4}\tan^{-1}\left(\frac{}{\sqrt{}}\right.$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.18, size = 192, normalized size = 1.01

$$\frac{4x(-1+x^2)(1+x^2)^2+3x(1+x^2+x^4)+15x(1+x^2)(1+x^2+x^4)-\sqrt[3]{-1}(1+x^2)^2\sqrt{1+\sqrt[3]{-1}x^2}\sqrt{1-(-1)^{2/3}x^2}\left(19E\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)+\left(-9+10i\sqrt{3}\right)F\left(i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)-18\sqrt[3]{-1}\Pi\left(\sqrt[3]{-1};i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)}{12(1+x^2)^2\sqrt{1+x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)^3*(1 + x^2 + x^4)^(3/2)),x]
```

```
[Out] (4*x*(-1 + x^2)*(1 + x^2)^2 + 3*x*(1 + x^2 + x^4) + 15*x*(1 + x^2)*(1 + x^2
 + x^4) - (-1)^(1/3)*(1 + x^2)^2*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/
3)*x^2]*(19*EllipticE[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + (-9 + (10*I)*S
qrt[3])*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] - 18*(-1)^(1/3)*Elli
pticPi[(-1)^(1/3), I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)]))/(12*(1 + x^2)^2*S
qrt[1 + x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 439, normalized size = 2.31

method	result
risch	$\frac{x(19x^6+37x^4+29x^2+14)}{12(x^2+1)^2\sqrt{x^4+x^2+1}}+\frac{19\sqrt{1-\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-2+}}{2}\right.\right.}{3\sqrt{-2+2i\sqrt{3}}\sqrt{x^4}}$
default	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2}+\frac{5x\sqrt{x^4+x^2+1}}{4(x^2+1)}-\frac{2\left(\frac{1}{6}x-\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}}-\frac{10\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}}{3\sqrt{-2+2i\sqrt{3}}}$
elliptic	$\frac{x\sqrt{x^4+x^2+1}}{4(x^2+1)^2}+\frac{5x\sqrt{x^4+x^2+1}}{4(x^2+1)}-\frac{2\left(\frac{1}{6}x-\frac{1}{6}x^3\right)}{\sqrt{x^4+x^2+1}}-\frac{10\sqrt{1+\frac{x^2}{2}-\frac{ix^2\sqrt{3}}{2}}\sqrt{1+\frac{x^2}{2}+\frac{ix^2\sqrt{3}}{2}}}{3\sqrt{-2+2i\sqrt{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)^2+5/4*x*(x^4+x^2+1)^(1/2)/(x^2+1)-2*(1/6*x-
1/6*x^3)/(x^4+x^2+1)^(1/2)-10/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2
*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*Ellip
ticF(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))+19/3/(-2+2*I*
3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(
1/2))^(1/2)/(x^4+x^2+1)^(1/2)/(1+I*3^(1/2))*EllipticF(1/2*x*(-2+2*I*3^(1/2)
)^(1/2),1/2*(-2+2*I*3^(1/2))^(1/2))-19/3/(-2+2*I*3^(1/2))^(1/2)*(1+1/2*x^2-
```

```
1/2*I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1
/2)/(1+I*3^(1/2))*EllipticE(1/2*x*(-2+2*I*3^(1/2))^(1/2),1/2*(-2+2*I*3^(1/2
))^(1/2))+3/2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*I*x^2*3^(1/2))^(1/2
)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*EllipticPi((-1/2+1/
2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^(1/2))^(1/2)/(-1
/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^{2}-x+1\right)\left(x^{2}+x+1\right)\right)^{\frac{3}{2}}\left(x^{2}+1\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**3/(x**4+x**2+1)**(3/2),x)
[Out] Integral(1/(((x**2 - x + 1)*(x**2 + x + 1))**(3/2)*(x**2 + 1)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^3/(x^4+x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + x^2 + 1)^(3/2)*(x^2 + 1)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^2+1\right)^3\,\left(x^4+x^2+1\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(3/2)),x)
[Out] int(1/((x^2 + 1)^3*(x^2 + x^4 + 1)^(3/2)), x)
```

3.244 $\int (d+ex^2)^4 (a+bx^2+cx^4) \, dx$

Optimal. Leaf size=135

$$ad^4x+\frac{1}{3}d^3(bd+4ae)x^3+\frac{1}{5}d^2\left(cd^2+4bde+6ae^2\right)x^5+\frac{2}{7}de\left(2cd^2+e(3bd+2ae)\right)x^7+\frac{1}{9}e^2\left(6cd^2+e(4bd+ae)\right)$$

```
[Out] a*d^4*x+1/3*d^3*(4*a*e+b*d)*x^3+1/5*d^2*(6*a*e^2+4*b*d*e+c*d^2)*x^5+2/7*d*e
*(2*c*d^2+e*(2*a*e+3*b*d))*x^7+1/9*e^2*(6*c*d^2+e*(a*e+4*b*d))*x^9+1/11*e^3
*(b*e+4*c*d)*x^11+1/13*c*e^4*x^13
```

Rubi [A]
time = 0.09, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1167}

$$\frac{1}{9}e^2x^9\left(e(ae+4bd)+6cd^2\right)+\frac{1}{5}d^2x^5\left(6ae^2+4bde+cd^2\right)+\frac{2}{7}dex^7\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{3}d^3x^3(4ae+bd)+ad^4x+\frac{1}{11}e^3x^{11}(be+4cd)+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)
/5 + (2*d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^7)/7 + (e^2*(6*c*d^2 + e*(4*b*d
 + a*e))*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(d+ex^2\right)^4\left(a+bx^2+cx^4\right)\,dx=\int \left(ad^4+d^3(bd+4ae)x^2+d^2\left(cd^2+4bde+6ae^2\right)x^4+2de\left(2cd^2+e(3\right.\right.$$

$$=ad^4x+\frac{1}{3}d^3(bd+4ae)x^3+\frac{1}{5}d^2\left(cd^2+4bde+6ae^2\right)x^5+\frac{2}{7}de\left(2cd^2+e(\right.$$

Mathematica [A]
time = 0.03, size = 135, normalized size = 1.00

$$ad^4x+\frac{1}{3}d^3(bd+4ae)x^3+\frac{1}{5}d^2\left(cd^2+4bde+6ae^2\right)x^5+\frac{2}{7}de\left(2cd^2+3bde+2ae^2\right)x^7+\frac{1}{9}e^2\left(6cd^2+4bde+ae^2\right)x^9+\frac{1}{11}e^3(4cd+be)x^{11}+\frac{1}{13}ce^4x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^3)/3 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^5)
/5 + (2*d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^7)/7 + (e^2*(6*c*d^2 + 4*b*d*e
+ a*e^2)*x^9)/9 + (e^3*(4*c*d + b*e)*x^11)/11 + (c*e^4*x^13)/13
```

Maple [A]
time = 0.15, size = 136, normalized size = 1.01

method	result
norman	$\frac{c\,e^{4} x^{13}}{13}+\left(\frac{1}{11} e^{4} b+\frac{4}{11} d\,e^{3} c\right) x^{11}+\left(\frac{1}{9} e^{4} a+\frac{4}{9} d\,e^{3} b+\frac{2}{3} d^{2} e^{2} c\right) x^{9}+\left(\frac{4}{7} d\,e^{3} a+\frac{6}{7} d^{2} e^{2} b+\frac{4}{7} d^{3} e c\right) x^{7}+\left(\frac{6}{5}\right.$
default	$\frac{c\,e^{4} x^{13}}{13}+\frac{\left(e^{4} b+4 d\,e^{3} c\right) x^{11}}{11}+\frac{\left(e^{4} a+4 d\,e^{3} b+6 d^{2} e^{2} c\right) x^{9}}{9}+\frac{\left(4 d\,e^{3} a+6 d^{2} e^{2} b+4 d^{3} e c\right) x^{7}}{7}+\frac{\left(6 d^{2} e^{2} a+4 d^{3} e b+d^{4} c\right) x^{5}}{5}+\frac{\left(4 d^{3} e a+d^{4}\right.}{3}$
gosper	$\frac{1}{13} c\,e^{4} x^{13}+\frac{1}{11} x^{11} e^{4} b+\frac{4}{11} c d\,e^{3} x^{11}+\frac{1}{9} x^{9} e^{4} a+\frac{4}{9} x^{9} d\,e^{3} b+\frac{2}{3} x^{9} d^{2} e^{2} c+\frac{4}{7} x^{7} d\,e^{3} a+\frac{6}{7} x^{7} d^{2} e^{2} b+\frac{4}{7} x^{7} d^{3}$
risch	$\frac{1}{13} c\,e^{4} x^{13}+\frac{1}{11} x^{11} e^{4} b+\frac{4}{11} c d\,e^{3} x^{11}+\frac{1}{9} x^{9} e^{4} a+\frac{4}{9} x^{9} d\,e^{3} b+\frac{2}{3} x^{9} d^{2} e^{2} c+\frac{4}{7} x^{7} d\,e^{3} a+\frac{6}{7} x^{7} d^{2} e^{2} b+\frac{4}{7} x^{7} d^{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c*e^4*x^13+1/11*(b*e^4+4*c*d*e^3)*x^11+1/9*(a*e^4+4*b*d*e^3+6*c*d^2*e^
2)*x^9+1/7*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^7+1/5*(6*a*d^2*e^2+4*b*d^3*e
+c*d^4)*x^5+1/3*(4*a*d^3*e+b*d^4)*x^3+a*d^4*x
```

Maxima [A]
time = 0.29, size = 129, normalized size = 0.96

$$\frac{1}{13}\,cx^{13}e^{4}+\frac{1}{11}\,\left(4\,cde^{3}+be^{4}\right)x^{11}+\frac{1}{9}\,\left(6\,cd^{2}e^{2}+4\,bde^{3}+ae^{4}\right)x^{9}+\frac{2}{7}\,\left(2\,cd^{3}e+3\,bd^{2}e^{2}+2\,ade^{3}\right)x^{7}+ad^{4}x+\frac{1}{5}\,\left(cd^{4}+4\,bd^{3}e+6\,ad^{2}e^{2}\right)x^{5}+\frac{1}{3}\,\left(bd^{4}+4\,ad^{3}e\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*x^13*e^4 + 1/11*(4*c*d*e^3 + b*e^4)*x^11 + 1/9*(6*c*d^2*e^2 + 4*b*d*
e^3 + a*e^4)*x^9 + 2/7*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^7 + a*d^4*x
+ 1/5*(c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^5 + 1/3*(b*d^4 + 4*a*d^3*e)*x^3
```

Fricas [A]
time = 0.34, size = 138, normalized size = 1.02

$$\frac{1}{5}\,cd^{4}x^{5}+\frac{1}{3}\,bd^{4}x^{3}+ad^{4}x+\frac{1}{1287}\,\left(99\,cx^{13}+117\,bx^{11}+143\,ax^{9}\right)e^{4}+\frac{4}{693}\,\left(63\,cdx^{11}+77\,bdx^{9}+99\,adx^{7}\right)e^{3}+\frac{2}{105}\,\left(35\,cd^{2}x^{9}+45\,bd^{2}x^{7}+63\,ad^{2}x^{5}\right)e^{2}+\frac{4}{105}\,\left(15\,cd^{3}x^{7}+21\,bd^{3}x^{5}+35\,ad^{3}x^{3}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/5*c*d^4*x^5 + 1/3*b*d^4*x^3 + a*d^4*x + 1/1287*(99*c*x^13 + 117*b*x^11 +
143*a*x^9)*e^4 + 4/693*(63*c*d*x^11 + 77*b*d*x^9 + 99*a*d*x^7)*e^3 + 2/105*
(35*c*d^2*x^9 + 45*b*d^2*x^7 + 63*a*d^2*x^5)*e^2 + 4/105*(15*c*d^3*x^7 + 21
*b*d^3*x^5 + 35*a*d^3*x^3)*e
```

Sympy [A]
time = 0.02, size = 156, normalized size = 1.16

$$ad^4x + \frac{ce^4x^{13}}{13} + x^{11}\left(\frac{be^4}{11} + \frac{4cde^3}{11}\right) + x^9\left(\frac{ae^4}{9} + \frac{4bde^3}{9} + \frac{2cd^2e^2}{3}\right) + x^7 \cdot \left(\frac{4ade^3}{7} + \frac{6bd^2e^2}{7} + \frac{4cd^3e}{7}\right) + x^5 \cdot \left(\frac{6ad^2e^2}{5} + \frac{4bd^3e}{5} + \frac{cd^4}{5}\right) + x^3 \cdot \left(\frac{4ad^3e}{3} + \frac{bd^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**13/13 + x**11*(b*e**4/11 + 4*c*d*e**3/11) + x**9*(a*e*
*4/9 + 4*b*d*e**3/9 + 2*c*d**2*e**2/3) + x**7*(4*a*d*e**3/7 + 6*b*d**2*e**2
/7 + 4*c*d**3*e/7) + x**5*(6*a*d**2*e**2/5 + 4*b*d**3*e/5 + c*d**4/5) + x**
3*(4*a*d**3*e/3 + b*d**4/3)
```

Giac [A]
time = 5.18, size = 142, normalized size = 1.05

$$\frac{1}{13}\,cx^{13}e^4 + \frac{4}{11}\,cdx^{11}e^3 + \frac{1}{11}\,bx^{11}e^4 + \frac{2}{3}\,cd^2x^9e^2 + \frac{4}{9}\,bdx^9e^3 + \frac{4}{7}\,cd^3x^7e + \frac{1}{9}\,ax^9e^4 + \frac{6}{7}\,bd^2x^7e^2 + \frac{1}{5}\,cd^4x^5 + \frac{4}{7}\,adx^7e^3 + \frac{4}{5}\,bd^3x^5e + \frac{6}{5}\,ad^2x^5e^2 + \frac{1}{3}\,bd^4x^3 + \frac{4}{3}\,ad^3x^3e + ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^4 + 4/11*c*d*x^11*e^3 + 1/11*b*x^11*e^4 + 2/3*c*d^2*x^9*e^2 +
 4/9*b*d*x^9*e^3 + 4/7*c*d^3*x^7*e + 1/9*a*x^9*e^4 + 6/7*b*d^2*x^7*e^2 + 1/
5*c*d^4*x^5 + 4/7*a*d*x^7*e^3 + 4/5*b*d^3*x^5*e + 6/5*a*d^2*x^5*e^2 + 1/3*b
*d^4*x^3 + 4/3*a*d^3*x^3*e + a*d^4*x
```

Mupad [B]
time = 0.06, size = 131, normalized size = 0.97

$$x^3\,\left(\frac{b\,d^4}{3}+\frac{4\,a\,e\,d^3}{3}\right)+x^{11}\,\left(\frac{b\,e^4}{11}+\frac{4\,c\,d\,e^3}{11}\right)+x^5\,\left(\frac{c\,d^4}{5}+\frac{4\,b\,d^3\,e}{5}+\frac{6\,a\,d^2\,e^2}{5}\right)+x^9\,\left(\frac{2\,c\,d^2\,e^2}{3}+\frac{4\,b\,d\,e^3}{9}+\frac{a\,e^4}{9}\right)+\frac{c\,e^4\,x^{13}}{13}+a\,d^4\,x+\frac{2\,d\,e\,x^7\,\left(2\,c\,d^2+3\,b\,d\,e+2\,a\,e^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((b*d^4)/3 + (4*a*d^3*e)/3) + x^11*((b*e^4)/11 + (4*c*d*e^3)/11) + x^5*
((c*d^4)/5 + (6*a*d^2*e^2)/5 + (4*b*d^3*e)/5) + x^9*((a*e^4)/9 + (2*c*d^2*e
^2)/3 + (4*b*d*e^3)/9) + (c*e^4*x^13)/13 + a*d^4*x + (2*d*e*x^7*(2*a*e^2 +
2*c*d^2 + 3*b*d*e))/7
```

3.245 $\int (d+ex^2)^3 (a+bx^2+cx^4) \, dx$

Optimal. Leaf size=103

$$ad^3x+\frac{1}{3}d^2(bd+3ae)x^3+\frac{1}{5}d\left(cd^2+3e(bd+ae)\right)x^5+\frac{1}{7}e\left(3cd^2+e(3bd+ae)\right)x^7+\frac{1}{9}e^2(3cd+be)x^9+\frac{1}{11}ce^3x^{11}$$

```
[Out] a*d^3*x+1/3*d^2*(3*a*e+b*d)*x^3+1/5*d*(c*d^2+3*e*(a*e+b*d))*x^5+1/7*e*(3*c*
d^2+e*(a*e+3*b*d))*x^7+1/9*e^2*(b*e+3*c*d)*x^9+1/11*c*e^3*x^11
```

Rubi [A]
time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1167}

$$\frac{1}{7}ex^7\left(e(ae+3bd)+3cd^2\right)+\frac{1}{5}dx^5\left(3e(ae+bd)+cd^2\right)+\frac{1}{3}d^2x^3(3ae+bd)+ad^3x+\frac{1}{9}e^2x^9(be+3cd)+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^5)/5 +
 (e*(3*c*d^2 + e*(3*b*d + a*e))*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e^3
*x^11)/11
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right) \, dx = \int \left(ad^3+d^2(bd+3ae)x^2+d\left(cd^2+3e(bd+ae)\right)x^4+e\left(3cd^2+e(3bd+\right.\right.$$
$$= ad^3x+\frac{1}{3}d^2(bd+3ae)x^3+\frac{1}{5}d\left(cd^2+3e(bd+ae)\right)x^5+\frac{1}{7}e\left(3cd^2+e(3bd\right.$$

Mathematica [A]
time = 0.02, size = 104, normalized size = 1.01

$$ad^3x+\frac{1}{3}d^2(bd+3ae)x^3+\frac{1}{5}d\left(cd^2+3bde+3ae^2\right)x^5+\frac{1}{7}e\left(3cd^2+3bde+ae^2\right)x^7+\frac{1}{9}e^2(3cd+be)x^9+\frac{1}{11}ce^3x^{11}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4),x]

[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^3)/3 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^5)/5 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^7)/7 + (e^2*(3*c*d + b*e)*x^9)/9 + (c*e^3*x^11)/11

Maple [A]
time = 0.14, size = 103, normalized size = 1.00

method	result
norman	$\frac{c\,e^3x^{11}}{11}+\left(\frac{1}{9}e^3b+\frac{1}{3}d\,e^2c\right)x^9+\left(\frac{1}{7}a\,e^3+\frac{3}{7}d\,e^2b+\frac{3}{7}c\,d^2e\right)x^7+\left(\frac{3}{5}d\,e^2a+\frac{3}{5}d^2eb+\frac{1}{5}c\,d^3\right)x^5+\left(a\,d^2\right.$
default	$\frac{c\,e^3x^{11}}{11}+\frac{(e^3b+3d\,e^2c)x^9}{9}+\frac{(a\,e^3+3d\,e^2b+3c\,d^2e)x^7}{7}+\frac{(3d\,e^2a+3d^2eb+c\,d^3)x^5}{5}+\frac{(3a\,d^2e+d^3b)x^3}{3}+a\,d^3x$
gosper	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{9}x^9e^3b+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7d\,e^2b+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{3}{5}x^5d^2eb+\frac{1}{5}x^5c\,d^3$
risch	$\frac{1}{11}c\,e^3x^{11}+\frac{1}{9}x^9e^3b+\frac{1}{3}cd\,e^2x^9+\frac{1}{7}x^7a\,e^3+\frac{3}{7}x^7d\,e^2b+\frac{3}{7}x^7c\,d^2e+\frac{3}{5}x^5d\,e^2a+\frac{3}{5}x^5d^2eb+\frac{1}{5}x^5c\,d^3$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^3*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/11*c*e^3*x^11+1/9*(b*e^3+3*c*d*e^2)*x^9+1/7*(a*e^3+3*b*d*e^2+3*c*d^2*e)*x^7+1/5*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^5+1/3*(3*a*d^2*e+b*d^3)*x^3+a*d^3*x

Maxima [A]
time = 0.28, size = 99, normalized size = 0.96

$$\frac{1}{11}\,cx^{11}e^3+\frac{1}{9}\left(3\,cde^2+be^3\right)x^9+\frac{1}{7}\left(3\,cd^2e+3\,bde^2+ae^3\right)x^7+\frac{1}{5}\left(cd^3+3\,bd^2e+3\,ade^2\right)x^5+ad^3x+\frac{1}{3}\left(bd^3+3\,ad^2e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] 1/11*c*x^11*e^3 + 1/9*(3*c*d*e^2 + b*e^3)*x^9 + 1/7*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*x^7 + 1/5*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^5 + a*d^3*x + 1/3*(b*d^3 + 3*a*d^2*e)*x^3

Fricas [A]
time = 0.34, size = 106, normalized size = 1.03

$$\frac{1}{5}\,cd^3x^5+\frac{1}{3}\,bd^3x^3+ad^3x+\frac{1}{693}\left(63\,cx^{11}+77\,bx^9+99\,ax^7\right)e^3+\frac{1}{105}\left(35\,cdx^9+45\,bdx^7+63\,adx^5\right)e^2+\frac{1}{35}\left(15\,cd^2x^7+21\,bd^2x^5+35\,ad^2x^3\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] 1/5*c*d^3*x^5 + 1/3*b*d^3*x^3 + a*d^3*x + 1/693*(63*c*x^11 + 77*b*x^9 + 99*a*x^7)*e^3 + 1/105*(35*c*d*x^9 + 45*b*d*x^7 + 63*a*d*x^5)*e^2 + 1/35*(15*c*d^2*x^7 + 21*b*d^2*x^5 + 35*a*d^2*x^3)*e

Sympy [A]
time = 0.01, size = 112, normalized size = 1.09

$$ad^3x + \frac{ce^3x^{11}}{11} + x^9\left(\frac{be^3}{9} + \frac{cde^2}{3}\right) + x^7\left(\frac{ae^3}{7} + \frac{3bde^2}{7} + \frac{3cd^2e}{7}\right) + x^5\cdot\left(\frac{3ade^2}{5} + \frac{3bd^2e}{5} + \frac{cd^3}{5}\right) + x^3\left(ad^2e + \frac{bd^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**11/11 + x**9*(b*e**3/9 + c*d*e**2/3) + x**7*(a*e**3/7
+ 3*b*d*e**2/7 + 3*c*d**2*e/7) + x**5*(3*a*d*e**2/5 + 3*b*d**2*e/5 + c*d**3
/5) + x**3*(a*d**2*e + b*d**3/3)
```

Giac [A]
time = 3.94, size = 108, normalized size = 1.05

$$\frac{1}{11}cx^{11}e^3 + \frac{1}{3}cdx^9e^2 + \frac{1}{9}bx^9e^3 + \frac{3}{7}cd^2x^7e + \frac{3}{7}bdx^7e^2 + \frac{1}{5}cd^3x^5 + \frac{1}{7}ax^7e^3 + \frac{3}{5}bd^2x^5e + \frac{3}{5}adx^5e^2 + \frac{1}{3}bd^3x^3 + ad^2x^3e + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^3 + 1/3*c*d*x^9*e^2 + 1/9*b*x^9*e^3 + 3/7*c*d^2*x^7*e + 3/7*b
*d*x^7*e^2 + 1/5*c*d^3*x^5 + 1/7*a*x^7*e^3 + 3/5*b*d^2*x^5*e + 3/5*a*d*x^5*
e^2 + 1/3*b*d^3*x^3 + a*d^2*x^3*e + a*d^3*x
```

Mupad [B]
time = 4.63, size = 101, normalized size = 0.98

$$x^3\left(\frac{b\,d^3}{3} + a\,e\,d^2\right) + x^9\left(\frac{b\,e^3}{9} + \frac{c\,d\,e^2}{3}\right) + x^5\left(\frac{c\,d^3}{5} + \frac{3\,b\,d^2\,e}{5} + \frac{3\,a\,d\,e^2}{5}\right) + x^7\left(\frac{3\,c\,d^2\,e}{7} + \frac{3\,b\,d\,e^2}{7} + \frac{a\,e^3}{7}\right) + \frac{c\,e^3\,x^{11}}{11} + a\,d^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((b*d^3)/3 + a*d^2*e) + x^9*((b*e^3)/9 + (c*d*e^2)/3) + x^5*((c*d^3)/5
+ (3*a*d*e^2)/5 + (3*b*d^2*e)/5) + x^7*((a*e^3)/7 + (3*b*d*e^2)/7 + (3*c*d^
2*e)/7) + (c*e^3*x^11)/11 + a*d^3*x
```

3.246 $\int (d+ex^2)^2 (a+bx^2+cx^4)\, dx$

Optimal. Leaf size=73

$$ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5+\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x+1/3*d*(2*a*e+b*d)*x^3+1/5*(c*d^2+e*(a*e+2*b*d))*x^5+1/7*e*(b*e+2*c*
d)*x^7+1/9*c*e^2*x^9
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1167}

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx=\int \left(ad^2+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^4+e(2cd+be)x^6+ce\right.$$
$$=ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5+\frac{1}{7}e(2cd+be)x^7+$$

Mathematica [A]
time = 0.01, size = 73, normalized size = 1.00

$$ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+2bde+ae^2\right)x^5+\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Maple [A]
time = 0.14, size = 70, normalized size = 0.96

method	result	size
default	$\frac{c\,e^2x^9}{9}+\frac{(e^2b+2cde)x^7}{7}+\frac{(a\,e^2+2deb+c\,d^2)x^5}{5}+\frac{(2ade+d^2b)x^3}{3}+a\,d^2x$	70
norman	$\frac{c\,e^2x^9}{9}+\left(\frac{1}{7}e^2b+\frac{2}{7}cde\right)x^7+\left(\frac{1}{5}a\,e^2+\frac{2}{5}deb+\frac{1}{5}c\,d^2\right)x^5+\left(\frac{2}{3}ade+\frac{1}{3}d^2b\right)x^3+a\,d^2x$	71
gosper	$\frac{1}{9}c\,e^2x^9+\frac{1}{7}x^7e^2b+\frac{2}{7}cde\,x^7+\frac{1}{5}x^5a\,e^2+\frac{2}{5}x^5deb+\frac{1}{5}x^5c\,d^2+\frac{2}{3}ade\,x^3+\frac{1}{3}x^3d^2b+a\,d^2x$	77
risch	$\frac{1}{9}c\,e^2x^9+\frac{1}{7}x^7e^2b+\frac{2}{7}cde\,x^7+\frac{1}{5}x^5a\,e^2+\frac{2}{5}x^5deb+\frac{1}{5}x^5c\,d^2+\frac{2}{3}ade\,x^3+\frac{1}{3}x^3d^2b+a\,d^2x$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x^5+1/3*(2*
a*d*e+b*d^2)*x^3+a*d^2*x
```

Maxima [A]
time = 0.29, size = 69, normalized size = 0.95

$$\frac{1}{9}\,cx^9e^2+\frac{1}{7}\left(2\,cde+be^2\right)x^7+\frac{1}{5}\left(cd^2+2\,bde+ae^2\right)x^5+ad^2x+\frac{1}{3}\left(bd^2+2\,ade\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*x^9*e^2 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x
^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

Fricas [A]
time = 0.34, size = 74, normalized size = 1.01

$$\frac{1}{5}\,cd^2x^5+\frac{1}{3}\,bd^2x^3+ad^2x+\frac{1}{315}\left(35\,cx^9+45\,bx^7+63\,ax^5\right)e^2+\frac{2}{105}\left(15\,cdx^7+21\,bdx^5+35\,adx^3\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/5*c*d^2*x^5 + 1/3*b*d^2*x^3 + a*d^2*x + 1/315*(35*c*x^9 + 45*b*x^7 + 63*a
*x^5)*e^2 + 2/105*(15*c*d*x^7 + 21*b*d*x^5 + 35*a*d*x^3)*e
```

Sympy [A]
time = 0.01, size = 78, normalized size = 1.07

$$ad^2x+\frac{ce^2x^9}{9}+x^7\left(\frac{be^2}{7}+\frac{2cde}{7}\right)+x^5\left(\frac{ae^2}{5}+\frac{2bde}{5}+\frac{cd^2}{5}\right)+x^3\cdot\left(\frac{2ade}{3}+\frac{bd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2
*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

Giac [A]
time = 3.78, size = 76, normalized size = 1.04

$$\frac{1}{9}cx^9e^2 + \frac{2}{7}cdx^7e + \frac{1}{7}bx^7e^2 + \frac{1}{5}cd^2x^5 + \frac{2}{5}bdx^5e + \frac{1}{5}ax^5e^2 + \frac{1}{3}bd^2x^3 + \frac{2}{3}adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/7*b*x^7*e^2 + 1/5*c*d^2*x^5 + 2/5*b*d*x^5
*e + 1/5*a*x^5*e^2 + 1/3*b*d^2*x^3 + 2/3*a*d*x^3*e + a*d^2*x
```

Mupad [B]
time = 4.59, size = 70, normalized size = 0.96

$$x^5\left(\frac{c\,d^2}{5}+\frac{2\,b\,d\,e}{5}+\frac{a\,e^2}{5}\right)+x^3\left(\frac{b\,d^2}{3}+\frac{2\,a\,e\,d}{3}\right)+x^7\left(\frac{b\,e^2}{7}+\frac{2\,c\,d\,e}{7}\right)+\frac{c\,e^2\,x^9}{9}+a\,d^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^5*((a*e^2)/5 + (c*d^2)/5 + (2*b*d*e)/5) + x^3*((b*d^2)/3 + (2*a*d*e)/3) +
 x^7*((b*e^2)/7 + (2*c*d*e)/7) + (c*e^2*x^9)/9 + a*d^2*x
```

3.247 $\int (d+ex^2)(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=42

$$adx+\frac{1}{3}(bd+ae)x^3+\frac{1}{5}(cd+be)x^5+\frac{1}{7}cex^7$$

```
[Out] a*d*x+1/3*(a*e+b*d)*x^3+1/5*(b*e+c*d)*x^5+1/7*c*e*x^7
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1167}

$$\frac{1}{3}x^3(ae+bd)+adx+\frac{1}{5}x^5(be+cd)+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex^2)(a+bx^2+cx^4)\,dx&=\int \left(ad+(bd+ae)x^2+(cd+be)x^4+cex^6\right)dx\\&=adx+\frac{1}{3}(bd+ae)x^3+\frac{1}{5}(cd+be)x^5+\frac{1}{7}cex^7\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$adx+\frac{1}{3}(bd+ae)x^3+\frac{1}{5}(cd+be)x^5+\frac{1}{7}cex^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^3)/3 + ((c*d + b*e)*x^5)/5 + (c*e*x^7)/7
```

Maple [A]
time = 0.05, size = 37, normalized size = 0.88

method	result	size
default	$adx+\frac{(ae+bd)x^3}{3}+\frac{(eb+cd)x^5}{5}+\frac{ce\,x^7}{7}$	37
norman	$\frac{ce\,x^7}{7}+\left(\frac{eb}{5}+\frac{cd}{5}\right)x^5+\left(\frac{ae}{3}+\frac{bd}{3}\right)x^3+adx$	39
gosper	$\frac{1}{7}ce\,x^7+\frac{1}{5}x^5eb+\frac{1}{5}cd\,x^5+\frac{1}{3}ae\,x^3+\frac{1}{3}bd\,x^3+adx$	41
risch	$\frac{1}{7}ce\,x^7+\frac{1}{5}x^5eb+\frac{1}{5}cd\,x^5+\frac{1}{3}ae\,x^3+\frac{1}{3}bd\,x^3+adx$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
[Out] a*d*x+1/3*(a*e+b*d)*x^3+1/5*(b*e+c*d)*x^5+1/7*c*e*x^7
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.93

$$\frac{1}{7}\,cx^7e+\frac{1}{5}\,(cd+be)x^5+\frac{1}{3}\,(bd+ae)x^3+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] 1/7*c*x^7*e + 1/5*(c*d + b*e)*x^5 + 1/3*(b*d + a*e)*x^3 + a*d*x
```

Fricas [A]
time = 0.35, size = 42, normalized size = 1.00

$$\frac{1}{5}\,cdx^5+\frac{1}{3}\,bdx^3+adx+\frac{1}{105}\,\left(15\,cx^7+21\,bx^5+35\,ax^3\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] 1/5*c*d*x^5 + 1/3*b*d*x^3 + a*d*x + 1/105*(15*c*x^7 + 21*b*x^5 + 35*a*x^3)*
e
```

Sympy [A]
time = 0.01, size = 39, normalized size = 0.93

$$adx+\frac{cex^7}{7}+x^5\left(\frac{be}{5}+\frac{cd}{5}\right)+x^3\left(\frac{ae}{3}+\frac{bd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d*x + c*e*x**7/7 + x**5*(b*e/5 + c*d/5) + x**3*(a*e/3 + b*d/3)
```

Giac [A]
time = 3.87, size = 43, normalized size = 1.02

$$\frac{1}{7}cx^7e + \frac{1}{5}cdx^5 + \frac{1}{5}bx^5e + \frac{1}{3}bdx^3 + \frac{1}{3}ax^3e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e + 1/5*c*d*x^5 + 1/5*b*x^5*e + 1/3*b*d*x^3 + 1/3*a*x^3*e + a*d*x
```

Mupad [B]
time = 0.04, size = 38, normalized size = 0.90

$$\frac{c\,e\,x^7}{7} + \left(\frac{b\,e}{5} + \frac{c\,d}{5}\right)\,x^5 + \left(\frac{a\,e}{3} + \frac{b\,d}{3}\right)\,x^3 + a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((a*e)/3 + (b*d)/3) + x^5*((b*e)/5 + (c*d)/5) + a*d*x + (c*e*x^7)/7
```

3.248 $\int \frac{a+bx^2+cx^4}{d+ex^2} dx$

Optimal. Leaf size=66

$$-\frac{(cd-be)x}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2-bde+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}$$

```
[Out] -(-b*e+c*d)*x/e^2+1/3*c*x^3/e+(a*e^2-b*d*e+c*d^2)*arctan(x*e^(1/2)/d^(1/2))
/e^(5/2)/d^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1167, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(ae^2-bde+cd^2)}{\sqrt{d}\,e^{5/2}}-\frac{x(cd-be)}{e^2}+\frac{cx^3}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
```

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{d+ex^2}\,dx = \int \left(-\frac{cd-be}{e^2}+\frac{cx^2}{e}+\frac{cd^2-bde+ae^2}{e^2(d+ex^2)}\right)dx$$
$$= -\frac{(cd-be)x}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2-bde+ae^2)\int\frac{1}{d+ex^2}\,dx}{e^2}$$
$$= -\frac{(cd-be)x}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2-bde+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.98

$$\frac{(-cd+be)x}{e^2}+\frac{cx^3}{3e}+\frac{(cd^2-bde+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2),x]
[Out] ((-(c*d) + b*e)*x)/e^2 + (c*x^3)/(3*e) + ((c*d^2 - b*d*e + a*e^2)*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(5/2))
```

Maple [A]
time = 0.13, size = 57, normalized size = 0.86

method	result
default	$\frac{\frac{1}{3}ce\,x^3+ebx-cdx}{e^2}+\frac{(a\,e^2-deb+c\,d^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e^2\sqrt{de}}$
risch	$\frac{c\,x^3}{3e}+\frac{bx}{e}-\frac{cdx}{e^2}-\frac{\ln\left(ex+\sqrt{-de}\right)a}{2\sqrt{-de}}+\frac{\ln\left(ex+\sqrt{-de}\right)db}{2e\sqrt{-de}}-\frac{\ln\left(ex+\sqrt{-de}\right)c\,d^2}{2e^2\sqrt{-de}}+\frac{\ln\left(-ex+\sqrt{-de}\right)a}{2\sqrt{-de}}-\frac{\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d),x,method=_RETURNVERBOSE)
[Out] 1/e^2*(1/3*c*e*x^3+e*b*x-c*d*x)+(a*e^2-b*d*e+c*d^2)/e^2/(d*e)^(1/2)*arctan(
e*x/(d*e)^(1/2))
```

Maxima [A]
time = 0.49, size = 55, normalized size = 0.83

$$\frac{(cd^2-bde+ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}}+\frac{1}{3}\left(cx^3e-3\,(cd-be)x\right)e^{(-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + 1/3*(c
*x^3*e - 3*(c*d - b*e)*x)*e^(-2)
```

Fricas [A]
time = 0.35, size = 155, normalized size = 2.35

$$\left[-\frac{\left(6\,cd^2xe+3\,(cd^2-bde+ae^2)\sqrt{-de}\,\log\left(\frac{x^2e-2\,\sqrt{-de}\,x-d}{x^2e+d}\right)-2\,(cdx^3+3\,bdx)e^2\right)e^{(-3)}}{6\,d},-\frac{\left(3\,cd^2xe-3\,(cd^2-bde+ae^2)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-(cdx^3+3\,bdx)e^2\right)e^{(-3)}}{3\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*c*d^2*x*e + 3*(c*d^2 - b*d*e + a*e^2)*sqrt(-d*e)*log((x^2*e - 2*sq
rt(-d*e)*x - d)/(x^2*e + d)) - 2*(c*d*x^3 + 3*b*d*x)*e^2)*e^(-3)/d, -1/3*(3
*c*d^2*x*e - 3*(c*d^2 - b*d*e + a*e^2)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^
(1/2) - (c*d*x^3 + 3*b*d*x)*e^2)*e^(-3)/d]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(58) = 116$.
time = 0.22, size = 117, normalized size = 1.77

$$\frac{cx^3}{3e}+x\left(\frac{b}{e}-\frac{cd}{e^2}\right)-\frac{\sqrt{-\frac{1}{de^5}}\,(ae^2-bde+cd^2)\log\left(-de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{de^5}}\,(ae^2-bde+cd^2)\log\left(de^2\sqrt{-\frac{1}{de^5}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d),x)
```

```
[Out] c*x**3/(3*e) + x*(b/e - c*d/e**2) - sqrt(-1/(d*e**5))*(a*e**2 - b*d*e + c*d
**2)*log(-d*e**2*sqrt(-1/(d*e**5)) + x)/2 + sqrt(-1/(d*e**5))*(a*e**2 - b*d
*e + c*d**2)*log(d*e**2*sqrt(-1/(d*e**5)) + x)/2
```

Giac [A]
time = 4.06, size = 56, normalized size = 0.85

$$\frac{(cd^2-bde+ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{\sqrt{d}}+\frac{1}{3}\left(cx^3e^2-3\,cdxe+3\,bxe^2\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) + 1/3*(c
*x^3*e^2 - 3*c*d*x*e + 3*b*x*e^2)*e^(-3)
```

Mupad [B]
time = 0.09, size = 57, normalized size = 0.86

$$x\left(\frac{b}{e}-\frac{c\,d}{e^2}\right)+\frac{c\,x^3}{3\,e}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{\sqrt{d}\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2),x)
```

```
[Out] x*(b/e - (c*d)/e^2) + (c*x^3)/(3*e) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + c
*d^2 - b*d*e))/(d^(1/2)*e^(5/2))
```

3.249 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=83

$$\frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)}-\frac{(3cd^2-e(bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1171, 396, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(3cd^2-e(ae+bd))}{2d^{3/2}e^{5/2}}+\frac{x(ae^2-bde+cd^2)}{2de^2\,(d+ex^2)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)} - \frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d} \\
&= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\int \frac{1}{d+ex^2}\,dx}{2de^2} \\
&= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 1.06

$$\frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)} - \frac{(3cd^2-bde-ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A]
time = 0.12, size = 79, normalized size = 0.95

method	result
default	$\frac{cx}{e^2} + \frac{\frac{(a\,e^2-deb+c\,d^2)x}{2d(e\,x^2+d)} + \frac{(a\,e^2+deb-3c\,d^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}}{e^2}$
risch	$\frac{cx}{e^2} + \frac{(a\,e^2-deb+c\,d^2)x}{2d\,e^2(e\,x^2+d)} - \frac{\ln\left(ex+\sqrt{-de}\right)a}{4\sqrt{-de}\,d} - \frac{\ln\left(ex+\sqrt{-de}\right)b}{4e\sqrt{-de}} + \frac{3d\ln\left(ex+\sqrt{-de}\right)c}{4e^2\sqrt{-de}} + \frac{\ln\left(-ex+\sqrt{-de}\right)a}{4\sqrt{-de}\,d} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*d
^2)/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.51, size = 74, normalized size = 0.89

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2 - bde + ae^2\right)x}{2\,\left(dx^2e^3 + d^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2 - b*d*e + a*e^2)*x/(d*x^2*e^3 + d^2*e^2)
```

Fricas [A]
time = 0.34, size = 266, normalized size = 3.20

$$\left[\frac{6\,cd^3xe + 2\,adxe^3 + \left(3\,cd^3 - ax^2e^3 - \left(bdx^2 + ad\right)e^2 + \left(3\,cd^2x^2 - bd^2\right)e\right)\sqrt{-de}\,\log\left(\frac{x^2e - 2\,\sqrt{-de}\,x - d}{x^2e + d}\right) + 2\,\left(2\,cd^2x^3 - bd^2x\right)e^2}{4\,\left(d^2x^2e^4 + d^3e^3\right)}, \frac{3\,cd^3xe + adxe^3 - \left(3\,cd^3 - ax^2e^3 - \left(bdx^2 + ad\right)e^2 + \left(3\,cd^2x^2 - bd^2\right)e\right)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}} + \left(2\,cd^2x^3 - bd^2x\right)e^2}{2\,\left(d^2x^2e^4 + d^3e^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(6*c*d^3*x*e + 2*a*d*x*e^3 + (3*c*d^3 - a*x^2*e^3 - (b*d*x^2 + a*d)*e^
2 + (3*c*d^2*x^2 - b*d^2)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x
^2*e + d)) + 2*(2*c*d^2*x^3 - b*d^2*x)*e^2)/(d^2*x^2*e^4 + d^3*e^3), 1/2*(3
*c*d^3*x*e + a*d*x*e^3 - (3*c*d^3 - a*x^2*e^3 - (b*d*x^2 + a*d)*e^2 + (3*c*
d^2*x^2 - b*d^2)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) + (2*c*d^2*x^
3 - b*d^2*x)*e^2)/(d^2*x^2*e^4 + d^3*e^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(75) = 150$.
time = 0.42, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x(ae^2 - bde + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2 + bde - 3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2 + bde - 3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Giac [A]
time = 4.03, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{(3\,cd^2 - bde - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{(cd^2x - bdxe + axe^2)e^{(-2)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

Mupad [B]
time = 4.67, size = 77, normalized size = 0.93

$$\frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2 + b\,d\,e + a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{2\,d\,\left(e^3\,x^2 + d\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.250 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx$

Optimal. Leaf size=115

$$\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d(d+ex^2)^2}-\frac{(5cd^2-e(bd+3ae))\,x}{8d^2e^2(d+ex^2)}+\frac{(3cd^2+e(bd+3ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

```
[Out] 1/4*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^2-1/8*(5*c*d^2-e*(3*a*e+b*d))*x/d^2/
e^2/(e*x^2+d)+1/8*(3*c*d^2+e*(3*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)
/e^(5/2)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1171, 393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(e(3ae+bd)+3cd^2)}{8d^{5/2}e^{5/2}}-\frac{x(5cd^2-e(3ae+bd))}{8d^2e^2(d+ex^2)}+\frac{x(ae^2-bde+cd^2)}{4de^2(d+ex^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(4*d*e^2*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d +
3*a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\,(d+ex^2)^2}-\frac{\int \frac{-3a+\frac{d(cd-be)}{e^2}-\frac{4cdx^2}{e}}{(d+ex^2)^2}\,dx}{4d}\\ &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\,(d+ex^2)^2}-\frac{(5cd^2-e(bd+3ae))\,x}{8d^2e^2\,(d+ex^2)}-\frac{\left(-\frac{4cd^2}{e}+e\left(-3a+\frac{d(cd-be)}{e^2}\right)\right)\int \frac{1}{d+ex^2}\,dx}{8d^2e}\\ &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\,(d+ex^2)^2}-\frac{(5cd^2-e(bd+3ae))\,x}{8d^2e^2\,(d+ex^2)}+\frac{(3cd^2+e(bd+3ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 110, normalized size = 0.96

$$\frac{x(-cd^2(3d+5ex^2)+e(bd(-d+ex^2)+ae(5d+3ex^2)))}{8d^2e^2\,(d+ex^2)^2}+\frac{(3cd^2+e(bd+3ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2))))
/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Maple [A]
time = 0.14, size = 107, normalized size = 0.93

method	result
default	$\frac{\frac{(3a\,e^2+deb-5c\,d^2)x^3}{8d^2e}+\frac{(5a\,e^2-deb-3c\,d^2)x}{8d\,e^2}}{(e\,x^2+d)^2}+\frac{(3a\,e^2+deb+3c\,d^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8d^2e^2\sqrt{de}}$
risch	$\frac{\frac{(3a\,e^2+deb-5c\,d^2)x^3}{8d^2e}+\frac{(5a\,e^2-deb-3c\,d^2)x}{8d\,e^2}}{(e\,x^2+d)^2}-\frac{3\ln\left(ex+\sqrt{-de}\right)a}{16\sqrt{-de}\,d^2}-\frac{\ln\left(ex+\sqrt{-de}\right)b}{16\sqrt{-de}\,ed}-\frac{3\ln\left(ex+\sqrt{-de}\right)c}{16\sqrt{-de}\,e^2}+\frac{3\ln\left(-e\right.}{16\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/d/e^2*x)
/(e*x^2+d)^2+1/8*(3*a*e^2+b*d*e+3*c*d^2)/d^2/e^2/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2))
```

Maxima [A]
time = 0.53, size = 110, normalized size = 0.96

$$\frac{(3\,cd^2+bde+3\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{(5\,cd^2e-bde^2-3\,ae^3)x^3+(3\,cd^3+bd^2e-5\,ade^2)x}{8\,(d^2x^4e^4+2\,d^3x^2e^3+d^4e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2)
- 1/8*((5*c*d^2*e - b*d*e^2 - 3*a*e^3)*x^3 + (3*c*d^3 + b*d^2*e - 5*a*d*e^2
)*x)/(d^2*x^4*e^4 + 2*d^3*x^2*e^3 + d^4*e^2)
```

Fricas [A]
time = 0.35, size = 389, normalized size = 3.38

$$\left[-\frac{6\,cd^4xe-6\,adx^3e^4+(3\,ax^4e^4+3\,cd^4+(bdx^4+6\,adx^2)e^3+(3\,cd^2x^4+2\,bd^2x^2+3\,ad^2)e^2+(6\,cd^3x^2+bd^3)e)\sqrt{-de}\,\log\left(\frac{x^2e-2\,\sqrt{-de}\,x-d}{x^2e+d}\right)-2\,(bd^2x^3+5\,ad^2x)e^3+2\,(5\,cd^3x^3+bd^3x)e^2}{16\,(d^3x^4e^5+2\,d^4x^2e^4+d^5e^3)},-\frac{3\,cd^4xe-3\,adx^3e^4-(3\,ax^4e^4+3\,cd^4+(bdx^4+6\,adx^2)e^3+(3\,cd^2x^4+2\,bd^2x^2+3\,ad^2)e^2+(6\,cd^3x^2+bd^3)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-(bd^2x^3+5\,ad^2x)e^3+(5\,cd^3x^3+bd^3x)e^2}{8\,(d^3x^4e^5+2\,d^4x^2e^4+d^5e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(6*c*d^4*x*e - 6*a*d*x^3*e^4 + (3*a*x^4*e^4 + 3*c*d^4 + (b*d*x^4 + 6
*a*d*x^2)*e^3 + (3*c*d^2*x^4 + 2*b*d^2*x^2 + 3*a*d^2)*e^2 + (6*c*d^3*x^2 +
b*d^3)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) - 2*(b*d
^2*x^3 + 5*a*d^2*x)*e^3 + 2*(5*c*d^3*x^3 + b*d^3*x)*e^2)/(d^3*x^4*e^5 + 2*d
^4*x^2*e^4 + d^5*e^3), -1/8*(3*c*d^4*x*e - 3*a*d*x^3*e^4 - (3*a*x^4*e^4 + 3
*c*d^4 + (b*d*x^4 + 6*a*d*x^2)*e^3 + (3*c*d^2*x^4 + 2*b*d^2*x^2 + 3*a*d^2)*
e^2 + (6*c*d^3*x^2 + b*d^3)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) -
(b*d^2*x^3 + 5*a*d^2*x)*e^3 + (5*c*d^3*x^3 + b*d^3*x)*e^2)/(d^3*x^4*e^5 + 2
*d^4*x^2*e^4 + d^5*e^3)]
```

Sympy [A]
time = 0.78, size = 196, normalized size = 1.70

$$-\frac{\sqrt{-\frac{1}{d^5e^5}}\cdot(3ae^2+bde+3cd^2)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^5e^5}}\cdot(3ae^2+bde+3cd^2)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{x^3\cdot(3ae^3+bde^2-5cd^2e)+x(5ade^2-bd^2e-3cd^3)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1/
(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*l
og(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*
c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e*
*3*x**2 + 8*d**2*e**4*x**4)
```

Giac [A]
time = 4.74, size = 101, normalized size = 0.88

$$\frac{(3\,cd^2+bde+3\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{(5\,cd^2x^3e-bdx^3e^2+3\,cd^3x-3\,ax^3e^3+bd^2xe-5\,adxe^2)e^{(-2)}}{8\,(x^2e+d)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2)
- 1/8*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^3*e^3 + b*d^2*x*e -
5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2)
```

Mupad [B]
time = 4.85, size = 112, normalized size = 0.97

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}}-\frac{\frac{x\,\left(3\,c\,d^2+b\,d\,e-5\,a\,e^2\right)}{8\,d\,e^2}-\frac{x^3\,\left(-5\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^2\,e}}{d^2+2\,d\,e\,x^2+e^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^3,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 3*c*d^2 + b*d*e))/(8*d^(5/2)*e^(5/2))
 - ((x*(3*c*d^2 - 5*a*e^2 + b*d*e))/(8*d*e^2) - (x^3*(3*a*e^2 - 5*c*d^2 + b
*d*e))/(8*d^2*e))/(d^2 + e^2*x^4 + 2*d*e*x^2)
```

3.251 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx$

Optimal. Leaf size=150

$$\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\,(d+ex^2)^3}-\frac{(7cd^2-e(bd+5ae))\,x}{24d^2e^2\,(d+ex^2)^2}+\frac{(cd^2+e(bd+5ae))\,x}{16d^3e^2\,(d+ex^2)}+\frac{(cd^2+e(bd+5ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

```
[Out] 1/6*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^3-1/24*(7*c*d^2-e*(5*a*e+b*d))*x/d^2
/e^2/(e*x^2+d)^2+1/16*(c*d^2+e*(5*a*e+b*d))*x/d^3/e^2/(e*x^2+d)+1/16*(c*d^2
+e*(5*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(5/2)
```

Rubi [A]
time = 0.12, antiderivative size = 154, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1171, 393, 205, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(e(5ae+bd)+cd^2)}{16d^{7/2}e^{5/2}}-\frac{x(7cd^2-e(5ae+bd))}{24d^2e^2\,(d+ex^2)^2}+\frac{x(ae^2-bde+cd^2)}{6de^2\,(d+ex^2)^3}+\frac{x(e(5ae+bd)+cd^2)}{16d^3e^2\,(d+ex^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^2)^3) - ((7*c*d^2 - e*(b*d +
5*a*e))*x)/(24*d^2*e^2*(d + e*x^2)^2) + ((c*d^2 + e*(b*d + 5*a*e))*x)/(16*d
^3*e^2*(d + e*x^2)) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(16*d^(7/2)*e^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{(d+ex^2)^4}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\,(d+ex^2)^3}-\frac{\int \frac{-5a+\frac{d(cd-be)}{e^2}-\frac{6cdx^2}{e}}{(d+ex^2)^3}\,dx}{6d}\\ &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\,(d+ex^2)^3}-\frac{(7cd^2-e(bd+5ae))\,x}{24d^2e^2\,(d+ex^2)^2}+\frac{(cd^2+e(bd+5ae))\int \frac{1}{(d+ex^2)^2}\,dx}{8d^2e^2}\\ &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\,(d+ex^2)^3}-\frac{(7cd^2-e(bd+5ae))\,x}{24d^2e^2\,(d+ex^2)^2}+\frac{(cd^2+e(bd+5ae))\,x}{16d^3e^2\,(d+ex^2)}+\frac{(cd^2+e(bd+5a}{16d^3}\\ &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{6d\,(d+ex^2)^3}-\frac{(7cd^2-e(bd+5ae))\,x}{24d^2e^2\,(d+ex^2)^2}+\frac{(cd^2+e(bd+5ae))\,x}{16d^3e^2\,(d+ex^2)}+\frac{(cd^2+e(bd+5a}{16d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 142, normalized size = 0.95

$$\frac{x(cd^2(-3d^2-8dex^2+3e^2x^4)+e(bd(-3d^2+8dex^2+3e^2x^4)+ae(33d^2+40dex^2+15e^2x^4)))}{48d^3e^2\,(d+ex^2)^3}+\frac{(cd^2+e(bd+5ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16d^{7/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^4,x]
```

```
[Out] (x*(c*d^2*(-3*d^2 - 8*d*e*x^2 + 3*e^2*x^4) + e*(b*d*(-3*d^2 + 8*d*e*x^2 + 3
*e^2*x^4) + a*e*(33*d^2 + 40*d*e*x^2 + 15*e^2*x^4))))/(48*d^3*e^2*(d + e*x^
2)^3) + ((c*d^2 + e*(b*d + 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)
*e^(5/2))
```

Maple [A]
time = 0.13, size = 130, normalized size = 0.87

method	result
default	$\frac{\frac{\left(5 a\,e^{2}+d e b+c\,d^{2}\right) x^{5}}{16 d^{3}}+\frac{\left(5 a\,e^{2}+d e b-c\,d^{2}\right) x^{3}}{6 d^{2} e}+\frac{\left(11 a\,e^{2}-d e b-c\,d^{2}\right) x}{16 d\,e^{2}}}{\left(e\,x^{2}+d\right)^{3}}+\frac{\left(5 a\,e^{2}+d e b+c\,d^{2}\right) \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{16 d^{3} e^{2} \sqrt{d e}}$
risch	$\frac{\frac{\left(5 a\,e^{2}+d e b+c\,d^{2}\right) x^{5}}{16 d^{3}}+\frac{\left(5 a\,e^{2}+d e b-c\,d^{2}\right) x^{3}}{6 d^{2} e}+\frac{\left(11 a\,e^{2}-d e b-c\,d^{2}\right) x}{16 d\,e^{2}}}{\left(e\,x^{2}+d\right)^{3}}-\frac{5 \ln\left(e x+\sqrt{-d e}\right) a}{32 \sqrt{-d e}\, d^{3}}-\frac{\ln\left(e x+\sqrt{-d e}\right) b}{32 \sqrt{-d e}\, e\,d^{2}}-\frac{\ln\left(e x+\sqrt{-\right.}}{32 \sqrt{-de}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*a*e^2+b*d*e+c*d^2)/d^3*x^5+1/6*(5*a*e^2+b*d*e-c*d^2)/d^2/e*x^3+1/1
6*(11*a*e^2-b*d*e-c*d^2)/d/e^2*x)/(e*x^2+d)^3+1/16*(5*a*e^2+b*d*e+c*d^2)/d^
3/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [A]
time = 0.52, size = 147, normalized size = 0.98

$$\frac{3\,(cd^2e^2+bde^3+5\,ae^4)x^5-8\,(cd^3e-bd^2e^2-5\,ade^3)x^3-3\,(cd^4+bd^3e-11\,ad^2e^2)x}{48\,(d^3x^6e^5+3\,d^4x^4e^4+3\,d^5x^2e^3+d^6e^2)}+\frac{(cd^2+bde+5\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{16\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^5 - 8*(c*d^3*e - b*d^2*e^2 - 5*a*
d*e^3)*x^3 - 3*(c*d^4 + b*d^3*e - 11*a*d^2*e^2)*x)/(d^3*x^6*e^5 + 3*d^4*x^4
*e^4 + 3*d^5*x^2*e^3 + d^6*e^2) + 1/16*(c*d^2 + b*d*e + 5*a*e^2)*arctan(x*e
^(1/2)/sqrt(d))*e^(-5/2)/d^(7/2)
```

Fricas [A]
time = 0.35, size = 535, normalized size = 3.57

$$\left[\frac{30\,adx^5e^5-6\,cd^5xe-3\,(5\,ax^6e^5+cd^5+(bdx^6+15\,adx^4)e^4+(cd^2x^6+3\,bd^2x^4+15\,ad^2x^2)e^3+(3\,cd^3x^4+3\,bd^3x^2+5\,ad^3)e^2+(3\,cd^4x^2+bd^4)e)\sqrt{-de}\,\log\left(\frac{x^2e-2\,\sqrt{-de}\,x-d}{x^2e+d}\right)+2\,(3\,bd^2x^5+40\,ad^2x^3)e^4+2\,(3\,cd^3x^5+8\,bd^3x^3+33\,ad^3x)e^3-2\,(8\,cd^4x^3+3\,bd^4x)e^2}{96\,(d^4x^6e^6+3\,d^5x^4e^5+3\,d^6x^2e^4+d^7e^3)}, \frac{15\,adx^5e^5-3\,cd^5xe+3\,(5\,ax^6e^5+cd^5+(bdx^6+15\,adx^4)e^4+(cd^2x^6+3\,bd^2x^4+15\,ad^2x^2)e^3+(3\,cd^3x^4+3\,bd^3x^2+5\,ad^3)e^2+(3\,cd^4x^2+bd^4)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}+(3\,bd^2x^5+40\,ad^2x^3)e^4+(3\,cd^3x^5+8\,bd^3x^3+33\,ad^3x)e^3-(8\,cd^4x^3+3\,bd^4x)e^2}{48\,(d^4x^6e^6+3\,d^5x^4e^5+3\,d^6x^2e^4+d^7e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(30*a*d*x^5*e^5 - 6*c*d^5*x*e - 3*(5*a*x^6*e^5 + c*d^5 + (b*d*x^6 + 1
5*a*d*x^4)*e^4 + (c*d^2*x^6 + 3*b*d^2*x^4 + 15*a*d^2*x^2)*e^3 + (3*c*d^3*x^
4 + 3*b*d^3*x^2 + 5*a*d^3)*e^2 + (3*c*d^4*x^2 + b*d^4)*e)*sqrt(-d*e)*log((x
^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) + 2*(3*b*d^2*x^5 + 40*a*d^2*x^3)*e^
4 + 2*(3*c*d^3*x^5 + 8*b*d^3*x^3 + 33*a*d^3*x)*e^3 - 2*(8*c*d^4*x^3 + 3*b*d
```

```
^4*x)*e^2)/(d^4*x^6*e^6 + 3*d^5*x^4*e^5 + 3*d^6*x^2*e^4 + d^7*e^3), 1/48*(1
5*a*d*x^5*e^5 - 3*c*d^5*x*e + 3*(5*a*x^6*e^5 + c*d^5 + (b*d*x^6 + 15*a*d*x^
4)*e^4 + (c*d^2*x^6 + 3*b*d^2*x^4 + 15*a*d^2*x^2)*e^3 + (3*c*d^3*x^4 + 3*b*
d^3*x^2 + 5*a*d^3)*e^2 + (3*c*d^4*x^2 + b*d^4)*e)*sqrt(d)*arctan(x*e^(1/2)/
sqrt(d))*e^(1/2) + (3*b*d^2*x^5 + 40*a*d^2*x^3)*e^4 + (3*c*d^3*x^5 + 8*b*d^
3*x^3 + 33*a*d^3*x)*e^3 - (8*c*d^4*x^3 + 3*b*d^4*x)*e^2)/(d^4*x^6*e^6 + 3*d
^5*x^4*e^5 + 3*d^6*x^2*e^4 + d^7*e^3)]
```

Sympy [A]
time = 1.43, size = 241, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{d^7e^5}}\cdot(5ae^2+bde+cd^2)\log\left(-d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{d^7e^5}}\cdot(5ae^2+bde+cd^2)\log\left(d^4e^2\sqrt{-\frac{1}{d^7e^5}}+x\right)}{32}+\frac{x^5\cdot(15ae^4+3bde^3+3cd^2e^2)+x^3\cdot(40ade^3+8bd^2e^2-8cd^3e)+x(33ad^2e^2-3bd^3e-3cd^4)}{48d^6e^2+144d^5e^3x^2+144d^4e^4x^4+48d^3e^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**4,x)
```

```
[Out] -sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(-d**4*e**2*sqrt(-1/(d
**7*e**5)) + x)/32 + sqrt(-1/(d**7*e**5))*(5*a*e**2 + b*d*e + c*d**2)*log(d
**4*e**2*sqrt(-1/(d**7*e**5)) + x)/32 + (x**5*(15*a*e**4 + 3*b*d*e**3 + 3*c
*d**2*e**2) + x**3*(40*a*d*e**3 + 8*b*d**2*e**2 - 8*c*d**3*e) + x*(33*a*d**
2*e**2 - 3*b*d**3*e - 3*c*d**4))/(48*d**6*e**2 + 144*d**5*e**3*x**2 + 144*d
**4*e**4*x**4 + 48*d**3*e**5*x**6)
```

Giac [A]
time = 4.75, size = 134, normalized size = 0.89

$$\frac{(cd^2+bde+5\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{16\,d^{\frac{7}{2}}}+\frac{(3\,cd^2x^5e^2+3\,bdx^5e^3-8\,cd^3x^3e+15\,ax^5e^4+8\,bd^2x^3e^2-3\,cd^4x+40\,adx^3e^3-3\,bd^3xe+33\,ad^2xe^2)e^{(-2)}}{48\,(x^2e+d)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] 1/16*(c*d^2 + b*d*e + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(7/2) +
 1/48*(3*c*d^2*x^5*e^2 + 3*b*d*x^5*e^3 - 8*c*d^3*x^3*e + 15*a*x^5*e^4 + 8*b
*d^2*x^3*e^2 - 3*c*d^4*x + 40*a*d*x^3*e^3 - 3*b*d^3*x*e + 33*a*d^2*x*e^2)*e
^(-2)/((x^2*e + d)^3*d^3)
```

Mupad [B]
time = 4.51, size = 144, normalized size = 0.96

$$\frac{\frac{x^5\,\left(c\,d^2+b\,d\,e+5\,a\,e^2\right)}{16\,d^3}-\frac{x\,\left(c\,d^2+b\,d\,e-11\,a\,e^2\right)}{16\,d\,e^2}+\frac{x^3\,\left(-c\,d^2+b\,d\,e+5\,a\,e^2\right)}{6\,d^2\,e}}{d^3+3\,d^2\,e\,x^2+3\,d\,e^2\,x^4+e^3\,x^6}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+b\,d\,e+5\,a\,e^2\right)}{16\,d^{7/2}\,e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^4,x)
```

```
[Out] ((x^5*(5*a*e^2 + c*d^2 + b*d*e))/(16*d^3) - (x*(c*d^2 - 11*a*e^2 + b*d*e))/
(16*d*e^2) + (x^3*(5*a*e^2 - c*d^2 + b*d*e))/(6*d^2*e))/(d^3 + e^3*x^6 + 3*
d^2*e*x^2 + 3*d*e^2*x^4) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2 + b*
d*e))/(16*d^(7/2)*e^(5/2))
```

3.252 $\int (d+ex^2)^3 (a+bx^2+cx^4)^2 \, dx$

Optimal. Leaf size=223

$$a^2d^3x+\frac{1}{3}ad^2(2bd+3ae)x^3+\frac{1}{5}d\big(b^2d^2+6abde+a\big(2cd^2+3ae^2\big)\big)\,x^5+\frac{1}{7}\big(2bcd^3+3b^2d^2e+6acd^2e+6abde^2+a$$

```
[Out] a^2*d^3*x+1/3*a*d^2*(3*a*e+2*b*d)*x^3+1/5*d*(b^2*d^2+6*a*b*d*e+a*(3*a*e^2+2
*c*d^2))*x^5+1/7*(a^2*e^3+6*a*b*d*e^2+6*a*c*d^2*e+3*b^2*d^2*e+2*b*c*d^3)*x^
7+1/9*(c^2*d^3+6*c*d*e*(a*e+b*d)+b*e^2*(2*a*e+3*b*d))*x^9+1/11*e*(3*c^2*d^2
+b^2*e^2+2*c*e*(a*e+3*b*d))*x^11+1/13*c*e^2*(2*b*e+3*c*d)*x^13+1/15*c^2*e^3
*x^15
```

Rubi [A]

time = 0.13, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{1167\}$

$$\frac{1}{7}x^7\big(a^2e^3+6abde^2+6acd^2e+3b^2d^2e+2bcd^3\big)+a^2d^3x+\frac{1}{11}ex^{11}\big(2ce(ae+3bd)+b^2e^2+3c^2d^2\big)+\frac{1}{5}dx^5\big(6abde+a\big(3ae^2+2cd^2\big)+b^2d^2\big)+\frac{1}{9}x^9\big(6cde(ae+bd)+be^2(2ae+3bd)+c^2d^3\big)+\frac{1}{3}ad^2x^3(3ae+2bd)+\frac{1}{13}ce^2x^{13}(2be+3cd)+\frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*
c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d +
2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 +
(c*e^2*(3*c*d + 2*b*e)*x^13)/13 + (c^2*e^3*x^15)/15
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+bx^2+cx^4\right)^2 \, dx = \int \left(a^2d^3+ad^2(2bd+3ae)x^2+d\big(b^2d^2+6abde+a\big(2cd^2+3ae^2\big)\big)\,x^4+\right.$$

$$=a^2d^3x+\frac{1}{3}ad^2(2bd+3ae)x^3+\frac{1}{5}d\big(b^2d^2+6abde+a\big(2cd^2+3ae^2\big)\big)\,x^5+$$

Mathematica [A]
time = 0.05, size = 223, normalized size = 1.00

$$a^2d^3x+\frac{1}{3}ad^2(2bd+3ae)x^3+\frac{1}{5}d\left(b^2d^2+6abde+a\left(2cd^2+3ae^2\right)\right)x^5+\frac{1}{7}\left(2bcd^3+3b^2d^2e+6acd^2e+6abde^2+a^2e^3\right)x^7+\frac{1}{9}\left(c^2d^3+6cde(bd+ae)+be^2(3bd+2ae)\right)x^9+\frac{1}{11}e\left(3c^2d^2+b^2e^2+2ce(3bd+ae)\right)x^{11}+\frac{1}{13}ce^2(3cd+2be)x^{13}+\frac{1}{15}c^2e^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^3)/3 + (d*(b^2*d^2 + 6*a*b*d*e + a*(2*
c*d^2 + 3*a*e^2))*x^5)/5 + ((2*b*c*d^3 + 3*b^2*d^2*e + 6*a*c*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*x^7)/7 + ((c^2*d^3 + 6*c*d*e*(b*d + a*e) + b*e^2*(3*b*d +
2*a*e))*x^9)/9 + (e*(3*c^2*d^2 + b^2*e^2 + 2*c*e*(3*b*d + a*e))*x^11)/11 +
(c*e^2*(3*c*d + 2*b*e)*x^13)/13 + (c^2*e^3*x^15)/15
```

Maple [A]
time = 0.14, size = 219, normalized size = 0.98

method	result
default	$\frac{c^2e^3x^{15}}{15}+\frac{(2e^3bc+3c^2d\,e^2)x^{13}}{13}+\frac{(3d^2e\,c^2+6d\,e^2bc+e^3(2ac+b^2))x^{11}}{11}+\frac{(c^2d^3+6d^2ebc+3d\,e^2(2ac+b^2)+2e^3ab)x^9}{9}+\frac{(2bc\,d^3+$
norman	$a^2d^3x+\left(d^2e\,a^2+\frac{2}{3}d^3ab\right)x^3+\left(\frac{3}{5}d\,e^2a^2+\frac{6}{5}d^2eab+\frac{2}{5}ac\,d^3+\frac{1}{5}b^2d^3\right)x^5+\left(\frac{1}{7}a^2e^3+\frac{6}{7}abd\,e^2+\frac{6}{7}a$
gosper	$a^2d^3x+a^2d^2e\,x^3+\frac{2}{3}x^3d^3ab+\frac{3}{5}x^5d\,e^2a^2+\frac{6}{5}x^5d^2eab+\frac{2}{5}x^5ac\,d^3+\frac{1}{5}x^5b^2d^3+\frac{1}{7}x^7a^2e^3+\frac{6}{7}x^7abd$
risch	$a^2d^3x+a^2d^2e\,x^3+\frac{2}{3}x^3d^3ab+\frac{3}{5}x^5d\,e^2a^2+\frac{6}{5}x^5d^2eab+\frac{2}{5}x^5ac\,d^3+\frac{1}{5}x^5b^2d^3+\frac{1}{7}x^7a^2e^3+\frac{6}{7}x^7abd$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*c^2*e^3*x^15+1/13*(2*b*c*e^3+3*c^2*d*e^2)*x^13+1/11*(3*d^2*e*c^2+6*d*e
^2*b*c+e^3*(2*a*c+b^2))*x^11+1/9*(c^2*d^3+6*d^2*e*b*c+3*d*e^2*(2*a*c+b^2)+2
*e^3*a*b)*x^9+1/7*(2*b*c*d^3+3*d^2*e*(2*a*c+b^2)+6*a*b*d*e^2+a^2*e^3)*x^7+1
/5*(d^3*(2*a*c+b^2)+6*d^2*e*a*b+3*d*e^2*a^2)*x^5+1/3*(3*a^2*d^2*e+2*a*b*d^3
)*x^3+a^2*d^3*x
```

Maxima [A]
time = 0.30, size = 220, normalized size = 0.99

$$\frac{1}{15}\,c^2x^{15}e^3+\frac{1}{13}\left(3\,c^2de^2+2\,bce^3\right)x^{13}+\frac{1}{11}\left(3\,c^2d^2e+6\,bcde^2+b^2e^3+2\,ace^3\right)x^{11}+\frac{1}{9}\left(c^2d^3+6\,bcd^2e+2\,abe^3+3\left(b^2e^2+2\,ace^2\right)d\right)x^9+\frac{1}{7}\left(2\,bcd^3+6\,abde^2+3\left(b^2e+2\,ace\right)d^2+a^2e^3\right)x^7+a^2d^3x+\frac{1}{5}\left(6\,abd^2e+\left(b^2+2\,ac\right)d^3+3\,a^2de^2\right)x^5+\frac{1}{3}\left(2\,abd^3+3\,a^2d^2e\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*c^2*x^15*e^3 + 1/13*(3*c^2*d*e^2 + 2*b*c*e^3)*x^13 + 1/11*(3*c^2*d^2*e
 + 6*b*c*d*e^2 + b^2*e^3 + 2*a*c*e^3)*x^11 + 1/9*(c^2*d^3 + 6*b*c*d^2*e + 2
*a*b*e^3 + 3*(b^2*e^2 + 2*a*c*e^2)*d)*x^9 + 1/7*(2*b*c*d^3 + 6*a*b*d*e^2 +
```

```
3*(b^2*e + 2*a*c*e)*d^2 + a^2*e^3)*x^7 + a^2*d^3*x + 1/5*(6*a*b*d^2*e + (b^
2 + 2*a*c)*d^3 + 3*a^2*d*e^2)*x^5 + 1/3*(2*a*b*d^3 + 3*a^2*d^2*e)*x^3
```

Fricas [A]
time = 0.34, size = 220, normalized size = 0.99

$$\frac{1}{9}c^2d^3x^9+\frac{2}{7}bcd^3x^7+\frac{1}{5}\left(b^2+2\,ac\right)d^3x^5+\frac{2}{3}abd^3x^3+a^2d^3x+\frac{1}{45045}\left(3003\,c^2x^{15}+6930\,bcx^{13}+4095\left(b^2+2\,ac\right)x^{11}+10010\,abx^9+6435\,a^2x^7\right)e^3+\frac{1}{15015}\left(3465\,c^2dx^{13}+8190\,bcdx^{11}+5005\left(b^2+2\,ac\right)dx^9+12870\,abdx^7+9009\,a^2dx^5\right)e^2+\frac{1}{1155}\left(315\,c^2d^2x^{11}+770\,bcd^2x^9+495\left(b^2+2\,ac\right)d^2x^7+1386\,abd^2x^5+1155\,a^2d^2x^3\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*d^3*x^9 + 2/7*b*c*d^3*x^7 + 1/5*(b^2 + 2*a*c)*d^3*x^5 + 2/3*a*b*d^3
*x^3 + a^2*d^3*x + 1/45045*(3003*c^2*x^15 + 6930*b*c*x^13 + 4095*(b^2 + 2*a
*c)*x^11 + 10010*a*b*x^9 + 6435*a^2*x^7)*e^3 + 1/15015*(3465*c^2*d*x^13 + 8
190*b*c*d*x^11 + 5005*(b^2 + 2*a*c)*d*x^9 + 12870*a*b*d*x^7 + 9009*a^2*d*x^
5)*e^2 + 1/1155*(315*c^2*d^2*x^11 + 770*b*c*d^2*x^9 + 495*(b^2 + 2*a*c)*d^2
*x^7 + 1386*a*b*d^2*x^5 + 1155*a^2*d^2*x^3)*e
```

Sympy [A]
time = 0.02, size = 272, normalized size = 1.22

$$a^2d^3x+\frac{c^2e^3x^{15}}{15}+x^{13}\cdot\left(\frac{2bce^3}{13}+\frac{3c^2de^2}{13}\right)+x^{11}\cdot\left(\frac{2ace^3}{11}+\frac{b^2e^3}{11}+\frac{6bcde^2}{11}+\frac{3c^2d^2e}{11}\right)+x^9\cdot\left(\frac{2abe^3}{9}+\frac{2acde^2}{3}+\frac{b^2de^2}{3}+\frac{2bcd^2e}{3}+\frac{c^2d^3}{9}\right)+x^7\left(\frac{a^2e^3}{7}+\frac{6abde^2}{7}+\frac{6acd^2e}{7}+\frac{3b^2d^2e}{7}+\frac{2bcd^3}{7}\right)+x^5\cdot\left(\frac{3a^2de^2}{5}+\frac{6abd^2e}{5}+\frac{2acd^3}{5}+\frac{b^2d^3}{5}\right)+x^3\left(a^2d^2e+\frac{2abd^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d**3*x + c**2*e**3*x**15/15 + x**13*(2*b*c*e**3/13 + 3*c**2*d*e**2/13)
 + x**11*(2*a*c*e**3/11 + b**2*e**3/11 + 6*b*c*d*e**2/11 + 3*c**2*d**2*e/11
) + x**9*(2*a*b*e**3/9 + 2*a*c*d*e**2/3 + b**2*d*e**2/3 + 2*b*c*d**2*e/3 +
c**2*d**3/9) + x**7*(a**2*e**3/7 + 6*a*b*d*e**2/7 + 6*a*c*d**2*e/7 + 3*b**2
*d**2*e/7 + 2*b*c*d**3/7) + x**5*(3*a**2*d*e**2/5 + 6*a*b*d**2*e/5 + 2*a*c*
d**3/5 + b**2*d**3/5) + x**3*(a**2*d**2*e + 2*a*b*d**3/3)
```

Giac [A]
time = 3.83, size = 255, normalized size = 1.14

$$\frac{1}{15}c^2x^{15}e^3+\frac{3}{13}c^2dx^{13}e^2+\frac{2}{13}bcx^{13}e^3+\frac{3}{11}c^2d^2x^{11}e+\frac{6}{11}bcdx^{11}e^2+\frac{1}{9}c^2d^3x^9+\frac{1}{11}b^2x^{11}e^3+\frac{2}{11}acx^{11}e^3+\frac{2}{3}bcd^2x^9e+\frac{1}{3}b^2dx^9e^2+\frac{2}{3}acdx^9e^2+\frac{2}{7}bcd^3x^7+\frac{2}{9}abx^9e^3+\frac{3}{7}b^2d^2x^7e+\frac{6}{7}acd^2x^7e+\frac{6}{7}abdx^7e^2+\frac{1}{5}b^2d^3x^5+\frac{2}{5}acd^3x^5+\frac{1}{7}a^2x^7e^3+\frac{6}{5}abd^2x^5e+\frac{3}{5}a^2dx^5e^2+\frac{2}{3}abd^3x^3+a^2d^2x^3e+a^2d^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/15*c^2*x^15*e^3 + 3/13*c^2*d*x^13*e^2 + 2/13*b*c*x^13*e^3 + 3/11*c^2*d^2*
x^11*e + 6/11*b*c*d*x^11*e^2 + 1/9*c^2*d^3*x^9 + 1/11*b^2*x^11*e^3 + 2/11*a
*c*x^11*e^3 + 2/3*b*c*d^2*x^9*e + 1/3*b^2*d*x^9*e^2 + 2/3*a*c*d*x^9*e^2 + 2
/7*b*c*d^3*x^7 + 2/9*a*b*x^9*e^3 + 3/7*b^2*d^2*x^7*e + 6/7*a*c*d^2*x^7*e +
6/7*a*b*d*x^7*e^2 + 1/5*b^2*d^3*x^5 + 2/5*a*c*d^3*x^5 + 1/7*a^2*x^7*e^3 + 6
```

```
/5*a*b*d^2*x^5*e + 3/5*a^2*d*x^5*e^2 + 2/3*a*b*d^3*x^3 + a^2*d^2*x^3*e + a^
2*d^3*x
```

Mupad [B]
time = 4.48, size = 220, normalized size = 0.99

$$x^7\left(\frac{a^2e^3}{7}+\frac{6abde^2}{7}+\frac{6cad^2e}{7}+\frac{3b^2d^2e}{7}+\frac{2cbd^3}{7}\right)+x^9\left(\frac{b^2de^2}{3}+\frac{2bcd^2e}{3}+\frac{2abe^3}{9}+\frac{c^2d^3}{9}+\frac{2acde^2}{3}\right)+x^5\left(\frac{3a^2de^2}{5}+\frac{6abd^2e}{5}+\frac{2cad^3}{5}+\frac{b^2d^3}{5}\right)+x^{11}\left(\frac{b^2e^3}{11}+\frac{6bcde^2}{11}+\frac{3c^2d^2e}{11}+\frac{2ace^3}{11}\right)+a^2d^3x+\frac{c^2e^3x^{15}}{15}+\frac{ad^2x^3(3ae+2bd)}{3}+\frac{ce^2x^{13}(2be+3cd)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^7*((a^2*e^3)/7 + (3*b^2*d^2*e)/7 + (2*b*c*d^3)/7 + (6*a*b*d*e^2)/7 + (6*a
*c*d^2*e)/7) + x^9*((c^2*d^3)/9 + (b^2*d*e^2)/3 + (2*a*b*e^3)/9 + (2*a*c*d*
e^2)/3 + (2*b*c*d^2*e)/3) + x^5*((b^2*d^3)/5 + (3*a^2*d*e^2)/5 + (2*a*c*d^3
)/5 + (6*a*b*d^2*e)/5) + x^11*((b^2*e^3)/11 + (3*c^2*d^2*e)/11 + (2*a*c*e^3
)/11 + (6*b*c*d*e^2)/11) + a^2*d^3*x + (c^2*e^3*x^15)/15 + (a*d^2*x^3*(3*a*
e + 2*b*d))/3 + (c*e^2*x^13*(2*b*e + 3*c*d))/13
```

3.253 $\int (d+ex^2)^2 (a+bx^2+cx^4)^2 \, dx$

Optimal. Leaf size=155

$$a^2d^2x+\frac{2}{3}ad(bd+ae)x^3+\frac{1}{5}\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)x^5+\frac{2}{7}\left(bcd^2+b^2de+2acde+abe^2\right)x^7+\frac{1}{9}\left(c^2d^2+b\right.$$

```
[Out] a^2*d^2*x+2/3*a*d*(a*e+b*d)*x^3+1/5*(b^2*d^2+4*a*b*d*e+a*(a*e^2+2*c*d^2))*x
^5+2/7*(a*b*e^2+2*a*c*d*e+b^2*d*e+b*c*d^2)*x^7+1/9*(c^2*d^2+b^2*e^2+2*c*e*(
a*e+2*b*d))*x^9+2/11*c*e*(b*e+c*d)*x^11+1/13*c^2*e^2*x^13
```

Rubi [A]
time = 0.10, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{1167\}$

$$a^2d^2x+\frac{1}{9}x^9\left(2ce(ae+2bd)+b^2e^2+c^2d^2\right)+\frac{2}{7}x^7\left(abe^2+2acde+b^2de+bcd^2\right)+\frac{1}{5}x^5\left(4abde+a\left(ae^2+2cd^2\right)+b^2d^2\right)+\frac{2}{3}adx^3(ae+bd)+\frac{2}{11}cex^{11}(be+cd)+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 4*a*b*d*e + a*(2*c*d^2
+ a*e^2))*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((
c^2*d^2 + b^2*e^2 + 2*c*e*(2*b*d + a*e))*x^9)/9 + (2*c*e*(c*d + b*e)*x^11)/
11 + (c^2*e^2*x^13)/13
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2d^2+2ad(bd+ae)x^2+\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)x^4+2\left(bc\right.\right.\\ &= a^2d^2x+\frac{2}{3}ad(bd+ae)x^3+\frac{1}{5}\left(b^2d^2+4abde+a\left(2cd^2+ae^2\right)\right)x^5+\frac{2}{7}\left(bc\right.\end{aligned}$$

Mathematica [A]
time = 0.03, size = 156, normalized size = 1.01

$$a^2d^2x+\frac{2}{3}ad(bd+ae)x^3+\frac{1}{5}\left(b^2d^2+2acd^2+4abde+a^2e^2\right)x^5+\frac{2}{7}\left(bcd^2+b^2de+2acde+abe^2\right)x^7+\frac{1}{9}\left(c^2d^2+4bcde+b^2e^2+2ace^2\right)x^9+\frac{2}{11}ce(cd+be)x^{11}+\frac{1}{13}c^2e^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^3)/3 + ((b^2*d^2 + 2*a*c*d^2 + 4*a*b*d*e +
 a^2*e^2)*x^5)/5 + (2*(b*c*d^2 + b^2*d*e + 2*a*c*d*e + a*b*e^2)*x^7)/7 + ((
c^2*d^2 + 4*b*c*d*e + b^2*e^2 + 2*a*c*e^2)*x^9)/9 + (2*c*e*(c*d + b*e)*x^11
)/11 + (c^2*e^2*x^13)/13
```

Maple [A]
time = 0.14, size = 155, normalized size = 1.00

method	result
default	$\frac{c^2e^2x^{13}}{13}+\frac{(2bc\,e^2+2c^2de)x^{11}}{11}+\frac{(c^2d^2+4bcde+e^2(2ac+b^2))x^9}{9}+\frac{(2bc\,d^2+2de(2ac+b^2)+2ab\,e^2)x^7}{7}+\frac{(d^2(2ac+b^2)+4abde+$
norman	$\frac{c^2e^2x^{13}}{13}+\left(\frac{2}{11}bc\,e^2+\frac{2}{11}c^2de\right)x^{11}+\left(\frac{2}{9}ac\,e^2+\frac{1}{9}e^2b^2+\frac{4}{9}bcde+\frac{1}{9}c^2d^2\right)x^9+\left(\frac{2}{7}ab\,e^2+\frac{4}{7}acde+\frac{2}{7}b^2c$
gosper	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}x^{11}bc\,e^2+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9ac\,e^2+\frac{1}{9}x^9e^2b^2+\frac{4}{9}x^9bcde+\frac{1}{9}x^9c^2d^2+\frac{2}{7}x^7ab\,e^2+\frac{4}{7}ac$
risch	$\frac{1}{13}c^2e^2x^{13}+\frac{2}{11}x^{11}bc\,e^2+\frac{2}{11}c^2de\,x^{11}+\frac{2}{9}x^9ac\,e^2+\frac{1}{9}x^9e^2b^2+\frac{4}{9}x^9bcde+\frac{1}{9}x^9c^2d^2+\frac{2}{7}x^7ab\,e^2+\frac{4}{7}ac$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c^2*e^2*x^13+1/11*(2*b*c*e^2+2*c^2*d*e)*x^11+1/9*(c^2*d^2+4*b*c*d*e+e^
2*(2*a*c+b^2))*x^9+1/7*(2*b*c*d^2+2*d*e*(2*a*c+b^2)+2*a*b*e^2)*x^7+1/5*(d^2
*(2*a*c+b^2)+4*a*b*d*e+e^2*a^2)*x^5+1/3*(2*a^2*d*e+2*a*b*d^2)*x^3+a^2*d^2*x
```

Maxima [A]
time = 0.29, size = 151, normalized size = 0.97

$$\frac{1}{13}\,c^2x^{13}e^2+\frac{2}{11}\,(c^2de+bce^2)x^{11}+\frac{1}{9}\,(c^2d^2+4\,bcde+b^2e^2+2\,ace^2)x^9+\frac{2}{7}\,(bcd^2+abe^2+(b^2e+2\,ace)d)x^7+\frac{1}{5}\,(4\,abde+(b^2+2\,ac)d^2+a^2e^2)x^5+a^2d^2x+\frac{2}{3}\,(abd^2+a^2de)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*x^13*e^2 + 2/11*(c^2*d*e + b*c*e^2)*x^11 + 1/9*(c^2*d^2 + 4*b*c*d*
e + b^2*e^2 + 2*a*c*e^2)*x^9 + 2/7*(b*c*d^2 + a*b*e^2 + (b^2*e + 2*a*c*e)*d
)*x^7 + 1/5*(4*a*b*d*e + (b^2 + 2*a*c)*d^2 + a^2*e^2)*x^5 + a^2*d^2*x + 2/3
*(a*b*d^2 + a^2*d*e)*x^3
```

Fricas [A]
time = 0.32, size = 157, normalized size = 1.01

$$\frac{1}{9}\,c^2d^2x^9+\frac{2}{7}\,bcd^2x^7+\frac{1}{5}\,(b^2+2\,ac)d^2x^5+\frac{2}{3}\,abd^2x^3+a^2d^2x+\frac{1}{45045}\,(3465\,c^2x^{13}+8190\,bcx^{11}+5005\,(b^2+2\,ac)x^9+12870\,abx^7+9009\,a^2x^5)e^2+\frac{2}{3465}\,(315\,c^2dx^{11}+770\,bcdx^9+495\,(b^2+2\,ac)dx^7+1386\,abdx^5+1155\,a^2dx^3)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*d^2*x^9 + 2/7*b*c*d^2*x^7 + 1/5*(b^2 + 2*a*c)*d^2*x^5 + 2/3*a*b*d^2
*x^3 + a^2*d^2*x + 1/45045*(3465*c^2*x^13 + 8190*b*c*x^11 + 5005*(b^2 + 2*a
*c)*x^9 + 12870*a*b*x^7 + 9009*a^2*x^5)*e^2 + 2/3465*(315*c^2*d*x^11 + 770*
b*c*d*x^9 + 495*(b^2 + 2*a*c)*d*x^7 + 1386*a*b*d*x^5 + 1155*a^2*d*x^3)*e
```

Sympy [A]
time = 0.02, size = 192, normalized size = 1.24

$$a^2d^2x + \frac{c^2e^2x^{13}}{13} + x^{11}\cdot\left(\frac{2bce^2}{11} + \frac{2c^2de}{11}\right) + x^9\cdot\left(\frac{2ace^2}{9} + \frac{b^2e^2}{9} + \frac{4bcde}{9} + \frac{c^2d^2}{9}\right) + x^7\cdot\left(\frac{2abe^2}{7} + \frac{4acde}{7} + \frac{2b^2de}{7} + \frac{2bcd^2}{7}\right) + x^5\left(\frac{a^2e^2}{5} + \frac{4abde}{5} + \frac{2acd^2}{5} + \frac{b^2d^2}{5}\right) + x^3\cdot\left(\frac{2a^2de}{3} + \frac{2abd^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d**2*x + c**2*e**2*x**13/13 + x**11*(2*b*c*e**2/11 + 2*c**2*d*e/11) +
x**9*(2*a*c*e**2/9 + b**2*e**2/9 + 4*b*c*d*e/9 + c**2*d**2/9) + x**7*(2*a*b
*e**2/7 + 4*a*c*d*e/7 + 2*b**2*d*e/7 + 2*b*c*d**2/7) + x**5*(a**2*e**2/5 +
4*a*b*d*e/5 + 2*a*c*d**2/5 + b**2*d**2/5) + x**3*(2*a**2*d*e/3 + 2*a*b*d**2
/3)
```

Giac [A]
time = 4.64, size = 181, normalized size = 1.17

$$\frac{1}{13}c^2x^{13}e^2 + \frac{2}{11}c^2dx^{11}e + \frac{2}{11}bcx^{11}e^2 + \frac{1}{9}c^2d^2x^9 + \frac{4}{9}bcdx^9e + \frac{1}{9}b^2x^9e^2 + \frac{2}{9}acx^9e^2 + \frac{2}{7}bcd^2x^7 + \frac{2}{7}b^2dx^7e + \frac{4}{7}acdx^7e + \frac{2}{7}abx^7e^2 + \frac{1}{5}b^2d^2x^5 + \frac{2}{5}acd^2x^5 + \frac{4}{5}abdx^5e + \frac{1}{5}a^2x^5e^2 + \frac{2}{3}abd^2x^3 + \frac{2}{3}a^2dx^3e + a^2d^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13*e^2 + 2/11*c^2*d*x^11*e + 2/11*b*c*x^11*e^2 + 1/9*c^2*d^2*x^9
 + 4/9*b*c*d*x^9*e + 1/9*b^2*x^9*e^2 + 2/9*a*c*x^9*e^2 + 2/7*b*c*d^2*x^7 +
2/7*b^2*d*x^7*e + 4/7*a*c*d*x^7*e + 2/7*a*b*x^7*e^2 + 1/5*b^2*d^2*x^5 + 2/5
*a*c*d^2*x^5 + 4/5*a*b*d*x^5*e + 1/5*a^2*x^5*e^2 + 2/3*a*b*d^2*x^3 + 2/3*a^
2*d*x^3*e + a^2*d^2*x
```

Mupad [B]
time = 4.52, size = 148, normalized size = 0.95

$$x^5\left(\frac{a^2\,e^2}{5} + \frac{4\,a\,b\,d\,e}{5} + \frac{2\,c\,a\,d^2}{5} + \frac{b^2\,d^2}{5}\right) + x^9\left(\frac{b^2\,e^2}{9} + \frac{4\,b\,c\,d\,e}{9} + \frac{c^2\,d^2}{9} + \frac{2\,a\,c\,e^2}{9}\right) + x^7\left(\frac{2\,b^2\,d\,e}{7} + \frac{2\,c\,b\,d^2}{7} + \frac{2\,a\,b\,e^2}{7} + \frac{4\,a\,c\,d\,e}{7}\right) + a^2\,d^2\,x + \frac{c^2\,e^2\,x^{13}}{13} + \frac{2\,a\,d\,x^3\,(a\,e + b\,d)}{3} + \frac{2\,c\,e\,x^{11}\,(b\,e + c\,d)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5*((a^2*e^2)/5 + (b^2*d^2)/5 + (2*a*c*d^2)/5 + (4*a*b*d*e)/5) + x^9*((b^2
*e^2)/9 + (c^2*d^2)/9 + (2*a*c*e^2)/9 + (4*b*c*d*e)/9) + x^7*((2*a*b*e^2)/7
 + (2*b*c*d^2)/7 + (2*b^2*d*e)/7 + (4*a*c*d*e)/7) + a^2*d^2*x + (c^2*e^2*x^
13)/13 + (2*a*d*x^3*(a*e + b*d))/3 + (2*c*e*x^11*(b*e + c*d))/11
```

3.254 $\int (d+ex^2)(a+bx^2+cx^4)^2\,dx$

Optimal. Leaf size=96

$$a^2dx+\frac{1}{3}a(2bd+ae)x^3+\frac{1}{5}\left(b^2d+2acd+2abe\right)x^5+\frac{1}{7}\left(2bcd+b^2e+2ace\right)x^7+\frac{1}{9}c(cd+2be)x^9+\frac{1}{11}c^2ex^{11}$$

```
[Out] a^2*d*x+1/3*a*(a*e+2*b*d)*x^3+1/5*(2*a*b*e+2*a*c*d+b^2*d)*x^5+1/7*(2*a*c*e+
b^2*e+2*b*c*d)*x^7+1/9*c*(2*b*e+c*d)*x^9+1/11*c^2*e*x^11
```

Rubi [A]
time = 0.05, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1167}

$$a^2dx+\frac{1}{7}x^7\left(2ace+b^2e+2bcd\right)+\frac{1}{5}x^5\left(2abe+2acd+b^2d\right)+\frac{1}{3}ax^3(ae+2bd)+\frac{1}{9}cx^9(2be+cd)+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^11)
/11
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex^2)\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2d+a(2bd+ae)x^2+\left(b^2d+2acd+2abe\right)x^4+\left(2bcd+b^2e+2ac\right.\right.\\ &= a^2dx+\frac{1}{3}a(2bd+ae)x^3+\frac{1}{5}\left(b^2d+2acd+2abe\right)x^5+\frac{1}{7}\left(2bcd+b^2e+2\right.\end{aligned}$$

Mathematica [A]
time = 0.01, size = 96, normalized size = 1.00

$$a^2dx+\frac{1}{3}a(2bd+ae)x^3+\frac{1}{5}\left(b^2d+2acd+2abe\right)x^5+\frac{1}{7}\left(2bcd+b^2e+2ace\right)x^7+\frac{1}{9}c(cd+2be)x^9+\frac{1}{11}c^2ex^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*d*x + (a*(2*b*d + a*e)*x^3)/3 + ((b^2*d + 2*a*c*d + 2*a*b*e)*x^5)/5 + (
(2*b*c*d + b^2*e + 2*a*c*e)*x^7)/7 + (c*(c*d + 2*b*e)*x^9)/9 + (c^2*e*x^11)
/11
```

Maple [A]
time = 0.08, size = 91, normalized size = 0.95

method	result
default	$\frac{c^{2} e\,x^{11}}{11}+\frac{\left(2 b c e+c^{2} d\right) x^{9}}{9}+\frac{\left(2 b c d+e\left(2 a c+b^{2}\right)\right) x^{7}}{7}+\frac{\left(d\left(2 a c+b^{2}\right)+2 a b e\right) x^{5}}{5}+\frac{\left(e\,a^{2}+2 a b d\right) x^{3}}{3}+a^{2} d x$
norman	$\frac{c^{2} e\,x^{11}}{11}+\left(\frac{2}{9} b c e+\frac{1}{9} c^{2} d\right) x^{9}+\left(\frac{2}{7} a c e+\frac{1}{7} b^{2} e+\frac{2}{7} b c d\right) x^{7}+\left(\frac{2}{5} a b e+\frac{2}{5} a c d+\frac{1}{5} b^{2} d\right) x^{5}+\left(\frac{1}{3} e\,a^{2}+\frac{2}{3} a b d\right.$
gosper	$\frac{1}{11} c^{2} e\,x^{11}+\frac{2}{9} x^{9} b c e+\frac{1}{9} c^{2} d\,x^{9}+\frac{2}{7} a c e\,x^{7}+\frac{1}{7} x^{7} b^{2} e+\frac{2}{7} x^{7} b c d+\frac{2}{5} x^{5} a b e+\frac{2}{5} a c d\,x^{5}+\frac{1}{5} x^{5} b^{2} d+\frac{1}{3} a^{2} e\,x$
risch	$\frac{1}{11} c^{2} e\,x^{11}+\frac{2}{9} x^{9} b c e+\frac{1}{9} c^{2} d\,x^{9}+\frac{2}{7} a c e\,x^{7}+\frac{1}{7} x^{7} b^{2} e+\frac{2}{7} x^{7} b c d+\frac{2}{5} x^{5} a b e+\frac{2}{5} a c d\,x^{5}+\frac{1}{5} x^{5} b^{2} d+\frac{1}{3} a^{2} e\,x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c^2*e*x^11+1/9*(2*b*c*e+c^2*d)*x^9+1/7*(2*b*c*d+e*(2*a*c+b^2))*x^7+1/5
*(d*(2*a*c+b^2)+2*a*b*e)*x^5+1/3*(a^2*e+2*a*b*d)*x^3+a^2*d*x
```

Maxima [A]
time = 0.31, size = 96, normalized size = 1.00

$$\frac{1}{11}\,c^{2}x^{11}e+\frac{1}{9}\left(c^{2}d+2\,bce\right)x^{9}+\frac{1}{7}\left(2\,bcd+b^{2}e+2\,ace\right)x^{7}+\frac{1}{5}\left(2\,abe+\left(b^{2}+2\,ac\right)d\right)x^{5}+a^{2}dx+\frac{1}{3}\left(2\,abd+a^{2}e\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11*e + 1/9*(c^2*d + 2*b*c*e)*x^9 + 1/7*(2*b*c*d + b^2*e + 2*a*c*
e)*x^7 + 1/5*(2*a*b*e + (b^2 + 2*a*c)*d)*x^5 + a^2*d*x + 1/3*(2*a*b*d + a^2
*e)*x^3
```

Fricas [A]
time = 0.37, size = 94, normalized size = 0.98

$$\frac{1}{9}\,c^{2}dx^{9}+\frac{2}{7}\,bcdx^{7}+\frac{1}{5}\left(b^{2}+2\,ac\right)dx^{5}+\frac{2}{3}\,abdx^{3}+a^{2}dx+\frac{1}{3465}\left(315\,c^{2}x^{11}+770\,bcx^{9}+495\left(b^{2}+2\,ac\right)x^{7}+1386\,abx^{5}+1155\,a^{2}x^{3}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*d*x^9 + 2/7*b*c*d*x^7 + 1/5*(b^2 + 2*a*c)*d*x^5 + 2/3*a*b*d*x^3 + a
^2*d*x + 1/3465*(315*c^2*x^11 + 770*b*c*x^9 + 495*(b^2 + 2*a*c)*x^7 + 1386*
a*b*x^5 + 1155*a^2*x^3)*e
```

Sympy [A]
time = 0.02, size = 107, normalized size = 1.11

$$a^2dx+\frac{c^2ex^{11}}{11}+x^9\cdot\left(\frac{2bce}{9}+\frac{c^2d}{9}\right)+x^7\cdot\left(\frac{2ace}{7}+\frac{b^2e}{7}+\frac{2bcd}{7}\right)+x^5\cdot\left(\frac{2abe}{5}+\frac{2acd}{5}+\frac{b^2d}{5}\right)+x^3\left(\frac{a^2e}{3}+\frac{2abd}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*d*x + c**2*e*x**11/11 + x**9*(2*b*c*e/9 + c**2*d/9) + x**7*(2*a*c*e/7
+ b**2*e/7 + 2*b*c*d/7) + x**5*(2*a*b*e/5 + 2*a*c*d/5 + b**2*d/5) + x**3*(a
**2*e/3 + 2*a*b*d/3)
```

Giac [A]
time = 4.19, size = 106, normalized size = 1.10

$$\frac{1}{11}c^2x^{11}e+\frac{1}{9}c^2dx^9+\frac{2}{9}bcx^9e+\frac{2}{7}bcdx^7+\frac{1}{7}b^2x^7e+\frac{2}{7}acx^7e+\frac{1}{5}b^2dx^5+\frac{2}{5}acdx^5+\frac{2}{5}abx^5e+\frac{2}{3}abdx^3+\frac{1}{3}a^2x^3e+a^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11*e + 1/9*c^2*d*x^9 + 2/9*b*c*x^9*e + 2/7*b*c*d*x^7 + 1/7*b^2*x
^7*e + 2/7*a*c*x^7*e + 1/5*b^2*d*x^5 + 2/5*a*c*d*x^5 + 2/5*a*b*x^5*e + 2/3*
a*b*d*x^3 + 1/3*a^2*x^3*e + a^2*d*x
```

Mupad [B]
time = 0.04, size = 90, normalized size = 0.94

$$x^5\left(\frac{d\,b^2}{5}+\frac{2\,a\,e\,b}{5}+\frac{2\,a\,c\,d}{5}\right)+x^7\left(\frac{e\,b^2}{7}+\frac{2\,c\,d\,b}{7}+\frac{2\,a\,c\,e}{7}\right)+x^3\left(\frac{e\,a^2}{3}+\frac{2\,b\,d\,a}{3}\right)+x^9\left(\frac{d\,c^2}{9}+\frac{2\,b\,e\,c}{9}\right)+\frac{c^2\,e\,x^{11}}{11}+a^2\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5*((b^2*d)/5 + (2*a*b*e)/5 + (2*a*c*d)/5) + x^7*((b^2*e)/7 + (2*a*c*e)/7
+ (2*b*c*d)/7) + x^3*((a^2*e)/3 + (2*a*b*d)/3) + x^9*((c^2*d)/9 + (2*b*c*e)
/9) + (c^2*e*x^11)/11 + a^2*d*x
```

3.255 $\int (a + bx^2 + cx^4)^2 \, dx$

Optimal. Leaf size=49

$$a^2x + \frac{2}{3}abx^3 + \frac{1}{5}\left(b^2 + 2ac\right)x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1104\}$

$$a^2x + \frac{1}{5}x^5\left(2ac + b^2\right) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a + bx^2 + cx^4\right)^2 \, dx &= \int \left(a^2 + 2abx^2 + b^2\left(1 + \frac{2ac}{b^2}\right)x^4 + 2bcx^6 + c^2x^8\right) dx \\ &= a^2x + \frac{2}{3}abx^3 + \frac{1}{5}\left(b^2 + 2ac\right)x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 49, normalized size = 1.00

$$a^2x + \frac{2}{3}abx^3 + \frac{1}{5}\left(b^2 + 2ac\right)x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Maple [A]
time = 0.01, size = 42, normalized size = 0.86

method	result	size
default	$a^{2}x+\frac{2ab\,x^{3}}{3}+\frac{\left(2ac+b^{2}\right)x^{5}}{5}+\frac{2bc\,x^{7}}{7}+\frac{c^{2}x^{9}}{9}$	42
norman	$\frac{c^{2}x^{9}}{9}+\frac{2bc\,x^{7}}{7}+\left(\frac{2ac}{5}+\frac{b^{2}}{5}\right)x^{5}+\frac{2ab\,x^{3}}{3}+a^{2}x$	43
gosper	$\frac{1}{9}c^{2}x^{9}+\frac{2}{7}bc\,x^{7}+\frac{2}{5}ac\,x^{5}+\frac{1}{5}b^{2}x^{5}+\frac{2}{3}ab\,x^{3}+a^{2}x$	44
risch	$\frac{1}{9}c^{2}x^{9}+\frac{2}{7}bc\,x^{7}+\frac{2}{5}ac\,x^{5}+\frac{1}{5}b^{2}x^{5}+\frac{2}{3}ab\,x^{3}+a^{2}x$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A]
time = 0.29, size = 45, normalized size = 0.92

$$\frac{1}{9}\,c^{2}x^{9}+\frac{2}{7}\,bcx^{7}+\frac{1}{5}\,b^{2}x^{5}+a^{2}x+\frac{2}{15}\,\left(3\,cx^{5}+5\,bx^{3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 + 5*b*x^3)*
a
```

Fricas [A]
time = 0.32, size = 41, normalized size = 0.84

$$\frac{1}{9}\,c^{2}x^{9}+\frac{2}{7}\,bcx^{7}+\frac{1}{5}\,\left(b^{2}+2\,ac\right)x^{5}+\frac{2}{3}\,abx^{3}+a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A]
time = 0.01, size = 48, normalized size = 0.98

$$a^{2}x+\frac{2abx^{3}}{3}+\frac{2bcx^{7}}{7}+\frac{c^{2}x^{9}}{9}+x^{5}\cdot\left(\frac{2ac}{5}+\frac{b^{2}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

Giac [A]
time = 4.00, size = 43, normalized size = 0.88

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5+\frac{2}{5}\,acx^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B]
time = 0.02, size = 42, normalized size = 0.86

$$a^2\,x+x^5\,\left(\frac{b^2}{5}+\frac{2\,a\,c}{5}\right)+\frac{c^2\,x^9}{9}+\frac{2\,a\,b\,x^3}{3}+\frac{2\,b\,c\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2,x)
```

```
[Out] a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7)
/7
```

3.256 $\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx$

Optimal. Leaf size=143

$$-\frac{(cd-be)\left(cd^2-e(bd-2ae)\right)x}{e^4}+\frac{\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^3}{3e^3}-\frac{c(cd-2be)x^5}{5e^2}+\frac{c^2x^7}{7e}+\frac{(cd^2-bde+ae^2}{\sqrt{d}}$$

```
[Out] -(-b*e+c*d)*(c*d^2-e*(-2*a*e+b*d))*x/e^4+1/3*(c^2*d^2+b^2*e^2-2*c*e*(-a*e+b
*d))*x^3/e^3-1/5*c*(-2*b*e+c*d)*x^5/e^2+1/7*c^2*x^7/e+(a*e^2-b*d*e+c*d^2)^2
*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1167, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(ae^2-bde+cd^2)^2}{\sqrt{d}\,e^{9/2}}+\frac{x^3\left(-2ce(bd-ae)+b^2e^2+c^2d^2\right)}{3e^3}-\frac{x(cd-be)\left(cd^2-e(bd-2ae)\right)}{e^4}-\frac{cx^5(cd-2be)}{5e^2}+\frac{c^2x^7}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] -(((c*d - b*e)*(c*d^2 - e*(b*d - 2*a*e))*x)/e^4) + ((c^2*d^2 + b^2*e^2 - 2*
c*e*(b*d - a*e))*x^3)/(3*e^3) - (c*(c*d - 2*b*e)*x^5)/(5*e^2) + (c^2*x^7)/(
7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*e^(
9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{d+ex^2}\,dx = \int \left(-\frac{(cd-be)\,(cd^2-e(bd-2ae))}{e^4} + \frac{(c^2d^2+b^2e^2-2ce(bd-ae))\,x^2}{e^3} - \frac{c(cd-2b}{e^2}\right.$$

$$= -\frac{(cd-be)\,(cd^2-e(bd-2ae))\,x}{e^4} + \frac{(c^2d^2+b^2e^2-2ce(bd-ae))\,x^3}{3e^3} - \frac{c(cd-2be)x}{5e^2}$$

$$= -\frac{(cd-be)\,(cd^2-e(bd-2ae))\,x}{e^4} + \frac{(c^2d^2+b^2e^2-2ce(bd-ae))\,x^3}{3e^3} - \frac{c(cd-2be)x}{5e^2}$$

Mathematica [A]
time = 0.04, size = 144, normalized size = 1.01

$$\frac{(-cd+be)\,(cd^2-bde+2ae^2)\,x}{e^4} + \frac{(c^2d^2-2bcde+b^2e^2+2ace^2)\,x^3}{3e^3} + \frac{c(-cd+2be)x^5}{5e^2} + \frac{c^2x^7}{7e} + \frac{(cd^2-bde+ae^2)^2\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2),x]
```

```
[Out] ((-(c*d) + b*e)*(c*d^2 - b*d*e + 2*a*e^2)*x)/e^4 + ((c^2*d^2 - 2*b*c*d*e +
b^2*e^2 + 2*a*c*e^2)*x^3)/(3*e^3) + (c*(-(c*d) + 2*b*e)*x^5)/(5*e^2) + (c^2
*x^7)/(7*e) + ((c*d^2 - b*d*e + a*e^2)^2*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt
[d]*e^(9/2))
```

Maple [A]
time = 0.15, size = 178, normalized size = 1.24

method	result
default	$\frac{\frac{c^2x^7e^3}{7}+\frac{((eb-cd)ce^2+e^3bc)x^5}{5}+\frac{((eb-cd)e^2b+ce(2ae^2-deb+cd^2))x^3}{3}+(eb-cd)(2ae^2-deb+cd^2)x}{e^4}+\frac{(a^2e^4-2abde^3+2acd^2e^2+b^2d^2e}{e^4}$
risch	$\frac{c^2x^7}{7e}+\frac{2x^5bc}{5e}-\frac{c^2dx^5}{5e^2}+\frac{x^3b^2}{3e}-\frac{2x^3dbc}{3e^2}+\frac{2x^3ac}{3e}+\frac{x^3d^2c^2}{3e^3}+\frac{2abx}{e}-\frac{2acdx}{e^2}-\frac{b^2dx}{e^2}+\frac{2bcd^2x}{e^3}-\frac{c^2d^3x}{e^4}-\frac{\ln(ex+}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/7*c^2*x^7*e^3+1/5*((b*e-c*d)*c*e^2+e^3*b*c)*x^5+1/3*((b*e-c*d)*e^2
*b+c*e*(2*a*e^2-b*d*e+c*d^2))*x^3+(b*e-c*d)*(2*a*e^2-b*d*e+c*d^2)*x)+(a^2*e
^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/e^4/(d*e)^(1/
2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [A]
time = 0.51, size = 171, normalized size = 1.20

$$\frac{\left(c^{2}d^{4}-2\,bcd^{3}e-2\,abde^{3}+\left(b^{2}e^{2}+2\,ace^{2}\right)d^{2}+a^{2}e^{4}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}}+\frac{1}{105}\left(15\,c^{2}x^{7}e^{3}-21\left(c^{2}de^{2}-2\,bce^{3}\right)x^{5}+35\left(c^{2}d^{2}e-2\,bcde^{2}+b^{2}e^{3}+2\,ace^{3}\right)x^{3}-105\left(c^{2}d^{3}-2\,bcd^{2}e-2\,abe^{3}+\left(b^{2}e^{2}+2\,ace^{2}\right)d\right)x\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + (b^2*e^2 + 2*a*c*e^2)*d^2 + a^2*e^4)
*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt(d) + 1/105*(15*c^2*x^7*e^3 - 21*(c
^2*d*e^2 - 2*b*c*e^3)*x^5 + 35*(c^2*d^2*e - 2*b*c*d*e^2 + b^2*e^3 + 2*a*c*e
^3)*x^3 - 105*(c^2*d^3 - 2*b*c*d^2*e - 2*a*b*e^3 + (b^2*e^2 + 2*a*c*e^2)*d)
*x)*e^(-4)
```

Fricas [A]
time = 0.35, size = 393, normalized size = 2.75

$$\left[-\frac{\left(210\,c^{2}d^{4}xe+105\left(c^{2}d^{4}-2\,bcd^{3}e-2\,abde^{3}+\left(b^{2}+2\,ac\right)d^{2}e^{2}+a^{2}e^{4}\right)\sqrt{-de}\log\left(\frac{x^{2}e-2\sqrt{-de}x-d}{x^{2}e+d}\right)-2\left(15\,c^{2}dx^{7}+42\,bcdx^{5}+35\left(b^{2}+2\,ac\right)dx^{3}+210\,abdx\right)e^{4}+14\left(3\,c^{2}d^{2}x^{5}+10\,bcd^{2}x^{3}+15\left(b^{2}+2\,ac\right)d^{2}x\right)e^{3}-70\left(c^{2}d^{3}x^{3}+6\,bcd^{3}x\right)e^{2}\right)e^{(-5)}}{210\,d},\ -\frac{\left(105\,c^{2}d^{4}xe-105\left(c^{2}d^{4}-2\,bcd^{3}e-2\,abde^{3}+\left(b^{2}+2\,ac\right)d^{2}e^{2}+a^{2}e^{4}\right)\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-\left(15\,c^{2}dx^{7}+42\,bcdx^{5}+35\left(b^{2}+2\,ac\right)dx^{3}+210\,abdx\right)e^{4}+7\left(3\,c^{2}d^{2}x^{5}+10\,bcd^{2}x^{3}+15\left(b^{2}+2\,ac\right)d^{2}x\right)e^{3}-35\left(c^{2}d^{3}x^{3}+6\,bcd^{3}x\right)e^{2}\right)e^{(-5)}}{105\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [-1/210*(210*c^2*d^4*x*e + 105*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + (b^2
+ 2*a*c)*d^2*e^2 + a^2*e^4)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^
2*e + d)) - 2*(15*c^2*d*x^7 + 42*b*c*d*x^5 + 35*(b^2 + 2*a*c)*d*x^3 + 210*a
*b*d*x)*e^4 + 14*(3*c^2*d^2*x^5 + 10*b*c*d^2*x^3 + 15*(b^2 + 2*a*c)*d^2*x)*
e^3 - 70*(c^2*d^3*x^3 + 6*b*c*d^3*x)*e^2)*e^(-5)/d, -1/105*(105*c^2*d^4*x*e
 - 105*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + (b^2 + 2*a*c)*d^2*e^2 + a^2*e
^4)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - (15*c^2*d*x^7 + 42*b*c*d*x^
5 + 35*(b^2 + 2*a*c)*d*x^3 + 210*a*b*d*x)*e^4 + 7*(3*c^2*d^2*x^5 + 10*b*c*d
^2*x^3 + 15*(b^2 + 2*a*c)*d^2*x)*e^3 - 35*(c^2*d^3*x^3 + 6*b*c*d^3*x)*e^2)*
e^(-5)/d]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(133) = 266.
time = 0.54, size = 371, normalized size = 2.59

$$\frac{c^{2}x^{7}}{7e}+x^{5}\cdot\left(\frac{2bc}{5e}-\frac{c^{2}d}{5e^{2}}\right)+x^{3}\cdot\left(\frac{2ac}{3e}+\frac{b^{2}}{3e}-\frac{2bcd}{3e^{2}}+\frac{c^{2}d^{2}}{3e^{3}}\right)+x\left(\frac{2ab}{e}-\frac{2acd}{e^{2}}-\frac{b^{2}d}{e^{2}}+\frac{2bcd^{2}}{e^{3}}-\frac{c^{2}d^{3}}{e^{4}}\right)-\frac{\sqrt{-\frac{1}{de^{9}}}\left(ae^{2}-bde+cd^{2}\right)^{2}\log\left(-\frac{de^{4}\sqrt{-\frac{1}{de^{9}}}\left(ae^{2}-bde+cd^{2}\right)^{2}}{a^{2}e^{4}-2abde^{3}+2acd^{2}e^{2}+b^{2}d^{2}e^{2}-2bcd^{3}e+c^{2}d^{4}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{de^{9}}}\left(ae^{2}-bde+cd^{2}\right)^{2}\log\left(\frac{de^{4}\sqrt{-\frac{1}{de^{9}}}\left(ae^{2}-bde+cd^{2}\right)^{2}}{a^{2}e^{4}-2abde^{3}+2acd^{2}e^{2}+b^{2}d^{2}e^{2}-2bcd^{3}e+c^{2}d^{4}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d),x)
```

```
[Out] c**2*x**7/(7*e) + x**5*(2*b*c/(5*e) - c**2*d/(5*e**2)) + x**3*(2*a*c/(3*e)
+ b**2/(3*e) - 2*b*c*d/(3*e**2) + c**2*d**2/(3*e**3)) + x*(2*a*b/e - 2*a*c*
```

```
d/e**2 - b**2*d/e**2 + 2*b*c*d**2/e**3 - c**2*d**3/e**4) - sqrt(-1/(d*e**9)
)*(a*e**2 - b*d*e + c*d**2)**2*log(-d*e**4*sqrt(-1/(d*e**9))*(a*e**2 - b*d*
e + c*d**2)**2/(a**2*e**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2
 - 2*b*c*d**3*e + c**2*d**4) + x)/2 + sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c
*d**2)**2*log(d*e**4*sqrt(-1/(d*e**9))*(a*e**2 - b*d*e + c*d**2)**2/(a**2*e
**4 - 2*a*b*d*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2
*d**4) + x)/2
```

<u>**Giac**</u> **[A]**
time = 3.16, size = 185, normalized size = 1.29

$$\frac{(c^2d^4 - 2\,bcd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,abde^3 + a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{\sqrt{d}} + \frac{1}{105}\left(15\,c^2x^7e^6 - 21\,c^2dx^5e^5 + 42\,bcx^5e^6 + 35\,c^2d^2x^3e^4 - 70\,bcdx^3e^5 - 105\,c^2d^3xe^3 + 35\,b^2x^3e^6 + 70\,acx^3e^6 + 210\,bcd^2xe^4 - 105\,b^2dxe^5 - 210\,acdxe^5 + 210\,abxe^6\right)e^{(-7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^
4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt(d) + 1/105*(15*c^2*x^7*e^6 - 21*
c^2*d*x^5*e^5 + 42*b*c*x^5*e^6 + 35*c^2*d^2*x^3*e^4 - 70*b*c*d*x^3*e^5 - 10
5*c^2*d^3*x*e^3 + 35*b^2*x^3*e^6 + 70*a*c*x^3*e^6 + 210*b*c*d^2*x*e^4 - 105
*b^2*d*x*e^5 - 210*a*c*d*x*e^5 + 210*a*b*x*e^6)*e^(-7)
```

<u>**Mupad**</u> **[B]**
time = 4.47, size = 229, normalized size = 1.60

$$x^3\left(\frac{b^2+2\,a\,c}{3\,e}+\frac{d\left(\frac{c^2\,d}{e^2}-\frac{2\,b\,c}{e}\right)}{3\,e}\right)-x\left(\frac{d\left(\frac{b^2+2\,a\,c}{e}+\frac{d\left(\frac{c^2\,d}{e^2}-\frac{2\,b\,c}{e}\right)}{e}\right)}{e}-\frac{2\,a\,b}{e}\right)-x^5\left(\frac{c^2\,d}{5\,e^2}-\frac{2\,b\,c}{5\,e}\right)+\frac{c^2\,x^7}{7\,e}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,{\left(c\,d^2-b\,d\,e+a\,e^2\right)}^2}{\sqrt{d}\,\left(a^2\,e^4-2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2-2\,b\,c\,d^3\,e+c^2\,d^4\right)}\right)\,{\left(c\,d^2-b\,d\,e+a\,e^2\right)}^2}{\sqrt{d}\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2),x)
```

```
[Out] x^3*((2*a*c + b^2)/(3*e) + (d*((c^2*d)/e^2 - (2*b*c)/e))/(3*e)) - x*((d*((2
*a*c + b^2)/e + (d*((c^2*d)/e^2 - (2*b*c)/e))/e))/e - (2*a*b)/e) - x^5*((c^
2*d)/(5*e^2) - (2*b*c)/(5*e)) + (c^2*x^7)/(7*e) + (atan((e^(1/2)*x*(a*e^2 +
 c*d^2 - b*d*e)^2)/(d^(1/2)*(a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3
- 2*b*c*d^3*e + 2*a*c*d^2*e^2)))*(a*e^2 + c*d^2 - b*d*e)^2)/(d^(1/2)*e^(9/2
))
```

3.257 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=166

$$\frac{(3c^2d^2+b^2e^2-2ce(2bd-ae))\,x}{e^4}-\frac{2c(cd-be)x^3}{3e^3}+\frac{c^2x^5}{5e^2}+\frac{(cd^2-bde+ae^2)^2\,x}{2de^4\,(d+ex^2)}-\frac{(cd^2-bde+ae^2)\,(7cd^2-e}{2d^{3/2}}$$

```
[Out] (3*c^2*d^2+b^2*e^2-2*c*e*(-a*e+2*b*d))*x/e^4-2/3*c*(-b*e+c*d)*x^3/e^3+1/5*c
^2*x^5/e^2+1/2*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)-1/2*(a*e^2-b*d*e+c*d
^2)*(7*c*d^2-e*(a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(9/2)
```

Rubi [A]
time = 0.19, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1171, 1824, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(ae^2-bde+cd^2)\,(7cd^2-e(ae+3bd))}{2d^{3/2}e^{9/2}}+\frac{x(-2ce(2bd-ae)+b^2e^2+3c^2d^2)}{e^4}+\frac{x(ae^2-bde+cd^2)^2}{2de^4\,(d+ex^2)}-\frac{2cx^3(cd-be)}{3e^3}+\frac{c^2x^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] ((3*c^2*d^2 + b^2*e^2 - 2*c*e*(2*b*d - a*e))*x)/e^4 - (2*c*(c*d - b*e)*x^3)
/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 - b*d*e + a*e^2)^2*x)/(2*d*e^4*(d +
e*x^2)) - ((c*d^2 - b*d*e + a*e^2)*(7*c*d^2 - e*(3*b*d + a*e))*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^2}\,dx = \frac{(cd^2-bde+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-a^2e^2\right)}{e^4}-\frac{2d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3}+\frac{2cd}{}}{d+ex^2}}{2d}$$

$$= \frac{(cd^2-bde+ae^2)^2 x}{2de^4(d+ex^2)} - \frac{\int \left(-\frac{2d(3c^2d^2+b^2e^2-2ce(2bd-ae))}{e^4}+\frac{4cd(cd-be)x^2}{e^3}-\frac{2c^2dx^4}{e^2}+\frac{7c^2d^4-}{}\right.}{2d}$$

$$= \frac{(3c^2d^2+b^2e^2-2ce(2bd-ae))x}{e^4} - \frac{2c(cd-be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2-bde+ae^2)^2 x}{2de^4(d+ex^2)} -$$

$$= \frac{(3c^2d^2+b^2e^2-2ce(2bd-ae))x}{e^4} - \frac{2c(cd-be)x^3}{3e^3} + \frac{c^2x^5}{5e^2} + \frac{(cd^2-bde+ae^2)^2 x}{2de^4(d+ex^2)} -$$

Mathematica [A]
time = 0.07, size = 183, normalized size = 1.10

$$\frac{(3c^2d^2+b^2e^2+2ce(-2bd+ae))x}{e^4}+\frac{2c(-cd+be)x^3}{3e^3}+\frac{c^2x^5}{5e^2}+\frac{(cd^2+e(-bd+ae))^2x}{2de^4(d+ex^2)}-\frac{(7c^2d^4+2cd^2e(-5bd+3ae)-e^2(-3b^2d^2+2abde+a^2e^2))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x]
```

```
[Out] ((3*c^2*d^2 + b^2*e^2 + 2*c*e*(-2*b*d + a*e))*x)/e^4 + (2*c*(-(c*d) + b*e)*
x^3)/(3*e^3) + (c^2*x^5)/(5*e^2) + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(2*d*e^
4*(d + e*x^2)) - ((7*c^2*d^4 + 2*c*d^2*e*(-5*b*d + 3*a*e) - e^2*(-3*b^2*d^2
 + 2*a*b*d*e + a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(9/2))
```

Maple [A]
time = 0.16, size = 213, normalized size = 1.28

method	result
default	$\frac{\frac{1}{5}x^5e^2c^2+\frac{2}{3}bc\,e^2x^3-\frac{2}{3}c^2de\,x^3+2ac\,e^2x+e^2b^2x-4bcdex+3c^2d^2x}{e^4}+\frac{\frac{\left(a^2e^4-2abd\,e^3+2ac\,d^2e^2+b^2d^2e^2-2bc\,d^3e+c^2d^4\right)x}{2d\left(e\,x^2+d\right)}+\frac{\left(a^2e^4+2abd\,e^3-\right.}{}}{e^4}$
risch	$\frac{c^2x^5}{5e^2}+\frac{2bc\,x^3}{3e^2}-\frac{2c^2d\,x^3}{3e^3}+\frac{2acx}{e^2}+\frac{b^2x}{e^2}-\frac{4bcdx}{e^3}+\frac{3c^2d^2x}{e^4}+\frac{(a^2e^4-2abd\,e^3+2ac\,d^2e^2+b^2d^2e^2-2bc\,d^3e+c^2d^4)x}{2d\,e^4(e\,x^2+d)}-\frac{\ln\left(ex\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/5*x^5*e^2*c^2+2/3*b*c*e^2*x^3-2/3*c^2*d*e*x^3+2*a*c*e^2*x+e^2*b^2*
x-4*b*c*d*e*x+3*c^2*d^2*x)+1/e^4*(1/2*(a^2*e^4-2*a*b*d*e^3+2*a*c*d^2*e^2+b^
2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/d*x/(e*x^2+d)+1/2*(a^2*e^4+2*a*b*d*e^3-6*a*c
*d^2*e^2-3*b^2*d^2*e^2+10*b*c*d^3*e-7*c^2*d^4)/d/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2)))
```

Maxima [A]
time = 0.51, size = 197, normalized size = 1.19

$$\frac{1}{15}\left(3\,c^2x^5e^2-10\,(c^2de-bce^2)x^3+15\,(3\,c^2d^2-4\,bcde+b^2e^2+2\,ace^2)x\right)e^{(-4)}-\frac{(7\,c^2d^4-10\,bcd^3e-2\,abde^3+3\,(b^2e^2+2\,ace^2)d^2-a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,d^{\frac{3}{2}}}+\frac{(c^2d^4-2\,bcd^3e-2\,abde^3+(b^2e^2+2\,ace^2)d^2+a^2e^4)x}{2\,(dx^2e^5+d^2e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*x^5*e^2 - 10*(c^2*d*e - b*c*e^2)*x^3 + 15*(3*c^2*d^2 - 4*b*c*d*
e + b^2*e^2 + 2*a*c*e^2)*x)*e^(-4) - 1/2*(7*c^2*d^4 - 10*b*c*d^3*e - 2*a*b*
d*e^3 + 3*(b^2*e^2 + 2*a*c*e^2)*d^2 - a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^
(-9/2)/d^(3/2) + 1/2*(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + (b^2*e^2 + 2*a*
c*e^2)*d^2 + a^2*e^4)*x/(d*x^2*e^5 + d^2*e^4)
```

Fricas [A]
time = 0.36, size = 595, normalized size = 3.58

$$\left[\frac{210\,c^2d^5xe+30\,a^2dxe^5+15\,(7\,c^2d^5-a^2x^2e^5-(2\,abdx^2+a^2d)e^4+(3\,(b^2+2\,ac)d^2x^2-2\,abd^2)e^3-(10\,bcd^3x^2-3\,(b^2+2\,ac)d^3)e^2+(7\,c^2d^4x^2-10\,bcd^4)e)\sqrt{-de}\,\log\left(\frac{x^2e-2\,\sqrt{-de}\,x-d}{x^2e+d}\right)+4\,(3\,c^2d^2x^7+10\,bcd^2x^5+15\,(b^2+2\,ac)d^2x^3-15\,abd^2x)e^4-2\,(14\,c^2d^3x^5+100\,bcd^3x^3-45\,(b^2+2\,ac)d^3x)e^3+20\,(7\,c^2d^4x^3-15\,bcd^4x)e^2}{60\,(d^2x^2e^6+d^3e^5)},\frac{105\,c^2d^5xe+15\,a^2dxe^5-15\,(7\,c^2d^5-a^2x^2e^5-(2\,abdx^2+a^2d)e^4+(3\,(b^2+2\,ac)d^2x^2-2\,abd^2)e^3-(10\,bcd^3x^2-3\,(b^2+2\,ac)d^3)e^2+(7\,c^2d^4x^2-10\,bcd^4)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}+2\,(3\,c^2d^2x^7+10\,bcd^2x^5+15\,(b^2+2\,ac)d^2x^3-15\,abd^2x)e^4-(14\,c^2d^3x^5+100\,bcd^3x^3-45\,(b^2+2\,ac)d^3x)e^3+10\,(7\,c^2d^4x^3-15\,bcd^4x)e^2}{30\,(d^2x^2e^6+d^3e^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(210*c^2*d^5*x*e + 30*a^2*d*x*e^5 + 15*(7*c^2*d^5 - a^2*x^2*e^5 - (2*
a*b*d*x^2 + a^2*d)*e^4 + (3*(b^2 + 2*a*c)*d^2*x^2 - 2*a*b*d^2)*e^3 - (10*b*
c*d^3*x^2 - 3*(b^2 + 2*a*c)*d^3)*e^2 + (7*c^2*d^4*x^2 - 10*b*c*d^4)*e)*sqrt
(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) + 4*(3*c^2*d^2*x^7 + 1
0*b*c*d^2*x^5 + 15*(b^2 + 2*a*c)*d^2*x^3 - 15*a*b*d^2*x)*e^4 - 2*(14*c^2*d^
3*x^5 + 100*b*c*d^3*x^3 - 45*(b^2 + 2*a*c)*d^3*x)*e^3 + 20*(7*c^2*d^4*x^3 -
 15*b*c*d^4*x)*e^2)/(d^2*x^2*e^6 + d^3*e^5), 1/30*(105*c^2*d^5*x*e + 15*a^2
*d*x*e^5 - 15*(7*c^2*d^5 - a^2*x^2*e^5 - (2*a*b*d*x^2 + a^2*d)*e^4 + (3*(b^
2 + 2*a*c)*d^2*x^2 - 2*a*b*d^2)*e^3 - (10*b*c*d^3*x^2 - 3*(b^2 + 2*a*c)*d^3
)*e^2 + (7*c^2*d^4*x^2 - 10*b*c*d^4)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e
^(1/2) + 2*(3*c^2*d^2*x^7 + 10*b*c*d^2*x^5 + 15*(b^2 + 2*a*c)*d^2*x^3 - 15*
a*b*d^2*x)*e^4 - (14*c^2*d^3*x^5 + 100*b*c*d^3*x^3 - 45*(b^2 + 2*a*c)*d^3*x
)*e^3 + 10*(7*c^2*d^4*x^3 - 15*b*c*d^4*x)*e^2)/(d^2*x^2*e^6 + d^3*e^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(156) = 312$.

time = 1.42, size = 484, normalized size = 2.92

$$\frac{c^2x^5}{5e^2} + x^3 \cdot \left(\frac{2bc}{3e^2} - \frac{2c^2d}{3e^3}\right) + x\left(\frac{2ac}{e^2} + \frac{b^2}{e^2} - \frac{4bcd}{e^3} + \frac{3c^2d^2}{e^4}\right) + \frac{x(a^2e^4 - 2abde^3 + 2acd^2e^2 + b^2d^2e^2 - 2bcd^3e + c^2d^4)}{2d^2e^4 + 2de^5x^2} - \frac{\sqrt{-\frac{1}{d^3e^9}}\,(ae^2 - bde + cd^2)\,(ae^2 + 3bde - 7cd^2)\log\left(-\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}\,(ae^2-bde+cd^2)\,(ae^2+3bde-7cd^2)}{a^2e^4+2abde^3-6acd^2e^2-3b^2d^2e^2+10bcd^3e-7c^2d^4} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^9}}\,(ae^2 - bde + cd^2)\,(ae^2 + 3bde - 7cd^2)\log\left(\frac{d^2e^4\sqrt{-\frac{1}{d^3e^9}}\,(ae^2-bde+cd^2)\,(ae^2+3bde-7cd^2)}{a^2e^4+2abde^3-6acd^2e^2-3b^2d^2e^2+10bcd^3e-7c^2d^4} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**2,x)
```

```
[Out] c**2*x**5/(5*e**2) + x**3*(2*b*c/(3*e**2) - 2*c**2*d/(3*e**3)) + x*(2*a*c/e
**2 + b**2/e**2 - 4*b*c*d/e**3 + 3*c**2*d**2/e**4) + x*(a**2*e**4 - 2*a*b*d
*e**3 + 2*a*c*d**2*e**2 + b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4)/(2*d**
2*e**4 + 2*d*e**5*x**2) - sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a
*e**2 + 3*b*d*e - 7*c*d**2)*log(-d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - b
*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)/(a**2*e**4 + 2*a*b*d*e**3 - 6*
a*c*d**2*e**2 - 3*b**2*d**2*e**2 + 10*b*c*d**3*e - 7*c**2*d**4) + x)/4 + sq
rt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*d*e - 7*c*d**2)*
log(d**2*e**4*sqrt(-1/(d**3*e**9))*(a*e**2 - b*d*e + c*d**2)*(a*e**2 + 3*b*
d*e - 7*c*d**2)/(a**2*e**4 + 2*a*b*d*e**3 - 6*a*c*d**2*e**2 - 3*b**2*d**2*e
**2 + 10*b*c*d**3*e - 7*c**2*d**4) + x)/4
```

Giac [A]

time = 3.30, size = 207, normalized size = 1.25

$$\frac{1}{15}\,(3\,c^2x^5e^8 - 10\,c^2dx^3e^7 + 10\,bcx^3e^8 + 45\,c^2d^2xe^6 - 60\,bcdxe^7 + 15\,b^2xe^8 + 30\,acxe^8)e^{(-10)} - \frac{(7\,c^2d^4 - 10\,bcd^3e + 3\,b^2d^2e^2 + 6\,acd^2e^2 - 2\,abde^3 - a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{(c^2d^4x - 2\,bcd^3xe + b^2d^2xe^2 + 2\,acd^2xe^2 - 2\,abdxe^3 + a^2xe^4)e^{(-4)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*c^2*x^5*e^8 - 10*c^2*d*x^3*e^7 + 10*b*c*x^3*e^8 + 45*c^2*d^2*x*e^6
- 60*b*c*d*x*e^7 + 15*b^2*x*e^8 + 30*a*c*x*e^8)*e^(-10) - 1/2*(7*c^2*d^4 -
10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 - 2*a*b*d*e^3 - a^2*e^4)*arcta
n(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(3/2) + 1/2*(c^2*d^4*x - 2*b*c*d^3*x*e + b^
2*d^2*x*e^2 + 2*a*c*d^2*x*e^2 - 2*a*b*d*x*e^3 + a^2*x*e^4)*e^(-4)/((x^2*e +
 d)*d)
```

Mupad [B]

time = 4.56, size = 293, normalized size = 1.77

$$x\left(\frac{b^2+2\,a\,c}{e^2}+\frac{2\,d\left(\frac{2\,c^2\,d}{e^3}-\frac{2\,b\,c}{e^2}\right)}{e}-\frac{c^2\,d^2}{e^4}\right)-x^3\left(\frac{2\,c^2\,d}{3\,e^3}-\frac{2\,b\,c}{3\,e^2}\right)+\frac{c^2\,x^5}{5\,e^2}+\frac{x\,(a^2\,e^4-2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2-2\,b\,c\,d^3\,e+c^2\,d^4)}{2\,d\,(e^5\,x^2+d\,e^4)}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x\,(c\,d^2-b\,d\,e+a\,e^2)\,(-7\,c\,d^2+3\,b\,d\,e+a\,e^2)}{\sqrt{d}\,(a^2\,e^4+2\,a\,b\,d\,e^3-6\,a\,c\,d^2\,e^2-3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-7\,c^2\,d^4)}\right)\,(c\,d^2-b\,d\,e+a\,e^2)\,(-7\,c\,d^2+3\,b\,d\,e+a\,e^2)}{2\,d^{3/2}\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^2,x)
```

```
[Out] x*((2*a*c + b^2)/e^2 + (2*d*((2*c^2*d)/e^3 - (2*b*c)/e^2))/e - (c^2*d^2)/e^
4) - x^3*((2*c^2*d)/(3*e^3) - (2*b*c)/(3*e^2)) + (c^2*x^5)/(5*e^2) + (x*(a^
2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^2))
/(2*d*(d*e^4 + e^5*x^2)) + (atan((e^(1/2)*x*(a*e^2 + c*d^2 - b*d*e)*(a*e^2
- 7*c*d^2 + 3*b*d*e))/(d^(1/2)*(a^2*e^4 - 7*c^2*d^4 - 3*b^2*d^2*e^2 + 2*a*b
*d*e^3 + 10*b*c*d^3*e - 6*a*c*d^2*e^2)))*(a*e^2 + c*d^2 - b*d*e)*(a*e^2 - 7
*c*d^2 + 3*b*d*e))/(2*d^(3/2)*e^(9/2))
```

3.258 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=201

$$-\frac{c(3cd-2be)x}{e^4}+\frac{c^2x^3}{3e^3}+\frac{(cd^2-bde+ae^2)^2\,x}{4de^4\,(d+ex^2)^2}-\frac{(13cd^2-5bde-3ae^2)\,(cd^2-bde+ae^2)\,x}{8d^2e^4\,(d+ex^2)}+\frac{(35c^2d^4-6cd^2e(5}{}$$

```
[Out] -c*(-2*b*e+3*c*d)*x/e^4+1/3*c^2*x^3/e^3+1/4*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(
e*x^2+d)^2-1/8*(-3*a*e^2-5*b*d*e+13*c*d^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/(e
*x^2+d)+1/8*(35*c^2*d^4-6*c*d^2*e*(-a*e+5*b*d)+e^2*(3*a^2*e^2+2*a*b*d*e+3*b
^2*d^2))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(9/2)
```

Rubi [A]
time = 0.26, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1171, 1828, 1167, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(e^2(3a^2e^2+2abde+3b^2d^2)-6cd^2e(5bd-ae)+35c^2d^4\right)}{8d^{5/2}e^{9/2}}-\frac{x(-3ae^2-5bde+13cd^2)\,(ae^2-bde+cd^2)}{8d^2e^4\,(d+ex^2)}+\frac{x(ae^2-bde+cd^2)^2}{4de^4\,(d+ex^2)^2}-\frac{cx(3cd-2be)}{e^4}+\frac{c^2x^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] -((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^3)/(3*e^3) + ((c*d^2 - b*d*e + a*e^2)
^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 5*b*d*e - 3*a*e^2)*(c*d^2 - b*
d*e + a*e^2)*x)/(8*d^2*e^4*(d + e*x^2)) + ((35*c^2*d^4 - 6*c*d^2*e*(5*b*d -
 a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(8*d^(5/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^3}\,dx = \frac{(cd^2-bde+ae^2)^2\,x}{4de^4\,(d+ex^2)^2} - \frac{\int \frac{\frac{(cd^2-bde-ae^2)(cd^2-bde+3ae^2)}{e^4} - \frac{4d(c^2d^2+b^2e^2-2ce(bd-ae))x^2}{e^3} + \frac{4cd(cd-2be)x}{e^2}}{(d+ex^2)^2}}{4d}$$

$$= \frac{(cd^2-bde+ae^2)^2\,x}{4de^4\,(d+ex^2)^2} - \frac{(13cd^2-5bde-3ae^2)(cd^2-bde+ae^2)\,x}{8d^2e^4\,(d+ex^2)} + \frac{\int \frac{11c^2d^4-2cd^2e(}{}}{}$$

$$= \frac{(cd^2-bde+ae^2)^2\,x}{4de^4\,(d+ex^2)^2} - \frac{(13cd^2-5bde-3ae^2)(cd^2-bde+ae^2)\,x}{8d^2e^4\,(d+ex^2)} + \frac{\int \left(-\frac{8cd^2(3cd}{e^4}\right.}{}$$

$$= -\frac{c(3cd-2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2-bde+ae^2)^2\,x}{4de^4\,(d+ex^2)^2} - \frac{(13cd^2-5bde-3ae^2)(cd^2-b}{8d^2e^4\,(d+ex^2)}$$

$$= -\frac{c(3cd-2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2-bde+ae^2)^2\,x}{4de^4\,(d+ex^2)^2} - \frac{(13cd^2-5bde-3ae^2)(cd^2-b}{8d^2e^4\,(d+ex^2)}$$

Mathematica [A]
time = 0.07, size = 217, normalized size = 1.08

$$\frac{c(-3cd+2be)x}{e^4} + \frac{c^2x^3}{3e^3} + \frac{(cd^2+e(-bd+ae))^2\,x}{4de^4\,(d+ex^2)^2} - \frac{(13c^2d^4-2cd^2e(9bd-5ae)+e^2(5b^2d^2-2abde-3a^2e^2))\,x}{8d^2e^4\,(d+ex^2)} + \frac{(35c^2d^4+6cd^2e(-5bd+ae)+e^2(3b^2d^2+2abde+3a^2e^2))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x]
```

```
[Out] (c*(-3*c*d + 2*b*e)*x)/e^4 + (c^2*x^3)/(3*e^3) + ((c*d^2 + e*(-(b*d) + a*e)
)^2*x)/(4*d*e^4*(d + e*x^2)^2) - ((13*c^2*d^4 - 2*c*d^2*e*(9*b*d - 5*a*e) +
 e^2*(5*b^2*d^2 - 2*a*b*d*e - 3*a^2*e^2))*x)/(8*d^2*e^4*(d + e*x^2)) + ((35
*c^2*d^4 + 6*c*d^2*e*(-5*b*d + a*e) + e^2*(3*b^2*d^2 + 2*a*b*d*e + 3*a^2*e^
2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(9/2))
```

Maple [A]
time = 0.15, size = 237, normalized size = 1.18

method	result
default	$\frac{c\left(\frac{1}{3}ce\,x^{3}+2ebx-3cdx\right)}{e^{4}}+\frac{\frac{\frac{e\left(3a^{2}e^{4}+2abd\,e^{3}-10ac\,d^{2}e^{2}-5b^{2}d^{2}e^{2}+18bc\,d^{3}e-13c^{2}d^{4}\right)x^{3}}{8d^{2}}+\frac{\left(5a^{2}e^{4}-2abd\,e^{3}-6ac\,d^{2}e^{2}-3b^{2}d^{2}e^{2}+14bc\,d^{3}e-11c^{2}\right.}{8d}}{\left(e\,x^{2}+d\right)^{2}}}{e^{4}}$
risch	$\frac{c^{2}x^{3}}{3e^{3}}+\frac{2cbx}{e^{3}}-\frac{3c^{2}dx}{e^{4}}+\frac{\frac{e\left(3a^{2}e^{4}+2abd\,e^{3}-10ac\,d^{2}e^{2}-5b^{2}d^{2}e^{2}+18bc\,d^{3}e-13c^{2}d^{4}\right)x^{3}}{8d^{2}}+\frac{\left(5a^{2}e^{4}-2abd\,e^{3}-6ac\,d^{2}e^{2}-3b^{2}d^{2}e^{2}+14bc\,d^{3}e-11c\right.}{8d}}{e^{4}\left(e\,x^{2}+d\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c/e^4*(1/3*c*e*x^3+2*e*b*x-3*c*d*x)+1/e^4*((1/8*e*(3*a^2*e^4+2*a*b*d*e^3-10
*a*c*d^2*e^2-5*b^2*d^2*e^2+18*b*c*d^3*e-13*c^2*d^4)/d^2*x^3+1/8*(5*a^2*e^4-
2*a*b*d*e^3-6*a*c*d^2*e^2-3*b^2*d^2*e^2+14*b*c*d^3*e-11*c^2*d^4)/d*x)/(e*x^
2+d)^2+1/8*(3*a^2*e^4+2*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2-30*b*c*d^3*e+
35*c^2*d^4)/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.51, size = 238, normalized size = 1.18

$$\frac{1}{3}\left(c^{2}x^{3}e-3\left(3\,c^{2}d-2\,bce\right)x\right)e^{(-4)}+\frac{\left(35\,c^{2}d^{4}-30\,bcd^{3}e+2\,abde^{3}+3\left(b^{2}e^{2}+2\,ace^{2}\right)d^{2}+3\,a^{2}e^{4}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(13\,c^{2}d^{4}e-18\,bcd^{3}e^{2}-2\,abde^{4}+5\left(b^{2}e^{3}+2\,ace^{3}\right)d^{2}-3\,a^{2}e^{5}\right)x^{3}+\left(11\,c^{2}d^{5}-14\,bcd^{4}e+2\,abd^{2}e^{3}+3\left(b^{2}e^{2}+2\,ace^{2}\right)d^{3}-5\,a^{2}de^{4}\right)x}{8\left(d^{2}x^{4}e^{6}+2\,d^{3}x^{2}e^{5}+d^{4}e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^3*e - 3*(3*c^2*d - 2*b*c*e)*x)*e^(-4) + 1/8*(35*c^2*d^4 - 30*b*c
*d^3*e + 2*a*b*d*e^3 + 3*(b^2*e^2 + 2*a*c*e^2)*d^2 + 3*a^2*e^4)*arctan(x*e^
(1/2)/sqrt(d))*e^(-9/2)/d^(5/2) - 1/8*((13*c^2*d^4*e - 18*b*c*d^3*e^2 - 2*a
*b*d*e^4 + 5*(b^2*e^3 + 2*a*c*e^3)*d^2 - 3*a^2*e^5)*x^3 + (11*c^2*d^5 - 14*
b*c*d^4*e + 2*a*b*d^2*e^3 + 3*(b^2*e^2 + 2*a*c*e^2)*d^3 - 5*a^2*d*e^4)*x)/(
d^2*x^4*e^6 + 2*d^3*x^2*e^5 + d^4*e^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(183) = 366.
time = 0.35, size = 792, normalized size = 3.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(210*c^2*d^6*x*e - 18*a^2*d*x^3*e^6 + 3*(35*c^2*d^6 + 3*a^2*x^4*e^6
+ 2*(a*b*d*x^4 + 3*a^2*d*x^2)*e^5 + (3*(b^2 + 2*a*c)*d^2*x^4 + 4*a*b*d^2*x^
2 + 3*a^2*d^2)*e^4 - 2*(15*b*c*d^3*x^4 - 3*(b^2 + 2*a*c)*d^3*x^2 - a*b*d^3)
*e^3 + (35*c^2*d^4*x^4 - 60*b*c*d^4*x^2 + 3*(b^2 + 2*a*c)*d^4)*e^2 + 10*(7*
c^2*d^5*x^2 - 3*b*c*d^5)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^
2*e + d)) - 6*(2*a*b*d^2*x^3 + 5*a^2*d^2*x)*e^5 - 2*(8*c^2*d^3*x^7 + 48*b*c
*d^3*x^5 - 15*(b^2 + 2*a*c)*d^3*x^3 - 6*a*b*d^3*x)*e^4 + 2*(56*c^2*d^4*x^5
- 150*b*c*d^4*x^3 + 9*(b^2 + 2*a*c)*d^4*x)*e^3 + 10*(35*c^2*d^5*x^3 - 18*b*
c*d^5*x)*e^2)/(d^3*x^4*e^7 + 2*d^4*x^2*e^6 + d^5*e^5), -1/24*(105*c^2*d^6*x
*e - 9*a^2*d*x^3*e^6 - 3*(35*c^2*d^6 + 3*a^2*x^4*e^6 + 2*(a*b*d*x^4 + 3*a^2
*d*x^2)*e^5 + (3*(b^2 + 2*a*c)*d^2*x^4 + 4*a*b*d^2*x^2 + 3*a^2*d^2)*e^4 - 2
*(15*b*c*d^3*x^4 - 3*(b^2 + 2*a*c)*d^3*x^2 - a*b*d^3)*e^3 + (35*c^2*d^4*x^4
 - 60*b*c*d^4*x^2 + 3*(b^2 + 2*a*c)*d^4)*e^2 + 10*(7*c^2*d^5*x^2 - 3*b*c*d^
5)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - 3*(2*a*b*d^2*x^3 + 5*a^2*
d^2*x)*e^5 - (8*c^2*d^3*x^7 + 48*b*c*d^3*x^5 - 15*(b^2 + 2*a*c)*d^3*x^3 - 6
*a*b*d^3*x)*e^4 + (56*c^2*d^4*x^5 - 150*b*c*d^4*x^3 + 9*(b^2 + 2*a*c)*d^4*x
)*e^3 + 5*(35*c^2*d^5*x^3 - 18*b*c*d^5*x)*e^2)/(d^3*x^4*e^7 + 2*d^4*x^2*e^6
 + d^5*e^5)]
```

Sympy [A]
time = 10.48, size = 398, normalized size = 1.98

$$\frac{c^2x^3}{3e^3}+x\left(\frac{2bc}{e^3}-\frac{3c^2d}{e^4}\right)-\frac{\sqrt{-\frac{1}{d^5e^9}}\cdot(3a^2e^4+2abde^3+6acd^2e^2+3b^2d^2e^2-30bcd^3e+35c^2d^4)\log\left(-d^3e^4\sqrt{-\frac{1}{d^5e^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^5e^9}}\cdot(3a^2e^4+2abde^3+6acd^2e^2+3b^2d^2e^2-30bcd^3e+35c^2d^4)\log\left(d^3e^4\sqrt{-\frac{1}{d^5e^9}}+x\right)}{16}+\frac{x^3\cdot(3a^2e^5+2abde^4-10acd^2e^3-5b^2d^2e^3+18bcd^3e^2-13c^2d^4e)+x(5a^2de^4-2abd^2e^3-6acd^3e^2-3b^2d^3e^2+14bcd^4e-11c^2d^5)}{8d^4e^4+16d^3e^5x^2+8d^2e^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**3,x)
```

```
[Out] c**2*x**3/(3*e**3) + x*(2*b*c/e**3 - 3*c**2*d/e**4) - sqrt(-1/(d**5*e**9))*
(3*a**2*e**4 + 2*a*b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e**2 - 30*b*c*d
**3*e + 35*c**2*d**4)*log(-d**3*e**4*sqrt(-1/(d**5*e**9)) + x)/16 + sqrt(-1
/(d**5*e**9))*(3*a**2*e**4 + 2*a*b*d*e**3 + 6*a*c*d**2*e**2 + 3*b**2*d**2*e
**2 - 30*b*c*d**3*e + 35*c**2*d**4)*log(d**3*e**4*sqrt(-1/(d**5*e**9)) + x)
/16 + (x**3*(3*a**2*e**5 + 2*a*b*d*e**4 - 10*a*c*d**2*e**3 - 5*b**2*d**2*e*
*3 + 18*b*c*d**3*e**2 - 13*c**2*d**4*e) + x*(5*a**2*d*e**4 - 2*a*b*d**2*e**
3 - 6*a*c*d**3*e**2 - 3*b**2*d**3*e**2 + 14*b*c*d**4*e - 11*c**2*d**5))/(8*
d**4*e**4 + 16*d**3*e**5*x**2 + 8*d**2*e**6*x**4)
```

Giac [A]
time = 4.73, size = 244, normalized size = 1.21

$$\frac{1}{3}\left(c^2x^3e^6-9\,c^2dxe^5+6\,bcxe^6\right)e^{(-9)}+\frac{\left(35\,c^2d^4-30\,bcd^3e+3\,b^2d^2e^2+6\,acd^2e^2+2\,abde^3+3\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(13\,c^2d^4x^3e-18\,bcd^3x^3e^2+11\,c^2d^5x+5\,b^2d^2x^3e^3+10\,acd^2x^3e^3-14\,bcd^4xe-2\,abdx^3e^4+3\,b^2d^3xe^2+6\,acd^3xe^2-3\,a^2x^3e^5+2\,abd^2xe^3-5\,a^2dxe^4\right)e^{(-4)}}{8\,\left(x^2e+d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/3*(c^2*x^3*e^6 - 9*c^2*d*x*e^5 + 6*b*c*x*e^6)*e^(-9) + 1/8*(35*c^2*d^4 -
30*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 2*a*b*d*e^3 + 3*a^2*e^4)*arc
tan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(5/2) - 1/8*(13*c^2*d^4*x^3*e - 18*b*c*d^
3*x^3*e^2 + 11*c^2*d^5*x + 5*b^2*d^2*x^3*e^3 + 10*a*c*d^2*x^3*e^3 - 14*b*c*
d^4*x*e - 2*a*b*d*x^3*e^4 + 3*b^2*d^3*x*e^2 + 6*a*c*d^3*x*e^2 - 3*a^2*x^3*e
^5 + 2*a*b*d^2*x*e^3 - 5*a^2*d*x*e^4)*e^(-4)/((x^2*e + d)^2*d^2)
```

Mupad [B]
time = 0.12, size = 257, normalized size = 1.28

$$\frac{c^2\,x^3}{3\,e^3}-x\,\left(\frac{3\,c^2\,d}{e^4}-\frac{2\,b\,c}{e^3}\right)-\frac{\frac{x\,\left(-5\,a^2\,e^4+2\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2-14\,b\,c\,d^3\,e+11\,c^2\,d^4\right)}{8\,d}-\frac{x^3\,\left(3\,a^2\,e^5+2\,a\,b\,d\,e^4-10\,a\,c\,d^2\,e^3-5\,b^2\,d^2\,e^3+18\,b\,c\,d^3\,e^2-13\,c^2\,d^4\,e\right)}{8\,d^2}}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,a^2\,e^4+2\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2-30\,b\,c\,d^3\,e+35\,c^2\,d^4\right)}{8\,d^{5/2}\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^3,x)
```

```
[Out] (c^2*x^3)/(3*e^3) - x*((3*c^2*d)/e^4 - (2*b*c)/e^3) - ((x*(11*c^2*d^4 - 5*a
^2*e^4 + 3*b^2*d^2*e^2 + 2*a*b*d*e^3 - 14*b*c*d^3*e + 6*a*c*d^2*e^2))/(8*d)
 - (x^3*(3*a^2*e^5 - 13*c^2*d^4*e - 5*b^2*d^2*e^3 + 2*a*b*d*e^4 - 10*a*c*d^
2*e^3 + 18*b*c*d^3*e^2))/(8*d^2))/(d^2*e^4 + e^6*x^4 + 2*d*e^5*x^2) + (atan
((e^(1/2)*x)/d^(1/2))*(3*a^2*e^4 + 35*c^2*d^4 + 3*b^2*d^2*e^2 + 2*a*b*d*e^3
 - 30*b*c*d^3*e + 6*a*c*d^2*e^2))/(8*d^(5/2)*e^(9/2))
```

3.259 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4} dx$

Optimal. Leaf size=250

$$\frac{c^2x}{e^4}+\frac{(cd^2-bde+ae^2)^2 x}{6de^4(d+ex^2)^3}-\frac{(19cd^2-7bde-5ae^2)(cd^2-bde+ae^2)x}{24d^2e^4(d+ex^2)^2}+\frac{(29c^2d^4-2cd^2e(11bd-ae)+e^2(b^2}{16d^3e^4(d+ex}$$

```
[Out] c^2*x/e^4+1/6*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)^3-1/24*(-5*a*e^2-7*b*
d*e+19*c*d^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/(e*x^2+d)^2+1/16*(29*c^2*d^4-2*
c*d^2*e*(-a*e+11*b*d)+e^2*(5*a^2*e^2+2*a*b*d*e+b^2*d^2))*x/d^3/e^4/(e*x^2+d
)-1/16*(35*c^2*d^4-2*c*d^2*e*(a*e+5*b*d)-e^2*(5*a^2*e^2+2*a*b*d*e+b^2*d^2))
*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(9/2)
```

Rubi [A]
time = 0.34, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1171, 1828, 396, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(-e^2(5a^2e^2+2abde+b^2d^2)-2cd^2e(ae+5bd)+35c^2d^4)}{16d^{7/2}e^{9/2}}+\frac{x(e^2(5a^2e^2+2abde+b^2d^2)-2cd^2e(11bd-ae)+29c^2d^4)}{16d^3e^4(d+ex^2)}-\frac{x(-5ae^2-7bde+19cd^2)(ae^2-bde+cd^2)}{24d^2e^4(d+ex^2)^2}+\frac{x(ae^2-bde+cd^2)^2}{6de^4(d+ex^2)^3}+\frac{c^2x}{e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]
```

```
[Out] (c^2*x)/e^4 + ((c*d^2 - b*d*e + a*e^2)^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((19*
c*d^2 - 7*b*d*e - 5*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(24*d^2*e^4*(d + e*x^
2)^2) + ((29*c^2*d^4 - 2*c*d^2*e*(11*b*d - a*e) + e^2*(b^2*d^2 + 2*a*b*d*e
+ 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2)) - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d
 + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(16*d^(7/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^4}\,dx = \frac{(cd^2-bde+ae^2)^2 x}{6de^4(d+ex^2)^3} - \frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-5a^2e^2\right)}{e^4} - \frac{6d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3} + \frac{6c\cdots}{}}{(d+ex^2)^3}}{6d}$$

$$= \frac{(cd^2-bde+ae^2)^2 x}{6de^4(d+ex^2)^3} - \frac{(19cd^2-7bde-5ae^2)(cd^2-bde+ae^2)x}{24d^2e^4(d+ex^2)^2} + \frac{\int \frac{3\left(5c^2d^4-2cd^2e(3\cdots\right.}{}}{}$$

$$= \frac{(cd^2-bde+ae^2)^2 x}{6de^4(d+ex^2)^3} - \frac{(19cd^2-7bde-5ae^2)(cd^2-bde+ae^2)x}{24d^2e^4(d+ex^2)^2} + \frac{(29c^2d^4-2cd\cdots}{}$$

$$= \frac{c^2x}{e^4} + \frac{(cd^2-bde+ae^2)^2 x}{6de^4(d+ex^2)^3} - \frac{(19cd^2-7bde-5ae^2)(cd^2-bde+ae^2)x}{24d^2e^4(d+ex^2)^2} + \frac{(29c^2d^4\cdots}{}$$

$$= \frac{c^2x}{e^4} + \frac{(cd^2-bde+ae^2)^2 x}{6de^4(d+ex^2)^3} - \frac{(19cd^2-7bde-5ae^2)(cd^2-bde+ae^2)x}{24d^2e^4(d+ex^2)^2} + \frac{(29c^2d^4\cdots}{}$$

Mathematica [A]

time = 0.10, size = 267, normalized size = 1.07

$$\frac{c^2x}{e^4} + \frac{(cd^2+e(-bd+ae))^2 x}{6de^4(d+ex^2)^3} - \frac{(19c^2d^4+2cd^2e(-13bd+7ae)+e^2(7b^2d^2-2abde-5a^2e^2))x}{24d^2e^4(d+ex^2)^2} + \frac{(29c^2d^4+2cd^2e(-11bd+ae)+e^2(b^2d^2+2abde+5a^2e^2))x}{16d^3e^4(d+ex^2)} - \frac{(35c^2d^4-2cd^2e(5bd+ae)-e^2(b^2d^2+2abde+5a^2e^2))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{16d^{7/2}e^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x]

[Out] (c^2*x)/e^4 + ((c*d^2 + e*(-(b*d) + a*e))^2*x)/(6*d*e^4*(d + e*x^2)^3) - ((
19*c^2*d^4 + 2*c*d^2*e*(-13*b*d + 7*a*e) + e^2*(7*b^2*d^2 - 2*a*b*d*e - 5*a
^2*e^2))*x)/(24*d^2*e^4*(d + e*x^2)^2) + ((29*c^2*d^4 + 2*c*d^2*e*(-11*b*d
+ a*e) + e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^2*e^2))*x)/(16*d^3*e^4*(d + e*x^2))
 - ((35*c^2*d^4 - 2*c*d^2*e*(5*b*d + a*e) - e^2*(b^2*d^2 + 2*a*b*d*e + 5*a^
2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(16*d^(7/2)*e^(9/2))

Maple [A]
time = 0.14, size = 285, normalized size = 1.14

method	result
default	$\frac{c^2 x}{e^4} + \frac{\frac{\frac{e^2\left(5a^2e^4+2abd\,e^3+2ac\,d^2e^2+b^2d^2e^2-22bc\,d^3e+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4+2abd\,e^3-2ac\,d^2e^2-b^2d^2e^2-10bc\,d^3e+17c^2d^4\right)x^3}{6d^2}+\frac{(11a^2e^4-2\cdots}{}}{\left(e\,x^2+d\right)^3}}{e^4}$
risch	$\frac{c^2 x}{e^4} + \frac{\frac{e^2\left(5a^2e^4+2abd\,e^3+2ac\,d^2e^2+b^2d^2e^2-22bc\,d^3e+29c^2d^4\right)x^5}{16d^3}+\frac{e\left(5a^2e^4+2abd\,e^3-2ac\,d^2e^2-b^2d^2e^2-10bc\,d^3e+17c^2d^4\right)x^3}{6d^2}+\frac{(11a^2e^4-2\cdots}{}}{e^4\left(e\,x^2+d\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x,method=_RETURNVERBOSE)

[Out] c^2*x/e^4+1/e^4*((1/16*e^2*(5*a^2*e^4+2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2
-22*b*c*d^3*e+29*c^2*d^4)/d^3*x^5+1/6*e*(5*a^2*e^4+2*a*b*d*e^3-2*a*c*d^2*e^
2-b^2*d^2*e^2-10*b*c*d^3*e+17*c^2*d^4)/d^2*x^3+1/16*(11*a^2*e^4-2*a*b*d*e^3
-2*a*c*d^2*e^2-b^2*d^2*e^2-10*b*c*d^3*e+19*c^2*d^4)/d*x)/(e*x^2+d)^3+1/16*(
5*a^2*e^4+2*a*b*d*e^3+2*a*c*d^2*e^2+b^2*d^2*e^2+10*b*c*d^3*e-35*c^2*d^4)/d^
3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))

Maxima [A]
time = 0.53, size = 288, normalized size = 1.15

$$c^2xe^{(-4)} + \frac{3\,(29\,c^2d^4e^2 - 22\,bcd^3e^3 + 2\,abde^5 + (b^2e^4 + 2\,ace^4)d^2 + 5\,a^2e^6)x^5 + 8\,(17\,c^2d^5e - 10\,bcd^4e^2 + 2\,abd^2e^4 - (b^2e^3 + 2\,ace^3)d^3 + 5\,a^2de^5)x^3 + 3\,(19\,c^2d^6 - 10\,bcd^5e - 2\,abd^3e^3 - (b^2e^2 + 2\,ace^2)d^4 + 11\,a^2d^2e^4)x}{48\,(d^3x^6e^7 + 3\,d^4x^4e^6 + 3\,d^5x^2e^5 + d^6e^4)} - \frac{(35\,c^2d^4 - 10\,bcd^3e - 2\,abde^3 - (b^2e^2 + 2\,ace^2)d^2 - 5\,a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{16\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="maxima")

[Out] c^2*x*e^(-4) + 1/48*(3*(29*c^2*d^4*e^2 - 22*b*c*d^3*e^3 + 2*a*b*d*e^5 + (b^
2*e^4 + 2*a*c*e^4)*d^2 + 5*a^2*e^6)*x^5 + 8*(17*c^2*d^5*e - 10*b*c*d^4*e^2
+ 2*a*b*d^2*e^4 - (b^2*e^3 + 2*a*c*e^3)*d^3 + 5*a^2*d*e^5)*x^3 + 3*(19*c^2*
d^6 - 10*b*c*d^5*e - 2*a*b*d^3*e^3 - (b^2*e^2 + 2*a*c*e^2)*d^4 + 11*a^2*d^2
*e^4)*x)/(d^3*x^6*e^7 + 3*d^4*x^4*e^6 + 3*d^5*x^2*e^5 + d^6*e^4) - 1/16*(35
*c^2*d^4 - 10*b*c*d^3*e - 2*a*b*d*e^3 - (b^2*e^2 + 2*a*c*e^2)*d^2 - 5*a^2*e
^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(7/2)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(233) = 466$.
time = 0.35, size = 1021, normalized size = 4.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(210*c^2*d^7*x*e + 30*a^2*d*x^5*e^7 + 3*(35*c^2*d^7 - 5*a^2*x^6*e^7 -
 (2*a*b*d*x^6 + 15*a^2*d*x^4)*e^6 - ((b^2 + 2*a*c)*d^2*x^6 + 6*a*b*d^2*x^4
+ 15*a^2*d^2*x^2)*e^5 - (10*b*c*d^3*x^6 + 3*(b^2 + 2*a*c)*d^3*x^4 + 6*a*b*d
^3*x^2 + 5*a^2*d^3)*e^4 + (35*c^2*d^4*x^6 - 30*b*c*d^4*x^4 - 3*(b^2 + 2*a*c
)*d^4*x^2 - 2*a*b*d^4)*e^3 + (105*c^2*d^5*x^4 - 30*b*c*d^5*x^2 - (b^2 + 2*a
*c)*d^5)*e^2 + 5*(21*c^2*d^6*x^2 - 2*b*c*d^6)*e)*sqrt(-d*e)*log((x^2*e - 2*
sqrt(-d*e)*x - d)/(x^2*e + d)) + 4*(3*a*b*d^2*x^5 + 20*a^2*d^2*x^3)*e^6 + 2
*(3*(b^2 + 2*a*c)*d^3*x^5 + 16*a*b*d^3*x^3 + 33*a^2*d^3*x)*e^5 + 4*(24*c^2*
d^4*x^7 - 33*b*c*d^4*x^5 - 4*(b^2 + 2*a*c)*d^4*x^3 - 3*a*b*d^4*x)*e^4 + 2*(
231*c^2*d^5*x^5 - 80*b*c*d^5*x^3 - 3*(b^2 + 2*a*c)*d^5*x)*e^3 + 20*(28*c^2*
d^6*x^3 - 3*b*c*d^6*x)*e^2)/(d^4*x^6*e^8 + 3*d^5*x^4*e^7 + 3*d^6*x^2*e^6 +
d^7*e^5), 1/48*(105*c^2*d^7*x*e + 15*a^2*d*x^5*e^7 - 3*(35*c^2*d^7 - 5*a^2*
x^6*e^7 - (2*a*b*d*x^6 + 15*a^2*d*x^4)*e^6 - ((b^2 + 2*a*c)*d^2*x^6 + 6*a*b
*d^2*x^4 + 15*a^2*d^2*x^2)*e^5 - (10*b*c*d^3*x^6 + 3*(b^2 + 2*a*c)*d^3*x^4
+ 6*a*b*d^3*x^2 + 5*a^2*d^3)*e^4 + (35*c^2*d^4*x^6 - 30*b*c*d^4*x^4 - 3*(b^
2 + 2*a*c)*d^4*x^2 - 2*a*b*d^4)*e^3 + (105*c^2*d^5*x^4 - 30*b*c*d^5*x^2 - (
b^2 + 2*a*c)*d^5)*e^2 + 5*(21*c^2*d^6*x^2 - 2*b*c*d^6)*e)*sqrt(d)*arctan(x*
e^(1/2)/sqrt(d))*e^(1/2) + 2*(3*a*b*d^2*x^5 + 20*a^2*d^2*x^3)*e^6 + (3*(b^2
 + 2*a*c)*d^3*x^5 + 16*a*b*d^3*x^3 + 33*a^2*d^3*x)*e^5 + 2*(24*c^2*d^4*x^7
- 33*b*c*d^4*x^5 - 4*(b^2 + 2*a*c)*d^4*x^3 - 3*a*b*d^4*x)*e^4 + (231*c^2*d^
5*x^5 - 80*b*c*d^5*x^3 - 3*(b^2 + 2*a*c)*d^5*x)*e^3 + 10*(28*c^2*d^6*x^3 -
3*b*c*d^6*x)*e^2)/(d^4*x^6*e^8 + 3*d^5*x^4*e^7 + 3*d^6*x^2*e^6 + d^7*e^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.42, size = 296, normalized size = 1.18

$$c^2 x e^{(-4)} - \frac{\left(35\,c^2 d^4 - 10\,bcd^3 e - b^2 d^2 e^2 - 2\,acd^2 e^2 - 2\,abde^3 - 5\,a^2 e^4\right) \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{9}{2}\right)}}{16\,d^{\frac{7}{2}}} + \frac{\left(87\,c^2 d^4 x^5 e^2 - 66\,bcd^3 x^5 e^3 + 136\,c^2 d^5 x^3 e + 3\,b^2 d^2 x^5 e^4 + 6\,acd^2 x^5 e^4 - 80\,bcd^4 x^3 e^2 + 57\,c^2 d^6 x + 6\,abdx^5 e^5 - 8\,b^2 d^3 x^3 e^3 - 16\,acd^3 x^3 e^3 - 30\,bcd^5 xe + 15\,a^2 x^5 e^6 + 16\,abd^2 x^3 e^4 - 3\,b^2 d^4 xe^2 - 6\,acd^4 xe^2 + 40\,a^2 dx^3 e^5 - 6\,abd^3 xe^3 + 33\,a^2 d^2 xe^4\right) e^{(-4)}}{48\,\left(x^2 e + d\right)^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^4,x, algorithm="giac")
```

```
[Out] c^2*x*e^(-4) - 1/16*(35*c^2*d^4 - 10*b*c*d^3*e - b^2*d^2*e^2 - 2*a*c*d^2*e^
2 - 2*a*b*d*e^3 - 5*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(7/2) + 1
/48*(87*c^2*d^4*x^5*e^2 - 66*b*c*d^3*x^5*e^3 + 136*c^2*d^5*x^3*e + 3*b^2*d^
2*x^5*e^4 + 6*a*c*d^2*x^5*e^4 - 80*b*c*d^4*x^3*e^2 + 57*c^2*d^6*x + 6*a*b*d
*x^5*e^5 - 8*b^2*d^3*x^3*e^3 - 16*a*c*d^3*x^3*e^3 - 30*b*c*d^5*x*e + 15*a^2
*x^5*e^6 + 16*a*b*d^2*x^3*e^4 - 3*b^2*d^4*x*e^2 - 6*a*c*d^4*x*e^2 + 40*a^2*
d*x^3*e^5 - 6*a*b*d^3*x*e^3 + 33*a^2*d^2*x*e^4)*e^(-4)/((x^2*e + d)^3*d^3)
```

Mupad [B]
time = 4.60, size = 308, normalized size = 1.23

$$\frac{\frac{x^5\,\left(5\,a^2\,e^6+2\,a\,b\,d\,e^5+2\,a\,c\,d^2\,e^4+b^2\,d^2\,e^4-22\,b\,c\,d^3\,e^3+29\,c^2\,d^4\,e^2\right)}{16\,d^3}-\frac{x\,\left(-11\,a^2\,e^4+2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-19\,c^2\,d^4\right)}{16\,d}+\frac{x^3\,\left(5\,a^2\,e^5+2\,a\,b\,d\,e^4-2\,a\,c\,d^2\,e^3-b^2\,d^2\,e^3-10\,b\,c\,d^3\,e^2+17\,c^2\,d^4\,e\right)}{6\,d^2}}{d^3\,e^4+3\,d^2\,e^5\,x^2+3\,d\,e^6\,x^4+e^7\,x^6}+\frac{c^2\,x}{e^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(5\,a^2\,e^4+2\,a\,b\,d\,e^3+2\,a\,c\,d^2\,e^2+b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-35\,c^2\,d^4\right)}{16\,d^{7/2}\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^4,x)
```

```
[Out] ((x^5*(5*a^2*e^6 + b^2*d^2*e^4 + 29*c^2*d^4*e^2 + 2*a*b*d*e^5 + 2*a*c*d^2*e
^4 - 22*b*c*d^3*e^3))/(16*d^3) - (x*(b^2*d^2*e^2 - 19*c^2*d^4 - 11*a^2*e^4
+ 2*a*b*d*e^3 + 10*b*c*d^3*e + 2*a*c*d^2*e^2))/(16*d) + (x^3*(5*a^2*e^5 + 1
7*c^2*d^4*e - b^2*d^2*e^3 + 2*a*b*d*e^4 - 2*a*c*d^2*e^3 - 10*b*c*d^3*e^2))/
(6*d^2))/(d^3*e^4 + e^7*x^6 + 3*d*e^6*x^4 + 3*d^2*e^5*x^2) + (c^2*x)/e^4 +
(atan((e^(1/2)*x)/d^(1/2))*(5*a^2*e^4 - 35*c^2*d^4 + b^2*d^2*e^2 + 2*a*b*d*
e^3 + 10*b*c*d^3*e + 2*a*c*d^2*e^2))/(16*d^(7/2)*e^(9/2))
```

3.260 $\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx$

Optimal. Leaf size=317

$$\frac{(cd^2-bde+ae^2)^2x}{8de^4(d+ex^2)^4}-\frac{(25cd^2-9bde-7ae^2)(cd^2-bde+ae^2)x}{48d^2e^4(d+ex^2)^3}+\frac{(163c^2d^4-2cd^2e(59bd-3ae)+e^2(3b^2d^2}{192d^3e^4(d+ex^2)^2}$$

```
[Out] 1/8*(a*e^2-b*d*e+c*d^2)^2*x/d/e^4/(e*x^2+d)^4-1/48*(-7*a*e^2-9*b*d*e+25*c*d
^2)*(a*e^2-b*d*e+c*d^2)*x/d^2/e^4/(e*x^2+d)^3+1/192*(163*c^2*d^4-2*c*d^2*e*
(-3*a*e+59*b*d)+e^2*(35*a^2*e^2+10*a*b*d*e+3*b^2*d^2))*x/d^3/e^4/(e*x^2+d)^
2-1/128*(93*c^2*d^4-2*c*d^2*e*(3*a*e+5*b*d)-e^2*(35*a^2*e^2+10*a*b*d*e+3*b^
2*d^2))*x/d^4/e^4/(e*x^2+d)+1/128*(35*c^2*d^4+2*c*d^2*e*(3*a*e+5*b*d)+e^2*(
35*a^2*e^2+10*a*b*d*e+3*b^2*d^2))*arctan(x*e^(1/2)/d^(1/2))/d^(9/2)/e^(9/2)
```

Rubi [A]
time = 0.40, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1171, 1828, 393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e^2(35a^2e^2+10abde+3b^2d^2)+2cd^2e(3ae+5bd)+35c^2d^4\right)}{128d^{9/2}e^{9/2}}-\frac{x(-e^2(35a^2e^2+10abde+3b^2d^2)-2cd^2e(3ae+5bd)+93c^2d^4)}{128d^4e^4(d+ex^2)}+\frac{x(e^2(35a^2e^2+10abde+3b^2d^2)-2cd^2e(59bd-3ae)+163c^2d^4)}{192d^3e^4(d+ex^2)^2}+\frac{x(ae^2-bde+cd^2)^2}{8de^4(d+ex^2)^4}-\frac{x(-7ae^2-9bde+25cd^2)(ae^2-bde+cd^2)}{48d^2e^4(d+ex^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)^2*x)/(8*d*e^4*(d + e*x^2)^4) - ((25*c*d^2 - 9*b*d*
e - 7*a*e^2)*(c*d^2 - b*d*e + a*e^2)*x)/(48*d^2*e^4*(d + e*x^2)^3) + ((163*
c^2*d^4 - 2*c*d^2*e*(59*b*d - 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2
*e^2))*x)/(192*d^3*e^4*(d + e*x^2)^2) - ((93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3
*a*e) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(128*d^4*e^4*(d + e*x
^2)) + ((35*c^2*d^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d
*e + 35*a^2*e^2))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(128*d^(9/2)*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^2}{(d+ex^2)^5}\,dx = \frac{(cd^2-bde+ae^2)^2 x}{8de^4(d+ex^2)^4} - \frac{\int \frac{\frac{c^2d^4-2cd^2e(bd-ae)+e^2\left(b^2d^2-2abde-7a^2e^2\right)}{e^4} - \frac{8d\left(c^2d^2+b^2e^2-2ce(bd-ae)\right)x^2}{e^3} + \ldots}{(d+ex^2)^4}}{8d}$$

$$= \frac{(cd^2-bde+ae^2)^2 x}{8de^4(d+ex^2)^4} - \frac{(25cd^2-9bde-7ae^2)(cd^2-bde+ae^2)x}{48d^2e^4(d+ex^2)^3} + \frac{\int \frac{19c^2d^4-2cd^2e(\ldots}{\ldots}}{\ldots}$$

$$= \frac{(cd^2-bde+ae^2)^2 x}{8de^4(d+ex^2)^4} - \frac{(25cd^2-9bde-7ae^2)(cd^2-bde+ae^2)x}{48d^2e^4(d+ex^2)^3} + \frac{(163c^2d^4-2\ldots}{\ldots}$$

$$= \frac{(cd^2-bde+ae^2)^2 x}{8de^4(d+ex^2)^4} - \frac{(25cd^2-9bde-7ae^2)(cd^2-bde+ae^2)x}{48d^2e^4(d+ex^2)^3} + \frac{(163c^2d^4-2\ldots}{\ldots}$$

$$= \frac{(cd^2-bde+ae^2)^2 x}{8de^4(d+ex^2)^4} - \frac{(25cd^2-9bde-7ae^2)(cd^2-bde+ae^2)x}{48d^2e^4(d+ex^2)^3} + \frac{(163c^2d^4-2\ldots}{\ldots}$$

Mathematica [A]
time = 0.14, size = 345, normalized size = 1.09

$$\frac{\frac{48d^{7/2}\sqrt{e}\,(cd^2+e(-bd+ae))^2x}{(d+ex^2)^4} - \frac{8d^{5/2}\sqrt{e}\,(25c^2d^4+2cd^2e(-17bd+9ae)+e^2(9b^2d^2-2abde-7a^2e^2))x}{(d+ex^2)^3} + \frac{2d^{3/2}\sqrt{e}\,(163c^2d^4+2cd^2e(-59bd+3ae)+e^2(3b^2d^2+10abde+35a^2e^2))x}{(d+ex^2)^2} - \frac{3\sqrt{d}\,\sqrt{e}\,(93c^2d^4-2cd^2e(5bd+3ae)-e^2(3b^2d^2+10abde+35a^2e^2))x}{d+ex^2} + 3(35c^2d^4+2cd^2e(5bd+3ae)+e^2(3b^2d^2+10abde+35a^2e^2))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{384d^{9/2}e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x]
```

```
[Out] ((48*d^(7/2)*Sqrt[e]*(c*d^2 + e*(-(b*d) + a*e))^2*x)/(d + e*x^2)^4 - (8*d^(
5/2)*Sqrt[e]*(25*c^2*d^4 + 2*c*d^2*e*(-17*b*d + 9*a*e) + e^2*(9*b^2*d^2 - 2
*a*b*d*e - 7*a^2*e^2))*x)/(d + e*x^2)^3 + (2*d^(3/2)*Sqrt[e]*(163*c^2*d^4 +
 2*c*d^2*e*(-59*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x
)/(d + e*x^2)^2 - (3*Sqrt[d]*Sqrt[e]*(93*c^2*d^4 - 2*c*d^2*e*(5*b*d + 3*a*e
) - e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))*x)/(d + e*x^2) + 3*(35*c^2*d
^4 + 2*c*d^2*e*(5*b*d + 3*a*e) + e^2*(3*b^2*d^2 + 10*a*b*d*e + 35*a^2*e^2))
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(384*d^(9/2)*e^(9/2))
```

Maple [A]
time = 0.15, size = 347, normalized size = 1.09

method	result
default	$\frac{\frac{\left(35 a^2 e^4+10 a b d e^3+6 a c d^2 e^2+3 b^2 d^2 e^2+10 b c d^3 e-93 c^2 d^4\right) x^7}{128 d^4 e}+\frac{\left(385 a^2 e^4+110 a b d e^3+66 a c d^2 e^2+33 b^2 d^2 e^2-146 b c d^3 e-511 c^2 d^4\right) x^5}{384 d^3 e^2}+\frac{(511 a^2 e^4+1 \cdots}{\cdots}}{\left(e x^2+d\right)^4}$
risch	$\frac{\frac{\left(35 a^2 e^4+10 a b d e^3+6 a c d^2 e^2+3 b^2 d^2 e^2+10 b c d^3 e-93 c^2 d^4\right) x^7}{128 d^4 e}+\frac{\left(385 a^2 e^4+110 a b d e^3+66 a c d^2 e^2+33 b^2 d^2 e^2-146 b c d^3 e-511 c^2 d^4\right) x^5}{384 d^3 e^2}+\frac{(511 a^2 e^4+1 \cdots}{\cdots}}{\left(e x^2+d\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (1/128*(35*a^2*e^4+10*a*b*d*e^3+6*a*c*d^2*e^2+3*b^2*d^2*e^2+10*b*c*d^3*e-93
*c^2*d^4)/d^4/e*x^7+1/384*(385*a^2*e^4+110*a*b*d*e^3+66*a*c*d^2*e^2+33*b^2*
d^2*e^2-146*b*c*d^3*e-511*c^2*d^4)/d^3/e^2*x^5+1/384*(511*a^2*e^4+146*a*b*d
*e^3-66*a*c*d^2*e^2-33*b^2*d^2*e^2-110*b*c*d^3*e-385*c^2*d^4)/d^2/e^3*x^3+1
/128*(93*a^2*e^4-10*a*b*d*e^3-6*a*c*d^2*e^2-3*b^2*d^2*e^2-10*b*c*d^3*e-35*c
^2*d^4)/d/e^4*x)/(e*x^2+d)^4+1/128*(35*a^2*e^4+10*a*b*d*e^3+6*a*c*d^2*e^2+3
*b^2*d^2*e^2+10*b*c*d^3*e+35*c^2*d^4)/d^4/e^4/(d*e)^(1/2)*arctan(e*x/(d*e)^
(1/2))
```

Maxima [A]
time = 0.52, size = 352, normalized size = 1.11

$$-\frac{3\,(93\,c^2d^4e^3-10\,bcd^3e^4-10\,abde^6-3\,(b^2e^5+2\,ace^5)d^2-35\,a^2e^7)x^7+(511\,c^2d^5e^2+146\,bcd^4e^3-110\,abd^2e^5-33\,(b^2e^4+2\,ace^4)d^3-385\,a^2de^6)x^5+(385\,c^2d^6e+110\,bcd^5e^2-146\,abd^3e^4+33\,(b^2e^3+2\,ace^3)d^4-511\,a^2d^2e^5)x^3+3\,(35\,c^2d^7+10\,bcd^6e+10\,abd^4e^3+3\,(b^2e^2+2\,ace^2)d^5-93\,a^2d^3e^4)x}{384\,(d^4x^8e^8+4\,d^5x^6e^7+6\,d^6x^4e^6+4\,d^7x^2e^5+d^8e^4)}+\frac{(35\,c^2d^4+10\,bcd^3e+10\,abde^3+3\,(b^2e^2+2\,ace^2)d^2+35\,a^2e^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{128\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="maxima")
```

```
[Out] -1/384*(3*(93*c^2*d^4*e^3 - 10*b*c*d^3*e^4 - 10*a*b*d*e^6 - 3*(b^2*e^5 + 2*
a*c*e^5)*d^2 - 35*a^2*e^7)*x^7 + (511*c^2*d^5*e^2 + 146*b*c*d^4*e^3 - 110*a
```

```
*b*d^2*e^5 - 33*(b^2*e^4 + 2*a*c*e^4)*d^3 - 385*a^2*d*e^6)*x^5 + (385*c^2*d
^6*e + 110*b*c*d^5*e^2 - 146*a*b*d^3*e^4 + 33*(b^2*e^3 + 2*a*c*e^3)*d^4 - 5
11*a^2*d^2*e^5)*x^3 + 3*(35*c^2*d^7 + 10*b*c*d^6*e + 10*a*b*d^4*e^3 + 3*(b^
2*e^2 + 2*a*c*e^2)*d^5 - 93*a^2*d^3*e^4)*x)/(d^4*x^8*e^8 + 4*d^5*x^6*e^7 +
6*d^6*x^4*e^6 + 4*d^7*x^2*e^5 + d^8*e^4) + 1/128*(35*c^2*d^4 + 10*b*c*d^3*e
 + 10*a*b*d*e^3 + 3*(b^2*e^2 + 2*a*c*e^2)*d^2 + 35*a^2*e^4)*arctan(x*e^(1/2
)/sqrt(d))*e^(-9/2)/d^(9/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(300) = 600$.

time = 0.36, size = 1267, normalized size = 4.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="fricas")
```

```
[Out] [-1/768*(210*c^2*d^8*x*e - 210*a^2*d*x^7*e^8 + 3*(35*a^2*x^8*e^8 + 35*c^2*d
^8 + 10*(a*b*d*x^8 + 14*a^2*d*x^6)*e^7 + (3*(b^2 + 2*a*c)*d^2*x^8 + 40*a*b*
d^2*x^6 + 210*a^2*d^2*x^4)*e^6 + 2*(5*b*c*d^3*x^8 + 6*(b^2 + 2*a*c)*d^3*x^6
 + 30*a*b*d^3*x^4 + 70*a^2*d^3*x^2)*e^5 + (35*c^2*d^4*x^8 + 40*b*c*d^4*x^6
+ 18*(b^2 + 2*a*c)*d^4*x^4 + 40*a*b*d^4*x^2 + 35*a^2*d^4)*e^4 + 2*(70*c^2*d
^5*x^6 + 30*b*c*d^5*x^4 + 6*(b^2 + 2*a*c)*d^5*x^2 + 5*a*b*d^5)*e^3 + (210*c
^2*d^6*x^4 + 40*b*c*d^6*x^2 + 3*(b^2 + 2*a*c)*d^6)*e^2 + 10*(14*c^2*d^7*x^2
 + b*c*d^7)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) - 1
0*(6*a*b*d^2*x^7 + 77*a^2*d^2*x^5)*e^7 - 2*(9*(b^2 + 2*a*c)*d^3*x^7 + 110*a
*b*d^3*x^5 + 511*a^2*d^3*x^3)*e^6 - 2*(30*b*c*d^4*x^7 + 33*(b^2 + 2*a*c)*d^
4*x^5 + 146*a*b*d^4*x^3 + 279*a^2*d^4*x)*e^5 + 2*(279*c^2*d^5*x^7 + 146*b*c
*d^5*x^5 + 33*(b^2 + 2*a*c)*d^5*x^3 + 30*a*b*d^5*x)*e^4 + 2*(511*c^2*d^6*x^
5 + 110*b*c*d^6*x^3 + 9*(b^2 + 2*a*c)*d^6*x)*e^3 + 10*(77*c^2*d^7*x^3 + 6*b
*c*d^7*x)*e^2)/(d^5*x^8*e^9 + 4*d^6*x^6*e^8 + 6*d^7*x^4*e^7 + 4*d^8*x^2*e^6
 + d^9*e^5), -1/384*(105*c^2*d^8*x*e - 105*a^2*d*x^7*e^8 - 3*(35*a^2*x^8*e^
8 + 35*c^2*d^8 + 10*(a*b*d*x^8 + 14*a^2*d*x^6)*e^7 + (3*(b^2 + 2*a*c)*d^2*x
^8 + 40*a*b*d^2*x^6 + 210*a^2*d^2*x^4)*e^6 + 2*(5*b*c*d^3*x^8 + 6*(b^2 + 2*
a*c)*d^3*x^6 + 30*a*b*d^3*x^4 + 70*a^2*d^3*x^2)*e^5 + (35*c^2*d^4*x^8 + 40*
b*c*d^4*x^6 + 18*(b^2 + 2*a*c)*d^4*x^4 + 40*a*b*d^4*x^2 + 35*a^2*d^4)*e^4 +
 2*(70*c^2*d^5*x^6 + 30*b*c*d^5*x^4 + 6*(b^2 + 2*a*c)*d^5*x^2 + 5*a*b*d^5)*
e^3 + (210*c^2*d^6*x^4 + 40*b*c*d^6*x^2 + 3*(b^2 + 2*a*c)*d^6)*e^2 + 10*(14
*c^2*d^7*x^2 + b*c*d^7)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - 5*(6
*a*b*d^2*x^7 + 77*a^2*d^2*x^5)*e^7 - (9*(b^2 + 2*a*c)*d^3*x^7 + 110*a*b*d^3
*x^5 + 511*a^2*d^3*x^3)*e^6 - (30*b*c*d^4*x^7 + 33*(b^2 + 2*a*c)*d^4*x^5 +
146*a*b*d^4*x^3 + 279*a^2*d^4*x)*e^5 + (279*c^2*d^5*x^7 + 146*b*c*d^5*x^5 +
 33*(b^2 + 2*a*c)*d^5*x^3 + 30*a*b*d^5*x)*e^4 + (511*c^2*d^6*x^5 + 110*b*c*
d^6*x^3 + 9*(b^2 + 2*a*c)*d^6*x)*e^3 + 5*(77*c^2*d^7*x^3 + 6*b*c*d^7*x)*e^2
)/(d^5*x^8*e^9 + 4*d^6*x^6*e^8 + 6*d^7*x^4*e^7 + 4*d^8*x^2*e^6 + d^9*e^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**2/(e*x**2+d)**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.52, size = 364, normalized size = 1.15

$$\frac{\left(35\,c^2d^4+10\,bcd^3e+3\,b^2d^2e^2+6\,acd^2e^2+10\,abde^3+35\,a^2e^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{128\,d^{\frac{9}{2}}}-\frac{\left(279\,c^2d^4x^7e^3-30\,bcd^3x^7e^4+511\,c^2d^5x^5e^2-9\,b^2d^2x^7e^5-18\,acd^2x^7e^5+146\,bcd^4x^5e^3+385\,c^2d^6x^3e-30\,abdx^7e^6-33\,b^2d^3x^5e^4-66\,acd^3x^5e^4+110\,bcd^5x^3e^2+105\,c^2d^7x-105\,a^2x^7e^7-110\,abd^2x^5e^5+33\,b^2d^4x^3e^3+66\,acd^4x^3e^3+30\,bcd^6xe-385\,a^2dx^5e^6-146\,abd^3x^3e^4+9\,b^2d^5xe^2+18\,acd^5xe^2-511\,a^2d^2x^3e^5+30\,abd^4xe^3-279\,a^2d^3xe^4\right)e^{(-4)}}{384\,\left(x^2e+d\right)^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^2/(e*x^2+d)^5,x, algorithm="giac")
```

```
[Out] 1/128*(35*c^2*d^4 + 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 6*a*c*d^2*e^2 + 10*a*b*d
*e^3 + 35*a^2*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/d^(9/2) - 1/384*(279*
c^2*d^4*x^7*e^3 - 30*b*c*d^3*x^7*e^4 + 511*c^2*d^5*x^5*e^2 - 9*b^2*d^2*x^7*
e^5 - 18*a*c*d^2*x^7*e^5 + 146*b*c*d^4*x^5*e^3 + 385*c^2*d^6*x^3*e - 30*a*b
*d*x^7*e^6 - 33*b^2*d^3*x^5*e^4 - 66*a*c*d^3*x^5*e^4 + 110*b*c*d^5*x^3*e^2
+ 105*c^2*d^7*x - 105*a^2*x^7*e^7 - 110*a*b*d^2*x^5*e^5 + 33*b^2*d^4*x^3*e^
3 + 66*a*c*d^4*x^3*e^3 + 30*b*c*d^6*x*e - 385*a^2*d*x^5*e^6 - 146*a*b*d^3*x
^3*e^4 + 9*b^2*d^5*x*e^2 + 18*a*c*d^5*x*e^2 - 511*a^2*d^2*x^3*e^5 + 30*a*b*
d^4*x*e^3 - 279*a^2*d^3*x*e^4)*e^(-4)/((x^2*e + d)^4*d^4)
```

Mupad [B]
time = 4.57, size = 375, normalized size = 1.18

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(35\,a^2\,e^4+10\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e+35\,c^2\,d^4\right)}{128\,d^{9/2}\,e^{9/2}}-\frac{\frac{x\,\left(-93\,a^2\,e^4+10\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e+35\,c^2\,d^4\right)}{128\,d\,e^4}-\frac{x^7\,\left(35\,a^2\,e^4+10\,a\,b\,d\,e^3+6\,a\,c\,d^2\,e^2+3\,b^2\,d^2\,e^2+10\,b\,c\,d^3\,e-93\,c^2\,d^4\right)}{128\,d^4\,e}+\frac{x^3\,\left(-511\,a^2\,e^4-146\,a\,b\,d\,e^3+66\,a\,c\,d^2\,e^2+33\,b^2\,d^2\,e^2+110\,b\,c\,d^3\,e+385\,c^2\,d^4\right)}{384\,d^2\,e^3}-\frac{x^5\,\left(385\,a^2\,e^4+110\,a\,b\,d\,e^3+66\,a\,c\,d^2\,e^2+33\,b^2\,d^2\,e^2-146\,b\,c\,d^3\,e-511\,c^2\,d^4\right)}{384\,d^3\,e^2}}{d^4+4\,d^3\,e\,x^2+6\,d^2\,e^2\,x^4+4\,d\,e^3\,x^6+e^4\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^2/(d + e*x^2)^5,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(35*a^2*e^4 + 35*c^2*d^4 + 3*b^2*d^2*e^2 + 10*a*
b*d*e^3 + 10*b*c*d^3*e + 6*a*c*d^2*e^2))/(128*d^(9/2)*e^(9/2)) - ((x*(35*c^
2*d^4 - 93*a^2*e^4 + 3*b^2*d^2*e^2 + 10*a*b*d*e^3 + 10*b*c*d^3*e + 6*a*c*d^
2*e^2))/(128*d*e^4) - (x^7*(35*a^2*e^4 - 93*c^2*d^4 + 3*b^2*d^2*e^2 + 10*a*
b*d*e^3 + 10*b*c*d^3*e + 6*a*c*d^2*e^2))/(128*d^4*e) + (x^3*(385*c^2*d^4 -
511*a^2*e^4 + 33*b^2*d^2*e^2 - 146*a*b*d*e^3 + 110*b*c*d^3*e + 66*a*c*d^2*e
^2))/(384*d^2*e^3) - (x^5*(385*a^2*e^4 - 511*c^2*d^4 + 33*b^2*d^2*e^2 + 110
*a*b*d*e^3 - 146*b*c*d^3*e + 66*a*c*d^2*e^2))/(384*d^3*e^2))/(d^4 + e^4*x^8
 + 4*d^3*e*x^2 + 4*d*e^3*x^6 + 6*d^2*e^2*x^4)
```

3.261 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=83

$$\frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)}-\frac{(3cd^2-e(bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1171, 396, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(3cd^2-e(ae+bd))}{2d^{3/2}e^{5/2}}+\frac{x(ae^2-bde+cd^2)}{2de^2\,(d+ex^2)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)} - \frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}$$
$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\int \frac{1}{d+ex^2}\,dx}{2de^2}$$
$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Mathematica [A]
time = 0.01, size = 88, normalized size = 1.06

$$\frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)} - \frac{(3cd^2-bde-ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A]
time = 0.10, size = 79, normalized size = 0.95

method	result
default	$\frac{cx}{e^2} + \frac{\frac{(a\,e^2-deb+c\,d^2)x}{2d(e\,x^2+d)} + \frac{(a\,e^2+deb-3c\,d^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}}{e^2}$
risch	$\frac{cx}{e^2} + \frac{(a\,e^2-deb+c\,d^2)x}{2d\,e^2(e\,x^2+d)} - \frac{\ln\left(ex+\sqrt{-de}\right)a}{4\sqrt{-de}\;d} - \frac{\ln\left(ex+\sqrt{-de}\right)b}{4e\sqrt{-de}} + \frac{3d\ln\left(ex+\sqrt{-de}\right)c}{4e^2\sqrt{-de}} + \frac{\ln\left(-ex+\sqrt{-de}\right)a}{4\sqrt{-de}\;d} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*d
^2)/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.51, size = 74, normalized size = 0.89

$$cxe^{(-2)} - \frac{(3\,cd^2 - bde - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{(cd^2 - bde + ae^2)x}{2\,(dx^2e^3 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2 - b*d*e + a*e^2)*x/(d*x^2*e^3 + d^2*e^2)
```

Fricas [A]
time = 0.36, size = 266, normalized size = 3.20

$$\left[\frac{6\,cd^3xe + 2\,adxe^3 + (3\,cd^3 - ax^2e^3 - (bdx^2 + ad)e^2 + (3\,cd^2x^2 - bd^2)e)\sqrt{-de}\,\log\left(\frac{x^2e - 2\,\sqrt{-de}\,x - d}{x^2e + d}\right) + 2\,(2\,cd^2x^3 - bd^2x)e^2}{4\,(d^2x^2e^4 + d^3e^3)}, \frac{3\,cd^3xe + adxe^3 - (3\,cd^3 - ax^2e^3 - (bdx^2 + ad)e^2 + (3\,cd^2x^2 - bd^2)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}} + (2\,cd^2x^3 - bd^2x)e^2}{2\,(d^2x^2e^4 + d^3e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(6*c*d^3*x*e + 2*a*d*x*e^3 + (3*c*d^3 - a*x^2*e^3 - (b*d*x^2 + a*d)*e^
2 + (3*c*d^2*x^2 - b*d^2)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x
^2*e + d)) + 2*(2*c*d^2*x^3 - b*d^2*x)*e^2)/(d^2*x^2*e^4 + d^3*e^3), 1/2*(3
*c*d^3*x*e + a*d*x*e^3 - (3*c*d^3 - a*x^2*e^3 - (b*d*x^2 + a*d)*e^2 + (3*c*
d^2*x^2 - b*d^2)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) + (2*c*d^2*x^
3 - b*d^2*x)*e^2)/(d^2*x^2*e^4 + d^3*e^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(75) = 150$.
time = 0.42, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x(ae^2 - bde + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2 + bde - 3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2 + bde - 3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Giac [A]
time = 3.67, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{(3\,cd^2 - bde - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{(cd^2x - bdxe + axe^2)e^{(-2)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

Mupad [B]
time = 0.00, size = 77, normalized size = 0.93

$$\frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2 + b\,d\,e + a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{2\,d\,\left(e^3\,x^2 + d\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.262 $\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=83

$$\frac{cx}{e^2}+\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)}-\frac{(3cd^2-e(bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1828, 396, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(3cd^2-e(ae+bd))}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\,(d+ex^2)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{a+x^2(b+cx^2)}{(d+ex^2)^2}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}$$

$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\int \frac{1}{d+ex^2}\,dx}{2de^2}$$

$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Mathematica [A]
time = 0.01, size = 88, normalized size = 1.06

$$\frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{2de^2(d+ex^2)} - \frac{(3cd^2-bde-ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2*(b + c*x^2))/(d + e*x^2)^2,x]
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A]
time = 0.13, size = 79, normalized size = 0.95

method	result
default	$\frac{cx}{e^2} + \frac{\frac{(a e^2-deb+c d^2)x}{2d(e x^2+d)}+\frac{(a e^2+deb-3c d^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}}{e^2}$
risch	$\frac{cx}{e^2} + \frac{(a e^2-deb+c d^2)x}{2d e^2(e x^2+d)} - \frac{\ln\left(ex+\sqrt{-de}\right)a}{4\sqrt{-de}\,d} - \frac{\ln\left(ex+\sqrt{-de}\right)b}{4e\sqrt{-de}} + \frac{3d\ln\left(ex+\sqrt{-de}\right)c}{4e^2\sqrt{-de}} + \frac{\ln\left(-ex+\sqrt{-de}\right)a}{4\sqrt{-de}\,d} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*d
^2)/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.50, size = 74, normalized size = 0.89

$$cxe^{(-2)} - \frac{(3\,cd^2 - bde - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{(cd^2 - bde + ae^2)x}{2\,(dx^2e^3 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2 - b*d*e + a*e^2)*x/(d*x^2*e^3 + d^2*e^2)
```

Fricas [A]
time = 0.37, size = 266, normalized size = 3.20

$$\left[\frac{6\,cd^3xe + 2\,adxe^3 + (3\,cd^3 - ax^2e^3 - (bdx^2 + ad)e^2 + (3\,cd^2x^2 - bd^2)e)\sqrt{-de}\,\log\left(\frac{x^2e - 2\sqrt{-de}\,x - d}{x^2e + d}\right) + 2\,(2\,cd^2x^3 - bd^2x)e^2}{4\,(d^2x^2e^4 + d^3e^3)}, \frac{3\,cd^3xe + adxe^3 - (3\,cd^3 - ax^2e^3 - (bdx^2 + ad)e^2 + (3\,cd^2x^2 - bd^2)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}} + (2\,cd^2x^3 - bd^2x)e^2}{2\,(d^2x^2e^4 + d^3e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(6*c*d^3*x*e + 2*a*d*x*e^3 + (3*c*d^3 - a*x^2*e^3 - (b*d*x^2 + a*d)*e^
2 + (3*c*d^2*x^2 - b*d^2)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x
^2*e + d)) + 2*(2*c*d^2*x^3 - b*d^2*x)*e^2)/(d^2*x^2*e^4 + d^3*e^3), 1/2*(3
*c*d^3*x*e + a*d*x*e^3 - (3*c*d^3 - a*x^2*e^3 - (b*d*x^2 + a*d)*e^2 + (3*c*
d^2*x^2 - b*d^2)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) + (2*c*d^2*x^
3 - b*d^2*x)*e^2)/(d^2*x^2*e^4 + d^3*e^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(75) = 150$.
time = 0.43, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x(ae^2 - bde + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2 + bde - 3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2 + bde - 3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x**2*(c*x**2+b))/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Giac [A]
time = 4.03, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{(3\,cd^2 - bde - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{(cd^2x - bdxe + axe^2)e^{(-2)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x^2*(c*x^2+b))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

Mupad [B]
time = 0.11, size = 77, normalized size = 0.93

$$\frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2 + b\,d\,e + a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{2\,d\,\left(e^3\,x^2 + d\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2*(b + c*x^2))/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.263 $\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=459

$$\frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))\,x}{c^3}+\frac{e^3(4cd-be)x^3}{3c^2}+\frac{e^4x^5}{5c}+\frac{\Big(e(2cd-be)\left(2c^2d^2+b^2e^2-2ce(bd+ae)\right)+\frac{2c}{\cdots}}{\cdots}$$

```
[Out] e^2*(6*c^2*d^2+b^2*e^2-c*e*(a*e+4*b*d))*x/c^3+1/3*e^3*(-b*e+4*c*d)*x^3/c^2+
1/5*e^4*x^5/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e
*(-b*e+2*c*d)*(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))+(2*c^4*d^4+b^4*e^4-4*b^2*
c*e^3*(a*e+b*d)-4*c^3*d^2*e*(3*a*e+b*d)+2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*
d^2))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e*(-b*e+2*c*d)*(2*c
^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))+(-2*c^4*d^4-b^4*e^4+4*b^2*c*e^3*(a*e+b*d)+4
*c^3*d^2*e*(3*a*e+b*d)-2*c^2*e^2*(a^2*e^2+6*a*b*d*e+3*b^2*d^2))/(-4*a*c+b^2
)^(1/2))/c^(7/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 1.00, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1184, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}+e(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)\right)}{\sqrt{2}\,c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e(2cd-be)\left(-2ce(ae+bd)+b^2e^2+2c^2d^2\right)-\frac{2c^2e^2\left(a^2e^2+6abde+3b^2d^2\right)-4b^2ce^3(ae+bd)-4c^3d^2e(3ae+bd)+b^4e^4+2c^4d^4}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\,c^{7/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{e^2x\left(-ce(ae+4bd)+b^2e^2+6c^2d^2\right)}{c^3}+\frac{e^3x^3(4cd-be)}{3c^2}+\frac{e^4x^5}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*
x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2*e^2 - 2
*c*e*(b*d + a*e)) + (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e) - 4*c^3*
d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*c^(7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e)*(2*c^2*d^2 + b^2
*e^2 - 2*c*e*(b*d + a*e)) - (2*c^4*d^4 + b^4*e^4 - 4*b^2*c*e^3*(b*d + a*e)
- 4*c^3*d^2*e*(b*d + 3*a*e) + 2*c^2*e^2*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{(d+ex^2)^4}{a+bx^2+cx^4}\,dx = \int \left(\frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))}{c^3} + \frac{e^3(4cd-be)x^2}{c^2} + \frac{e^4x^4}{c} + \frac{c^3d^4-6ac^2d^2e^2-a\cdots}{\cdots}\right.$$

$$= \frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))\,x}{c^3} + \frac{e^3(4cd-be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\int \frac{c^3d^4-6ac^2d^2e^2-ab^2e^4+ace\cdots}{\cdots}}{\cdots}$$

$$= \frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))\,x}{c^3} + \frac{e^3(4cd-be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\left(e(2cd-be)\left(2c^2d^2+\cdots\right.\right.}{\cdots}$$

$$= \frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))\,x}{c^3} + \frac{e^3(4cd-be)x^3}{3c^2} + \frac{e^4x^5}{5c} + \frac{\left(e(2cd-be)\left(2c^2d^2+\cdots\right.\right.}{\cdots}$$

Mathematica [A]
time = 0.44, size = 570, normalized size = 1.24

$$\frac{e^2(6c^2d^2+b^2e^2-ce(4bd+ae))\,x}{c^3}+\frac{e^3(4cd-be)x^3}{3c^2}+\frac{e^4x^5}{5c}+\frac{\left(2c^4d^4+b^3\left(b-\sqrt{b^2-4ac}\right)e^4+4c^3d^2e\left(-bd+\sqrt{b^2-4ac}\,d-3ae\right)+2bce^3\left(-2b^2d+2b\sqrt{b^2-4ac}\,d-2abe+a\sqrt{b^2-4ac}\,e\right)+2c^2e^2\left(3b^2d^2-3bd\left(\sqrt{b^2-4ac}\,d-2ae\right)+ae\left(-2\sqrt{b^2-4ac}\,d+ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{7/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(2c^4d^4+b^3\left(b+\sqrt{b^2-4ac}\right)e^4-4c^3d^2e\left(bd+\sqrt{b^2-4ac}\,d+3ae\right)-2bce^3\left(2b^2d+a\sqrt{b^2-4ac}\,e+2b\left(\sqrt{b^2-4ac}\,d+ae\right)\right)+2c^2e^2\left(3b^2d^2+ae\left(2\sqrt{b^2-4ac}\,d+ae\right)+3bd\left(\sqrt{b^2-4ac}\,d+2ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{7/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(6*c^2*d^2 + b^2*e^2 - c*e*(4*b*d + a*e))*x)/c^3 + (e^3*(4*c*d - b*e)*
x^3)/(3*c^2) + (e^4*x^5)/(5*c) + ((2*c^4*d^4 + b^3*(b - Sqrt[b^2 - 4*a*c])*
e^4 + 4*c^3*d^2*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*b*c*e^3*(-2*b^
2*d + 2*b*Sqrt[b^2 - 4*a*c]*d - 2*a*b*e + a*Sqrt[b^2 - 4*a*c]*e) + 2*c^2*e^
```

```
2*(3*b^2*d^2 - 3*b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + a*e*(-2*Sqrt[b^2 - 4*a
*c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c^4*d^4
+ b^3*(b + Sqrt[b^2 - 4*a*c])*e^4 - 4*c^3*d^2*e*(b*d + Sqrt[b^2 - 4*a*c]*d
+ 3*a*e) - 2*b*c*e^3*(2*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 2*b*(Sqrt[b^2 - 4*a
*c]*d + a*e)) + 2*c^2*e^2*(3*b^2*d^2 + a*e*(2*Sqrt[b^2 - 4*a*c]*d + a*e) +
3*b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

Maple [A]
time = 0.12, size = 621, normalized size = 1.35

method	result
risch	$\frac{e^4x^5}{5c}-\frac{e^4bx^3}{3c^2}+\frac{4de^3x^3}{3c}-\frac{e^4ax}{c^2}+\frac{e^4b^2x}{c^3}-\frac{4e^3bdx}{c^2}+\frac{6e^2d^2x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(e\left(2abce^3-4ac^2de^2-b^3e^3 \ldots\right.\right.}{\ldots}}{\ldots}$ $-\frac{\left(2abce^4\sqrt{-4ac+b^2}-4ac^2de^3\sqrt{-4ac+b^2} \ldots\right.}{\ldots}$
default	$-\frac{e^2\left(-\frac{1}{5}x^5e^2c^2+\frac{1}{3}bce^2x^3-\frac{4}{3}c^2dex^3+ace^2x-e^2b^2x+4bcdex-6c^2d^2x\right)}{c^3}+\frac{\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/c^3*(-1/5*x^5*e^2*c^2+1/3*b*c*e^2*x^3-4/3*c^2*d*e*x^3+a*c*e^2*x-e^2*b^
2*x+4*b*c*d*e*x-6*c^2*d^2*x)+4/c^2*(-1/8*(2*a*b*c*e^4*(-4*a*c+b^2)^(1/2)-4*
a*c^2*d*e^3*(-4*a*c+b^2)^(1/2)-b^3*e^4*(-4*a*c+b^2)^(1/2)+4*b^2*c*d*e^3*(-4
*a*c+b^2)^(1/2)-6*b*c^2*d^2*e^2*(-4*a*c+b^2)^(1/2)+4*c^3*d^3*e*(-4*a*c+b^2)
^(1/2)+2*a^2*c^2*e^4-4*a*b^2*e^4*c+12*a*b*c^2*d*e^3-12*a*c^3*d^2*e^2+b^4*e^
4-4*b^3*c*d*e^3+6*b^2*c^2*d^2*e^2-4*b*c^3*e*d^3+2*c^4*d^4)/c/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(2*a*b*c*e^4*(-4*a*c+b^2)^(1/2)-4*a*c^2*d*e^
3*(-4*a*c+b^2)^(1/2)-b^3*e^4*(-4*a*c+b^2)^(1/2)+4*b^2*c*d*e^3*(-4*a*c+b^2)^
(1/2)-6*b*c^2*d^2*e^2*(-4*a*c+b^2)^(1/2)+4*c^3*d^3*e*(-4*a*c+b^2)^(1/2)-2*a
^2*c^2*e^4+4*a*b^2*e^4*c-12*a*b*c^2*d*e^3+12*a*c^3*d^2*e^2-b^4*e^4+4*b^3*c*
d*e^3-6*b^2*c^2*d^2*e^2+4*b*c^3*e*d^3-2*c^4*d^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*x^5*e^4 + 5*(4*c^2*d*e^3 - b*c*e^4)*x^3 + 15*(6*c^2*d^2*e^2 - 4
*b*c*d*e^3 + b^2*e^4 - a*c*e^4)*x)/c^3 + integrate((c^3*d^4 - 6*a*c^2*d^2*e
^2 + 4*a*b*c*d*e^3 - a*b^2*e^4 + a^2*c*e^4 + (4*c^3*d^3*e - 6*b*c^2*d^2*e^2
 - b^3*e^4 + 2*a*b*c*e^4 + 4*(b^2*c*e^3 - a*c^2*e^3)*d)*x^2)/(c*x^4 + b*x^2
 + a), x)/c^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 15935 vs. $2(426) = 852$.
time = 31.23, size = 15935, normalized size = 34.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/30*(180*c^2*d^2*x*e^2 - 15*sqrt(1/2)*c^3*sqrt(-(b*c^7*d^8 - 16*a*c^7*d^7*
e + 28*a*b*c^6*d^6*e^2 - 56*(a*b^2*c^5 - 2*a^2*c^6)*d^5*e^3 + 70*(a*b^3*c^4
 - 3*a^2*b*c^5)*d^4*e^4 - 56*(a*b^4*c^3 - 4*a^2*b^2*c^4 + 2*a^3*c^5)*d^3*e^
5 + 28*(a*b^5*c^2 - 5*a^2*b^3*c^3 + 5*a^3*b*c^4)*d^2*e^6 - 8*(a*b^6*c - 6*a
^2*b^4*c^2 + 9*a^3*b^2*c^3 - 2*a^4*c^4)*d*e^7 + (a*b^7 - 7*a^2*b^5*c + 14*a
^3*b^3*c^2 - 7*a^4*b*c^3)*e^8 + (a*b^2*c^7 - 4*a^2*c^8)*sqrt((c^14*d^16 - 5
6*a*c^13*d^14*e^2 + 112*a*b*c^12*d^13*e^3 - 28*(5*a*b^2*c^11 - 33*a^2*c^12)
*d^12*e^4 + 112*(a*b^3*c^10 - 30*a^2*b*c^11)*d^11*e^5 - 56*(a*b^4*c^9 - 129
*a^2*b^2*c^10 + 71*a^3*c^11)*d^10*e^6 + 16*(a*b^5*c^8 - 690*a^2*b^3*c^9 + 8
85*a^3*b*c^10)*d^9*e^7 - 2*(a*b^6*c^7 - 6375*a^2*b^4*c^8 + 13530*a^3*b^2*c^
9 - 3235*a^4*c^10)*d^8*e^8 - 224*(51*a^2*b^5*c^7 - 155*a^3*b^3*c^8 + 90*a^4
*b*c^9)*d^7*e^9 + 56*(143*a^2*b^6*c^6 - 573*a^3*b^4*c^7 + 558*a^4*b^2*c^8 -
 71*a^5*c^9)*d^6*e^10 - 112*(39*a^2*b^7*c^5 - 195*a^3*b^5*c^6 + 272*a^4*b^3
*c^7 - 87*a^5*b*c^8)*d^5*e^11 + 28*(65*a^2*b^8*c^4 - 390*a^3*b^6*c^5 + 715*
a^4*b^4*c^6 - 395*a^5*b^2*c^7 + 33*a^6*c^8)*d^4*e^12 - 112*(5*a^2*b^9*c^3 -
 35*a^3*b^7*c^4 + 80*a^4*b^5*c^5 - 65*a^5*b^3*c^6 + 14*a^6*b*c^7)*d^3*e^13
+ 8*(15*a^2*b^10*c^2 - 120*a^3*b^8*c^3 + 330*a^4*b^6*c^4 - 360*a^5*b^4*c^5
+ 135*a^6*b^2*c^6 - 7*a^7*c^7)*d^2*e^14 - 16*(a^2*b^11*c - 9*a^3*b^9*c^2 +
29*a^4*b^7*c^3 - 40*a^5*b^5*c^4 + 22*a^6*b^3*c^5 - 3*a^7*b*c^6)*d*e^15 + (a
^2*b^12 - 10*a^3*b^10*c + 37*a^4*b^8*c^2 - 62*a^5*b^6*c^3 + 46*a^6*b^4*c^4
- 12*a^7*b^2*c^5 + a^8*c^6)*e^16)/(a^2*b^2*c^14 - 4*a^3*c^15)))/(a*b^2*c^7
- 4*a^2*c^8))*log(2*c^11*d^16*x - 8*b*c^10*d^15*x*e + 12*(b^2*c^9 - 4*a*c^1
0)*d^14*x*e^2 - 8*(b^3*c^8 - 39*a*b*c^9)*d^13*x*e^3 + 2*(b^4*c^7 - 450*a*b^
2*c^8 - 36*a^2*c^9)*d^12*x*e^4 + 312*(5*a*b^3*c^7 + a^2*b*c^8)*d^11*x*e^5 -
 44*(42*a*b^4*c^6 + 15*a^2*b^2*c^7 - 4*a^3*c^8)*d^10*x*e^6 + 88*(18*a*b^5*c
^5 + 12*a^2*b^3*c^6 - 11*a^3*b*c^7)*d^9*x*e^7 - 198*(5*a*b^6*c^4 + 7*a^2*b^
4*c^5 - 10*a^3*b^2*c^6 - 2*a^4*c^7)*d^8*x*e^8 + 88*(5*a*b^7*c^3 + 15*a^2*b^
```

```
5*c^4 - 22*a^3*b^3*c^5 - 17*a^4*b*c^6)*d^7*x*e^9 - 44*(3*a*b^8*c^2 + 19*a^2
*b^6*c^3 - 20*a^3*b^4*c^4 - 51*a^4*b^2*c^5 - 4*a^5*c^6)*d^6*x*e^10 + 24*(a*
b^9*c + 14*a^2*b^7*c^2 - a^3*b^5*c^3 - 75*a^4*b^3*c^4 - 17*a^5*b*c^5)*d^5*x
*e^11 - 2*(a*b^10 + 39*a^2*b^8*c + 88*a^3*b^6*c^2 - 415*a^4*b^4*c^3 - 234*a
^5*b^2*c^4 + 36*a^6*c^5)*d^4*x*e^12 + 8*(a^2*b^9 + 10*a^3*b^7*c - 21*a^4*b^
5*c^2 - 49*a^5*b^3*c^3 + 21*a^6*b*c^4)*d^3*x*e^13 - 12*(a^3*b^8 + a^4*b^6*c
 - 14*a^5*b^4*c^2 + 5*a^6*b^2*c^3 + 4*a^7*c^4)*d^2*x*e^14 + 8*(a^4*b^7 - 3*
a^5*b^5*c - 2*a^6*b^3*c^2 + 5*a^7*b*c^3)*d*x*e^15 - 2*(a^5*b^6 - 5*a^6*b^4*
c + 6*a^7*b^2*c^2 - a^8*c^3)*x*e^16 + sqrt(1/2)*((b^2*c^10 - 4*a*c^11)*d^12
 - 34*(a*b^2*c^9 - 4*a^2*c^10)*d^10*e^2 + 60*(a*b^3*c^8 - 4*a^2*b*c^9)*d^9*
e^3 - (71*a*b^4*c^7 - 523*a^2*b^2*c^8 + 956*a^3*c^9)*d^8*e^4 + 56*(a*b^5*c^
6 - 14*a^2*b^3*c^7 + 40*a^3*b*c^8)*d^7*e^5 - 28*(a*b^6*c^5 - 31*a^2*b^4*c^6
 + 125*a^3*b^2*c^7 - 68*a^4*c^8)*d^6*e^6 + 8*(a*b^7*c^4 - 92*a^2*b^5*c^5 +
481*a^3*b^3*c^6 - 516*a^4*b*c^7)*d^5*e^7 - (a*b^8*c^3 - 471*a^2*b^6*c^4 + 2
966*a^3*b^4*c^5 - 4631*a^4*b^2*c^6 + 956*a^5*c^7)*d^4*e^8 - 8*(27*a^2*b^7*c
^3 - 195*a^3*b^5*c^4 + 394*a^4*b^3*c^5 - 184*a^5*b*c^6)*d^3*e^9 + 2*(33*a^2
*b^8*c^2 - 267*a^3*b^6*c^3 + 662*a^4*b^4*c^4 - 505*a^5*b^2*c^5 + 68*a^6*c^6
)*d^2*e^10 - 4*(3*a^2*b^9*c - 27*a^3*b^7*c^2 + 80*a^4*b^5*c^3 - 87*a^5*b^3*
c^4 + 28*a^6*b*c^5)*d*e^11 + (a^2*b^10 - 10*a^3*b^8*c + 35*a^4*b^6*c^2 - 51
*a^5*b^4*c^3 + 29*a^6*b^2*c^4 - 4*a^7*c^5)*e^12 - ((a*b^3*c^10 - 4*a^2*b*c^
11)*d^4 - 8*(a^2*b^2*c^10 - 4*a^3*c^11)*d^3*e + 6*(a^2*b^3*c^9 - 4*a^3*b*c^
10)*d^2*e^2 - 4*(a^2*b^4*c^8 - 6*a^3*b^2*c^9 + 8*a^4*c^10)*d*e^3 + (a^2*b^5
*c^7 - 7*a^3*b^3*c^8 + 12*a^4*b*c^9)*e^4)*sqrt((c^14*d^16 - 56*a*c^13*d^14*
e^2 + 112*a*b*c^12*d^13*e^3 - 28*(5*a*b^2*c^11 - 33*a^2*c^12)*d^12*e^4 + 11
2*(a*b^3*c^10 - 30*a^2*b*c^11)*d^11*e^5 - 56*(a*b^4*c^9 - 129*a^2*b^2*c^10
+ 71*a^3*c^11)*d^10*e^6 + 16*(a*b^5*c^8 - 690*a^2*b^3*c^9 + 885*a^3*b*c^10)
*d^9*e^7 - 2*(a*b^6*c^7 - 6375*a^2*b^4*c^8 + 13530*a^3*b^2*c^9 - 3235*a^4*c
^10)*d^8*e^8 - 224*(51*a^2*b^5*c^7 - 155*a^3*b^3*c^8 + 90*a^4*b*c^9)*d^7*e^
9 + 56*(143*a^2*b^6*c^6 - 573*a^3*b^4*c^7 + 558*a^4*b^2*c^8 - 71*a^5*c^9)*d
^6*e^10 - 112*(39*a^2*b^7*c^5 - 195*a^3*b^5*c^6 + 272*a^4*b^3*c^7 - 87*a^5*
b*c^8)*d^5*e^11 + 28*(65*a^2*b^8*c^4 - 390*a^3*b^6*c^5 + 715*a^4*b^4*c^6 -
395*a^5*b^2*c^7 + 33*a^6*c^8)*d^4*e^12 - 112*(5*a^2*b^9*c^3 - 35*a^3*b^7*c^
4 + 80*a^4*b^5*c^5 - 65*a^5*b^3*c^6 + 14*a^6*b*c^7)*d^3*e^13 + 8*(15*a^2*b^
10*c^2 - 120*a^3*b^8*c^3 + 330*a^4*b^6*c^4 - 360*a^5*b^4*c^5 + 135*a^6*b^2*
c^6 - 7*a^7*c^7)*d^2*e^14 - 16*(a^2*b^11*c - 9*a^3*b^9*c^2 + 29*a^4*b^7*c^3
 - 40*a^5*b^5*c^4 + 22*a^6*b^3*c^5 - 3*a^7*b*c^6)*d*e^15 + (a^2*b^12 - 10*a
^3*b^10*c + 37*a^4*b^8*c^2 - 62*a^5*b^6*c^3 + 46*a^6*b^4*c^4 - 12*a^7*b^2*c
^5 + a^8*c^6)*e^16)/(a^2*b^2*c^14 - 4*a^3*c^15)))*sqrt(-(b*c^7*d^8 - 16*a*c
^7*d^7*e + 28*a*b*c^6*d^6*e^2 - 56*(a*b^2*c^5 - 2*a^2*c^6)*d^5*e^3 + 70*(a*
b^3*c^4 - 3*a^2*b*c^5)*d^4*e^4 - 56*(a*b^4*c^3 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 9285 vs. $2(426) = 852$.
time = 6.68, size = 9285, normalized size = 20.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(4*(2*b^4*c^5 - 16*a*b^2*c^6 + 32*a^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*c^6 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6)*c^2*d^3*e + 2*(s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^6 + 2*b^4*c^6 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^7 + 8*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^2*c^7 - 16*a*b^2*c^7 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*c^8 + 32*a^2*c^8 - 2*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7)*d
^4*abs(c) - 6*(2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)
*c^2*d^2*e^2 + 2*(2*b^3*c^8 - 8*a*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b*c^8 - 2*(b^2 - 4*a*c)*b*c^8)*d^4 + 4*(2*b^6*c^3 - 18*a*b^4*c^4 +
48*a^2*b^2*c^5 - 32*a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^6*c + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
```

```
^2*b^2*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^
3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 5*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*
a*c)*b^4*c^3 + 10*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*d*
e^3 - 12*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^3*c^5 + 2*a*b^4*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^3*c^6 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 16*a^2*b^2*c^6 - 4*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^7 + 32*a^3*c^7 - 2*(b^2 - 4*a*c)*a*b^2*c
^5 + 8*(b^2 - 4*a*c)*a^2*c^6)*d^2*abs(c)*e^2 - 4*(2*b^4*c^7 - 8*a*b^2*c^8 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^7 - 2*(b^2 - 4*a*c)*b^2*c
^7)*d^3*e - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)
*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*c^2*e^4 + 8*(sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^5*c
^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^2 - 4*a*c)*a*...
```

Mupad [B]
time = 9.31, size = 2500, normalized size = 5.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^4/(a + b*x^2 + c*x^4),x)
```

[Out] x*((b*((b*e^4)/c^2 - (4*d*e^3)/c))/c - (a*e^4)/c^2 + (6*d^2*e^2)/c) - x^3*(
(b*e^4)/(3*c^2) - (4*d*e^3)/(3*c)) + atan(((((16*a*c^8*d^4 + 16*a^3*c^6*e^4
 - 4*b^2*c^7*d^4 + 4*a*b^4*c^4*e^4 - 20*a^2*b^2*c^5*e^4 - 96*a^2*c^7*d^2*e^
2 - 16*a*b^3*c^5*d*e^3 + 64*a^2*b*c^6*d*e^3 + 24*a*b^2*c^6*d^2*e^2)/c^5 - (
2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c
- b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 2
8*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8
- 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d
^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*
c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^
5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^
2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2
)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7
*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a
*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c
^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d
^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*
e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a
*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2
*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2
)^3)^(1/2) - 28*a*b^4*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*
d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^
(1/2) + 8*a*b^5*c*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(
-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2
))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b
^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*
a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8
+ a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*
d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^
2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b
^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a
^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c
^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*
(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70
*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*
b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*
c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^
2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) +
24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c
 - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4
*c^2*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 112*a^2*b*c^4*d^3*e^5*(-(4*a*c - b^
2)^3)^(1/2) - 32*a^2*b^3*c^2*d*e^7*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^5*c*d*e
^7*(-(4*a*c - b^2)^3)^(1/2) + 84*a^2*b^2*c^3*d^2*e^6*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^3*c^9 + a*b^4*c^7 - 8*a^2*b^2*c^8)))^(1/2) - (2*x*(b^8*e^8 + 2
*c^8*d^8 + 2*a^4*c^4*e^8 - 56*a*c^7*d^6*e^2 + 20*a^2*b^4*c^2*e^8 - 16*a^3*b

^2*c^3*e^8 + 140*a^2*c^6*d^4*e^4 - 56*a^3*c^5*d^2*e^6 + 28*b^2*c^6*d^6*e^2 - 56*b^3*c^5*d^5*e^3 + 70*b^4*c^4*d^4*e^4 - 56*b^5*c^3*d^3*e^5 + 28*b^6*c^2*d^2*e^6 - 8*a*b^6*c*e^8 - 8*b*c^7*d^7*e - 8*b^7*c*d*e^7 + 252*a^2*b^2*c^4*d^2*e^6 + 168*a*b*c^6*d^5*e^3 + 56*a*b^5*c^2*d*e^7 + 56*a^3*b*c^4*d*e^7 - 280*a*b^2*c^5*d^4*e^4 + 280*a*b^3*c^4*d^3*e^5 - 168*a*b^4*c^3*d^2*e^6 - 280*a^2*b*c^5*d^3*e^5 - 112*a^2*b^3*c^3*d*e^7))/c^5)*(-(a*b^9*e^8 + b^3*c^7*d^8 + c^7*d^8*(-(4*a*c - b^2)^3)^(1/2) - a*b^6*e^8*(-(4*a*c - b^2)^3)^(1/2) - 11*a^2*b^7*c*e^8 + 28*a^5*b*c^4*e^8 + 64*a^2*c^8*d^7*e - 64*a^5*c^5*d*e^7 + 42*a^3*b^5*c^2*e^8 - 63*a^4*b^3*c^3*e^8 + a^4*c^3*e^8*(-(4*a*c - b^2)^3)^(1/2) - 448*a^3*c^7*d^5*e^3 + 448*a^4*c^6*d^3*e^5 - 4*a*b*c^8*d^8 - 8*a*b^8*c*d*e^7 - 6*a^3*b^2*c^2*e^8*(-(4*a*c - b^2)^3)^(1/2) + 336*a^2*b^2*c^6*d^5*e^3 - 490*a^2*b^3*c^5*d^4*e^4 + 448*a^2*b^4*c^4*d^3*e^5 - 252*a^2*b^5*c^3*d^2*e^6 - 1008*a^3*b^2*c^5*d^3*e^5 + 700*a^3*b^3*c^4*d^2*e^6 + 70*a^2*c^5*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) - 28*a^3*c^4*d^2*e^6*(-(4*a*c - b^2)^3)^(1/2) - 16*a*b^2*c^7*d^7*e + 5*a^2*b^4*c*e^8*(-(4*a*c - b^2)^3)^(1/2) + 28*a*b^3*c^6*d^6*e^2 - 56*a*b^4*c^5*d^5*e^3 + 70*a*b^5*c^4*d^4*e^4 - 56*a*b^6*c^3*d^3*e^5 + 28*a*b^7*c^2*d^2*e^6 - 112*a^2*b*c^7*d^6*e^2 + 80*a^2*b^6*c^2*d*e^7 + 840*a^3*b*c^6*d^4*e^4 - 264*a^3*b^4*c^3*d*e^7 - 560*a^4*b*c^5*d^2*e^6 + 304*a^4*b^2*c^4*d*e^7 - 28*a*c^6*d^6*e^2*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b*c^5*d^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^3*b*c^3*d*e^7*(-(4*a*c - b^2)^3)^(1/2) - 70*a*b^2*c^4*d^4*e^4*(-(4*a*c - b^2)^3)^(1/2) + 56*a*b^3*c^3*d^3*e^5*(-(4*a*c - b^2)^3)^(1/2) - 28*a*b^4*c^2*d...

3.264 $\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=316

$$\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}+\frac{\left(e(3c^2d^2+b^2e^2-ce(3bd+ae))+\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3ae))}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] e^2*(-b*e+3*c*d)*x/c^2+1/3*e^3*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*(e*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))+(-b*e+2*c*d)*(c
^2*d^2+b^2*e^2-c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*(e*(3*c^2*d^2+b^2*e^2-c*e*(a*e+3*b*d))-(-b*e+2*c*d)*(c^2*d^2+b^2*e^2-
c*e*(3*a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))
^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1184, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e(-ce(ae+3bd)+b^2e^2+3c^2d^2)+\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e(-ce(ae+3bd)+b^2e^2+3c^2d^2)-\frac{(2cd-be)(-ce(3ae+bd)+b^2e^2+c^2d^2)}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{e^2x(3cd-be)}{c^2}+\frac{e^3x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*(3*c*d - b*e)*x)/c^2 + (e^3*x^3)/(3*c) + ((e*(3*c^2*d^2 + b^2*e^2 - c*
e*(3*b*d + a*e)) + ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)))/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(3*c^2*d^2 + b^2*e^2 -
 c*e*(3*b*d + a*e)) - ((2*c*d - b*e)*(c^2*d^2 + b^2*e^2 - c*e*(b*d + 3*a*e)
))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{a+bx^2+cx^4}\,dx = \int \left(\frac{e^2(3cd-be)}{c^2}+\frac{e^3x^2}{c}+\frac{c^2d^3-3acde^2+abe^3+e(3c^2d^2+b^2e^2-ce(3bd+ae))\,x}{c^2\,(a+bx^2+cx^4)}\right.$$

$$=\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}+\frac{\int \frac{c^2d^3-3acde^2+abe^3+e(3c^2d^2+b^2e^2-ce(3bd+ae))x^2}{a+bx^2+cx^4}\,dx}{c^2}$$

$$=\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}+\frac{\left(e(3c^2d^2+b^2e^2-ce(3bd+ae))-\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3a}{\sqrt{b^2-4ac}}\right.}{2c^2}$$

$$=\frac{e^2(3cd-be)x}{c^2}+\frac{e^3x^3}{3c}+\frac{\left(e(3c^2d^2+b^2e^2-ce(3bd+ae))+\frac{(2cd-be)(c^2d^2+b^2e^2-ce(bd+3a}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.35, size = 402, normalized size = 1.27

$$\frac{6\sqrt{c}\,e^2(3cd-be)x+2c^{3/2}e^3x^3+\frac{3\sqrt{2}\left(2c^3d^3+b^2\left(-b+\sqrt{b^2-4ac}\right)e^3+3c^2de\left(-bd+\sqrt{b^2-4ac}\,d-2ae\right)+ce^2\left(3b^2d-3b\sqrt{b^2-4ac}\,d+3abe-a\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\left(-2c^3d^3+b^2\left(b+\sqrt{b^2-4ac}\right)e^3+3c^2de\left(bd+\sqrt{b^2-4ac}\,d+2ae\right)-ce^2\left(3b^2d+a\sqrt{b^2-4ac}\,e+3b\left(\sqrt{b^2-4ac}\,d+ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*Sqrt[c]*e^2*(3*c*d - b*e)*x + 2*c^(3/2)*e^3*x^3 + (3*Sqrt[2]*(2*c^3*d^3
+ b^2*(-b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(-(b*d) + Sqrt[b^2 - 4*a*c]*
d - 2*a*e) + c*e^2*(3*b^2*d - 3*b*Sqrt[b^2 - 4*a*c]*d + 3*a*b*e - a*Sqrt[b^
2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(-2*c^3*d^3 + b^2
*(b + Sqrt[b^2 - 4*a*c])*e^3 + 3*c^2*d*e*(b*d + Sqrt[b^2 - 4*a*c]*d + 2*a*e
```

```
) - c*e^2*(3*b^2*d + a*Sqrt[b^2 - 4*a*c]*e + 3*b*(Sqrt[b^2 - 4*a*c]*d + a*e
)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*
a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

Maple [A]
time = 0.15, size = 402, normalized size = 1.27

method	result
risch	$\frac{e^{3} x^{3}}{3 c}-\frac{e^{3} b x}{c^{2}}+\frac{3 d\, e^{2} x}{c}+\frac{\sum_{\_R =\mathit{RootOf}\left(c\,\_Z^{4}+\_Z^{2} b +a \right)}\frac{\left(e \left(-a c\, e^{2}+e^{2} b^{2}-3 b c d e +3 c^{2} d^{2}\right) \_R^{2}+e^{3} a b -3 a c d\, e^{2}+c^{2} d^{3}\right) \ln \left(x -\_R \right)}{2 c\,\_R^{3}+\_R b}}{2 c^{2}}$
default	$-\frac{e^{2} \left(-\frac{1}{3} c e\, x^{3}+e b x -3 c d x \right)}{c^{2}}+\frac{-\frac{\left(-e^{3} a c \sqrt{-4 a c +b^{2}}+b^{2} e^{3} \sqrt{-4 a c +b^{2}}-3 d\, e^{2} b c \sqrt{-4 a c +b^{2}}+3 d^{2} e\, c^{2} \sqrt{-4 a c +b^{2}}\right.}{2 c \sqrt{-4 a c +b^{2}}\, \sqrt{(}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2/c^2*(-1/3*c*e*x^3+e*b*x-3*c*d*x)+4/c*(-1/8*(-e^3*a*c*(-4*a*c+b^2)^(1/2
)+b^2*e^3*(-4*a*c+b^2)^(1/2)-3*d*e^2*b*c*(-4*a*c+b^2)^(1/2)+3*d^2*e*c^2*(-4
*a*c+b^2)^(1/2)+3*a*b*c*e^3-6*a*c^2*d*e^2-b^3*e^3+3*b^2*c*d*e^2-3*b*c^2*d^2
*e+2*c^3*d^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-e^3*a*c*(-4*
a*c+b^2)^(1/2)+b^2*e^3*(-4*a*c+b^2)^(1/2)-3*d*e^2*b*c*(-4*a*c+b^2)^(1/2)+3*
d^2*e*c^2*(-4*a*c+b^2)^(1/2)-3*a*b*c*e^3+6*a*c^2*d*e^2+b^3*e^3-3*b^2*c*d*e^
2+3*b*c^2*d^2*e-2*c^3*d^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^3*e^3 + 3*(3*c*d*e^2 - b*e^3)*x)/c^2 + integrate((c^2*d^3 - 3*a*c*
d*e^2 + (3*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3 - a*c*e^3)*x^2 + a*b*e^3)/(c*x
^4 + b*x^2 + a), x)/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9383 vs. $2(287) = 574$.
time = 6.08, size = 9383, normalized size = 29.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(18*c*d*x*e^2 + 3*sqrt(1/2)*c^2*sqrt(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*
a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^
2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 -
 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 -
 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8
*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c
^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15
*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^
3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4
*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2
 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^
4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2
*c^5 - 4*a^2*c^6))*log(-2*c^8*d^12*x + 6*b*c^7*d^11*x*e - 6*(b^2*c^6 - 4*a*
c^7)*d^10*x*e^2 + 2*(b^3*c^5 - 59*a*b*c^6)*d^9*x*e^3 + 18*(13*a*b^2*c^5 + 3
*a^2*c^6)*d^8*x*e^4 - 36*(7*a*b^3*c^4 + 5*a^2*b*c^5)*d^7*x*e^5 + 84*(2*a*b^
4*c^3 + 3*a^2*b^2*c^4)*d^6*x*e^6 - 36*(2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*
c^4)*d^5*x*e^7 + 18*(a*b^6*c + 7*a^2*b^4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d
^4*x*e^8 - 2*(a*b^7 + 21*a^2*b^5*c + 10*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*x*e
^9 + 6*(a^2*b^6 + 4*a^3*b^4*c - 9*a^4*b^2*c^2 - 4*a^5*c^3)*d^2*x*e^10 - 6*(
a^3*b^5 - a^4*b^3*c - 3*a^5*b*c^2)*d*x*e^11 + 2*(a^4*b^4 - 3*a^5*b^2*c + a^
6*c^2)*x*e^12 + sqrt(1/2)*((b^2*c^7 - 4*a*c^8)*d^9 - 18*(a*b^2*c^6 - 4*a^2*
c^7)*d^7*e^2 + 21*(a*b^3*c^5 - 4*a^2*b*c^6)*d^6*e^3 - 15*(a*b^4*c^4 - 8*a^2
*b^2*c^5 + 16*a^3*c^6)*d^5*e^4 + 3*(2*a*b^5*c^3 - 37*a^2*b^3*c^4 + 116*a^3*
b*c^5)*d^4*e^5 - (a*b^6*c^2 - 72*a^2*b^4*c^3 + 318*a^3*b^2*c^4 - 184*a^4*c^
5)*d^3*e^6 - 3*(11*a^2*b^5*c^2 - 61*a^3*b^3*c^3 + 68*a^4*b*c^4)*d^2*e^7 + 3
*(3*a^2*b^6*c - 19*a^3*b^4*c^2 + 29*a^4*b^2*c^3 - 4*a^5*c^4)*d*e^8 - (a^2*b
^7 - 7*a^3*b^5*c + 13*a^4*b^3*c^2 - 4*a^5*b*c^3)*e^9 - ((a*b^3*c^7 - 4*a^2*
b*c^8)*d^3 - 6*(a^2*b^2*c^7 - 4*a^3*c^8)*d^2*e + 3*(a^2*b^3*c^6 - 4*a^3*b*c
^7)*d*e^2 - (a^2*b^4*c^5 - 6*a^3*b^2*c^6 + 8*a^4*c^7)*e^3)*sqrt((c^10*d^12
- 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^
8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*
c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 1
5*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c
^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^
4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^
2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b
^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a^2*b^2*c^10 - 4*a^3*c^11)))*sqrt(
-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2*c
^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b^2
*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^2*
c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3 -
```

```
 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^7*
e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*b^
3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*a^
4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*e^
9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*e^
10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11 +
(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/(a
^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))) - 3*sqrt(1/2)*c^2*sqr
t(-(b*c^5*d^6 - 12*a*c^5*d^5*e + 15*a*b*c^4*d^4*e^2 - 20*(a*b^2*c^3 - 2*a^2
*c^4)*d^3*e^3 + 15*(a*b^3*c^2 - 3*a^2*b*c^3)*d^2*e^4 - 6*(a*b^4*c - 4*a^2*b
^2*c^2 + 2*a^3*c^3)*d*e^5 + (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e^6 + (a*b^
2*c^5 - 4*a^2*c^6)*sqrt((c^10*d^12 - 30*a*c^9*d^10*e^2 + 40*a*b*c^8*d^9*e^3
 - 15*(2*a*b^2*c^7 - 17*a^2*c^8)*d^8*e^4 + 12*(a*b^3*c^6 - 52*a^2*b*c^7)*d^
7*e^5 - 2*(a*b^4*c^5 - 428*a^2*b^2*c^6 + 226*a^3*c^7)*d^6*e^6 - 60*(13*a^2*
b^3*c^5 - 16*a^3*b*c^6)*d^5*e^7 + 15*(33*a^2*b^4*c^4 - 68*a^3*b^2*c^5 + 17*
a^4*c^6)*d^4*e^8 - 20*(11*a^2*b^5*c^3 - 33*a^3*b^3*c^4 + 20*a^4*b*c^5)*d^3*
e^9 + 6*(11*a^2*b^6*c^2 - 44*a^3*b^4*c^3 + 44*a^4*b^2*c^4 - 5*a^5*c^5)*d^2*
e^10 - 12*(a^2*b^7*c - 5*a^3*b^5*c^2 + 7*a^4*b^3*c^3 - 2*a^5*b*c^4)*d*e^11
+ (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^12)/
(a^2*b^2*c^10 - 4*a^3*c^11)))/(a*b^2*c^5 - 4*a^2*c^6))*log(-2*c^8*d^12*x +
6*b*c^7*d^11*x*e - 6*(b^2*c^6 - 4*a*c^7)*d^10*x*e^2 + 2*(b^3*c^5 - 59*a*b*c
^6)*d^9*x*e^3 + 18*(13*a*b^2*c^5 + 3*a^2*c^6)*d^8*x*e^4 - 36*(7*a*b^3*c^4 +
 5*a^2*b*c^5)*d^7*x*e^5 + 84*(2*a*b^4*c^3 + 3*a^2*b^2*c^4)*d^6*x*e^6 - 36*(
2*a*b^5*c^2 + 6*a^2*b^3*c^3 - a^3*b*c^4)*d^5*x*e^7 + 18*(a*b^6*c + 7*a^2*b^
4*c^2 - 2*a^3*b^2*c^3 - 3*a^4*c^4)*d^4*x*e^8 - 2*(a*b^7 + 21*a^2*b^5*c + 10
*a^3*b^3*c^2 - 55*a^4*b*c^3)*d^3*x*e^9 + 6*(a^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6407 vs. $2(287) = 574$.
time = 4.71, size = 6407, normalized size = 20.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(3*(2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d^2*e + 2*(s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^5 + 2*b^4*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 8*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^2*c^6 - 16*a*b^2*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*c^7 + 32*a^2*c^7 - 2*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6)*d
^3*abs(c) - 3*(2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c
^2*d*e^2 + 2*(2*b^3*c^7 - 8*a*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b*c^7 - 2*(b^2 - 4*a*c)*b*c^7)*d^3 + (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2
*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2
 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*c^2*e^3 - 6*(sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4
*a*c)*a^2*c^5)*d*abs(c)*e^2 - 3*(2*b^4*c^6 - 8*a*b^2*c^7 - sqrt(2)*sqrt(b^2
```

```
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^6)*d^2*e + 2*(sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^
4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 +
 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c)*e^3 + 3*(2*b^5*c^5 - 12*a*b^3*c^6 + 16*a
^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^
3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^
5 + 4*(b^2 - 4*a*c)*a*b*c^6)*d*e^2 - (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2
*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 +
7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - ...
```

Mupad [B]
time = 7.29, size = 2500, normalized size = 7.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3
- 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^6
*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6
*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2
)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^
2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c -
 b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1
/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 -
60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a
^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*d
^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/
2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-(4*a
*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2))/c^
```

3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^
6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^
6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^
2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a
^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c
- b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(
1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 -
60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*
a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*
d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1
/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2) -
(2*x*(b^6*e^6 + 2*c^6*d^6 - 2*a^3*c^3*e^6 - 30*a*c^5*d^4*e^2 + 9*a^2*b^2*c^
2*e^6 + 30*a^2*c^4*d^2*e^4 + 15*b^2*c^4*d^4*e^2 - 20*b^3*c^3*d^3*e^3 + 15*b
^4*c^2*d^2*e^4 - 6*a*b^4*c*e^6 - 6*b*c^5*d^5*e - 6*b^5*c*d*e^5 + 60*a*b*c^4
*d^3*e^3 + 30*a*b^3*c^2*d*e^5 - 30*a^2*b*c^3*d*e^5 - 60*a*b^2*c^3*d^2*e^4))
/c^3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4
e^6(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2
*c^6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c -
b^2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 12
0*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a
*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3
)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^
4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 1
08*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c
^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)
^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2)
*1i - (((16*a*c^6*d^3 - 4*b^2*c^5*d^3 - 4*a*b^3*c^3*e^3 + 16*a^2*b*c^4*e^3
- 48*a^2*c^5*d*e^2 + 12*a*b^2*c^4*d*e^2)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^6
*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^6
*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^2
)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^
2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c -
b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1
/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 -
60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a
^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*d
^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/
2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^3*c*d*e^5*(-(4*a
c - b^2)^3)^(1/2))/(8(16*a^3*c^7 + a*b^4*c^5 - 8*a^2*b^2*c^6)))^(1/2))/c^
3)*(-(a*b^7*e^6 + b^3*c^5*d^6 - c^5*d^6*(-(4*a*c - b^2)^3)^(1/2) + a*b^4*e^
6*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^5*c*e^6 - 20*a^4*b*c^3*e^6 + 48*a^2*c^
6*d^5*e + 48*a^4*c^4*d*e^5 + 25*a^3*b^3*c^2*e^6 + a^3*c^2*e^6*(-(4*a*c - b^

2)^3)^(1/2) - 160*a^3*c^5*d^3*e^3 - 4*a*b*c^6*d^6 - 6*a*b^6*c*d*e^5 + 120*a^2*b^2*c^4*d^3*e^3 - 105*a^2*b^3*c^3*d^2*e^4 - 15*a^2*c^3*d^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^5*d^5*e - 3*a^2*b^2*c*e^6*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^3*c^4*d^4*e^2 - 20*a*b^4*c^3*d^3*e^3 + 15*a*b^5*c^2*d^2*e^4 - 60*a^2*b*c^5*d^4*e^2 + 48*a^2*b^4*c^2*d*e^5 + 180*a^3*b*c^4*d^2*e^4 - 108*a^3*b^2*c^3*d*e^5 + 15*a*c^4*d^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a*b*c^3*d^3*e^3*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d*e^5*(-(4*a*c - b^2)^3)^(1/2) + 15*a*b^2*c^2*d^2*e^4*(-(4*a*c - b^2)^3)^(...

3.265 $\int \frac{\left(d+ex^2\right)^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=238

$$\frac{e^2x}{c}+\frac{\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+\cdots}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b-\cdots}}$$

```
[Out] e^2*x/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e*(-b*e
+2*c*d)+(2*c^2*d^2+b^2*e^2-2*c*e*(a*e+b*d))/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*(e*(-b*e+2*c*d)+(-2*c^2*d^2-b^2*e^2+2*c*e*(a*e+b*d))/(-4*
a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1184, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}+e(2cd-be)\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e(2cd-be)-\frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{e^2x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^2*x)/c + ((e*(2*c*d - b*e) + (2*c^2*d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e*(2*c*d - b*e) - (2*c^2*
d^2 + b^2*e^2 - 2*c*e*(b*d + a*e))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^2}{a+bx^2+cx^4}\,dx &= \int \left(\frac{e^2}{c}+\frac{cd^2-ae^2+e(2cd-be)x^2}{c\,(a+bx^2+cx^4)}\right)dx \\
&= \frac{e^2x}{c}+\frac{\int \frac{cd^2-ae^2+e(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{e^2x}{c}+\frac{\left(e(2cd-be)-\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}+\frac{\left(e(2cd-be)\right.\cdots}{\cdots} \\
&= \frac{e^2x}{c}+\frac{\left(e(2cd-be)+\frac{2c^2d^2+b^2e^2-2ce(bd+ae)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e(2cd-\right.\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 269, normalized size = 1.13

$$\frac{2\sqrt{c}\,e^2x+\frac{\sqrt{2}\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*e^2*x + (Sqrt[2]*(2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*
c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqr
t[2]*(2*c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4
*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

Maple [A]
time = 0.11, size = 249, normalized size = 1.05

method	result
risch	$\frac{e^2x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(e(-eb+2cd)\_R^2-a\,e^2+c\,d^2\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{e^2x}{c}-\frac{\left(-e^2b\sqrt{-4ac+b^2}+2cde\sqrt{-4ac+b^2}-2ac\,e^2+e^2b^2-2bcde+2c^2d^2\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+\right.}}}\right.}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*x/c-1/2*(-e^2*b*(-4*a*c+b^2)^(1/2)+2*c*d*e*(-4*a*c+b^2)^(1/2)-2*a*c*e^2
+e^2*b^2-2*b*c*d*e+2*c^2*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/
2*(-e^2*b*(-4*a*c+b^2)^(1/2)+2*c*d*e*(-4*a*c+b^2)^(1/2)+2*a*c*e^2-e^2*b^2+2
*b*c*d*e-2*c^2*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] x*e^2/c + integrate((c*d^2 + (2*c*d*e - b*e^2)*x^2 - a*e^2)/(c*x^4 + b*x^2
+ a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4573 vs. $2(209) = 418$.
time = 1.11, size = 4573, normalized size = 19.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*
(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2*
c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^
3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*
```

d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*c^5*d^8*x - 4*b*c^4*d^7*x*e + 28*a*b*c^3*d^5*x*e^3 + 2*(b^2*c^3 - 4*a*c^4)*d^6*x*e^2 - 10*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*x*e^4 + 12*(a*b^3*c + 3*a^2*b*c^2)*d^3*x*e^5 - 2*(a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*x*e^6 + 4*(a^2*b^3 + a^3*b*c)*d*x*e^7 - 2*(a^3*b^2 - a^4*c)*x*e^8 + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 - ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*c^5*d^8*x - 4*b*c^4*d^7*x*e + 28*a*b*c^3*d^5*x*e^3 + 2*(b^2*c^3 - 4*a*c^4)*d^6*x*e^2 - 10*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*x*e^4 + 12*(a*b^3*c + 3*a^2*b*c^2)*d^3*x*e^5 - 2*(a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*x*e^6 + 4*(a^2*b^3 + a^3*b*c)*d*x*e^7 - 2*(a^3*b^2 - a^4*c)*x*e^8 - sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 - ((a*b^3*c^4 - 4*a^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) + sqrt(1/2)*c*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^

```
3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c
^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^
4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*c^5*
d^8*x - 4*b*c^4*d^7*x*e + 28*a*b*c^3*d^5*x*e^3 + 2*(b^2*c^3 - 4*a*c^4)*d^6*
x*e^2 - 10*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*x*e^4 + 12*(a*b^3*c + 3*a^2*b*c^2)
*d^3*x*e^5 - 2*(a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*x*e^6 + 4*(a^2*b^3 + a
^3*b*c)*d*x*e^7 - 2*(a^3*b^2 - a^4*c)*x*e^8 + sqrt(1/2)*((b^2*c^4 - 4*a*c^5
)*d^6 - 7*(a*b^2*c^3 - 4*a^2*c^4)*d^4*e^2 + 4*(a*b^3*c^2 - 4*a^2*b*c^3)*d^3
*e^3 - (a*b^4*c - 11*a^2*b^2*c^2 + 28*a^3*c^3)*d^2*e^4 - 4*(a^2*b^3*c - 4*a
^3*b*c^2)*d*e^5 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*e^6 + ((a*b^3*c^4 - 4
*a^2*b*c^5)*d^2 - 4*(a^2*b^2*c^4 - 4*a^3*c^5)*d*e + (a^2*b^3*c^3 - 4*a^3*b*
c^4)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c
^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*
c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a
^4*c^2)*e^8)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e +
 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^
4 - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^
5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4107 vs. 2(209) = 418.
time = 4.24, size = 4107, normalized size = 17.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] x*e^2/c + 1/8*(2*(2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*d*e
```

+ 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 16*a*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^6 + 32*a^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*d^2*abs(c) - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*e^2 + 2*(2*b^3*c^6 - 8*a*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b*c^6)*d^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c)*e^2 - 2*(2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*d*e + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*e^2)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*(2*(2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqr

```
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 +
 8*(b^2 - 4*a*c)*a*c^4)*c^2*d*e - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 16*a*b^2*c^5 -
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^6 - 32*a^2*c^6 + 2*(b^2 - 4*a
*c)*b^2*c^4 - 8*(b^2 - 4*a*c)*a*c^5)*d^2*abs(c) - (2*b^5*c^2 - 16*a*b^3*c^3
 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*...
```

Mupad [B]
time = 6.48, size = 2500, normalized size = 10.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*x^2 + c*x^4),x)
```

```
[Out] atan(((((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/c
 - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*
a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4
+ 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e
- 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e +
6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2
*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*e^4 + b^3
*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^
(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(
1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^
3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2
*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(
-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2
) - (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*c^
2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^
3))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^
2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c
*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b
*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a
^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^
```

3 - 8*a^2*b^2*c^4)))^(1/2)*1i - (((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i)/((2*(2*c^3*d^5*e - a^2*b*e^6 - b^3*d^2*e^4 + 4*a*c^2*d^3*e^3 - 5*b*c^2*d^4*e^2 + 4*b^2*c*d^3*e^3 + 2*a*b^2*d*e^5 + 2*a^2*c*d*e^5 - 6*a*b*c*d^2*e^4))/c + (((16*a*c^4*d^2 - 16*a^2*c^3*e^2 - 4*b^2*c^3*d^2 + 4*a*b^2*c^2*e^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2 - 24*a^2*b*c^3*d^2*e^2 + 24*a^2*b^2*c^2*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(b^4*e^4 + 2*c^4*d^4 + 2*a^2*c^2*e^4 - 12*a*c^3*d^2*e^2 + 6*b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*b*c^3*d^3*e - 4*b^3*c*d*e^3 + 12*a*b*c^2*d*e^3))/c)*(-(a*b^5*e^4 + b^3*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^3)^(1/2) - a*b^2*e^4*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*e^4 + 12*a^3*b*c^2*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^3)^(1/2) + 32*a^2*c^4*d^3*e - 32*a^3*c^3*d*e^3 - 4*a*b*c^4*d^4 - 4*a*b^4*c*d*e^3 - 8*a*b^2*c^3*d^3*e + 6*a*b^3*c^2*d^2*e^2...

3.266 $\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=174

$$\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*d)/
(-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a*c+b^2
)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((e -
(2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}-\frac{1}{2}\sqrt{}}$$

$$=\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{}{\sqrt{b-}}\right.}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-}}}$$

Mathematica [A]
time = 0.09, size = 172, normalized size = 0.99

$$\frac{\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4),x]
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b^2
 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqrt
[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.04, size = 164, normalized size = 0.94

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(\_R^2 e+d\right)\ln\left(x-\_R\right)}{2c\_R^3+\_R b}\right)}{2}$
default	$4c\left(-\frac{\left(e\sqrt{-4ac+b^2}-eb+2cd\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(e\sqrt{-4ac+b^2}+eb-2cd\right)\sqrt{}}{8c\sqrt{-4ac+b^2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/8*(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))+1/8*(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1505 vs. 2(145) = 290.
time = 0.50, size = 1505, normalized size = 8.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*
sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-2*c^2*d^4*x + 2*b*c*d^3*x*e - 2*a*b*d*x*e^3 + 2*a^2*x
*e^4 + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3
*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^
2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a
*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^
2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(b*c
*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^
2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*
c^2*d^4*x + 2*b*c*d^3*x*e - 2*a*b*d*x*e^3 + 2*a^2*x*e^4 - sqrt(1/2)*((b^2*c
 - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*
(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^
2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^
2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))
```

```
/(a*b^2*c - 4*a^2*c^2))) + 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e
^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^
2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*c^2*d^4*x + 2*b*c*d^3*x*
e - 2*a*b*d*x*e^3 + 2*a^2*x*e^4 + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2
 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*
e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqr
t(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2
*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)))
 - 1/2*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^
2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*
b^2*c - 4*a^2*c^2))*log(-2*c^2*d^4*x + 2*b*c*d^3*x*e - 2*a*b*d*x*e^3 + 2*a^
2*x*e^4 - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*
b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c
*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e
+ a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(145) = 290$.
time = 5.22, size = 1402, normalized size = 8.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)
```

```
*b*c^2)*d - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2
- 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2
+ a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*
c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b
^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arcta
n(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*
a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

Mupad [B]
time = 5.38, size = 2500, normalized size = 14.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^
2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2
+ a*b^4*c)))^(1/2) - 4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*
c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*
d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^
2*b^2*c^2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4
*b*c^2*d*e))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*
d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d
*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i +
((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^
2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*
```

c)))^(1/2) + 4*b^2*c^2*d - 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 4*b^2*c^2*d - 16*a*c^3*d)*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*c^2*d^2*e + 2*a*c*e^3 - 2*b*c*d*e^2))*(-(a*b^3*e^2 + a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 - c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i - atan((((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i + ((x*(8*b^3*c^2 - 32*a*b*c^3)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 4*b^2*c^2*d - 16*a*c^3*d)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b

```
*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a
*b^4*c)))^(1/2) + x*(4*c^3*d^2 - 4*a*c^2*e^2 + 2*b^2*c*e^2 - 4*b*c^2*d*e))*
(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*
c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((x*(8*b^3*c^
2 - 32*a*b*c^3)*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 +
 c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^
2*d*e - 4*a*b^2*c*d*e)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) -
4*b^2*c^2*d + 16*a*c^3*d)*(-(a*b^3*e^2 - a*e^2*...
```

3.267 $\int \frac{1}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*
a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*2^(1/2)*c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1107, 211}

$$\frac{\sqrt{2}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{a+bx^2+cx^4}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.05, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\,\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-1),x]
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [A]
time = 0.02, size = 117, normalized size = 0.78

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$	38
default	$4c\left(-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} - \frac{\sqrt{2}\,\mathrm{arctan}\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/4/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. 2(114) = 228.
time = 0.47, size = 613, normalized size = 4.09

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

Sympy [A]
time = 0.65, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac-2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1024 vs. $2(114) = 228$.
time = 4.58, size = 1024, normalized size = 6.83

$$\frac{\left(\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^4-8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ab^2c-2\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^3c-2b^4c+16\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,a^2c^2+8\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,abc^2+\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^2c^2+16ab^2c^2-2b^3c^2-4\sqrt{2}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ac^3-32a^2c^3+8abc^3+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^3-4\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,abc-2\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^2c+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,bc^2+2(b^2-4ac)b^2c-8(b^2-4ac)ac^2+2(b^2-4ac)bc^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{c}}}\right)}{4\,(ab^4-8a^2b^2c-2ab^3c+16a^3c^2+8a^2bc^2+ab^2c^2-4a^2c^3)|c|}+\frac{\left(\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^4-8\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,ab^2c-2\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^3c+2b^4c+16\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,a^2c^2+8\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,abc^2+\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^2c^2-16ab^2c^2-2b^3c^2-4\sqrt{2}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,ac^3+32a^2c^3+8abc^3+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^3-4\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,abc-2\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^2c+\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,bc^2-2(b^2-4ac)b^2c+8(b^2-4ac)ac^2+2(b^2-4ac)bc^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{c}}}\right)}{4\,(ab^4-8a^2b^2c-2ab^3c+16a^3c^2+8a^2bc^2+ab^2c^2-4a^2c^3)|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^
2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs
(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 -
4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4
 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c
^3)*abs(c))
```

Mupad [**B**]
time = 0.51, size = 763, normalized size = 5.09

$$-\mathrm{atan}\left(\frac{b^4\,x\,1{}\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1{}\mathrm{i}+a^2\,c^2\,x\,16{}\mathrm{i}-a\,b^2\,c\,x\,8{}\mathrm{i}}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2{}\mathrm{i}-\mathrm{atan}\left(\frac{b^4\,x\,1{}\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1{}\mathrm{i}+a^2\,c^2\,x\,16{}\mathrm{i}-a\,b^2\,c\,x\,8{}\mathrm{i}}{4\,a\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*
a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
- 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2
) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(b^3 + (
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 +
 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i
)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(((b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

3.268 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=254

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}-\frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$

```
[Out] e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/d^(1/2)-1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*a*
c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e+2*c
*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1184, 211, 1180}

$$-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{cd-be-cex^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{cd-be-cex^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)}$$

$$= -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(e+\frac{2cd}{\sqrt{b^2-}}\right.}{\sqrt{2}\sqrt{b+}}$$

Mathematica [A]
time = 0.18, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(-2cd+be+\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{\sqrt{c}\left(2cd-be+\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) +
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Maple [A]
time = 0.20, size = 215, normalized size = 0.85

method	result
default	$\frac{4c\left(-\frac{\left(-e\sqrt{-4ac+b^2}-eb+2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(eb-2cd-e\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{ar}}{8\sqrt{-4ac+b^2}\,\sqrt{(b}}\right.}{a\,e^2-deb+c\,d^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)*c*(-1/8*(-e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(e*b-2*c*d-e*(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)))+e^2/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(e
*x/(d*e)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*d^2 - b*d*e + a*e^2)*sqrt(d)) - integ
rate((c*x^2*e - c*d + b*e)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7850 vs. $2(213) = 426$.
time = 12.98, size = 15733, normalized size = 61.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c^2*d^2 - 2*(b^2*c - 2*a*c
^2)*d*e + (b^3 - 3*a*b*c)*e^2 + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c -
```

4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4)*sqrt((c^4*d^4 - 4*b*c^3*d^3*e + 2*(3*b^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^4)/((a^2*b^2*c^4 - 4*a^3*c^5)*d^8 - 4*(a^2*b^3*c^3 - 4*a^3*b*c^4)*d^7*e + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^6*e^2 - 4*(a^2*b^5*c - a^3*b^3*c^2 - 12*a^4*b*c^3)*d^5*e^3 + (a^2*b^6 + 8*a^3*b^4*c - 42*a^4*b^2*c^2 - 24*a^5*c^3)*d^4*e^4 - 4*(a^3*b^5 - a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^5 + 2*(3*a^4*b^4 - 10*a^5*b^2*c - 8*a^6*c^2)*d^2*e^6 - 4*(a^5*b^3 - 4*a^6*b*c)*d*e^7 + (a^6*b^2 - 4*a^7*c)*e^8)))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4))*log(-2*c^4*d^2*x + 4*b*c^3*d*x*e - 2*(b^2*c^2 - a*c^3)*x*e^2 + sqrt(1/2)*((b^2*c^3 - 4*a*c^4)*d^3 - 3*(b^3*c^2 - 4*a*b*c^3)*d^2*e + (3*b^4*c - 13*a*b^2*c^2 + 4*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^3 - ((a*b^3*c^3 - 4*a^2*b*c^4)*d^5 - (3*a*b^4*c^2 - 14*a^2*b^2*c^3 + 8*a^3*c^4)*d^4*e + (3*a*b^5*c - 14*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^3*e^2 - (a*b^6 - 2*a^2*b^4*c - 12*a^3*b^2*c^2 + 16*a^4*c^3)*d^2*e^3 + (2*a^2*b^5 - 11*a^3*b^3*c + 12*a^4*b*c^2)*d*e^4 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^5)*sqrt((c^4*d^4 - 4*b*c^3*d^3*e + 2*(3*b^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^4)/((a^2*b^2*c^4 - 4*a^3*c^5)*d^8 - 4*(a^2*b^3*c^3 - 4*a^3*b*c^4)*d^7*e + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^6*e^2 - 4*(a^2*b^5*c - a^3*b^3*c^2 - 12*a^4*b*c^3)*d^5*e^3 + (a^2*b^6 + 8*a^3*b^4*c - 42*a^4*b^2*c^2 - 24*a^5*c^3)*d^4*e^4 - 4*(a^3*b^5 - a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^5 + 2*(3*a^4*b^4 - 10*a^5*b^2*c - 8*a^6*c^2)*d^2*e^6 - 4*(a^5*b^3 - 4*a^6*b*c)*d*e^7 + (a^6*b^2 - 4*a^7*c)*e^8)))*sqrt(-(b*c^2*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^3 - 3*a*b*c)*e^2 + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4)*sqrt((c^4*d^4 - 4*b*c^3*d^3*e + 2*(3*b^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^4)/((a^2*b^2*c^4 - 4*a^3*c^5)*d^8 - 4*(a^2*b^3*c^3 - 4*a^3*b*c^4)*d^7*e + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^6*e^2 - 4*(a^2*b^5*c - a^3*b^3*c^2 - 12*a^4*b*c^3)*d^5*e^3 + (a^2*b^6 + 8*a^3*b^4*c - 42*a^4*b^2*c^2 - 24*a^5*c^3)*d^4*e^4 - 4*(a^3*b^5 - a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^5 + 2*(3*a^4*b^4 - 10*a^5*b^2*c - 8*a^6*c^2)*d^2*e^6 - 4*(a^5*b^3 - 4*a^6*b*c)*d*e^7 + (a^6*b^2 - 4*a^7*c)*e^8)))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c^2*d^2 - 2*(b^2*c - 2*a*c^2)*d*e + (b^3 - 3*a*b*c)*e^2 + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4)*sqrt((c^4*d^4 - 4*b*c^3*d^3*e + 2*(3*b^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^4)/((a^2*b^2*c^4 - 4*a^3*c^5)*d^8 - 4*(a^2*b^3*c^3 - 4*a^3*b*c^4)*d^7*e + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^6*e^2 - 4*(a^2*b^5*c - a^3*b^3*c^2 - 12*a^4*b*c^3)*d^5*e^3 + (a^2*b^6 + 8*a^3*

```
b^4*c - 42*a^4*b^2*c^2 - 24*a^5*c^3)*d^4*e^4 - 4*(a^3*b^5 - a^4*b^3*c - 12*
a^5*b*c^2)*d^3*e^5 + 2*(3*a^4*b^4 - 10*a^5*b^2*c - 8*a^6*c^2)*d^2*e^6 - 4*(
a^5*b^3 - 4*a^6*b*c)*d*e^7 + (a^6*b^2 - 4*a^7*c)*e^8)))/((a*b^2*c^2 - 4*a^2
*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*
c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4))*lo
g(-2*c^4*d^2*x + 4*b*c^3*d*x*e - 2*(b^2*c^2 - a*c^3)*x*e^2 - sqrt(1/2)*((b^
2*c^3 - 4*a*c^4)*d^3 - 3*(b^3*c^2 - 4*a*b*c^3)*d^2*e + (3*b^4*c - 13*a*b^2*
c^2 + 4*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^3 - ((a*b^3*c^3
- 4*a^2*b*c^4)*d^5 - (3*a*b^4*c^2 - 14*a^2*b^2*c^3 + 8*a^3*c^4)*d^4*e + (3*
a*b^5*c - 14*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^3*e^2 - (a*b^6 - 2*a^2*b^4*c - 12
*a^3*b^2*c^2 + 16*a^4*c^3)*d^2*e^3 + (2*a^2*b^5 - 11*a^3*b^3*c + 12*a^4*b*c
^2)*d*e^4 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^5)*sqrt((c^4*d^4 - 4*b*c^
3*d^3*e + 2*(3*b^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4
- 2*a*b^2*c + a^2*c^2)*e^4)/((a^2*b^2*c^4 - 4*a^3*c^5)*d^8 - 4*(a^2*b^3*c^3
 - 4*a^3*b*c^4)*d^7*e + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^6*
e^2 - 4*(a^2*b^5*c - a^3*b^3*c^2 - 12*a^4*b*c^3)*d^5*e^3 + (a^2*b^6 + 8*a^3
*b^4*c - 42*a^4*b^2*c^2 - 24*a^5*c^3)*d^4*e^4 - 4*(a^3*b^5 - a^4*b^3*c - 12
*a^5*b*c^2)*d^3*e^5 + 2*(3*a^4*b^4 - 10*a^5*b^2*c - 8*a^6*c^2)*d^2*e^6 - 4*
(a^5*b^3 - 4*a^6*b*c)*d*e^7 + (a^6*b^2 - 4*a^7*c)*e^8)))*sqrt(-(b*c^2*d^2 -
 2*(b^2*c - 2*a*c^2)*d*e + (b^3 - 3*a*b*c)*e^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7650 vs. $2(213) = 426$.
time = 6.12, size = 7650, normalized size = 30.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^
```

```
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e -
 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c^3 - 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 +
8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^2*c^4 + 16*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*c^5 - 32*a^2*c^5 + 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c
^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2
 - 4*a*c)*a*b*c^4)*d^3*e^2 + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2
*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*
a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e -
(2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 - 2*
(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 7*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 2
*b^6*c + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 6*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c^2 + 14*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 3*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*b^4*c -
 6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d*abs(c*d^2 - b*d*e +
 a*e^2)*e^2 - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
```

```
t(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e +
a*e^2)^2*e + 2*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^
2*b*c^3)*d*e^4 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -...
```

Mupad [B]
time = 9.45, size = 2500, normalized size = 9.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2
 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3
*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2
 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3
*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*
b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d
^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2
*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*
b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 +
b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^
2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 -
 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^
3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^
2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*
```

c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*

```
a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 1
6*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b...
```

3.269 $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=429

$$\frac{e^2x}{2d\,(cd^2-bde+ae^2)\,(d+ex^2)}+\frac{\sqrt{c}\,\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\right.}{\sqrt{2}\,\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,\left(cd^2-bde+ae^2\right)^2}$$

```
[Out] 1/2*e^2*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)+1/2*e^(3/2)*arctan(x*e^(1/2)/d^(1
/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)+e^(3/2)*(-b*e+2*c*d)*arctan(x*e^(1/2)/d^(1
/2))/(a*e^2-b*d*e+c*d^2)^2/d^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-2*c*e*(b
*d+a*e+d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(-4*a*c+b^2)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2))*c^(1/2)*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(b*d
+a*e-d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(-4*a*c+b^2)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.97, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1184, 205, 211, 1180}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)^2}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)^2}+\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(2cd-be)}{\sqrt{d}\,(ae^2-bde+cd^2)^2}+\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}(ae^2-bde+cd^2)}+\frac{e^2x}{2d\,(d+ex^2)(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(2*d*(c*d^2 - b*d*e + a*e^2)*(d + e*x^2)) + (Sqrt[c]*(2*c^2*d^2 + b
*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4
*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (Sqrt[c]*(2*
c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d
+ a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (
e^(3/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e
+ a*e^2)^2) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c*d^2 - b*d
*e + a*e^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)}\,dx = \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^2)^2} - \frac{e^2(-2cd+be)}{(cd^2-bde+ae^2)^2(d+ex^2)} + \frac{c^2d^2 \cdots}{\cdots}\right.$$

$$= \frac{\int \frac{c^2d^2+b^2e^2-ce(2bd+ae)-ce(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{(cd^2-bde+ae^2)^2} + \frac{(e^2(2cd-be))\int \frac{1}{d+ex^2}\,dx}{(cd^2-bde+ae^2)^2} + \frac{e^2\int \cdots}{cd^2-\cdots}$$

$$= \frac{e^2x}{2d(cd^2-bde+ae^2)(d+ex^2)} + \frac{e^{3/2}(2cd-be)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)^2} + \frac{e^{\cdots}}{2d(c\cdots}$$

$$= \frac{e^2x}{2d(cd^2-bde+ae^2)(d+ex^2)} + \frac{\sqrt{c}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-\cdots\right.}{\sqrt{2}\sqrt{b^2-4ac}\cdots}$$

Mathematica [A]

time = 0.49, size = 354, normalized size = 0.83

$$\frac{\frac{e^2(cd^2+e(-bd+ae))x}{d(d+ex^2)}+\frac{\sqrt{2}\sqrt{c}\left(2c^2d^2+b\left(b+\sqrt{b^2-4ac}\right)e^2-2ce\left(bd+\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-2c^2d^2+b\left(-b+\sqrt{b^2-4ac}\right)e^2+2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}+\frac{e^{3/2}(5cd^2+e(-3bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}}}{2\left(cd^2+e(-bd+ae)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((e^2*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^2)) + (Sqrt[2]*Sqrt[c]*(2*c
^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d +
 a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-2*c^2*d^2 + b*(-b
 + Sqrt[b^2 - 4*a*c])*e^2 + 2*c*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]) + (e^(3/2)*(5*c*d^2 + e*(-3*b*d + a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/d^(3/2))/(2*(c*d^2 + e*(-(b*d) + a*e))^2)
```

Maple [A]
time = 0.28, size = 345, normalized size = 0.80

method	result
default	$\frac{4c\left(-\frac{\left(e^2b\sqrt{-4ac+b^2}-2cde\sqrt{-4ac+b^2}-2ac\,e^2+e^2b^2-2bcde+2c^2d^2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{(a\,e^2-deb+c}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)^2*c*(-1/8*(e^2*b*(-4*a*c+b^2)^(1/2)-2*c*d*e*(-4*a*c+b
^2)^(1/2)-2*a*c*e^2+e^2*b^2-2*b*c*d*e+2*c^2*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))+1/8*(e^2*b*(-4*a*c+b^2)^(1/2)-2*c*d*e*(-4*a*c+b^2)^(1/2)+2*a
*c*e^2-e^2*b^2+2*b*c*d*e-2*c^2*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+
e^2/(a*e^2-b*d*e+c*d^2)^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2
-3*b*d*e+5*c*d^2)/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] 1/2*(5*c*d^2*e^2 - 3*b*d*e^3 + a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^2*d^5 - 2*b*c*d^4*e - 2*a*b*d^2*e^3 + (b^2*e^2 + 2*a*c*e^2)*d^3 + a^2*d*e^4)*sqrt(d)) + 1/2*x*e^2/(c*d^4 - b*d^3*e + a*d^2*e^2 + (c*d^3*e - b*d^2*e^2 + a*d*e^3)*x^2) - integrate(-(c^2*d^2 - 2*b*c*d*e - (2*c^2*d*e - b*c*e^2)*x^2 + b^2*e^2 - a*c*e^2)/(c*x^4 + b*x^2 + a), x)/(c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + (b^2*e^2 + 2*a*c*e^2)*d^2 + a^2*e^4)

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a),x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2357 vs. $2(365) = 730$.
time = 5.37, size = 2357, normalized size = 5.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] 1/2*(5*c*d^2*e^2 - 3*b*d*e^3 + a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^2*d^5 - 2*b*c*d^4*e + b^2*d^3*e^2 + 2*a*c*d^3*e^2 - 2*a*b*d^2*e^3 + a^2*d*e^4)*sqrt(d)) + 1/2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 16*a*b^2*c^4 - 2*b^3*c^4 - 4*s

qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 32*a^2*c^5 + 8*a*b*c^5 + sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 + 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(
b^2 - 4*a*c)*a*c^4 + 2*(b^2 - 4*a*c)*b*c^4)*arctan(2*sqrt(1/2)*x/sqrt((b*c^
2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2
*b*e^4 + sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 -
2*a*b^2*d*e^3 + a^2*b*e^4)^2 - 4*(a*c^2*d^4 - 2*a*b*c*d^3*e + a*b^2*d^2*e^2
 + 2*a^2*c*d^2*e^2 - 2*a^2*b*d*e^3 + a^3*e^4)*(c^3*d^4 - 2*b*c^2*d^3*e + b^
2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/(c^3*d^4 - 2*b
*c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))
/(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a
*b^2*c^4 - 4*a^2*c^5)*d^2*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2
 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 - (a*b^4*c - 8*a^
2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)
*sqrt(b^2 - 4*a*c))*d*abs(c)*e + (a*b^6 - 10*a^2*b^4*c - 2*a*b^5*c + 32*a^3
*b^2*c^2 + 12*a^2*b^3*c^2 + a*b^4*c^2 - 32*a^4*c^3 - 16*a^3*b*c^3 - 6*a^2*b
^2*c^3 + 8*a^3*c^4 - (a*b^5 - 8*a^2*b^3*c - 2*a*b^4*c + 16*a^3*b*c^2 + 8*a^
2*b^2*c^2 + a*b^3*c^2 - 4*a^2*b*c^3)*sqrt(b^2 - 4*a*c))*abs(c)*e^2) + 1/2*(
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*
c^3 + 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^2*c^4 - 16*a*b^2*c^4 - 2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*c^5 + 32*a^2*c^5 + 8*a*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4 + 2
*(b^2 - 4*a*c)*b*c^4)*arctan(2*sqrt(1/2)*x/sqrt((b*c^2*d^4 - 2*b^2*c*d^3*e
 + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*e^4 - sqrt((b*c^2*d
^4 - 2*b^2*c*d^3*e + b^3*d^2*e^2 + 2*a*b*c*d^2*e^2 - 2*a*b^2*d*e^3 + a^2*b*
e^4)^2 - 4*(a*c^2*d^4 - 2*a*b*c*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2
*a^2*b*d*e^3 + a^3*e^4)*(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*
d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/(c^3*d^4 - 2*b*c^2*d^3*e + b^2*c*d^2
*e^2 + 2*a*c^2*d^2*e^2 - 2*a*b*c*d*e^3 + a^2*c*e^4)))/(2*(a*b^4*c^2 - 8*a^2
*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*
d^2*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^
2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 - (a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^
2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*sqrt(b^2 - 4*a*c))*d*
abs(c)*e + (a*b^6 - 10*a^2*b^4*c - 2*a*b^5*c + 32*a^3*b^2*c^2 + 12*a^2*b^3*
c^2 + a*b^4*c^2 - 32*a^4*c^3 - 16*a^3*b*c^3 - 6*a^2*b^2*c^3 + 8*a^3*c^4 - (
a*b^5 - 8*a^2*b^3*c - 2*a*b^4*c + 16*a^3*b*c^2 + 8*a^2*b^2*c^2 + a*b^3*c^2
 - 4*a^2*b*c^3)*sqrt(b^2 - 4*a*c))*abs(c)*e^2) + 1/2*x*e^2/((c*d^3 - b*d^2*e

```
 + a*d*e^2)*(x^2*e + d))
```

Mupad [B]
time = 10.28, size = 2500, normalized size = 5.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan(((((x*(54*c^9*d^6*e^5 - 2*a^3*c^6*e^11 - 22*a*c^8*d^4*e^7 - 118*b*c^8
*d^5*e^6 + a^2*b^2*c^5*e^11 - 14*a^2*c^7*d^2*e^9 + 107*b^2*c^7*d^4*e^7 - 48
*b^3*c^6*d^3*e^8 + 9*b^4*c^5*d^2*e^9 + 20*a*b*c^7*d^3*e^8 - 6*a*b^3*c^5*d*e
^10 + 10*a^2*b*c^6*d*e^10 + 4*a*b^2*c^6*d^2*e^9))/(2*(c^4*d^10 + a^4*d^2*e^
8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a
^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*
b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 -
 12*a^2*b*c*d^5*e^5)) - (((2*a^2*b^6*c^2*e^13 - 200*a*c^9*d^8*e^5 - 8*a^5*c
^5*e^13 - 14*a^3*b^4*c^3*e^13 + 26*a^4*b^2*c^4*e^13 + 480*a^2*c^8*d^6*e^7 +
 784*a^3*c^7*d^4*e^9 + 96*a^4*c^6*d^2*e^11 + 50*b^2*c^8*d^8*e^5 - 240*b^3*c
^7*d^7*e^6 + 466*b^4*c^6*d^6*e^7 - 464*b^5*c^5*d^5*e^8 + 246*b^6*c^4*d^4*e^
9 - 64*b^7*c^3*d^3*e^10 + 6*b^8*c^2*d^2*e^11 + 4*a^2*b^2*c^6*d^4*e^9 + 672*
a^2*b^3*c^5*d^3*e^10 - 354*a^2*b^4*c^4*d^2*e^11 + 464*a^3*b^2*c^5*d^2*e^11
+ 960*a*b*c^8*d^7*e^6 - 8*a*b^7*c^2*d*e^12 - 96*a^4*b*c^5*d*e^12 - 1984*a*b
^2*c^7*d^6*e^7 + 2072*a*b^3*c^6*d^5*e^8 - 1034*a*b^4*c^5*d^4*e^9 + 160*a*b^
5*c^4*d^3*e^10 + 34*a*b^6*c^3*d^2*e^11 - 864*a^2*b*c^7*d^5*e^8 + 40*a^2*b^5
*c^3*d*e^12 - 1152*a^3*b*c^6*d^3*e^10 - 8*a^3*b^3*c^4*d*e^12)/(2*(c^4*d^10
+ a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d
^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*
d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2
*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - ((-d^3*e^3)^(1/2)*((x*(32*c^11*d^13*e^2
 + 48*a^6*b*c^4*e^15 + 96*a*c^10*d^11*e^4 - 64*a^6*c^5*d*e^14 - 160*b*c^10*
d^12*e^3 + 4*a^4*b^5*c^2*e^15 - 28*a^5*b^3*c^3*e^15 - 2048*a^2*c^9*d^9*e^6
- 4416*a^3*c^8*d^7*e^8 - 2528*a^4*c^7*d^5*e^10 - 288*a^5*c^6*d^3*e^12 + 336
*b^2*c^9*d^11*e^4 - 268*b^3*c^8*d^10*e^5 - 360*b^4*c^7*d^9*e^6 + 1260*b^5*c
^6*d^8*e^7 - 1568*b^6*c^5*d^7*e^8 + 1036*b^7*c^4*d^6*e^9 - 360*b^8*c^3*d^5*
e^10 + 52*b^9*c^2*d^4*e^11 - 7584*a^2*b^2*c^7*d^7*e^8 - 536*a^2*b^3*c^6*d^6
*e^9 + 5936*a^2*b^4*c^5*d^5*e^10 - 3552*a^2*b^5*c^4*d^4*e^11 + 464*a^2*b^6*
c^3*d^3*e^12 + 104*a^2*b^7*c^2*d^2*e^13 - 12768*a^3*b^2*c^6*d^5*e^10 + 3720
*a^3*b^3*c^5*d^4*e^11 + 1280*a^3*b^4*c^4*d^3*e^12 - 648*a^3*b^5*c^3*d^2*e^1
3 - 4272*a^4*b^2*c^5*d^3*e^12 + 740*a^4*b^3*c^4*d^2*e^13 - 848*a*b*c^9*d^10
*e^5 + 3632*a*b^2*c^8*d^9*e^6 - 7852*a*b^3*c^7*d^8*e^7 + 8864*a*b^4*c^6*d^7
*e^8 - 4936*a*b^5*c^5*d^6*e^9 + 816*a*b^6*c^4*d^5*e^10 + 356*a*b^7*c^3*d^4*
e^11 - 128*a*b^8*c^2*d^3*e^12 + 7216*a^2*b*c^8*d^8*e^7 + 12896*a^3*b*c^7*d^
6*e^9 - 32*a^3*b^6*c^2*d*e^14 + 5696*a^4*b*c^6*d^4*e^11 + 216*a^4*b^4*c^3*d
```

*e^14 + 752*a^5*b*c^5*d^2*e^13 - 336*a^5*b^2*c^4*d*e^14))/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) + (((128*a*c^11*d^15*e^2 - 256*a^8*c^4*d*e^16 - 256*a^2*c^10*d^13*e^4 - 3456*a^3*c^9*d^11*e^6 - 8960*a^4*c^8*d^9*e^8 - 10880*a^5*c^7*d^7*e^10 - 6912*a^6*c^6*d^5*e^12 - 2176*a^7*c^5*d^3*e^14 - 32*b^2*c^10*d^15*e^2 + 256*b^3*c^9*d^14*e^3 - 896*b^4*c^8*d^13*e^4 + 1792*b^5*c^7*d^12*e^5 - 2240*b^6*c^6*d^11*e^6 + 1792*b^7*c^5*d^10*e^7 - 896*b^8*c^4*d^9*e^8 + 256*b^9*c^3*d^8*e^9 - 32*b^10*c^2*d^7*e^10 + 2848*a^2*b^2*c^8*d^11*e^6 - 12160*a^2*b^3*c^7*d^10*e^7 + 18480*a^2*b^4*c^6*d^9*e^8 - 12864*a^2*b^5*c^5*d^8*e^9 + 3008*a^2*b^6*c^4*d^7*e^10 + 832*a^2*b^7*c^3*d^6*e^11 - 400*a^2*b^8*c^2*d^5*e^12 - 17920*a^3*b^2*c^7*d^9*e^8 + 1280*a^3*b^3*c^6*d^8*e^9 + 14240*a^3*b^4*c^5*d^7*e^10 - 9824*a^3*b^5*c^4*d^6*e^11 + 1120*a^3*b^6*c^3*d^5*e^12 + 480*a^3*b^7*c^2*d^4*e^13 - 33760*a^4*b^2*c^6*d^7*e^10 + 7680*a^4*b^3*c^5*d^6*e^11 + 7520*a^4*b^4*c^4*d^5*e^12 - 2880*a^4*b^5*c^3*d^4*e^13 - 320*a^4*b^6*c^2*d^3*e^14 - 20672*a^5*b^2*c^5*d^5*e^12 + 896*a^5*b^3*c^4*d^4*e^13 + 2384*a^5*b^4*c^3*d^3*e^14 + 112*a^5*b^5*c^2*d^2*e^15 - 3872*a^6*b^2*c^4*d^3*e^14 - 896*a^6*b^3*c^3*d^2*e^15 - 1024*a*b*c^10*d^14*e^3 + 3648*a*b^2*c^9*d^13*e^4 - 7296*a*b^3*c^8*d^12*e^5 + 8464*a*b^4*c^7*d^11*e^6 - 5008*a*b^5*c^6*d^10*e^7 + 224*a*b^6*c^5*d^9*e^8 + 1632*a*b^7*c^4*d^8*e^9 - 944*a*b^8*c^3*d^7*e^10 + 176*a*b^9*c^2*d^6*e^11 + 512*a^2*b*c^9*d^12*e^5 + 14080*a^3*b*c^8*d^10*e^7 + 30720*a^4*b*c^7*d^8*e^9 + 28160*a^5*b*c^6*d^6*e^11 + 11776*a^6*b*c^5*d^4*e^13 - 16*a^6*b^4*c^2*d*e^16 + 1792*a^7*b*c^4*d^2*e^15 + 128*a^7*b^2*c^3*d*e^16)/(2*(c^4*d^10 + a^4*d^2*e^8 + b^4*d^6*e^4 - 4*a*b^3*d^5*e^5 - 4*a^3*b*d^3*e^7 + 4*a*c^3*d^8*e^2 + 4*a^3*c*d^4*e^6 - 4*b^3*c*d^7*e^3 + 6*a^2*b^2*d^4*e^6 + 6*a^2*c^2*d^6*e^4 + 6*b^2*c^2*d^8*e^2 - 4*b*c^3*d^9*e - 12*a*b*c^2*d^7*e^3 + 12*a*b^2*c*d^6*e^4 - 12*a^2*b*c*d^5*e^5)) - (x*(-d^3*e^3)^(1/2)*(a*e^2 + 5*c*d^2 - 3*b*d*e)*(1024*a^2*c^11*d^16*e^3 + 5120*a^3*c^10*d^14*e^5 + 9216*a^4*c^9*d^12*e^7 + 5120*a^5*c^8*d^10*e^9 - 5120*a^6*c^7*d^8*e^11 - 9216*a^7*c^6*d^6*e^13 - 5120*a^8*c^5*d^4*e^15 - 1024*a^9*c^4*d^2*e^17 - 64*b^3*c^10*d^17*e^2 + ...

3.270 $\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=563

$$\frac{x\left(c\left(b^2d^3-2ad(cd^2-3ae^2)-\frac{abe(3cd^2+ae^2)}{c}\right)-\left(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd^2+3ae^2)\right)x^2\right)}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{\left(ab^3e^3\right.}{}$$

```
[Out] 1/2*x*(c*(b^2*d^3-2*a*d*(-3*a*e^2+c*d^2)-a*b*e*(a*e^2+3*c*d^2)/c)-(a*b^2*e^
3+2*a*c*e*(-a*e^2+3*c*d^2)-b*c*d*(3*a*e^2+c*d^2))*x^2)/a/c/(-4*a*c+b^2)/(c*
x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(a*
b^3*e^3+6*a*c*(a*e^2+c*d^2)*(2*c*d+e*(-4*a*c+b^2)^(1/2))-b^2*(c^2*d^3-3*a*c
*d*e^2+a*e^3*(-4*a*c+b^2)^(1/2))-b*c*(c*d^2*(12*a*e+d*(-4*a*c+b^2)^(1/2))+a
*e^2*(8*a*e+3*d*(-4*a*c+b^2)^(1/2))))/a/c^(3/2)/(-4*a*c+b^2)^(3/2)*2^(1/2)/
(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2))*(a*b^3*e^3+6*a*c*(a*e^2+c*d^2)*(2*c*d-e*(-4*a*c+b^2)^(1/2))-b^
2*(c^2*d^3-3*a*c*d*e^2-a*e^3*(-4*a*c+b^2)^(1/2))+b*c*(c*d^2*(-12*a*e+d*(-4*
a*c+b^2)^(1/2))+a*e^2*(-8*a*e+3*d*(-4*a*c+b^2)^(1/2))))/a/c^(3/2)/(-4*a*c+b
^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 2.33, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1219, 1180, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(ab^3e^3-b^2\left(ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac(ae^2+cd^2)\left(e\sqrt{b^2-4ac}+2cd\right)-bc\left(cd^2\left(d\sqrt{b^2-4ac}+12ae\right)+ae^2\left(3d\sqrt{b^2-4ac}+8ae\right)\right)\right)}{2\sqrt{2}\,ac^{3/2}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(ab^3e^3-b^2\left(-ae^3\sqrt{b^2-4ac}-3acde^2+c^2d^3\right)+6ac(ae^2+cd^2)\left(2cd-e\sqrt{b^2-4ac}\right)+bc\left(cd^2\left(d\sqrt{b^2-4ac}-12ae\right)+ae^2\left(3d\sqrt{b^2-4ac}-8ae\right)\right)\right)}{2\sqrt{2}\,ac^{3/2}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(c\left(-\frac{abe(ae^2+3cd^2)}{c}-2ad(cd^2-3ae^2)+b^2d^3\right)-x^2\left(ab^2e^3-bcd(3ae^2+cd^2)+2ace(3cd^2-ae^2)\right)\right)}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(c*(b^2*d^3 - 2*a*d*(c*d^2 - 3*a*e^2) - (a*b*e*(3*c*d^2 + a*e^2))/c) - (
a*b^2*e^3 + 2*a*c*e*(3*c*d^2 - a*e^2) - b*c*d*(c*d^2 + 3*a*e^2))*x^2))/(2*a
*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((a*b^3*e^3 + 6*a*c*(2*c*d + Sqrt[b
^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt[b^2 -
4*a*c]*e^3) - b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sqrt[b^2
- 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ ((a*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) - b^2*(
c^2*d^3 - 3*a*c*d*e^2 - a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4
*a*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*c^(3/2)*(b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{(a+bx^2+cx^4)^2}\,dx = \frac{x\Big(c\Big(b^2d^3-2ad(cd^2-3ae^2)-\frac{abe(3cd^2+ae^2)}{c}\Big)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$= \frac{x\Big(c\Big(b^2d^3-2ad(cd^2-3ae^2)-\frac{abe(3cd^2+ae^2)}{c}\Big)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

$$= \frac{x\Big(c\Big(b^2d^3-2ad(cd^2-3ae^2)-\frac{abe(3cd^2+ae^2)}{c}\Big)-(ab^2e^3+2ace(3cd^2-ae^2)-bcd(cd}{2ac\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Mathematica [A]
time = 1.00, size = 540, normalized size = 0.96

$$\frac{\frac{2\sqrt{c}\,x\left(b^2\left(cd^3-ae^3x^2\right)+b\left(-a^2e^3+c^2d^3x^2-3acde\left(d-ex^2\right)\right)+2ac\left(ae^2\left(3d+ex^2\right)-cd^2\left(d+3ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(-ab^3e^3-6ac\left(2cd+\sqrt{b^2-4ac}\,e\right)\left(cd^2+ae^2\right)+b^2\left(c^2d^3-3acde^2+a\sqrt{b^2-4ac}\,e^3\right)+bc\left(ae^2\left(3\sqrt{b^2-4ac}\,d+8ae\right)+cd^2\left(\sqrt{b^2-4ac}\,d+12ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(ab^3e^3+6ac\left(2cd-\sqrt{b^2-4ac}\,e\right)\left(cd^2+ae^2\right)+b^2\left(-c^2d^3+3acde^2+a\sqrt{b^2-4ac}\,e^3\right)+bc\left(cd^2\left(\sqrt{b^2-4ac}\,d-12ae\right)+ae^2\left(3\sqrt{b^2-4ac}\,d-8ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*x*(b^2*(c*d^3 - a*e^3*x^2) + b*(-(a^2*e^3) + c^2*d^3*x^2 - 3*a*
c*d*e*(d - e*x^2)) + 2*a*c*(a*e^2*(3*d + e*x^2) - c*d^2*(d + 3*e*x^2))))/((
b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*e^3) - 6*a*c*(2*c*d +
 Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(c^2*d^3 - 3*a*c*d*e^2 + a*Sqrt
[b^2 - 4*a*c]*e^3) + b*c*(a*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 8*a*e) + c*d^2*(Sq
rt[b^2 - 4*a*c]*d + 12*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a
*b^3*e^3 + 6*a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e)*(c*d^2 + a*e^2) + b^2*(-(c^2
*d^3) + 3*a*c*d*e^2 + a*Sqrt[b^2 - 4*a*c]*e^3) + b*c*(c*d^2*(Sqrt[b^2 - 4*a
*c]*d - 12*a*e) + a*e^2*(3*Sqrt[b^2 - 4*a*c]*d - 8*a*e)))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[
b^2 - 4*a*c]]))/(4*a*c^(3/2))
```

Maple [A]

time = 0.13, size = 628, normalized size = 1.12

method	result
risch	$\frac{-\frac{\left(2a^{2}c\,e^{3}-a\,b^{2}e^{3}+3abcd\,e^{2}-6a\,c^{2}d^{2}e+b\,c^{2}d^{3}\right)x^{3}}{2ac\left(4ac-b^{2}\right)}+\frac{\left(a^{2}b\,e^{3}-6a^{2}cd\,e^{2}+3abc\,d^{2}e+2a\,c^{2}d^{3}-b^{2}c\,d^{3}\right)x}{2c\left(4ac-b^{2}\right)a}}{c\,x^{4}+b\,x^{2}+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+\_Z^{2}b+a\right)}\left(\frac{\left(6a \cdots\right.}{\cdots}\right.}{\cdots}$
default	$\frac{-\frac{\left(2a^{2}c\,e^{3}-a\,b^{2}e^{3}+3abcd\,e^{2}-6a\,c^{2}d^{2}e+b\,c^{2}d^{3}\right)x^{3}}{2ac\left(4ac-b^{2}\right)}+\frac{\left(a^{2}b\,e^{3}-6a^{2}cd\,e^{2}+3abc\,d^{2}e+2a\,c^{2}d^{3}-b^{2}c\,d^{3}\right)x}{2c\left(4ac-b^{2}\right)a}}{c\,x^{4}+b\,x^{2}+a}+\frac{-\frac{\left(6a^{2}c\,e^{3}\sqrt{-4ac+b^{2}}-a\,b^{2}e^{3}\cdots\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(2*a^2*c*e^3-a*b^2*e^3+3*a*b*c*d*e^2-6*a*c^2*d^2*e+b*c^2*d^3)/a/c/(4*
a*c-b^2)*x^3+1/2/c*(a^2*b*e^3-6*a^2*c*d*e^2+3*a*b*c*d^2*e+2*a*c^2*d^3-b^2*c
*d^3)/(4*a*c-b^2)/a*x)/(c*x^4+b*x^2+a)+2/a/(4*a*c-b^2)*(-1/8*(6*a^2*c*e^3*(
-4*a*c+b^2)^(1/2)-a*b^2*e^3*(-4*a*c+b^2)^(1/2)-3*a*b*c*d*e^2*(-4*a*c+b^2)^(
1/2)+6*a*c^2*d^2*e*(-4*a*c+b^2)^(1/2)-b*c^2*d^3*(-4*a*c+b^2)^(1/2)-8*a^2*b*
e^3*c+12*a^2*c^2*d*e^2+a*b^3*e^3+3*a*b^2*c*d*e^2-12*a*b*c^2*d^2*e+12*a*c^3*
d^3-b^2*c^2*d^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(6*a^2*c*e^
3*(-4*a*c+b^2)^(1/2)-a*b^2*e^3*(-4*a*c+b^2)^(1/2)-3*a*b*c*d*e^2*(-4*a*c+b^2
)^(1/2)+6*a*c^2*d^2*e*(-4*a*c+b^2)^(1/2)-b*c^2*d^3*(-4*a*c+b^2)^(1/2)+8*a^2
*b*e^3*c-12*a^2*c^2*d*e^2-a*b^3*e^3-3*a*b^2*c*d*e^2+12*a*b*c^2*d^2*e-12*a*c
```

```
^3*d^3+b^2*c^2*d^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)*
x^3 - (3*a*b*c*d^2*e - 6*a^2*c*d*e^2 - (b^2*c - 2*a*c^2)*d^3 + a^2*b*e^3)*x
)/(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b
*c^2)*x^2) + 1/2*integrate((3*a*b*c*d^2*e - 6*a^2*c*d*e^2 + (b^2*c - 6*a*c^
2)*d^3 + a^2*b*e^3 + (b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 + a*b^2*e^3
 - 6*a^2*c*e^3)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a^2*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11934 vs. 2(499) = 998.
time = 15.07, size = 11934, normalized size = 21.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c^2*d^3*x^3 + 2*(b^2*c - 2*a*c^2)*d^3*x - sqrt(1/2)*(a^2*b^2*c - 4
*a^3*c^2 + (a*b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2)*sqrt(
-((b^5*c^3 - 15*a*b^3*c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^
4 - 24*a^3*c^5)*d^5*e - 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^
3*b^2*c^3 + 36*a^4*c^4)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4
+ 6*(a^3*b^4*c - 6*a^4*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*
c + 60*a^5*b*c^2)*e^6 + (a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64
*a^6*c^6)*sqrt(-(108*a^3*b*c^6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 -
 18*a*b^2*c^7 + 81*a^2*c^8)*d^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18
*(a^2*b^2*c^6 + 9*a^3*c^7)*d^10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4
 + 12*(a^3*b^3*c^4 - 18*a^4*b*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^
4 + 162*a^5*c^5)*d^6*e^6 + 12*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a
^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 1
2*(a^6*b^3*c - 9*a^7*b*c^2)*d*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*
e^12)/(a^6*b^6*c^6 - 12*a^7*b^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b
^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6))*log(-(5*b^4*c^6 - 8
1*a*b^2*c^7 + 324*a^2*c^8)*d^12*x + 3*(3*b^5*c^5 - 65*a*b^3*c^6 + 324*a^2*b
*c^7)*d^11*x*e - 3*(b^6*c^4 - 42*a*b^4*c^5 + 252*a^2*b^2*c^6 + 432*a^3*c^7)
*d^10*x*e^2 - (b^7*c^3 + 3*a*b^5*c^4 + 33*a^2*b^3*c^5 - 2916*a^3*b*c^6)*d^9
```

*x*e^3 - 9*(a*b^6*c^3 - 15*a^2*b^4*c^4 + 195*a^3*b^2*c^5 + 180*a^4*c^6)*d^8
*x*e^4 + 162*(a^3*b^3*c^4 + 12*a^4*b*c^5)*d^7*x*e^5 - 162*(a^4*b^3*c^3 + 12
*a^5*b*c^4)*d^5*x*e^7 + 9*(a^3*b^6*c - 15*a^4*b^4*c^2 + 195*a^5*b^2*c^3 + 1
80*a^6*c^4)*d^4*x*e^8 + (a^3*b^7 + 3*a^4*b^5*c + 33*a^5*b^3*c^2 - 2916*a^6*
b*c^3)*d^3*x*e^9 + 3*(a^4*b^6 - 42*a^5*b^4*c + 252*a^6*b^2*c^2 + 432*a^7*c^
3)*d^2*x*e^10 - 3*(3*a^5*b^5 - 65*a^6*b^3*c + 324*a^7*b*c^2)*d*x*e^11 + (5*
a^6*b^4 - 81*a^7*b^2*c + 324*a^8*c^2)*x*e^12 + 1/2*sqrt(1/2)*((b^8*c^4 - 23
*a*b^6*c^5 + 190*a^2*b^4*c^6 - 672*a^3*b^2*c^7 + 864*a^4*c^8)*d^9 + 9*(a*b^
7*c^4 - 15*a^2*b^5*c^5 + 72*a^3*b^3*c^6 - 112*a^4*b*c^7)*d^8*e + 3*(a^2*b^6
*c^4 + 28*a^3*b^4*c^5 - 272*a^4*b^2*c^6 + 576*a^5*c^7)*d^7*e^2 + (a^2*b^7*c
^3 - 80*a^3*b^5*c^4 + 592*a^4*b^3*c^5 - 1152*a^5*b*c^6)*d^6*e^3 + 15*(a^3*b
^6*c^3 - 8*a^4*b^4*c^4 + 16*a^5*b^2*c^5)*d^5*e^4 - 6*(a^3*b^7*c^2 - 17*a^4*
b^5*c^3 + 88*a^5*b^3*c^4 - 144*a^6*b*c^5)*d^4*e^5 - (a^3*b^8*c - 5*a^4*b^6*
c^2 + 100*a^5*b^4*c^3 - 816*a^6*b^2*c^4 + 1728*a^7*c^5)*d^3*e^6 - 3*(a^4*b^
7*c - 32*a^5*b^5*c^2 + 208*a^6*b^3*c^3 - 384*a^7*b*c^4)*d^2*e^7 - 54*(a^6*b
^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)*d*e^8 - (a^5*b^7 - 17*a^6*b^5*c + 88*a
^7*b^3*c^2 - 144*a^8*b*c^3)*e^9 - ((a^3*b^9*c^4 - 20*a^4*b^7*c^5 + 144*a^5*
b^5*c^6 - 448*a^6*b^3*c^7 + 512*a^7*b*c^8)*d^3 + 3*(a^4*b^8*c^4 - 8*a^5*b^6
*c^5 + 128*a^7*b^2*c^7 - 256*a^8*c^8)*d^2*e - 12*(a^5*b^7*c^4 - 12*a^6*b^5*
c^5 + 48*a^7*b^3*c^6 - 64*a^8*b*c^7)*d*e^2 - (a^5*b^8*c^3 - 24*a^6*b^6*c^4
 + 192*a^7*b^4*c^5 - 640*a^8*b^2*c^6 + 768*a^9*c^7)*e^3)*sqrt(-(108*a^3*b*c^
6*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d
^12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^
10*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b
*c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12
*(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e
^8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d
*e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b
^4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))*sqrt(-((b^5*c^3 - 15*a*b^3*c^4 + 60
*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e - 3*(3*a
^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4)*d^3*e^
3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4*b^2*c^2
 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^4*b^3*c + 60*a^5*b*c^2)*e^6 + (a^3*b
^6*c^3 - 12*a^4*b^4*c^4 + 48*a^5*b^2*c^5 - 64*a^6*c^6)*sqrt(-(108*a^3*b*c^6
*d^9*e^3 + 108*a^6*b*c^3*d^3*e^9 - (b^4*c^6 - 18*a*b^2*c^7 + 81*a^2*c^8)*d^
12 - 12*(a*b^3*c^6 - 9*a^2*b*c^7)*d^11*e - 18*(a^2*b^2*c^6 + 9*a^3*c^7)*d^1
0*e^2 - 9*(2*a^3*b^2*c^5 - 9*a^4*c^6)*d^8*e^4 + 12*(a^3*b^3*c^4 - 18*a^4*b*
c^5)*d^7*e^5 + 2*(a^3*b^4*c^3 + 18*a^4*b^2*c^4 + 162*a^5*c^5)*d^6*e^6 + 12*
(a^4*b^3*c^3 - 18*a^5*b*c^4)*d^5*e^7 - 9*(2*a^5*b^2*c^3 - 9*a^6*c^4)*d^4*e^
8 - 18*(a^6*b^2*c^2 + 9*a^7*c^3)*d^2*e^10 - 12*(a^6*b^3*c - 9*a^7*b*c^2)*d*
e^11 - (a^6*b^4 - 18*a^7*b^2*c + 81*a^8*c^2)*e^12)/(a^6*b^6*c^6 - 12*a^7*b^
4*c^7 + 48*a^8*b^2*c^8 - 64*a^9*c^9)))/(a^3*b^6*c^3 - 12*a^4*b^4*c^4 + 48*a
^5*b^2*c^5 - 64*a^6*c^6))) + sqrt(1/2)*(a^2*b^2*c - 4*a^3*c^2 + (a*b^2*c^2
 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2)*sqrt(-((b^5*c^3 - 15*a*b^3*
c^4 + 60*a^2*b*c^5)*d^6 + 6*(a*b^4*c^3 - 6*a^2*b^2*c^4 - 24*a^3*c^5)*d^5*e

```
- 3*(3*a^2*b^3*c^3 - 92*a^3*b*c^4)*d^4*e^2 - 8*(11*a^3*b^2*c^3 + 36*a^4*c^4
)*d^3*e^3 - 3*(3*a^3*b^3*c^2 - 92*a^4*b*c^3)*d^2*e^4 + 6*(a^3*b^4*c - 6*a^4
*b^2*c^2 - 24*a^5*c^3)*d*e^5 + (a^3*b^5 - 15*a^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8983 vs. 2(499) = 998.
time = 5.56, size = 8983, normalized size = 15.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c^2*d^3*x^3 - 6*a*c^2*d^2*x^3*e + 3*a*b*c*d*x^3*e^2 + b^2*c*d^3*x -
2*a*c^2*d^3*x - a*b^2*x^3*e^3 + 2*a^2*c*x^3*e^3 - 3*a*b*c*d^2*x*e + 6*a^2*c
*d*x*e^2 - a^2*b*x*e^3)/((c*x^4 + b*x^2 + a)*(a*b^2*c - 4*a^2*c^2)) + 1/16*
((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 -
2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^3 - 6*(2*a*b^2*c^4 - 8*a^2
*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a
*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^6*c^3 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 2*a*b^6*c^4 + 64*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 20*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^5 + 28*a^2*b^4*c^5 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^
6 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 10*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 128*a^3*b^2*c^6 + 24*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 192*a^4*c^7 + 2*(b^2 - 4*a*c)*a*b^4*c^
4 - 20*(b^2 - 4*a*c)*a^2*b^2*c^5 + 48*(b^2 - 4*a*c)*a^3*c^6)*d^3*abs(a*b^2*
c - 4*a^2*c^2) + 3*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
```

```
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^
2*d*e^2 + 6*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 - 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*a^2*b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*
c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 16*a^3*b^3*c^5
- 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*a^4*b*c^6 + 2*(b
^2 - 4*a*c)*a^2*b^3*c^4 - 8*(b^2 - 4*a*c)*a^3*b*c^5)*d^2*abs(a*b^2*c - 4*a^
2*c^2)*e + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 224*a^4*b^3*c^8 - 384*a^5*b*c^
9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^4 +
 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^5 -
112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 -
 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 + 19
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^7 + 96*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 + 16*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 48*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 2*(b^
2 - 4*a*c)*a^2*b^5*c^6 + 32*(b^2 - 4*a*c)*a^3*b^3*c^7 - 96*(b^2 - 4*a*c)*a^
4*b*c^8)*d^3 + (2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^
2*c^3)*(a*b^2*c - 4*a^2*c^2)^2*e^3 - 12*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 -
 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 2*a^3*b^4*c^4 + 16
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^2*c^5 + 16*a^4*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6
 - 32*a^5*c^6 + 2*(b^2 - 4*a*c)*a^3*b^2*c^4 - 8*(b^2 - 4*a*c)*a^4*c^5)*d*ab
s(a*b^2*c - 4*a^2*c^2)*e^2 + 12*(2*a^3*b^6*c^6 - 16*a^4*b^4*c^7 + 32*a^5*b^
2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c
^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^
5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5
 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6
 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(...
```

Mupad [B]
time = 8.79, size = 2500, normalized size = 4.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^3*(b*c^2*d^3 - a*b^2*e^3 + 2*a^2*c*e^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^
2))/(2*a*c*(4*a*c - b^2)) - (x*(2*a*c^2*d^3 + a^2*b*e^3 - b^2*c*d^3 - 6*a^2
*c*d*e^2 + 3*a*b*c*d^2*e))/(2*a*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - ata
n(((((6144*a^5*c^7*d^3 + 16*a*b^8*c^3*d^3 - 1024*a^6*b*c^5*e^3 + 6144*a^6*c
^6*d*e^2 - 288*a^2*b^6*c^4*d^3 + 1920*a^3*b^4*c^5*d^3 - 5632*a^4*b^2*c^6*d^
3 + 16*a^3*b^7*c^2*e^3 - 192*a^4*b^5*c^3*e^3 + 768*a^5*b^3*c^4*e^3 - 3072*a
^5*b*c^6*d^2*e + 48*a^2*b^7*c^3*d^2*e - 576*a^3*b^5*c^4*d^2*e - 96*a^3*b^6*
c^3*d*e^2 + 2304*a^4*b^3*c^5*d^2*e + 1152*a^4*b^4*c^4*d*e^2 - 4608*a^5*b^2*
c^5*d*e^2)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) -
 (x*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 -
 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e
^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c
^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^
6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b
^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) -
18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 8
8*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 -
768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e
^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*
d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6
*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4
*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b
^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*
b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)
^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*
b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*
b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*
a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)
))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 -
9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^
6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^
6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6
 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^
5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 1
8432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88
*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 7
```

68*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108*a^4*b^8*c^2*d*e^5 + 4608*a^5*b^2*c^7*d^5*e - 576*a^5*b^6*c^3*d*e^5 + 17664*a^6*b*c^7*d^4*e^2 + 384*a^6*b^4*c^4*d*e^5 + 17664*a^7*b*c^6*d^2*e^4 + 4608*a^7*b^2*c^5*d*e^5 + 6*a*b*c^3*d^5*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b*c*d*e^5*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^5*c^3*e^6 - 72*a^2*c^6*d^6 - a^2*b^6*e^6 - b^4*c^4*d^6 + 14*a*b^2*c^5*d^6 + 16*a^3*b^4*c*e^6 - 74*a^4*b^2*c^2*e^6 - 72*a^3*c^5*d^4*e^2 + 72*a^4*c^4*d^2*e^4 - 102*a^2*b^2*c^4*d^4*e^2 + 44*a^2*b^3*c^3*d^3*e^3 + 9*a^2*b^4*c^2*d^2*e^4 - 174*a^3*b^2*c^3*d^2*e^4 - 6*a*b^3*c^4*d^5*e + 120*a^2*b*c^5*d^5*e - 6*a^2*b^5*c*d*e^5 + 24*a^4*b*c^3*d*e^5 + 144*a^3*b*c^4*d^3*e^3 + 42*a^3*b^3*c^2*d*e^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2))))*((27*a*b^9*c^4*d^6 - b^11*c^3*d^6 - a^3*b^11*e^6 + 3840*a^5*b*c^8*d^6 - 9*a*c^4*d^6*(-(4*a*c - b^2)^9)^(1/2) + 27*a^4*b^9*c*e^6 + 3840*a^8*b*c^5*e^6 + 9*a^4*c*e^6*(-(4*a*c - b^2)^9)^(1/2) - 9216*a^6*c^8*d^5*e - 9216*a^8*c^6*d*e^5 - 288*a^2*b^7*c^5*d^6 + 1504*a^3*b^5*c^6*d^6 - 3840*a^4*b^3*c^7*d^6 - a^3*b^2*e^6*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*e^6 + 1504*a^6*b^5*c^3*e^6 - 3840*a^7*b^3*c^4*e^6 + b^2*c^3*d^6*(-(4*a*c - b^2)^9)^(1/2) - 18432*a^7*c^7*d^3*e^3 + 9*a^2*b^9*c^3*d^4*e^2 - 384*a^3*b^7*c^4*d^4*e^2 + 88*a^3*b^8*c^3*d^3*e^3 + 9*a^3*b^9*c^2*d^2*e^4 + 3744*a^4*b^5*c^5*d^4*e^2 - 768*a^4*b^6*c^4*d^3*e^3 - 384*a^4*b^7*c^3*d^2*e^4 - 13824*a^5*b^3*c^6*d^4*e^2 + 768*a^5*b^4*c^5*d^3*e^3 + 3744*a^5*b^5*c^4*d^2*e^4 + 8192*a^6*b^2*c^6*d^3*e^3 - 13824*a^6*b^3*c^5*d^2*e^4 - 9*a^2*c^3*d^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 9*a^3*c^2*d^2*e^4*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^10*c^3*d^5*e - 6*a^3*b^10*c*d*e^5 + 108*a^2*b^8*c^4*d^5*e - 576*a^3*b^6*c^5*d^5*e + 384*a^4*b^4*c^6*d^5*e + 108...

3.271 $\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=386

$$\frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{\left(bcd^2-4acde+abe^2+\frac{8abcde+b^2(cd^2-ae^2)-4ac(3}{\sqrt{b^2-4ac}}\right.}{2\sqrt{2}\,a\sqrt{c}\,(b^2-4ac)\,\sqrt{}}$$

```
[Out] 1/2*x*(b^2*d^2-2*a*b*d*e-2*a*(-a*e^2+c*d^2)+(a*b*e^2-4*a*c*d*e+b*c*d^2)*x^2
)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2))*(b*c*d^2-4*a*c*d*e+a*b*e^2+(8*a*b*c*d*e+b^2*(-a*e^2+c*d^2)
-4*a*c*(a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/
(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2))*(b*c*d^2-4*a*c*d*e+a*b*e^2+(-8*a*b*c*d*e-b^2*(-a*e^2+c*d^2)+4*
a*c*(a*e^2+3*c*d^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.39, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1219, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd^2-ae^2)+8abcde-4ac(ae^2+3cd^2)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)}{2\sqrt{2}\,a\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{b^2(cd^2-ae^2)+8abcde-4ac(ae^2+3cd^2)}{\sqrt{b^2-4ac}}+abe^2-4acde+bcd^2\right)}{2\sqrt{2}\,a\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(x^2(abe^2-4acde+bcd^2)-2abde-2a(cd^2-ae^2)+b^2d^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d^2 - 2*a*b*d*e - 2*a*(c*d^2 - a*e^2) + (b*c*d^2 - 4*a*c*d*e + a*b*
e^2)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d^2 - 4*a*c*d*e
+ a*b*e^2 + (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^2 + a*e^2))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d^2
 - 4*a*c*d*e + a*b*e^2 - (8*a*b*c*d*e + b^2*(c*d^2 - a*e^2) - 4*a*c*(3*c*d^
2 + a*e^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{(a+bx^2+cx^4)^2}\,dx = \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{-b^2d^2-2abde+2a(3c \cdots}{\cdots}}{2 \cdots}$$

$$= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\Big(bcd^2-4acde+ \cdots}{\cdots}$$

$$= \frac{x(b^2d^2-2abde-2a(cd^2-ae^2)+(bcd^2-4acde+abe^2)\,x^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\Big(bcd^2-4acde+ \cdots}{\cdots}$$

Mathematica [A]
time = 0.69, size = 415, normalized size = 1.08

$$\frac{\frac{2x\left(b^2d^2+2a^2e^2+bcd^2x^2+abe\left(-2d+ex^2\right)-2acd\left(d+2ex^2\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(b^2(cd^2-ae^2)-4ac\left(3cd^2+e\left(\sqrt{b^2-4ac}\,d+ae\right)\right)+b\left(a\sqrt{b^2-4ac}\,e^2+cd\left(\sqrt{b^2-4ac}\,d+8ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2(-cd^2+ae^2)+b\left(a\sqrt{b^2-4ac}\,e^2+cd\left(\sqrt{b^2-4ac}\,d-8ae\right)\right)+4ac\left(3cd^2+e\left(-\sqrt{b^2-4ac}\,d+ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d^2 + 2*a^2*e^2 + b*c*d^2*x^2 + a*b*e*(-2*d + e*x^2) - 2*a*c*d*(
d + 2*e*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d^2 -
 a*e^2) - 4*a*c*(3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d + a*e)) + b*(a*Sqrt[b^2 -
 4*a*c]*e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d + 8*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[2]*(b^2*(-(c*d^2) + a*e^2) + b*(a*Sqrt[b^2 - 4*a*c]*
e^2 + c*d*(Sqrt[b^2 - 4*a*c]*d - 8*a*e)) + 4*a*c*(3*c*d^2 + e*(-(Sqrt[b^2 -
 4*a*c]*d) + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [A]
time = 0.12, size = 427, normalized size = 1.11

method	result
risch	$\frac{-\frac{\left(a b\, e^{2}-4 a c d e+b c\, d^{2}\right) x^{3}}{2 a \left(4 a c-b^{2}\right)}-\frac{\left(2 e^{2} a^{2}-2 a b d e-2 a c\, d^{2}+b^{2} d^{2}\right) x}{2 a \left(4 a c-b^{2}\right)}}{c\, x^{4}+b\, x^{2}+a}+\frac{\sum_{\_R =\mathrm{RootOf}\left(c\, \_Z^{4}+\_Z^{2} b+a\right)}\frac{\left(-\frac{\left(a b\, e^{2}-4 a c d e+b c\, d^{2}\right) \_R^{2}}{4 a c-b^{2}}+\frac{2 e^{2} a^{2}-2 a b d e \cdots}{4 a c \cdots}\right.}{2 c\, \_R^{3}+\_R b \cdots}}{4 a}$
default	$\frac{-\frac{\left(a b\, e^{2}-4 a c d e+b c\, d^{2}\right) x^{3}}{2 a \left(4 a c-b^{2}\right)}-\frac{\left(2 e^{2} a^{2}-2 a b d e-2 a c\, d^{2}+b^{2} d^{2}\right) x}{2 a \left(4 a c-b^{2}\right)}}{c\, x^{4}+b\, x^{2}+a}+\frac{2 c \left(-\frac{\left(-a b\, e^{2} \sqrt{-4 a c+b^{2}}+4 a c d e \sqrt{-4 a c+b^{2}}-b c\, d^{2} \sqrt{-4 a c \cdots}\right.}{8 c \sqrt{-4 a \cdots}}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/a*(a*b*e^2-4*a*c*d*e+b*c*d^2)/(4*a*c-b^2)*x^3-1/2*(2*a^2*e^2-2*a*b*d*
e-2*a*c*d^2+b^2*d^2)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/a/(4*a*c-b^2)*c*(-1
/8*(-a*b*e^2*(-4*a*c+b^2)^(1/2)+4*a*c*d*e*(-4*a*c+b^2)^(1/2)-b*c*d^2*(-4*a*
c+b^2)^(1/2)+4*a^2*c*e^2+a*b^2*e^2-8*a*b*c*d*e+12*a*c^2*d^2-b^2*c*d^2)/c/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-a*b*e^2*(-4*a*c+b^2)^(1/2)+4*a
*c*d*e*(-4*a*c+b^2)^(1/2)-b*c*d^2*(-4*a*c+b^2)^(1/2)-4*a^2*c*e^2-a*b^2*e^2+
8*a*b*c*d*e-12*a*c^2*d^2+b^2*c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d^2 - 4*a*c*d*e + a*b*e^2)*x^3 - (2*a*b*d*e - (b^2 - 2*a*c)*d^2 -
 2*a^2*e^2)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*
a^2*b*c)*x^2) + 1/2*integrate((2*a*b*d*e + (b^2 - 6*a*c)*d^2 + (b*c*d^2 - 4
*a*c*d*e + a*b*e^2)*x^2 - 2*a^2*e^2)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2
*c)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7250 vs. $2(342) = 684$.
time = 2.77, size = 7250, normalized size = 18.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*d^2*x^3 + 2*(b^2 - 2*a*c)*d^2*x + sqrt(1/2)*((a*b^2*c - 4*a^2*c^
2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*
b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*
e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^
3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c
^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - (b^4*c^2
 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 4*a^5*c*d^2*e^6 - 8*(a*b^3*c^2 - 9*a^2*
b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 - a^6*e^8 + 2*(a^3*b^2*
c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 6
4*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*lo
g((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^8*x - 2*(3*b^5*c^2 - 65*a*b^3*
c^3 + 324*a^2*b*c^4)*d^7*x*e + (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 43
2*a^3*c^4)*d^6*x*e^2 + 2*(3*a*b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*x
*e^3 + (3*a^2*b^4*c + 150*a^3*b^2*c^2 + 152*a^4*c^3)*d^4*x*e^4 - 10*(a^3*b^
3*c + 12*a^4*b*c^2)*d^3*x*e^5 - (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d^2*x
*e^6 - 2*(a^4*b^3 + 12*a^5*b*c)*d*x*e^7 + (3*a^5*b^2 + 4*a^6*c)*x*e^8 + 1/2
*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864
*a^4*c^5)*d^6 + 6*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^
4)*d^5*e + 2*(2*a^2*b^6*c - a^3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4
*e^2 - 12*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 1
8*a^4*b^4*c + 96*a^5*b^2*c^2 - 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*b^
3*c + 16*a^6*b*c^2)*d*e^5 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 - ((
a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*
c^5)*d^2 + 2*(a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*d*
e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*sqr
t(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - (b^4*c^2 - 18*a*b^2*c^3 + 81
*a^2*c^4)*d^8 - 4*a^5*c*d^2*e^6 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a
^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 - a^6*e^8 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*
e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(
```

```
(b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24
*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4
*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4 + (a^3*b^6*c - 12*a^4*b^4*c^2
+ 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*
e^5 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 4*a^5*c*d^2*e^6 - 8*(a*b^
3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^2 - a^6*e^8
 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a
^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 6
4*a^6*c^4))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (
a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^4 +
4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d^3*e - 2*(a^2*b^3*c - 52*a^3*b*c^
2)*d^2*e^2 - 8*(3*a^3*b^2*c + 4*a^4*c^2)*d*e^3 + (a^3*b^3 + 12*a^4*b*c)*e^4
 + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt(-(16*a^3
*b*c^2*d^5*e^3 + 8*a^4*b*c*d^3*e^5 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*
d^8 - 4*a^5*c*d^2*e^6 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2
 + 3*a^3*c^3)*d^6*e^2 - a^6*e^8 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*
b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a
^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log((5*b^4*c^3 - 81*a*b^2*c^4 +
324*a^2*c^5)*d^8*x - 2*(3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^7*x*e +
 (b^6*c - 51*a*b^4*c^2 + 336*a^2*b^2*c^3 + 432*a^3*c^4)*d^6*x*e^2 + 2*(3*a*
b^5*c - 27*a^2*b^3*c^2 - 244*a^3*b*c^3)*d^5*x*e^3 + (3*a^2*b^4*c + 150*a^3*
b^2*c^2 + 152*a^4*c^3)*d^4*x*e^4 - 10*(a^3*b^3*c + 12*a^4*b*c^2)*d^3*x*e^5
- (a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d^2*x*e^6 - 2*(a^4*b^3 + 12*a^5*b*c
)*d*x*e^7 + (3*a^5*b^2 + 4*a^6*c)*x*e^8 - 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*
c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^6 + 6*(a*b^7*c - 1
5*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^5*e + 2*(2*a^2*b^6*c - a^
3*b^4*c^2 - 88*a^4*b^2*c^3 + 240*a^5*c^4)*d^4*e^2 - 12*(a^3*b^5*c - 8*a^4*b
^3*c^2 + 16*a^5*b*c^3)*d^3*e^3 - (a^3*b^6 - 18*a^4*b^4*c + 96*a^5*b^2*c^2 -
 160*a^6*c^3)*d^2*e^4 - 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d*e^5 + 2*
(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^6 - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 1
44*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d^2 + 2*(a^4*b^8*c - 8*a^
5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*d*e - 4*(a^5*b^7*c - 12*a^6*b^5*
c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^2)*sqrt(-(16*a^3*b*c^2*d^5*e^3 + 8*a
^4*b*c*d^3*e^5 - (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^8 - 4*a^5*c*d^2*e^
6 - 8*(a*b^3*c^2 - 9*a^2*b*c^3)*d^7*e - 12*(a^2*b^2*c^2 + 3*a^3*c^3)*d^6*e^
2 - a^6*e^8 + 2*(a^3*b^2*c - 11*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4
*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2
*b*c^3)*d^4 + 4*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 6390 vs. $2(342) = 684$.
time = 5.50, size = 6390, normalized size = 16.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d^2*x^3 - 4*a*c*d*x^3*e + a*b*x^3*e^2 + b^2*d^2*x - 2*a*c*d^2*x -
2*a*b*d*x*e + 2*a^2*x*e^2)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*(
(2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(
b^2 - 4*a*c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d^2 - 4*(2*a*b^2*c^3 - 8*a^2*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*
b^2 - 4*a^2*c)^2*d*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c -
 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 2*a*b^6*c^2 + 64*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 28*a^2*b
^4*c^3 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 48*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^4 - 128*a^3*b^2*c^4 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^5 + 192*a^4*c^5 + 2*(b^2 - 4*a*c)*a*b^4*c^2 - 20*(b^2 - 4*a
*c)*a^2*b^2*c^3 + 48*(b^2 - 4*a*c)*a^3*c^4)*d^2*abs(a*b^2 - 4*a^2*c) + (2*a
*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2
*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2*e^2 + 4*(sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*a^2*b^
5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + 8*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^3*c^3 + 16*a^3*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^4 - 32*a^4*b*c^4 + 2*(b^2 - 4*a*c)*a^2*b^3*c^2 - 8*(b^2 - 4*
a*c)*a^3*b*c^3)*d*abs(a*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^3 - 40*a^3*b^5*c^4
```

```
+ 224*a^4*b^3*c^5 - 384*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^7*c + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^5*b*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^2*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^3*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*b^5*c^3 + 32*(b^2 - 4*a*c)*a^3
*b^3*c^4 - 96*(b^2 - 4*a*c)*a^4*b*c^5)*d^2 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*
c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*a^3*b^4*c^2
 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^3 + 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^2*c^3 + 16*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*c^4 - 32*a^5*c^4 + 2*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a*c)*a^4*c^3)
*abs(a*b^2 - 4*a^2*c)*e^2 + 8*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b^2*
c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c +
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 1
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*(b^2
 - 4*a*c)*a^3*b^4*c^3 + 8*(b^2 - 4*a*c)*a^4*b^2*c^4)*d*e - (2*a^3*b^7*c^2 -
 8*a^4*b^5*c^3 - 32*a^5*b^3*c^4 + 128*a^6*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^6*b*c^3 - 32*sqrt(2)*sqrt...
```

Mupad [B]
time = 9.84, size = 2500, normalized size = 6.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b
^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^3*b^6*c^2*e
```

^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(72*a^2*c^5*d^4 + 8*a^4*c^3*e^4 + b^4*c^3*d^4 - 14*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 + 2*a^3*b^2*c^2*e^4 + 16*a^3*c^4*d^2*e^2 + 44*a^2*b^2*c^3*d^2*e^2 + 4*a*b^3*c^3*d^3*e - 80*a^2*b*c^4*d^3*e - 16*a^3*b*c^3*d*e^3 - 12*a^2*b^3*c^2*d*e^3))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e - 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i - (((6144*a^5*c^6*d^2 + 2048*a^6*c^5*e^2 + 16*a*b^8*c^2*d^2 - 288*a^2*b^6*c^3*d^2 + 1920*a^3*b^4*c^4*d^2 - 5632*a^4*b^2*c^5*d^2 - 32*a^

```
3*b^6*c^2*e^2 + 384*a^4*b^4*c^3*e^2 - 1536*a^5*b^2*c^4*e^2 - 2048*a^5*b*c^5
*d*e + 32*a^2*b^7*c^2*d*e - 384*a^3*b^5*c^3*d*e + 1536*a^4*b^3*c^4*d*e)/(8*
(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11*c*d^4
+ a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^2*d^4 - 3840*
a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 768*a^7*b*c^4*e^4 -
b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e + 2048*a^7*c^5*d*e^
3 + 288*a^2*b^7*c^3*d^4 - 1504*a^3*b^5*c^4*d^4 + 3840*a^4*b^3*c^5*d^4 - 96*
a^5*b^5*c^2*e^4 + 512*a^6*b^3*c^3*e^4 + 4*a*b^10*c*d^3*e + 128*a^3*b^7*c^2*
d^2*e^2 - 1344*a^4*b^5*c^3*d^2*e^2 + 5120*a^5*b^3*c^4*d^2*e^2 - 24*a^3*b^8*
c*d*e^3 - 72*a^2*b^8*c^2*d^3*e - 2*a^2*b^9*c*d^2*e^2 + 384*a^3*b^6*c^3*d^3*
e - 256*a^4*b^4*c^4*d^3*e + 256*a^4*b^6*c^2*d*e^3 - 3072*a^5*b^2*c^5*d^3*e
- 768*a^5*b^4*c^3*d*e^3 - 6656*a^6*b*c^5*d^2*e^2 + 2*a^2*c*d^2*e^2*(-(4*a*c
 - b^2)^9)^(1/2) - 4*a*b*c*d^3*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^
7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 384
0*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2
+ 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*
c)))*(-(b^11*c*d^4 + a^3*b^9*e^4 + a^3*e^4*(-(4*a*c - b^2)^9)^(1/2) - 27*a*
b^9*c^2*d^4 - 3840*a^5*b*c^6*d^4 + 9*a*c^2*d^4*(-(4*a*c - b^2)^9)^(1/2) - 7
68*a^7*b*c^4*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^9)^(1/2) + 6144*a^6*c^6*d^3*e
+ 2048*a^7*c^5*d*e^3 + 288*a^2*b^7*c^3*d^4 - 15...
```

3.272 $\int \frac{d+ex^2}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=293

$$\frac{x(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(bd-2ae+\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}}{\quad}$$

```
[Out] 1/2*x*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*d-
2*a*e+(4*a*b*e-12*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*c^(1/2)*(b*d-2*a*e+(-4*a*b*e+12*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2)
)/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1192, 1180, 211}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{4abe-12acd+b^2d}{\sqrt{b^2-4ac}}-2ae+bd\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(cx^2(bd-2ae)-abe-2acd+b^2d)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[c]*(b*d - 2*a*e + (b^2*d - 12*a*c*d + 4*a*b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b*d - 2*a
*e - (b^2*d - 12*a*c*d + 4*a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^2}{\left(a+bx^2+cx^4\right)^2}\,dx &= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-b^2d+6acd-abe-c(bd-2ae)x^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}\\ &= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(c\left(bd-2ae-\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\cdots}{\frac{b}{2}+\frac{1}{2}\sqrt{b\cdots}}}{4a\left(b^2-4ac\right)}\\ &= \frac{x(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(bd-2ae+\frac{b^2d-12acd+4abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\cdots\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-\cdots}}}\end{aligned}$$

Mathematica [A]
time = 0.47, size = 310, normalized size = 1.06

$$\frac{\frac{2x\left(b^2d+b\left(-ae+cdx^2\right)-2ac\left(d+ex^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b^2d+b\left(\sqrt{b^2-4ac}\,d+4ae\right)-2a\left(6cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2d+12acd+b\sqrt{b^2-4ac}\,d-4abe-2a\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d + b*(-(a*e) + c*d*x^2) - 2*a*c*(d + e*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2 - 4*a*c]*d + 4*a
*e) - 2*a*(6*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) +
(Sqrt[2]*Sqrt[c]*(-(b^2*d) + 12*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - 4*a*b*e - 2
```

```
*a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [A]
time = 0.09, size = 464, normalized size = 1.58

method	result
risch	$\frac{\frac{c(2ae-bd)x^3}{2a(4ac-b^2)}+\frac{(abe+2acd-b^2d)x}{2a(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{c(2ae-bd)\_R^2}{4ac-b^2}-\frac{abe-6acd+b^2d}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_Rb}}{4a}$
default	$16c^2\left(\frac{\frac{\left(d\sqrt{-4ac+b^2}+2ae-bd\right)\sqrt{-4ac+b^2}\,x}{16ac\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}+\frac{\left(8a^2ce+6ab^2e-28abcd+3b^3d+12\sqrt{-4ac+b^2}\,acd-3\sqrt{-4ac+b\right.}{16a\left(4ac+3b^2\right)\sqrt{(}}}{4c(4ac-b^2)\sqrt{-4ac+b^2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*(1/16*(d*(-4*a*c+b^2)^(1/2)+2*
a*e-b*d)*(-4*a*c+b^2)^(1/2)/a/c*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/
16*(8*a^2*c*e+6*a*b^2*e-28*a*b*c*d+3*b^3*d+12*(-4*a*c+b^2)^(1/2)*a*c*d-3*(-
4*a*c+b^2)^(1/2)*b^2*d)*(2*b+(-4*a*c+b^2)^(1/2))/a/(4*a*c+3*b^2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)))+1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*(1/16*(-b*d+2*a*e-d*(-4*a*c+
b^2)^(1/2))*(-4*a*c+b^2)^(1/2)/a/c*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))
+1/16*(12*(-4*a*c+b^2)^(1/2)*a*c*d-3*(-4*a*c+b^2)^(1/2)*b^2*d-8*a^2*c*e-6*a
*b^2*e+28*a*b*c*d-3*b^3*d)*(-2*b+(-4*a*c+b^2)^(1/2))/a/(4*a*c+3*b^2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d - 2*a*c*e)*x^3 - (a*b*e - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2
*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate(((
b*c*d - 2*a*c*e)*x^2 + a*b*e + (b^2 - 6*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*
b^2 - 4*a^2*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4539 vs. $2(257) = 514$.
time = 0.92, size = 4539, normalized size = 15.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*d*x^3 + 2*(b^2 - 2*a*c)*d*x - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 +
 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*s
qrt((4*a^3*b*d*e^3 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + a^4*e^4 + 4*(a*b
^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^
4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2 - 64*a^6*c^3))*log(-(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d^4*x + (3*
b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*x*e + 3*(3*a*b^4*c - 28*a^2*b^2*c
^2)*d^2*x*e^2 + (9*a^2*b^3*c - 20*a^3*b*c^2)*d*x*e^3 + (3*a^3*b^2*c + 4*a^4
*c^2)*x*e^4 + 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*
b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3*b^3*c^2 - 112
*a^4*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^2 - 32*a^5*c^3
)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 - ((a^3*b^9 - 20*a^4*b
^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d + (a^4*b^8 - 8*
a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d*e^3 + (b^4 -
18*a*b^2*c + 81*a^2*c^2)*d^4 + a^4*e^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a
^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)))*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^
2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + (b^4 - 18*a*b^2*c +
81*a^2*c^2)*d^4 + a^4*e^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^
3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3
*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c
- 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5
 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*
e + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + a^
4*e^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(-(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^
```

```
4)*d^4*x + (3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*x*e + 3*(3*a*b^4*c
- 28*a^2*b^2*c^2)*d^2*x*e^2 + (9*a^2*b^3*c - 20*a^3*b*c^2)*d*x*e^3 + (3*a^3
*b^2*c + 4*a^4*c^2)*x*e^4 - 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c + 190*a^2*b^4*
c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*b^5*c + 72*a^3
*b^3*c^2 - 112*a^4*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c + 32*a^4*b^2*c^
2 - 32*a^5*c^3)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*e^3 - ((a^3*
b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*d +
 (a^4*b^8 - 8*a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*sqrt((4*a^3*b*d
*e^3 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + a^4*e^4 + 4*(a*b^3 - 9*a^2*b*c
)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b
^2*c^2 - 64*a^9*c^3)))*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*
b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 + (a^3*b^6
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + (b^4 -
 18*a*b^2*c + 81*a^2*c^2)*d^4 + a^4*e^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(
a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a
^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1
/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^
2)*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c -
24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*e^2 - (a^3*b^6 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((4*a^3*b*d*e^3 + (b^4 - 18*a*b^2*c + 81*a^2
*c^2)*d^4 + a^4*e^4 + 4*(a*b^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d
^2*e^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(-(5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*d^4*x + (3*b^5*c - 65*a*b^3*c^2 + 324*a^2*b*c^3)*d^3*x*e +
 3*(3*a*b^4*c - 28*a^2*b^2*c^2)*d^2*x*e^2 + (9*a^2*b^3*c - 20*a^3*b*c^2)*d*
x*e^3 + (3*a^3*b^2*c + 4*a^4*c^2)*x*e^4 + 1/2*sqrt(1/2)*((b^8 - 23*a*b^6*c
+ 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*d^3 + 3*(a*b^7 - 15*a^2*
b^5*c + 72*a^3*b^3*c^2 - 112*a^4*b*c^3)*d^2*e + 3*(a^2*b^6 - 10*a^3*b^4*c +
 32*a^4*b^2*c^2 - 32*a^5*c^3)*d*e^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2
)*e^3 + ((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*
a^7*b*c^4)*d + (a^4*b^8 - 8*a^5*b^6*c + 128*a^7*b^2*c^3 - 256*a^8*c^4)*e)*s
qrt((4*a^3*b*d*e^3 + (b^4 - 18*a*b^2*c + 81*a^2*c^2)*d^4 + a^4*e^4 + 4*(a*b
^3 - 9*a^2*b*c)*d^3*e + 6*(a^2*b^2 - 3*a^3*c)*d^2*e^2)/(a^6*b^6 - 12*a^7*b^
4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-((b^5 - 15*a*b^3*c + 60*a^2*b*c^
2)*d^2 + 2*(a*b^4 - 6*a^2*b^2*c - 24*a^3*c^2)*d*e + (a^2*b^3 + 12*a^3*b*c)*
e^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4433 vs. $2(257) = 514$.

time = 5.20, size = 4433, normalized size = 15.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d*x^3 - 2*a*c*x^3*e + b^2*d*x - 2*a*c*d*x - a*b*x*e)/((c*x^4 + b*x
^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d - 2*
(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*
c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^
2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2
 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(
b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*d*abs(a*b^2 - 4*a^2*c)
 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*
b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3 + 2*(b^2 - 4*a*c)*a^2*b
^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*abs(a*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^2 -
 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

```
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 -
4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*d + 4*(2*a^3*b^6*c^2 - 16*
a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*e)*
arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 -
 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^
3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*
b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2
 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*d - 2*(2*a*b^
2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a
*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*a
^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 1...
```

Mupad [B]
time = 9.39, size = 2500, normalized size = 8.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4
*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*
a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2)) - (x*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)
^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*
```

c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*
a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2
 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e
 + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b
*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
 + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5
)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*
c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 +
 a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840
*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1
504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b
^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6
*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1
536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4
096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7
*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 +
 b^4*c^3*d^2 - 14*a*b^2*c^4*d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40
*a^2*b*c^4*d*e))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^
2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(
1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*
c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2
+ 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2)
 + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*
c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3
 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6*d - 28
8*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*
e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c^5
e)/(8(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11
*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c -
b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288
*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5
*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)
^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128
*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2
))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 -
16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c
^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c - b^2)^9)^
(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 - 768*a^6*b*c
^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d^2 + 3840*a
^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a*b^9*c*d^2
- 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2*b^8*c*d*e
+ 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d*e + 2*a*b*
d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c

```
+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)
))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 - 14*a*b^2*c^4*
d^2 + 10*a^2*b^2*c^3*e^2 + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2*(-(4*a*c
 - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5*d^2 -
768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288*a^2*b^7*c^2*d^2 - 1504*a^3*b^5*c^3*d
^2 + 3840*a^4*b^3*c^4*d^2 - 96*a^4*b^5*c^2*e^2 + 512*a^5*b^3*c^3*e^2 - 27*a
*b^9*c*d^2 - 9*a*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 3072*a^6*c^5*d*e - 36*a^2
*b^8*c*d*e + 192*a^3*b^6*c^2*d*e - 128*a^4*b^4*c^3*d*e - 1536*a^5*b^2*c^4*d
*e + 2*a*b*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*
a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a
^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^5*c^6*d - 288*a^2*b^6*c^3*d + 1920*a^3*b
^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*
a^4*b^3*c^4*e + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11*d^2 + a^2*b^9*e^2 + a^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c
^5*d^2 - 768*a^6*b*c^4*e^2 + 2*a*b^10*d*e + 288...
```

3.273 $\int \frac{1}{\left(a+bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=252

$$\frac{x(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(b^2-12a\right.}{2\sqrt{}}$$

```
[Out] 1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/
2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)
^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1106, 1180, 211}

$$\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(-2ac+b^2+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx^2+cx^4)^2}\,dx = \frac{x(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{b^2-2ac-2(b^2-4ac)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\,(b^2-4ac)}$$

$$= \frac{x(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}}}{4a\,(b^2-4ac)^{3/2}}$$

$$= \frac{x(b^2-2ac+bcx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{b-\sqrt{}}}\right.}{2\sqrt{2}\,a\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.27, size = 243, normalized size = 0.96

$$\frac{\frac{2x(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [A]
time = 0.06, size = 320, normalized size = 1.27

method	result
risch	$\frac{-\frac{b c x^3}{2a(4ac-b^2)}+\frac{(2ac-b^2)x}{2a(4ac-b^2)}}{c x^4+b x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2 b+a)}\frac{\left(-\frac{bc\_R^2}{4ac-b^2}+\frac{6ac-b^2}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_Rb}}{4a}$
default	$16c^2\left(-\frac{\frac{\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16a c^2\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}+\frac{\left(b\sqrt{-4ac+b^2}+12ac-b^2\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{16ac\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{4\sqrt{-4ac+b^2}\,(4ac-b^2)}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(-1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*(1/16/a/c^2*(b*(-4*a*c+b^2)^(1/
2)+4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/16*(b*(-4*a*c+b^2)
^(1/2)+12*a*c-b^2)/a/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^
2)*(-1/16/a/c^2*(4*a*c-b^2-b*(-4*a*c+b^2)^(1/2))*x/(x^2+1/2*b/c-1/2/c*(-4*a
*c+b^2)^(1/2))-1/16*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))/a/c*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. $2(206) = 412$.
time = 0.44, size = 2309, normalized size = 9.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*

```
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. 2(206) = 412.
time = 4.63, size = 2682, normalized size = 10.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1
/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*
b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^
3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c
)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
```

*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) + 1/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c))

Mupad [B]
time = 6.26, size = 2500, normalized size = 9.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 +
 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4
*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
 - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 38
40*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840
*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*
b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
- b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 384
0*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2
 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2
)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4
*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 409
6*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b
^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a
^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-
(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1
504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c
^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^
5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^
5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11
+ b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
```

```
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*
b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16
*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 24
0*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(
1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4
- 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^
8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(10
24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2
*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*
b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27
*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 2
4*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5...
```

3.274 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=660

$$\frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)\,x^2)}{2a\,(b^2-4ac)\,(cd^2-bde+ae^2)\,(a+bx^2+cx^4)}-\frac{\sqrt{c}\,e^2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{\sqrt{2}\,\sqrt{b-\sqrt{b^2-4ac}}\,\left(cd^2-bde+ae^2\right)^2}$$

```
[Out] 1/2*x*(b^2*c*d-2*a*c^2*d-b^3*e+3*a*b*c*e+c*(2*a*c*e-b^2*e+b*c*d)*x^2)/a/(-4
*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)+e^(7/2)*arctan(x*e^(1/2)/d^(1
/2))/(a*e^2-b*d*e+c*d^2)^2/d^(1/2)-1/2*e^2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*
d*e+c*d^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*d-b^2*e+2*a*c*e+(8*a*b*c*e-1
2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*
d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*e^2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(
a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1
/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*d-b^2*e+2*a*c*e+(-8*
a*b*c*e+12*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/(a*e^2
-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 1.87, antiderivative size = 660, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1252, 211, 1192, 1180}

$$-\frac{\sqrt{c}\,e^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\,\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)^2}-\frac{\sqrt{c}\,e^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\,\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)^2}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{8abce-12ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)}{2\sqrt{2}\,a\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}\,\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{8abce-12ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+2ace+b^2(-e)+bcd\right)}{2\sqrt{2}\,a\,(b^2-4ac)\,\sqrt{\sqrt{b^2-4ac}+b}\,\left(ae^2-bde+cd^2\right)}+\frac{e^{7/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,\left(ae^2-bde+cd^2\right)^2}+\frac{x\left(cx^2(2ace+b^2(-e)+bcd)+3abce-2ac^2d+b^3(-e)+b^2cd\right)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (x*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e + c*(b*c*d - b^2*e + 2*a*c*e)*x
^2))/(2*a*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x^2 + c*x^4)) - (Sqr
t[c]*e^2*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 -
b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b^2*e + 2*a*c*e + (b^2*c*d - 12*a*c^2
*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*e^2*(e + (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) + (Sqrt[c]*(b*c*d - b
^2*e + 2*a*c*e - (b^2*c*d - 12*a*c^2*d - b^3*e + 8*a*b*c*e)/Sqrt[b^2 - 4*a*
```

```
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(
b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)^2}\,dx = \int \left(\frac{e^4}{(cd^2-bde+ae^2)^2(d+ex^2)} + \frac{cd-be-cex^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)^2} - \right.$$

$$= -\frac{e^2\int \frac{-cd+be+cex^2}{a+bx^2+cx^4}\,dx}{(cd^2-bde+ae^2)^2} + \frac{e^4\int \frac{1}{d+ex^2}\,dx}{(cd^2-bde+ae^2)^2} + \frac{\int \frac{cd-be-cex^2}{(a+bx^2+cx^4)^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)x^2)}{2a(b^2-4ac)(cd^2-bde+ae^2)(a+bx^2+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\right.}{\sqrt{d}\,(cd^2-bd}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)x^2)}{2a(b^2-4ac)(cd^2-bde+ae^2)(a+bx^2+cx^4)} - \frac{\sqrt{c}\,e^2\left(e-\frac{}{\sqrt{}}\right.}{\sqrt{2}\sqrt{b}}$$

$$= \frac{x(b^2cd-2ac^2d-b^3e+3abce+c(bcd-b^2e+2ace)x^2)}{2a(b^2-4ac)(cd^2-bde+ae^2)(a+bx^2+cx^4)} - \frac{\sqrt{c}\,e^2\left(e-\frac{}{\sqrt{}}\right.}{\sqrt{2}\sqrt{b}}$$

Mathematica [A]
time = 1.74, size = 708, normalized size = 1.07

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*(c*d^2 + e*(-(b*d) + a*e))*x*(b^3*e - b*c*(3*a*e + c*d*x^2) + 2*a*c^2*(
d - e*x^2) + b^2*c*(-d + e*x^2)))/(a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) +
(Sqrt[2]*Sqrt[c]*(b^4*d*e^2 + 2*a*c*(-6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c]*e^3
 + c*d*e*(Sqrt[b^2 - 4*a*c]*d - 14*a*e)) + b^3*e*(-2*c*d^2 + e*(Sqrt[b^2 -
4*a*c]*d - 3*a*e)) + b^2*(c^2*d^3 - 3*a*Sqrt[b^2 - 4*a*c]*e^3 - c*d*e*(2*Sq
rt[b^2 - 4*a*c]*d + 3*a*e)) + b*c*(a*e^2*(-(Sqrt[b^2 - 4*a*c]*d) + 16*a*e)
+ c*d^2*(Sqrt[b^2 - 4*a*c]*d + 20*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[2]*Sqrt[c]*(-(b^4*d*e^2) - b^2*(c^2*d^3 + 3*a*Sqrt[b^2 - 4*a*c]*e^3
 + c*d*e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e)) + b^3*e*(2*c*d^2 + e*(Sqrt[b^2 -
4*a*c]*d + 3*a*e)) + 2*a*c*(6*c^2*d^3 + 5*a*Sqrt[b^2 - 4*a*c]*e^3 + c*d*e*(
Sqrt[b^2 - 4*a*c]*d + 14*a*e)) + b*c*(c*d^2*(Sqrt[b^2 - 4*a*c]*d - 20*a*e)
- a*e^2*(Sqrt[b^2 - 4*a*c]*d + 16*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

```
+ (4*e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d])/(4*(c*d^2 + e*(-(b*d) +
a*e))^2)
```

Maple [A]
time = 0.42, size = 850, normalized size = 1.29

method	result
default	$-\frac{\frac{\frac{c\left(2a^{2}c\,e^{3}-a\,b^{2}e^{3}-abcd\,e^{2}+2a\,c^{2}d^{2}e+b^{3}d\,e^{2}-2b^{2}c\,d^{2}e+b\,c^{2}d^{3}\right)x^{3}}{2a\left(4ac-b^{2}\right)}+\frac{\left(3a^{2}b\,e^{3}c-2a^{2}c^{2}d\,e^{2}-a\,b^{3}e^{3}-2a\,b^{2}cd\,e^{2}+5ab\,c^{2}d^{2}e-2a\,c^{3}d^{3}+b^{4}d\,e^{2}-2b\right.}{2a\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/(a*e^2-b*d*e+c*d^2)^2*((1/2*c*(2*a^2*c*e^3-a*b^2*e^3-a*b*c*d*e^2+2*a*c^2
*d^2*e+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)/a/(4*a*c-b^2)*x^3+1/2*(3*a^2*b*c*
e^3-2*a^2*c^2*d*e^2-a*b^3*e^3-2*a*b^2*c*d*e^2+5*a*b*c^2*d^2*e-2*a*c^3*d^3+b
^4*d*e^2-2*b^3*c*d^2*e+b^2*c^2*d^3)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/a/(4
*a*c-b^2)*c*(-1/8*(10*a^2*c*e^3*(-4*a*c+b^2)^(1/2)-3*a*b^2*e^3*(-4*a*c+b^2)
^(1/2)-a*b*c*d*e^2*(-4*a*c+b^2)^(1/2)+2*a*c^2*d^2*e*(-4*a*c+b^2)^(1/2)+b^3*
d*e^2*(-4*a*c+b^2)^(1/2)-2*b^2*c*d^2*e*(-4*a*c+b^2)^(1/2)+b*c^2*d^3*(-4*a*c
+b^2)^(1/2)+16*a^2*b*e^3*c-28*a^2*c^2*d*e^2-3*a*b^3*e^3-3*a*b^2*c*d*e^2+20*
a*b*c^2*d^2*e-12*a*c^3*d^3+b^4*d*e^2-2*b^3*c*d^2*e+b^2*c^2*d^3)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(10*a^2*c*e^3*(-4*a*c+b^2)^(1/2)-3*a*b^2*
e^3*(-4*a*c+b^2)^(1/2)-a*b*c*d*e^2*(-4*a*c+b^2)^(1/2)+2*a*c^2*d^2*e*(-4*a*c
+b^2)^(1/2)+b^3*d*e^2*(-4*a*c+b^2)^(1/2)-2*b^2*c*d^2*e*(-4*a*c+b^2)^(1/2)+b
*c^2*d^3*(-4*a*c+b^2)^(1/2)-16*a^2*b*e^3*c+28*a^2*c^2*d*e^2+3*a*b^3*e^3+3*a
*b^2*c*d*e^2-20*a*b*c^2*d^2*e+12*a*c^3*d^3-b^4*d*e^2+2*b^3*c*d^2*e-b^2*c^2*
d^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))+e^4/(a*e^2-b*d*e+c*d^2)^2/(d*e
)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c^2*d^4 - 2*b*c*d^3*e - 2*a*b*d*e^3 + (
b^2*e^2 + 2*a*c*e^2)*d^2 + a^2*e^4)*sqrt(d)) + 1/2*((b*c^2*d - b^2*c*e + 2*
a*c^2*e)*x^3 - (b^3*e - 3*a*b*c*e - (b^2*c - 2*a*c^2)*d)*x)/(a^3*b^2*e^2 -
4*a^4*c*e^2 + (a^2*b^2*c*e^2 - 4*a^3*c^2*e^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^2
- (a*b^3*c*e - 4*a^2*b*c^2*e)*d)*x^4 + (a^2*b^2*c - 4*a^3*c^2)*d^2 + (a^2*b
^3*e^2 - 4*a^3*b*c*e^2 + (a*b^3*c - 4*a^2*b*c^2)*d^2 - (a*b^4*e - 4*a^2*b^2
*c*e)*d)*x^2 - (a^2*b^3*e - 4*a^3*b*c*e)*d) + 1/2*integrate(-(3*a*b^3*e^3 -
 13*a^2*b*c*e^3 - (b^2*c^2 - 6*a*c^3)*d^3 + (2*b^3*c*e - 11*a*b*c^2*e)*d^2
- (b*c^3*d^3 - 3*a*b^2*c*e^3 + 10*a^2*c^2*e^3 - 2*(b^2*c^2*e - a*c^3*e)*d^2
 + (b^3*c*e^2 - a*b*c^2*e^2)*d)*x^2 - (b^4*e^2 - 2*a*b^2*c*e^2 - 14*a^2*c^2
*e^2)*d)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2*e^4 - 4*a^4*c*e^4 + (a*b^2*c^2 -
4*a^2*c^3)*d^4 - 2*(a*b^3*c*e - 4*a^2*b*c^2*e)*d^3 + (a*b^4*e^2 - 2*a^2*b^2
*c*e^2 - 8*a^3*c^2*e^2)*d^2 - 2*(a^2*b^3*e^3 - 4*a^3*b*c*e^3)*d)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 36949 vs. 2(587) = 1174.
time = 12.94, size = 36949, normalized size = 55.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*((2*a^2*b^7*c^8 - 40*a^3*b^5*c^9 + 224*a^4*b^3*c^10 - 384*a^5*b*c^11
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 2
```

0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 11
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 3
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^8 + 192*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 + 96*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 + 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^9 - 48*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^10 - 2*(b^2
 - 4*a*c)*a^2*b^5*c^8 + 32*(b^2 - 4*a*c)*a^3*b^3*c^9 - 96*(b^2 - 4*a*c)*a^4
*b*c^10)*d^11 - 2*(6*a^2*b^8*c^7 - 116*a^3*b^6*c^8 + 640*a^4*b^4*c^9 - 1088
*a^5*b^2*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^8*c^5 + 58*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b^6*c^6 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^7*c^6 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^4*b^4*c^7 - 92*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b^5*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^6*c^7 + 544*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b^2*c^8 + 272*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^4*b^3*c^8 + 46*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^4*c^8 - 136*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^2*c^9 - 6*(b^2 - 4*a*c)*a^2*b^6*c^7 + 92*(b^2 - 4*a*c)*a^3*b^4*c^8
 - 272*(b^2 - 4*a*c)*a^4*b^2*c^9)*d^10*e + (30*a^2*b^9*c^6 - 542*a^3*b^7*c^
7 + 2744*a^4*b^5*c^8 - 3616*a^5*b^3*c^9 - 2432*a^6*b*c^10 - 15*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 + 271*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 30*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - 1372*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^6 - 422*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 15*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 1808*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^7 + 1056*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 211*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 1216*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^8 + 608*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 528*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 304*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^9 - 30*(b
^2 - 4*a*c)*a^2*b^7*c^6 + 422*(b^2 - 4*a*c)*a^3*b^5*c^7 - 1056*(b^2 - 4*a*c
)*a^4*b^3*c^8 - 608*(b^2 - 4*a*c)*a^5*b*c^9)*d^9*e^2 - (40*a^2*b^10*c^5 - 6
34*a^3*b^8*c^6 + 2448*a^4*b^6*c^7 + 608*a^5*b^4*c^8 - 11264*a^6*b^2*c^9 - 2
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^3 +
317*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^4 +
 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 -
 1224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^5
 - 474*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^

```
5 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^
5 - 304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c
^6 + 552*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*
c^6 + 237*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6
*c^6 + 5632*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b
^2*c^7 + 2816*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5
*b^3*c^7 - 276*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b^4*c^7 - 1408*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^5*b^2*c^8 - 40*(b^2 - 4*a*c)*a^2*b^8*c^5 + 474*(b^2 - 4*a*c)*a^3*b^6*c^6
- 552*(b^2 - 4*a*c)*a^4*b^4*c^7 - 2816*(b^2 - 4*a*c)*a^5*b^2*c^8)*d^8*e^3 -
 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 14*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^5*c^5 - 2*a*b^6*c^5 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^2*c^6 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^6 + 28*a^2*b^4*c^6 - 96*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b*c^7 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^
7 - 128*a^3*b^2*c^7 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^8 +
192*a^4*c^8 + 2*(b^2 - 4*a*c)*a*b^4*c^5 - 20*(b^2 - 4*a*c)*a^2*b^2*c^6 + 48
*(b^2 - 4*a*c)*a^3*c^7)*d^7*abs(a*b^2*c^2*d^4 -...
```

Mupad [B]
time = 16.46, size = 2500, normalized size = 3.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - atan(((((((1048576*a^13*c^8*e^16 + 256*a^7*b^12*c^2*e^16 - 6144*a^8*b^10*
c^3*e^16 + 61440*a^9*b^8*c^4*e^16 - 327680*a^10*b^6*c^5*e^16 + 983040*a^11*
b^4*c^6*e^16 - 1572864*a^12*b^2*c^7*e^16 - 196608*a^6*c^15*d^14*e^2 - 91750
4*a^7*c^14*d^12*e^4 - 589824*a^8*c^13*d^10*e^6 + 3932160*a^9*c^12*d^8*e^8 +
 10158080*a^10*c^11*d^6*e^10 + 10616832*a^11*c^10*d^4*e^12 + 5308416*a^12*c
^9*d^2*e^14 - 2816*a^2*b^8*c^11*d^14*e^2 + 22656*a^2*b^9*c^10*d^13*e^3 - 78
848*a^2*b^10*c^9*d^12*e^4 + 154112*a^2*b^11*c^8*d^11*e^5 - 182784*a^2*b^12*
c^7*d^10*e^6 + 130816*a^2*b^13*c^6*d^9*e^7 - 50176*a^2*b^14*c^5*d^8*e^8 + 4
608*a^2*b^15*c^4*d^7*e^9 + 3328*a^2*b^16*c^3*d^6*e^10 - 896*a^2*b^17*c^2*d^
5*e^11 + 24576*a^3*b^6*c^12*d^14*e^2 - 198656*a^3*b^7*c^11*d^13*e^3 + 68454
4*a^3*b^8*c^10*d^12*e^4 - 1291520*a^3*b^9*c^9*d^11*e^5 + 1403776*a^3*b^10*c
^8*d^10*e^6 - 798336*a^3*b^11*c^7*d^9*e^7 + 89856*a^3*b^12*c^6*d^8*e^8 + 15
5136*a^3*b^13*c^5*d^7*e^9 - 77440*a^3*b^14*c^4*d^6*e^10 + 5504*a^3*b^15*c^3
*d^5*e^11 + 2560*a^3*b^16*c^2*d^4*e^12 - 106496*a^4*b^4*c^13*d^14*e^2 + 864
256*a^4*b^5*c^12*d^13*e^3 - 2924544*a^4*b^6*c^11*d^12*e^4 + 5181440*a^4*b^7
*c^10*d^11*e^5 - 4686080*a^4*b^8*c^9*d^10*e^6 + 1045376*a^4*b^9*c^8*d^9*e^7
```

+ 1900544*a^4*b^10*c^7*d^8*e^8 - 1732096*a^4*b^11*c^6*d^7*e^9 + 390400*a^4*b^12*c^5*d^6*e^10 + 112000*a^4*b^13*c^4*d^5*e^11 - 40960*a^4*b^14*c^3*d^4*e^12 - 3840*a^4*b^15*c^2*d^3*e^13 + 229376*a^5*b^2*c^14*d^14*e^2 - 1867776*a^5*b^3*c^13*d^13*e^3 + 6078464*a^5*b^4*c^12*d^12*e^4 - 9297920*a^5*b^5*c^11*d^11*e^5 + 4055040*a^5*b^6*c^10*d^10*e^6 + 7788544*a^5*b^7*c^9*d^9*e^7 - 12657664*a^5*b^8*c^8*d^8*e^8 + 6130176*a^5*b^9*c^7*d^7*e^9 + 734080*a^5*b^10*c^6*d^6*e^10 - 1442560*a^5*b^11*c^5*d^5*e^11 + 168960*a^5*b^12*c^4*d^4*e^12 + 78080*a^5*b^13*c^3*d^3*e^13 + 3200*a^5*b^14*c^2*d^2*e^14 - 4587520*a^6*b^2*c^13*d^12*e^4 + 3080192*a^6*b^3*c^12*d^11*e^5 + 12001280*a^6*b^4*c^11*d^10*e^6 - 31076352*a^6*b^5*c^10*d^9*e^7 + 27475968*a^6*b^6*c^9*d^8*e^8 - 2088960*a^6*b^7*c^8*d^7*e^9 - 12205312*a^6*b^8*c^7*d^6*e^10 + 6043520*a^6*b^9*c^6*d^5*e^11 + 631808*a^6*b^10*c^5*d^4*e^12 - 610304*a^6*b^11*c^4*d^3*e^13 - 71936*a^6*b^12*c^3*d^2*e^14 - 21725184*a^7*b^2*c^12*d^10*e^6 + 30801920*a^7*b^3*c^11*d^9*e^7 - 8028160*a^7*b^4*c^10*d^8*e^8 - 32260096*a^7*b^5*c^9*d^7*e^9 + 37101568*a^7*b^6*c^8*d^6*e^10 - 7182336*a^7*b^7*c^7*d^5*e^11 - 7609856*a^7*b^8*c^6*d^4*e^12 + 2112256*a^7*b^9*c^5*d^3*e^13 + 661632*a^7*b^10*c^4*d^2*e^14 - 30146560*a^8*b^2*c^11*d^8*e^8 + 55050240*a^8*b^3*c^10*d^7*e^9 - 34365440*a^8*b^4*c^9*d^6*e^10 - 16429056*a^8*b^5*c^8*d^5*e^11 + 24600576*a^8*b^6*c^7*d^4*e^12 - 1683456*a^8*b^7*c^6*d^3*e^13 - 3151616*a^8*b^8*c^5*d^2*e^14 - 10977280*a^9*b^2*c^10*d^6*e^10 + 47022080*a^9*b^3*c^9*d^5*e^11 - 30621696*a^9*b^4*c^8*d^4*e^12 - 9232384*a^9*b^5*c^7*d^3*e^13 + 7970816*a^9*b^6*c^6*d^2*e^14 + 4325376*a^10*b^2*c^9*d^4*e^12 + 25493504*a^10*b^3*c^8*d^3*e^13 - 9117696*a^10*b^4*c^7*d^2*e^14 + 491520*a^11*b^2*c^8*d^2*e^14 - 4947968*a^12*b*c^8*d*e^15 + 128*a*b^10*c^10*d^14*e^2 - 1024*a*b^11*c^9*d^13*e^3 + 3584*a*b^12*c^8*d^12*e^4 - 7168*a*b^13*c^7*d^11*e^5 + 8960*a*b^14*c^6*d^10*e^6 - 7168*a*b^15*c^5*d^9*e^7 + 3584*a*b^16*c^4*d^8*e^8 - 1024*a*b^17*c^3*d^7*e^9 + 128*a*b^18*c^2*d^6*e^10 + 1605632*a^6*b*c^14*d^13*e^3 - 1408*a^6*b^13*c^2*d*e^15 + 7012352*a^7*b*c^13*d^11*e^5 + 33152*a^7*b^11*c^3*d*e^15 + 7045120*a^8*b*c^12*d^9*e^7 - 324480*a^8*b^9*c^4*d*e^15 - 9830400*a^9*b*c^11*d^7*e^9 + 1689600*a^9*b^7*c^5*d*e^15 - 25722880*a^10*b*c^10*d^5*e^11 - 4935680*a^10*b^5*c^6*d*e^15 - 19202048*a^11*b*c^9*d^3*e^13 + 7667712*a^11*b^3*c^7*d*e^15)/(16*(a^6*b^8*e^8 + 256*a^6*c^8*d^8 + 256*a^10*c^4*e^8 - 16*a^7*b^6*c*e^8 - 4*a^5*b^9*d*e^7 + a^2*b^8*c^4*d^8 - 16*a^3*b^6*c^5*d^8 + 96*a^4*b^4*c^6*d^8 - 256*a^5*b^2*c^7*d^8 + 96*a^8*b^4*c^2*e^8 - 256*a^9*b^2*c^3*e^8 + a^2*b^12*d^4*e^4 - 4*a^3*b^11*d^3*e^5 + 6*a^4*b^10*d^2*e^6 + 1024*a^7*c^7*d^6*e^2 + 1536*a^8*c^6*d^4*e^4 + 1024*a^9*c^5*d^2*e^6 + 6*a^2*b^10*c^2*d^6*e^2 - 92*a^3*b^8*c^3*d^6*e^2 + 52*a^3*b^9*c^2*d^5*e^3 + 512*a^4*b^6*c^4*d^6*e^2 - 192*a^4*b^7*c^3*d^5*e^3 - 90*a^4*b^8*c^2*d^4*e^4 - 1152*a^5*b^4*c^5*d^6*e^2 - 128*a^5*b^5*c^4*d^5*e^3 + 800*a^5*b^6*c^3*d^4*e^4 - 192*a^5*b^7*c^2*d^3*e^5 + 512*a^6*b^2*c^6*d^6*e^2 + 2048*a^6*b^3*c^5*d^5*e^3 - 2240*a^6*b^4*c^4*d^4*e^4 - 128*a^6*b^5*c^3*d^3*e^5 + 512*a^6*b^6*c^2*d^2*e^6 + 1536*a^7*b^2*c^5*d^4*e^4 + 2048*a^7*b^3*c^4*d^3*e^5 - 1152*a^7*b^4*c^3*d^2*e^6 + 512*a^8*b^2*c^4*d^2*e^6 - 1024*a^6*b*c^7*d^7*e + 64*a^6*b^7*c*d*e^7 - 1024*a^9*b*c^4*d*e^7 - 4*a^2*b^9*c^3*d^7*e - 4*a^2*b^11*c*d^5*e^3 + 64*a^3*b^7*c^4*d^7*e - 4*a^3*b^10*c*d^4*e^4 - 384*a^4*b^5*c^5*d^7*e + 52*a

```
^4*b^9*c*d^3*e^5 + 1024*a^5*b^3*c^6*d^7*e - 92*a^5*b^8*c*d^2*e^6 - 3072*a^7
*b*c^6*d^5*e^3 - 384*a^7*b^5*c^2*d*e^7 - 3072*a^8*b*c^5*d^3*e^5 + 1024*a^8*
b^3*c^3*d*e^7)) - (x*((27*a*b^9*c^5*d^6 - b^11*c^4*d^6 - b^15*d^2*e^4 - 9*a
^2*b^13*e^6 + 3840*a^5*b*c^9*d^6 - 9*a*c^5*d^6*(-(4*a*c - b^2)^9)^(1/2) + 2
13*a^3*b^11*c*e^6 - 26880*a^8*b*c^6*e^6 + 3072*a^6*c^9*d^5*e + 35840*a^8*c^
7*d*e^5 + 4*b^12*c^3*d^5*e + 4*b^14*c*d^3*e^3 -...
```

3.275 $\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=1077

$$\frac{e^4x}{2d\left(cd^2-bde+ae^2\right)^2\left(d+ex^2\right)}+\frac{x(abce(2cd-be)+\left(b^2-2ac\right)\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)-c(2b^2cde-4}{2a\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)^2\left(a+bx^2+\right.}$$

```
[Out] 1/2*e^4*x/d/(a*e^2-b*d*e+c*d^2)^2/(e*x^2+d)+1/2*x*(a*b*c*e*(-b*e+2*c*d)+(-2
*a*c+b^2)*(c^2*d^2+b^2*e^2-c*e*(a*e+2*b*d))-c*(2*b^2*c*d*e-4*a*c^2*d*e-b^3*
e^2-b*c*(-3*a*e^2+c*d^2))*x^2)/a/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)^2/(c*x^4+
b*x^2+a)+1/2*e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)^
2+2*e^(7/2)*(-b*e+2*c*d)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)^3/d^
(1/2)+e^2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^
(1/2)*(3*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-c*e*(3*b*d+a*e+2*d*(-4*a*c+b^
2)^(1/2)))/(a*e^2-b*d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b
^4*e^2-b^3*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-4*a*c^2*(3*c*d^2-e*(3*a*e+d*(-4*a
*c+b^2)^(1/2)))-b*c*(3*a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(16*a*e+d*(-4*a*c+b^2)^
(1/2)))+b^2*c*(c*d^2-e*(9*a*e+2*d*(-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)^(3/2
)/(a*e^2-b*d*e+c*d^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-e^2*arctan(x*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)*(3*c^2*d^2+b*e
^2*(b-(-4*a*c+b^2)^(1/2))-c*e*(3*b*d+a*e-2*d*(-4*a*c+b^2)^(1/2)))/(a*e^2-b*
d*e+c*d^2)^3/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^4*e^2-b^3*e*(2*c*d+
e*(-4*a*c+b^2)^(1/2))+b*c*(3*a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-16*a*e+d*(-4*a*
c+b^2)^(1/2)))-4*a*c^2*(3*c*d^2+e*(-3*a*e+d*(-4*a*c+b^2)^(1/2)))+b^2*c*(c*d
^2+e*(-9*a*e+2*d*(-4*a*c+b^2)^(1/2))))/a/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*
d^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 8.62, antiderivative size = 1077, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1252, 205, 211, 1192, 1180}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (e^4*x)/(2*d*(c*d^2 - b*d*e + a*e^2)^2*(d + e*x^2)) + (x*(a*b*c*e*(2*c*d -
b*e) + (b^2 - 2*a*c)*(c^2*d^2 + b^2*e^2 - c*e*(2*b*d + a*e)) - c*(2*b^2*c*d
*e - 4*a*c^2*d*e - b^3*e^2 - b*c*(c*d^2 - 3*a*e^2))*x^2))/(2*a*(b^2 - 4*a*c
```

```
)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*e^2*(3*
c^2*d^2 + b*(b + Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d + 2*Sqrt[b^2 - 4*a*c]*
d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^
2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^3) + (Sqrt[c
]*(b^4*e^2 - b^3*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) - 4*a*c^2*(3*c*d^2 - e*(Sq
rt[b^2 - 4*a*c]*d + 3*a*e)) + b^2*c*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 9*a
*e)) - b*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 16*a*e))
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^
2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)^2) - (
Sqrt[2]*Sqrt[c]*e^2*(3*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - c*e*(3*b*d
 - 2*Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*
e + a*e^2)^3) - (Sqrt[c]*(b^4*e^2 - b^3*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + b
*c*(3*a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 16*a*e)) + b^2*c
*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 9*a*e)) - 4*a*c^2*(3*c*d^2 + e*(Sqrt[b
^2 - 4*a*c]*d - 3*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)^2) + (2*e^(7/2)*(2*c*d - b*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2)^3) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(3/2)*(c*d^2 - b*d*e + a*e^2)^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
```

```
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1252

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2(a+bx^2+cx^4)^2}\,dx = \int \left(\frac{e^4}{(cd^2-bde+ae^2)^2(d+ex^2)^2} - \frac{2e^4(-2cd+be)}{(cd^2-bde+ae^2)^3(d+ex^2)} + \frac{c^2}{}\right.$$

$$= \frac{e^2\int \frac{3c^2d^2+2b^2e^2-ce(5bd+ae)-2ce(2cd-be)x^2}{a+bx^2+cx^4}\,dx}{(cd^2-bde+ae^2)^3} + \frac{(2e^4(2cd-be))\int \frac{1}{d+ex^2}\,dx}{(cd^2-bde+ae^2)^3} +$$

$$= \frac{e^4x}{2d(cd^2-bde+ae^2)^2(d+ex^2)} + \frac{x(abce(2cd-be)+(b^2-2ac)(c^2d^2}{2a(b^2}$$

$$= \frac{e^4x}{2d(cd^2-bde+ae^2)^2(d+ex^2)} + \frac{x(abce(2cd-be)+(b^2-2ac)(c^2d^2}{2a(b^2}$$

$$= \frac{e^4x}{2d(cd^2-bde+ae^2)^2(d+ex^2)} + \frac{x(abce(2cd-be)+(b^2-2ac)(c^2d^2}{2a(b^2}$$

Mathematica [A]
time = 3.56, size = 1020, normalized size = 0.95

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x]
```

[Out] ((2*e^4*x)/(d*(c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x^2)) - (2*x*(b^4*e^2 + b^3*c*e*(-2*d + e*x^2) + 2*a*c^2*(a*e^2 - c*d*(d - 2*e*x^2)) + b^2*c*(-4*a*e^2 + c*d*(d - 2*e*x^2)) + b*c^2*(c*d^2*x^2 - 3*a*e*(-2*d + e*x^2))))/(a*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^5*d*e^3 + b^3*e*(c*d - Sqrt[b^2 - 4*a*c]*e)*(3*c*d^2 + 5*a*e^2) + b^4*e^2*(-3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - 5*a*e)) - 4*a*c^2*(-3*c^2*d^4 + c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 12*a*e) + a*e^3*(9*Sqrt[b^2 - 4*a*c]*d + 7*a*e)) - b*c*(-19*a^2*Sqrt[b^2 - 4*a*c]*e^4 + 2*a*c*d*e^2*(-3*Sqrt[b^2 - 4*a*c]*d + 26*a*e) + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d + 28*a*e)) + b^2*c*(-(c^2*d^4) + 3*c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*e^3*(7*Sqrt[b^2 - 4*a*c]*d + 29*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))^3) - (Sqrt[2]*Sqrt[c]*(b^5*d*e^3 + b^3*e*(c*d + Sqrt[b^2 - 4*a*c]*e)*(3*c*d^2 + 5*a*e^2) - b^2*c*(c^2*d^4 + a*e^3*(7*Sqrt[b^2 - 4*a*c]*d - 29*a*e) + 3*c*d^2*e*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - b^4*e^2*(3*c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d + 5*a*e)) + 4*a*c^2*(3*c^2*d^4 + a*e^3*(9*Sqrt[b^2 - 4*a*c]*d - 7*a*e) + c*d^2*e*(Sqrt[b^2 - 4*a*c]*d + 12*a*e)) + b*c*(-19*a^2*Sqrt[b^2 - 4*a*c]*e^4 + c^2*d^3*(Sqrt[b^2 - 4*a*c]*d - 28*a*e) - 2*a*c*d*e^2*(3*Sqrt[b^2 - 4*a*c]*d + 26*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))^3) + (2*e^(7/2)*(9*c*d^2 + e*(-5*b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + e*(-(b*d) + a*e))^3))/4

Maple [A]
time = 0.64, size = 1250, normalized size = 1.16

method	result
default	$-\frac{-\frac{c\left(3a^2bc\,e^4-4a^2c^2d\,e^3-a\,b^3e^4-b^2cd\,e^3a+6b\,c^2d^2e^2a-4c^3d^3ea+b^4d\,e^3-3b^3c\,d^2e^2+3b^2c^2d^3e-b\,c^3d^4\right)x^3}{2a\left(4ac-b^2\right)}+\frac{\left(2a^3c^2e^4-4a^2b^2c\,e^4+4a^2b\,c^2d\,e^3+\right.}{}}{c\,x^4+b\,x^2+a}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -1/(a*e^2-b*d*e+c*d^2)^3*((-1/2*c*(3*a^2*b*c*e^4-4*a^2*c^2*d*e^3-a*b^3*e^4-a*b^2*c*d*e^3+6*a*b*c^2*d^2*e^2-4*a*c^3*d^3*e+b^4*d*e^3-3*b^3*c*d^2*e^2+3*b^2*c^2*d^3*e-b*c^3*d^4)/a/(4*a*c-b^2)*x^3+1/2*(2*a^3*c^2*e^4-4*a^2*b^2*c*e^4+4*a^2*b*c^2*d*e^3+a*b^4*e^4+2*a*b^3*c*d*e^3-9*a*b^2*c^2*d^2*e^2+8*a*b*c^3*d^3*e-2*a*c^4*d^4-b^5*d*e^3+3*b^4*c*d^2*e^2-3*b^3*c^2*d^3*e+b^2*c^3*d^4)/a

```
/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/a/(4*a*c-b^2)*c*(-1/8*(-19*a^2*b*c*e^4*(-
4*a*c+b^2)^(1/2)+36*a^2*c^2*d*e^3*(-4*a*c+b^2)^(1/2)+5*a*b^3*e^4*(-4*a*c+b^
2)^(1/2)-7*b^2*c*d*e^3*a*(-4*a*c+b^2)^(1/2)-6*b*c^2*d^2*e^2*a*(-4*a*c+b^2)^
(1/2)+4*c^3*d^3*e*a*(-4*a*c+b^2)^(1/2)-b^4*d*e^3*(-4*a*c+b^2)^(1/2)+3*b^3*c
*d^2*e^2*(-4*a*c+b^2)^(1/2)-3*b^2*c^2*d^3*e*(-4*a*c+b^2)^(1/2)+b*c^3*d^4*(-
4*a*c+b^2)^(1/2)+28*a^3*c^2*e^4-29*a^2*b^2*c*e^4+52*a^2*b*c^2*d*e^3-48*a^2*
c^3*d^2*e^2+5*a*b^4*e^4-5*a*b^3*c*d*e^3-12*a*b^2*c^2*d^2*e^2+28*a*b*c^3*d^3
*e-12*a*c^4*d^4-b^5*d*e^3+3*b^4*c*d^2*e^2-3*b^3*c^2*d^3*e+b^2*c^3*d^4)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-19*a^2*b*c*e^4*(-4*a*c+b^2)^(1/2
)+36*a^2*c^2*d*e^3*(-4*a*c+b^2)^(1/2)+5*a*b^3*e^4*(-4*a*c+b^2)^(1/2)-7*b^2*
c*d*e^3*a*(-4*a*c+b^2)^(1/2)-6*b*c^2*d^2*e^2*a*(-4*a*c+b^2)^(1/2)+4*c^3*d^3
*e*a*(-4*a*c+b^2)^(1/2)-b^4*d*e^3*(-4*a*c+b^2)^(1/2)+3*b^3*c*d^2*e^2*(-4*a*
c+b^2)^(1/2)-3*b^2*c^2*d^3*e*(-4*a*c+b^2)^(1/2)+b*c^3*d^4*(-4*a*c+b^2)^(1/2
)-28*a^3*c^2*e^4+29*a^2*b^2*c*e^4-52*a^2*b*c^2*d*e^3+48*a^2*c^3*d^2*e^2-5*a
*b^4*e^4+5*a*b^3*c*d*e^3+12*a*b^2*c^2*d^2*e^2-28*a*b*c^3*d^3*e+12*a*c^4*d^4
+b^5*d*e^3-3*b^4*c*d^2*e^2+3*b^3*c^2*d^3*e-b^2*c^3*d^4)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))))+e^4/(a*e^2-b*d*e+c*d^2)^3*(1/2*(a*e^2-b*d*e+c*d^2)/d*x
/(e*x^2+d)+1/2*(a*e^2-5*b*d*e+9*c*d^2)/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(9*c*d^2*e^4 - 5*b*d*e^5 + a*e^6)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((
c^3*d^7 - 3*b*c^2*d^6*e + 3*(b^2*c*e^2 + a*c^2*e^2)*d^5 - 3*a^2*b*d^2*e^5 -
 (b^3*e^3 + 6*a*b*c*e^3)*d^4 + a^3*d*e^6 + 3*(a*b^2*e^4 + a^2*c*e^4)*d^3)*s
qrt(d)) + 1/2*((b*c^3*d^3*e + a*b^2*c*e^4 - 4*a^2*c^2*e^4 - 2*(b^2*c^2*e^2
- 2*a*c^3*e^2)*d^2 + (b^3*c*e^3 - 3*a*b*c^2*e^3)*d)*x^5 + (b*c^3*d^4 + a*b^
3*e^4 - 4*a^2*b*c*e^4 - (b^2*c^2*e - 2*a*c^3*e)*d^3 - (b^3*c*e^2 - 3*a*b*c^
2*e^2)*d^2 + (b^4*e^3 - 4*a*b^2*c*e^3 + 2*a^2*c^2*e^3)*d)*x^3 + ((b^2*c^2 -
 2*a*c^3)*d^4 + a^2*b^2*e^4 - 4*a^3*c*e^4 - 2*(b^3*c*e - 3*a*b*c^2*e)*d^3 +
 (b^4*e^2 - 4*a*b^2*c*e^2 + 2*a^2*c^2*e^2)*d^2)*x)/((a^2*b^2*c^2 - 4*a^3*c^
3)*d^6 + ((a*b^2*c^3*e - 4*a^2*c^4*e)*d^5 - 2*(a*b^3*c^2*e^2 - 4*a^2*b*c^3*
e^2)*d^4 + (a*b^4*c*e^3 - 2*a^2*b^2*c^2*e^3 - 8*a^3*c^3*e^3)*d^3 - 2*(a^2*b
^3*c*e^4 - 4*a^3*b*c^2*e^4)*d^2 + (a^3*b^2*c*e^5 - 4*a^4*c^2*e^5)*d)*x^6 -
2*(a^2*b^3*c*e - 4*a^3*b*c^2*e)*d^5 + (a^2*b^4*e^2 - 2*a^3*b^2*c*e^2 - 8*a^
4*c^2*e^2)*d^4 + ((a*b^2*c^3 - 4*a^2*c^4)*d^6 - (a*b^3*c^2*e - 4*a^2*b*c^3*
e)*d^5 - (a*b^4*c*e^2 - 6*a^2*b^2*c^2*e^2 + 8*a^3*c^3*e^2)*d^4 + (a*b^5*e^3
```

```
 - 4*a^2*b^3*c*e^3)*d^3 - (2*a^2*b^4*e^4 - 9*a^3*b^2*c*e^4 + 4*a^4*c^2*e^4)
*d^2 + (a^3*b^3*e^5 - 4*a^4*b*c*e^5)*d)*x^4 - 2*(a^3*b^3*e^3 - 4*a^4*b*c*e^
3)*d^3 + (a^4*b^2*e^4 - 4*a^5*c*e^4)*d^2 + ((a*b^3*c^2 - 4*a^2*b*c^3)*d^6 -
 (2*a*b^4*c*e - 9*a^2*b^2*c^2*e + 4*a^3*c^3*e)*d^5 + (a*b^5*e^2 - 4*a^2*b^3
*c*e^2)*d^4 - (a^2*b^4*e^3 - 6*a^3*b^2*c*e^3 + 8*a^4*c^2*e^3)*d^3 - (a^3*b^
3*e^4 - 4*a^4*b*c*e^4)*d^2 + (a^4*b^2*e^5 - 4*a^5*c*e^5)*d)*x^2) + 1/2*inte
grate((5*a*b^4*e^4 - 24*a^2*b^2*c*e^4 + 14*a^3*c^2*e^4 + (b^2*c^3 - 6*a*c^4
)*d^4 - (3*b^3*c^2*e - 16*a*b*c^3*e)*d^3 + 3*(b^4*c*e^2 - 3*a*b^2*c^2*e^2 -
 8*a^2*c^3*e^2)*d^2 + (b*c^4*d^4 + 5*a*b^3*c*e^4 - 19*a^2*b*c^2*e^4 - (3*b^
2*c^3*e - 4*a*c^4*e)*d^3 + 3*(b^3*c^2*e^2 - 2*a*b*c^3*e^2)*d^2 - (b^4*c*e^3
 + 7*a*b^2*c^2*e^3 - 36*a^2*c^3*e^3)*d)*x^2 - (b^5*e^3 + 6*a*b^3*c*e^3 - 44
*a^2*b*c^2*e^3)*d)/(c*x^4 + b*x^2 + a), x)/((a*b^2*c^3 - 4*a^2*c^4)*d^6 + a
^4*b^2*e^6 - 4*a^5*c*e^6 - 3*(a*b^3*c^2*e - 4*a^2*b*c^3*e)*d^5 + 3*(a*b^4*c
*e^2 - 3*a^2*b^2*c^2*e^2 - 4*a^3*c^3*e^2)*d^4 - (a*b^5*e^3 + 2*a^2*b^3*c*e^
3 - 24*a^3*b*c^2*e^3)*d^3 + 3*(a^2*b^4*e^4 - 3*a^3*b^2*c*e^4 - 4*a^4*c^2*e^
4)*d^2 - 3*(a^3*b^3*e^5 - 4*a^4*b*c*e^5)*d)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 64316 vs. $2(962) = 1924$.
time = 16.32, size = 64316, normalized size = 59.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

[Out] 1/2*(9*c*d^2*e^4 - 5*b*d*e^5 + a*e^6)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/((c^3*d^7 - 3*b*c^2*d^6*e + 3*b^2*c*d^5*e^2 + 3*a*c^2*d^5*e^2 - b^3*d^4*e^3 - 6*a*b*c*d^4*e^3 + 3*a*b^2*d^3*e^4 + 3*a^2*c*d^3*e^4 - 3*a^2*b*d^2*e^5 + a^3*d*e^6)*sqrt(d)) + 1/16*((2*a^2*b^7*c^11 - 40*a^3*b^5*c^12 + 224*a^4*b^3*c^13 - 384*a^5*b*c^14 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^9 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^10 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^10 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^11 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^11 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^12 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^12 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^12 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^13 - 2*(b^2 - 4*a*c)*a^2*b^5*c^11 + 32*(b^2 - 4*a*c)*a^3*b^3*c^12 - 96*(b^2 - 4*a*c)*a^4*b*c^13)*d^16 - (18*a^2*b^8*c^10 - 344*a^3*b^6*c^11 + 1888*a^4*b^4*c^12 - 3200*a^5*b^2*c^13 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^8 + 172*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^9 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^9 - 944*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^10 - 272*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^10 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^10 + 1600*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^11 + 800*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^11 + 136*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^11 - 400*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^12 - 18*(b^2 - 4*a*c)*a^2*b^6*c^10 + 272*(b^2 - 4*a*c)*a^3*b^4*c^11 - 800*(b^2 - 4*a*c)*a^4*b^2*c^12)*d^15*e + 6*(12*a^2*b^9*c^9 - 214*a^3*b^7*c^10 + 1096*a^4*b^5*c^11 - 1568*a^5*b^3*c^12 - 640*a^6*b*c^13 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^7 + 107*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^8 - 548*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^9 - 166*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^9 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^9 + 784*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^10 + 432*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^10 + 83*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^10 + 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^11 + 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^11 - 216*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^11 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^12 - 12*(b^2 - 4*a*c)*a^2*b^7*c^9 + 166*(b^2 - 4*a*c)*a^3*b^5*c^10 - 432*(b^2 - 4*a*c)*a^4*b^3*c^11 - 160*(b^2 - 4*a*c)*a^5*b*c^12)*d^14*e^2 - 7*(24*a^2*b^10*c^8 - 386*a^3*b^8*c^9 +

```
 1688*a^4*b^6*c^10 - 1120*a^5*b^4*c^11 - 3968*a^6*b^2*c^12 - 12*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^6 + 193*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^7 + 24*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^7 - 844*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^8 - 290*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^8 - 12*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^8 + 560*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^9 + 528*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^9 + 145*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^9 + 1984*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^10 + 992*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^10 - 26
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^10 -
496*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^11
- 24*(b^2 - 4*a*c)*a^2*b^8*c^8 + 290*(b^2 - 4*a*c)*a^3*b^6*c^9 - 528*(b^2 -
 4*a*c)*a^4*b^4*c^10 - 992*(b^2 - 4*a*c)*a^5*b^2*c^11)*d^13*e^3 + (252*a^2*
b^11*c^7 - 3450*a^3*b^9*c^8 + 10148*a^4*b^7*c^9 + 19024*a^5*b^5*c^10 - 7865
6*a^6*b^3*c^11 - 14080*a^7*b*c^12 - 126*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^11*c^5 + 1725*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^6 + 252*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^6 - 5074*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^7 - 244...
```

Mupad [B]
time = 17.81, size = 2500, normalized size = 2.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log(root(128723189760*a^14*b^4*c^9*d^13*e^14*z^6 + 128723189760*a^12
*b^4*c^11*d^17*e^10*z^6 - 8432455680*a^11*b^12*c^4*d^11*e^16*z^6 - 84324556
80*a^7*b^12*c^8*d^19*e^8*z^6 + 12673351680*a^11*b^11*c^5*d^12*e^15*z^6 + 12
673351680*a^8*b^11*c^8*d^18*e^9*z^6 - 72637480960*a^12*b^9*c^6*d^12*e^15*z^
6 - 72637480960*a^9*b^9*c^9*d^18*e^9*z^6 - 21048344576*a^9*b^12*c^6*d^15*e^
12*z^6 - 16609443840*a^17*b^3*c^7*d^8*e^19*z^6 - 16609443840*a^10*b^3*c^14*
d^22*e^5*z^6 + 145332633600*a^13*b^5*c^9*d^14*e^13*z^6 + 145332633600*a^12*
b^5*c^10*d^16*e^11*z^6 + 123740356608*a^14*b^5*c^8*d^12*e^15*z^6 + 12374035
6608*a^11*b^5*c^11*d^18*e^9*z^6 + 3460300800*a^17*b^5*c^5*d^6*e^21*z^6 + 34
60300800*a^8*b^5*c^14*d^24*e^3*z^6 - 7751073792*a^15*b^7*c^5*d^8*e^19*z^6 -
 7751073792*a^8*b^7*c^12*d^22*e^5*z^6 + 12041846784*a^14*b^7*c^6*d^10*e^17*
z^6 + 12041846784*a^9*b^7*c^11*d^20*e^7*z^6 - 325545099264*a^14*b^3*c^10*d^
14*e^13*z^6 - 325545099264*a^13*b^3*c^11*d^16*e^11*z^6 - 3330539520*a^13*b^
10*c^4*d^9*e^18*z^6 - 3330539520*a^7*b^10*c^10*d^21*e^6*z^6 + 157789716480*
```

a^12*b^7*c^8*d^14*e^13*z^6 + 157789716480*a^11*b^7*c^9*d^16*e^11*z^6 + 37492359168*a^11*b^10*c^6*d^13*e^14*z^6 + 37492359168*a^9*b^10*c^8*d^17*e^10*z^6 + 301989888*a^8*b^3*c^16*d^26*e*z^6 - 7266631680*a^17*b^4*c^6*d^7*e^20*z^6 - 7266631680*a^9*b^4*c^14*d^23*e^4*z^6 - 201326592*a^20*b*c^6*d^4*e^23*z^6 - 188743680*a^7*b^5*c^15*d^26*e*z^6 + 45747339264*a^13*b^8*c^6*d^11*e^16*z^6 + 45747339264*a^9*b^8*c^10*d^19*e^8*z^6 - 74612736*a^10*b^16*c*d^9*e^18*z^6 - 2768240640*a^16*b^7*c^4*d^6*e^21*z^6 - 2768240640*a^7*b^7*c^13*d^24*e^3*z^6 + 69746688*a^11*b^15*c*d^8*e^19*z^6 + 62914560*a^6*b^7*c^14*d^26*e*z^6 + 2752020480*a^10*b^13*c^4*d^12*e^15*z^6 + 2752020480*a^7*b^13*c^7*d^18*e^9*z^6 + 55148544*a^9*b^17*c*d^10*e^17*z^6 - 45957120*a^12*b^14*c*d^7*e^20*z^6 - 2724986880*a^14*b^9*c^4*d^8*e^19*z^6 - 2724986880*a^7*b^9*c^11*d^22*e^5*z^6 - 25952256*a^8*b^18*c*d^11*e^16*z^6 + 21086208*a^13*b^13*c*d^6*e^21*z^6 - 11796480*a^5*b^9*c^13*d^26*e*z^6 - 6438912*a^14*b^12*c*d^5*e^22*z^6 + 5406720*a^7*b^19*c*d^12*e^15*z^6 + 1622016*a^6*b^20*c*d^13*e^14*z^6 - 1523712*a^5*b^21*c*d^14*e^13*z^6 + 1179648*a^15*b^11*c*d^4*e^23*z^6 + 1179648*a^4*b^11*c^12*d^26*e*z^6 + 442368*a^4*b^22*c*d^15*e^12*z^6 - 98304*a^16*b^10*c*d^3*e^24*z^6 - 49152*a^3*b^23*c*d^16*e^11*z^6 - 49152*a^3*b^13*c^11*d^26*e*z^6 + 6897106944*a^9*b^13*c^5*d^14*e^13*z^6 + 6897106944*a^8*b^13*c^6*d^16*e^11*z^6 - 2422210560*a^16*b^6*c^5*d^7*e^20*z^6 - 2422210560*a^8*b^6*c^13*d^23*e^4*z^6 + 255785435136*a^14*b^2*c^11*d^15*e^12*z^6 + 41004564480*a^15*b^4*c^8*d^11*e^16*z^6 + 41004564480*a^11*b^4*c^12*d^19*e^8*z^6 + 2270822400*a^13*b^11*c^3*d^8*e^19*z^6 + 2270822400*a^6*b^11*c^10*d^22*e^5*z^6 + 23677108224*a^14*b^8*c^5*d^9*e^18*z^6 + 23677108224*a^8*b^8*c^11*d^21*e^6*z^6 + 212600881152*a^15*b^2*c^10*d^13*e^14*z^6 + 212600881152*a^13*b^2*c^12*d^17*e^10*z^6 + 75157733376*a^15*b^5*c^7*d^10*e^17*z^6 + 75157733376*a^10*b^5*c^12*d^20*e^7*z^6 - 251217838080*a^13*b^6*c^8*d^13*e^14*z^6 - 251217838080*a^11*b^6*c^10*d^17*e^10*z^6 - 1952907264*a^14*b^10*c^3*d^7*e^20*z^6 - 1952907264*a^6*b^10*c^11*d^23*e^4*z^6 - 27691057152*a^13*b^9*c^5*d^10*e^17*z^6 - 27691057152*a^8*b^9*c^10*d^20*e^7*z^6 - 1902673920*a^8*b^15*c^4*d^14*e^13*z^6 - 1902673920*a^7*b^15*c^5*d^16*e^11*z^6 + 10465050624*a^10*b^11*c^6*d^14*e^13*z^6 + 10465050624*a^9*b^11*c^7*d^16*e^11*z^6 + 1613905920*a^9*b^14*c^4*d^13*e^14*z^6 + 1613905920*a^7*b^14*c^6*d^17*e^10*z^6 - 33218887680*a^17*b*c^9*d^10*e^17*z^6 - 33218887680*a^12*b*c^14*d^20*e^7*z^6 + 1524695040*a^10*b^14*c^3*d^11*e^16*z^6 + 1524695040*a^6*b^14*c^7*d^19*e^8*z^6 - 1472200704*a^18*b^4*c^5*d^5*e^22*z^6 - 1472200704*a^8*b^4*c^15*d^25*e^2*z^6 - 83047219200*a^16*b^3*c^8*d^10*e^17*z^6 - 83047219200*a^11*b^3*c^13*d^20*e^7*z^6 + 44291850240*a^17*b^2*c^8*d^9*e^18*z^6 + 44291850240*a^11*b^2*c^14*d^21*e^6*z^6 + 1308131328*a^8*b^14*c^5*d^15*e^12*z^6 - 201326592*a^9*b*c^17*d^26*e*z^6 + 48530718720*a^12*b^8*c^7*d^13*e^14*z^6 + 48530718720*a^10*b^8*c^9*d^17*e^10*z^6 - 1242644480*a^12*b^12*c^3*d^9*e^18*z^6 - 1242644480*a^6*b^12*c^9*d^21*e^6*z^6 + 9813196800*a^12*b^10*c^5*d^11*e^16*z^6 + 9813196800*a^8*b^10*c^9*d^19*e^8*z^6 - 93012885504*a^15*b*c^11*d^14*e^13*z^6 - 93012885504*a^14*b*c^12*d^16*e^11*z^6 + 177305812992*a^13*b^4*c^10*d^15*e^12*z^6 + 52730658816*a^10*b^10*c^7*d^15*e^12*z^6 - 1180106752*a^9*b^15*c^3*d^12*e^15*z^6 - 1180106752*a^6*b^15*c^6*d^18*e^9*z^6 + 1023672320*a^15*b^9*c^3*d^6*e^2

1*z^6 + 1023672320*a^6*b^9*c^12*d^24*e^3*z^6 + 975175680*a^17*b^6*c^4*d^5*e^22*z^6 + 975175680*a^7*b^6*c^14*d^25*e^2*z^6 - 11072962560*a^18*b*c^8*d^8*e^19*z^6 - 11072962560*a^11*b*c^15*d^22*e^5*z^6 + 9412018176*a^18*b^2*c^7*d^7*e^20*z^6 + 9412018176*a^10*b^2*c^15*d^23*e^4*z^6 + 805306368*a^19*b^2*c^6*d^5*e^22*z^6 + 805306368*a^9*b^2*c^16*d^25*e^2*z^6 - 133809831936*a^14*b^6*c^7*d^11*e^16*z^6 - 133809831936*a^10*b^6*c^11*d^19*e^8*z^6 - 2214592512*a^19*b*c^7*d^6*e^21*z^6 - 2214592512*a^10*b*c^16*d^24*e^3*z^6 + 82216747008*a^13*b^7*c^7*d^12*e^15*z^6 + 82216747008*a^10*...

3.276 $\int (d+ex^2)^{5/2}(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=215

$$\frac{d^2(3cd^2-10bde+80ae^2)\,x\sqrt{d+ex^2}}{256e^2}+\frac{d(3cd^2-10bde+80ae^2)\,x(d+ex^2)^{3/2}}{384e^2}+\frac{(3cd^2-10bde+80ae^2)\,x(\ldots}{480e^2}$$

```
[Out] 1/384*d*(80*a*e^2-10*b*d*e+3*c*d^2)*x*(e*x^2+d)^(3/2)/e^2+1/480*(80*a*e^2-1
0*b*d*e+3*c*d^2)*x*(e*x^2+d)^(5/2)/e^2-1/80*(-10*b*e+3*c*d)*x*(e*x^2+d)^(7/
2)/e^2+1/10*c*x^3*(e*x^2+d)^(7/2)/e+1/256*d^3*(80*a*e^2-10*b*d*e+3*c*d^2)*a
rctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/256*d^2*(80*a*e^2-10*b*d*e+3*c*
d^2)*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A]
time = 0.10, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1173, 396, 201, 223, 212}

$$\frac{x(d+ex^2)^{5/2}(80ae^2-10bde+3cd^2)}{480e^2}+\frac{dx(d+ex^2)^{3/2}(80ae^2-10bde+3cd^2)}{384e^2}+\frac{d^2x\sqrt{d+ex^2}(80ae^2-10bde+3cd^2)}{256e^2}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)(80ae^2-10bde+3cd^2)}{256e^{5/2}}-\frac{x(d+ex^2)^{7/2}(3cd-10be)}{80e^2}+\frac{cx^3(d+ex^2)^{7/2}}{10e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d^2*(3*c*d^2 - 10*b*d*e + 80*a*e^2)*x*Sqrt[d + e*x^2])/(256*e^2) + (d*(3*c
*d^2 - 10*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(3/2))/(384*e^2) + ((3*c*d^2 - 10
*b*d*e + 80*a*e^2)*x*(d + e*x^2)^(5/2))/(480*e^2) - ((3*c*d - 10*b*e)*x*(d
+ e*x^2)^(7/2))/(80*e^2) + (c*x^3*(d + e*x^2)^(7/2))/(10*e) + (d^3*(3*c*d^2
 - 10*b*d*e + 80*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(256*e^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(d+ex^2\right)^{5/2}\left(a+bx^2+cx^4\right)\,dx &= \frac{cx^3(d+ex^2)^{7/2}}{10e}+\frac{\int \left(d+ex^2\right)^{5/2}\left(10ae-(3cd-10be)x^2\right)\,dx}{10e}\\
&= -\frac{(3cd-10be)x(d+ex^2)^{7/2}}{80e^2}+\frac{cx^3(d+ex^2)^{7/2}}{10e}-\frac{1}{80}\Bigg(-80a-\frac{d(3cd-}{e^2}\\
&= \frac{1}{480}\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{5/2}-\frac{(3cd-10be)x(d+ex^2)^{7/}}{80e^2}\\
&= \frac{1}{384}d\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\left(d+ex^2\right)^{3/2}+\frac{1}{480}\Bigg(80a+\frac{d(3cd-10b}{e^2}\\
&= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{384}d\Bigg(80a+\frac{d(3cd-10l}{e^2}\\
&= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{384}d\Bigg(80a+\frac{d(3cd-10l}{e^2}\\
&= \frac{1}{256}d^2\left(80a+\frac{d(3cd-10be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{384}d\Bigg(80a+\frac{d(3cd-10l}{e^2}
\end{aligned}$$

Mathematica [A]
time = 0.31, size = 181, normalized size = 0.84

$$\frac{\sqrt{e}\,x\sqrt{d+ex^2}\left(c(-45d^4+30d^3ex^2+744d^2e^2x^4+1008de^3x^6+384e^4x^8)+10e(8ae(33d^2+26dex^2+8e^2x^4)+b(15d^3+118d^2ex^2+136de^2x^4+48e^3x^6))\right)-15(3cd^5-10d^3e(bd-8ae))\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{3840e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(c*(-45*d^4 + 30*d^3*e*x^2 + 744*d^2*e^2*x^4 + 1
008*d*e^3*x^6 + 384*e^4*x^8) + 10*e*(8*a*e*(33*d^2 + 26*d*e*x^2 + 8*e^2*x^4
) + b*(15*d^3 + 118*d^2*e*x^2 + 136*d*e^2*x^4 + 48*e^3*x^6))) - 15*(3*c*d^5
 - 10*d^3*e*(b*d - 8*a*e))*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(3840*e^(5/
2))
```

Maple [A]
time = 0.12, size = 277, normalized size = 1.29

method	result
risch	$\frac{x\left(384e^4 c\,x^8+480e^4 b\,x^6+1008d\,e^3 c\,x^6+640a\,e^4 x^4+1360bd\,e^3 x^4+744c\,d^2 e^2 x^4+2080d\,e^3 a\,x^2+1180d^2 e^2 b\,x^2+30d^3 ec\,x^2+2640d^2 e^2 a+\right.}{3840e^2}$

default	$c\left(\frac{x^3(e x^2+d)^{\frac{7}{2}}}{10e}-\frac{3d\left(\frac{x(e x^2+d)^{\frac{7}{2}}}{8e}-\frac{d\left(\frac{x(e x^2+d)^{\frac{5}{2}}}{6}+\frac{5d\left(\frac{x(e x^2+d)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right)}{4}\right)}{6}\right)}{8e}\right)}{10e}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/10*x^3*(e*x^2+d)^(7/2)/e-3/10*d/e*(1/8*x*(e*x^2+d)^(7/2)/e-1/8*d/e*(1/
6*x*(e*x^2+d)^(5/2)+5/6*d*(1/4*x*(e*x^2+d)^(3/2)+3/4*d*(1/2*x*(e*x^2+d)^(1/
2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))))+b*(1/8*x*(e*x^2+d)^(7/2
)/e-1/8*d/e*(1/6*x*(e*x^2+d)^(5/2)+5/6*d*(1/4*x*(e*x^2+d)^(3/2)+3/4*d*(1/2*
x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))))+a*(1/6*x*
(e*x^2+d)^(5/2)+5/6*d*(1/4*x*(e*x^2+d)^(3/2)+3/4*d*(1/2*x*(e*x^2+d)^(1/2)+1
/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))
```

Maxima [A]
time = 0.30, size = 258, normalized size = 1.20

$$\frac{1}{10}\left(x^{2}e+d\right)^{\frac{7}{2}}cx^{3}e^{(-1)}+\frac{3}{256}cd^{5}\operatorname{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}-\frac{3}{80}\left(x^{2}e+d\right)^{\frac{7}{2}}cdxe^{(-2)}+\frac{1}{160}\left(x^{2}e+d\right)^{\frac{5}{2}}cd^{2}xe^{(-2)}+\frac{1}{128}\left(x^{2}e+d\right)^{\frac{3}{2}}cd^{3}xe^{(-2)}+\frac{3}{256}\sqrt{x^{2}e+d}\,cd^{4}xe^{(-2)}-\frac{5}{128}bd^{4}\operatorname{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}+\frac{1}{8}\left(x^{2}e+d\right)^{\frac{7}{2}}bxe^{(-1)}-\frac{1}{48}\left(x^{2}e+d\right)^{\frac{5}{2}}bdxe^{(-1)}-\frac{5}{192}\left(x^{2}e+d\right)^{\frac{3}{2}}bd^{2}xe^{(-1)}-\frac{5}{128}\sqrt{x^{2}e+d}\,bd^{3}xe^{(-1)}+\frac{5}{16}ad^{3}\operatorname{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}+\frac{1}{6}\left(x^{2}e+d\right)^{\frac{5}{2}}ax+\frac{5}{24}\left(x^{2}e+d\right)^{\frac{3}{2}}adx+\frac{5}{16}\sqrt{x^{2}e+d}\,ad^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/10*(x^2*e + d)^(7/2)*c*x^3*e^(-1) + 3/256*c*d^5*arcsinh(x*e^(1/2)/sqrt(d)
)*e^(-5/2) - 3/80*(x^2*e + d)^(7/2)*c*d*x*e^(-2) + 1/160*(x^2*e + d)^(5/2)*
c*d^2*x*e^(-2) + 1/128*(x^2*e + d)^(3/2)*c*d^3*x*e^(-2) + 3/256*sqrt(x^2*e
+ d)*c*d^4*x*e^(-2) - 5/128*b*d^4*arcsinh(x*e^(1/2)/sqrt(d))*e^(-3/2) + 1/8
*(x^2*e + d)^(7/2)*b*x*e^(-1) - 1/48*(x^2*e + d)^(5/2)*b*d*x*e^(-1) - 5/192
*(x^2*e + d)^(3/2)*b*d^2*x*e^(-1) - 5/128*sqrt(x^2*e + d)*b*d^3*x*e^(-1) +
5/16*a*d^3*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + 1/6*(x^2*e + d)^(5/2)*a*x
+ 5/24*(x^2*e + d)^(3/2)*a*d*x + 5/16*sqrt(x^2*e + d)*a*d^2*x
```

Fricas [A]
time = 0.51, size = 180, normalized size = 0.84

$$\frac{1}{7680}\left(15\left(3cd^{5}-10bd^{4}e+80ad^{3}e^{2}\right)e^{\frac{1}{2}}\log\left(-2x^{2}e-2\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}-d\right)-2\left(45cd^{4}xe-32\left(12cx^{9}+15bx^{7}+20ax^{5}\right)e^{5}-16\left(63cdx^{7}+85bdx^{5}+130adx^{3}\right)e^{4}-4\left(186cd^{2}x^{5}+295bd^{2}x^{3}+660ad^{2}x\right)e^{3}-30\left(cd^{3}x^{3}+5bd^{3}x\right)e^{2}\right)\sqrt{x^{2}e+d}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7680*(15*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*e^(1/2)*log(-2*x^2*e - 2*s
qrt(x^2*e + d)*x*e^(1/2) - d) - 2*(45*c*d^4*x*e - 32*(12*c*x^9 + 15*b*x^7 +
 20*a*x^5)*e^5 - 16*(63*c*d*x^7 + 85*b*d*x^5 + 130*a*d*x^3)*e^4 - 4*(186*c*
d^2*x^5 + 295*b*d^2*x^3 + 660*a*d^2*x)*e^3 - 30*(c*d^3*x^3 + 5*b*d^3*x)*e^2
)*sqrt(x^2*e + d))*e^(-3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(209) = 418$.
time = 170.57, size = 505, normalized size = 2.35

$$\frac{ad^{\frac{5}{2}}x\sqrt{1+\frac{ex^{2}}{d}}}{2}+\frac{3ad^{\frac{5}{2}}x}{16\sqrt{1+\frac{ex^{2}}{d}}}+\frac{35ad^{\frac{3}{2}}ex^{3}}{48\sqrt{1+\frac{ex^{2}}{d}}}+\frac{17a\sqrt{d}\,e^{2}x^{5}}{24\sqrt{1+\frac{ex^{2}}{d}}}+\frac{5ad^{3}\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16\sqrt{e}}+\frac{ae^{3}x^{7}}{6\sqrt{d}\sqrt{1+\frac{ex^{2}}{d}}}+\frac{5bd^{\frac{7}{2}}x}{128e\sqrt{1+\frac{ex^{2}}{d}}}+\frac{133bd^{\frac{5}{2}}x^{3}}{384\sqrt{1+\frac{ex^{2}}{d}}}+\frac{127bd^{\frac{3}{2}}ex^{5}}{192\sqrt{1+\frac{ex^{2}}{d}}}+\frac{23b\sqrt{d}\,e^{2}x^{7}}{48\sqrt{1+\frac{ex^{2}}{d}}}-\frac{5bd^{4}\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{128e^{\frac{3}{2}}}+\frac{be^{3}x^{9}}{8\sqrt{d}\sqrt{1+\frac{ex^{2}}{d}}}-\frac{3cd^{\frac{9}{2}}x}{256e^{2}\sqrt{1+\frac{ex^{2}}{d}}}-\frac{cd^{\frac{7}{2}}x^{3}}{256e\sqrt{1+\frac{ex^{2}}{d}}}+\frac{129cd^{\frac{5}{2}}x^{5}}{640\sqrt{1+\frac{ex^{2}}{d}}}+\frac{73cd^{\frac{3}{2}}ex^{7}}{160\sqrt{1+\frac{ex^{2}}{d}}}+\frac{29c\sqrt{d}\,e^{2}x^{9}}{80\sqrt{1+\frac{ex^{2}}{d}}}+\frac{3cd^{5}\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{256e^{\frac{5}{2}}}+\frac{ce^{3}x^{11}}{10\sqrt{d}\sqrt{1+\frac{ex^{2}}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(5/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**(5/2)*x*sqrt(1 + e*x**2/d)/2 + 3*a*d**(5/2)*x/(16*sqrt(1 + e*x**2/d))
+ 35*a*d**(3/2)*e*x**3/(48*sqrt(1 + e*x**2/d)) + 17*a*sqrt(d)*e**2*x**5/(24
*sqrt(1 + e*x**2/d)) + 5*a*d**3*asinh(sqrt(e)*x/sqrt(d))/(16*sqrt(e)) + a*e
**3*x**7/(6*sqrt(d)*sqrt(1 + e*x**2/d)) + 5*b*d**(7/2)*x/(128*e*sqrt(1 + e*
x**2/d)) + 133*b*d**(5/2)*x**3/(384*sqrt(1 + e*x**2/d)) + 127*b*d**(3/2)*e*
x**5/(192*sqrt(1 + e*x**2/d)) + 23*b*sqrt(d)*e**2*x**7/(48*sqrt(1 + e*x**2/
d)) - 5*b*d**4*asinh(sqrt(e)*x/sqrt(d))/(128*e**(3/2)) + b*e**3*x**9/(8*sqr
t(d)*sqrt(1 + e*x**2/d)) - 3*c*d**(9/2)*x/(256*e**2*sqrt(1 + e*x**2/d)) - c
*d**(7/2)*x**3/(256*e*sqrt(1 + e*x**2/d)) + 129*c*d**(5/2)*x**5/(640*sqrt(1
 + e*x**2/d)) + 73*c*d**(3/2)*e*x**7/(160*sqrt(1 + e*x**2/d)) + 29*c*sqrt(d
)*e**2*x**9/(80*sqrt(1 + e*x**2/d)) + 3*c*d**5*asinh(sqrt(e)*x/sqrt(d))/(25
6*e**(5/2)) + c*e**3*x**11/(10*sqrt(d)*sqrt(1 + e*x**2/d))
```

Giac [A]
time = 3.01, size = 180, normalized size = 0.84

$$-\frac{1}{256}\left(3\,cd^5 - 10\,bd^4e + 80\,ad^3e^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e+d}\right|\right) + \frac{1}{3840}\left(2\left(4\left(6\left(8\,cx^2e^2 + \left(21\,cde^9 + 10\,be^{10}\right)e^{(-8)}\right)x^2 + \left(93\,cd^2e^8 + 170\,bde^9 + 80\,ae^{10}\right)e^{(-8)}\right)x^2 + 5\left(3\,cd^3e^7 + 118\,bd^2e^8 + 208\,ade^9\right)e^{(-8)}\right)x^2 - 15\left(3\,cd^4e^6 - 10\,bd^3e^7 - 176\,ad^2e^8\right)e^{(-8)}\right)\sqrt{x^2e+d}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(5/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/256*(3*c*d^5 - 10*b*d^4*e + 80*a*d^3*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) +
sqrt(x^2*e + d))) + 1/3840*(2*(4*(6*(8*c*x^2*e^2 + (21*c*d*e^9 + 10*b*e^10)
*e^(-8))*x^2 + (93*c*d^2*e^8 + 170*b*d*e^9 + 80*a*e^10)*e^(-8))*x^2 + 5*(3*
c*d^3*e^7 + 118*b*d^2*e^8 + 208*a*d*e^9)*e^(-8))*x^2 - 15*(3*c*d^4*e^6 - 10
*b*d^3*e^7 - 176*a*d^2*e^8)*e^(-8))*sqrt(x^2*e + d)*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x^2+d\right)^{5/2}\,\left(c\,x^4+b\,x^2+a\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(5/2)*(a + b*x^2 + c*x^4), x)
```

3.277 $\int \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=175

$$\frac{d(3cd^2-8bde+48ae^2)\,x\sqrt{d+ex^2}}{128e^2}+\frac{(3cd^2-8bde+48ae^2)\,x(d+ex^2)^{3/2}}{192e^2}-\frac{(3cd-8be)x(d+ex^2)^{5/2}}{48e^2}+\frac{cx^3(}{}$$

```
[Out] 1/192*(48*a*e^2-8*b*d*e+3*c*d^2)*x*(e*x^2+d)^(3/2)/e^2-1/48*(-8*b*e+3*c*d)*
x*(e*x^2+d)^(5/2)/e^2+1/8*c*x^3*(e*x^2+d)^(5/2)/e+1/128*d^2*(48*a*e^2-8*b*d
*e+3*c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/128*d*(48*a*e^2-8*
b*d*e+3*c*d^2)*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A]
time = 0.08, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1173, 396, 201, 223, 212}

$$\frac{x(d+ex^2)^{3/2}(48ae^2-8bde+3cd^2)}{192e^2}+\frac{dx\sqrt{d+ex^2}(48ae^2-8bde+3cd^2)}{128e^2}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)(48ae^2-8bde+3cd^2)}{128e^{5/2}}-\frac{x(d+ex^2)^{5/2}(3cd-8be)}{48e^2}+\frac{cx^3(d+ex^2)^{5/2}}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d*(3*c*d^2 - 8*b*d*e + 48*a*e^2)*x*Sqrt[d + e*x^2])/(128*e^2) + ((3*c*d^2
- 8*b*d*e + 48*a*e^2)*x*(d + e*x^2)^(3/2))/(192*e^2) - ((3*c*d - 8*b*e)*x*(
d + e*x^2)^(5/2))/(48*e^2) + (c*x^3*(d + e*x^2)^(5/2))/(8*e) + (d^2*(3*c*d^
2 - 8*b*d*e + 48*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(128*e^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$\int \left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)\,dx = \frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}+\frac{\int \left(d+ex^2\right)^{3/2}\left(8ae-(3cd-8be)x^2\right)\,dx}{8e}$$

$$= -\frac{(3cd-8be)x\left(d+ex^2\right)^{5/2}}{48e^2}+\frac{cx^3\left(d+ex^2\right)^{5/2}}{8e}-\frac{1}{48}\left(-48a-\frac{d(3cd-\cdots}{e^2}\right.$$

$$= \frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\left(d+ex^2\right)^{3/2}-\frac{(3cd-8be)x\left(d+ex^2\right)^{5/2}}{48e^2}\cdots$$

$$= \frac{1}{128}d\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)\cdots$$

$$= \frac{1}{128}d\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)\cdots$$

$$= \frac{1}{128}d\left(48a+\frac{d(3cd-8be)}{e^2}\right)x\sqrt{d+ex^2}+\frac{1}{192}\left(48a+\frac{d(3cd-8be)}{e^2}\right)\cdots$$

Mathematica [A]
time = 0.22, size = 148, normalized size = 0.85

$$\frac{\sqrt{e}\,x\sqrt{d+ex^2}\left(c\left(-9d^3+6d^2ex^2+72de^2x^4+48e^3x^6\right)+8e\left(6ae\left(5d+2ex^2\right)+b\left(3d^2+14dex^2+8e^2x^4\right)\right)\right)-3\left(3cd^4-8d^2e(bd-6ae)\right)\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{384e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[e]*x*Sqrt[d + e*x^2]*(c*(-9*d^3 + 6*d^2*e*x^2 + 72*d*e^2*x^4 + 48*e^3
*x^6) + 8*e*(6*a*e*(5*d + 2*e*x^2) + b*(3*d^2 + 14*d*e*x^2 + 8*e^2*x^4))) -
 3*(3*c*d^4 - 8*d^2*e*(b*d - 6*a*e))*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(
384*e^(5/2))
```

Maple [A]

time = 0.11, size = 229, normalized size = 1.31

method	result
risch	$\frac{x\left(48e^3 c x^6+64e^3 b x^4+72d e^2 c x^4+96a e^3 x^2+112d e^2 b x^2+6c d^2 e x^2+240d e^2 a+24d^2 e b-9c d^3\right)\sqrt{e x^2+d}}{384e^2}+\frac{3d^2\ln\left(x\sqrt{e}+\sqrt{\right.}}{8\sqrt{}}$
default	$c\left(\frac{x^3\left(e x^2+d\right)^{\frac{5}{2}}}{8e}-\frac{3d\left(\frac{x\left(e x^2+d\right)^{\frac{5}{2}}}{6e}-\frac{d\left(\frac{x\left(e x^2+d\right)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2\sqrt{e}}\right)}{4}\right)}{6e}\right)}{8e}\right)+b\left(\frac{x(e}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/8*x^3*(e*x^2+d)^(5/2)/e-3/8*d/e*(1/6*x*(e*x^2+d)^(5/2)/e-1/6*d/e*(1/4*
x*(e*x^2+d)^(3/2)+3/4*d*(1/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(
e*x^2+d)^(1/2))))))+b*(1/6*x*(e*x^2+d)^(5/2)/e-1/6*d/e*(1/4*x*(e*x^2+d)^(3/
2)+3/4*d*(1/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))
)))+a*(1/4*x*(e*x^2+d)^(3/2)+3/4*d*(1/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(
x*e^(1/2)+(e*x^2+d)^(1/2))))
```

Maxima [A]

time = 0.30, size = 203, normalized size = 1.16

$$\frac{1}{8}\left(x^2e+d\right)^{\frac{5}{2}}cx^3e^{(-1)}+\frac{3}{128}\,cd^4\operatorname{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}-\frac{1}{16}\left(x^2e+d\right)^{\frac{5}{2}}cdxe^{(-2)}+\frac{1}{64}\left(x^2e+d\right)^{\frac{3}{2}}cd^2xe^{(-2)}+\frac{3}{128}\sqrt{x^2e+d}\,cd^3xe^{(-2)}-\frac{1}{16}\,bd^3\operatorname{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}+\frac{1}{6}\left(x^2e+d\right)^{\frac{5}{2}}bxe^{(-1)}-\frac{1}{24}\left(x^2e+d\right)^{\frac{3}{2}}bdxe^{(-1)}-\frac{1}{16}\sqrt{x^2e+d}\,bd^2xe^{(-1)}+\frac{3}{8}\,ad^2\operatorname{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}+\frac{1}{4}\left(x^2e+d\right)^{\frac{3}{2}}ax+\frac{3}{8}\sqrt{x^2e+d}\,adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*(x^2*e + d)^(5/2)*c*x^3*e^(-1) + 3/128*c*d^4*arcsinh(x*e^(1/2)/sqrt(d))
*e^(-5/2) - 1/16*(x^2*e + d)^(5/2)*c*d*x*e^(-2) + 1/64*(x^2*e + d)^(3/2)*c*
d^2*x*e^(-2) + 3/128*sqrt(x^2*e + d)*c*d^3*x*e^(-2) - 1/16*b*d^3*arcsinh(x*
e^(1/2)/sqrt(d))*e^(-3/2) + 1/6*(x^2*e + d)^(5/2)*b*x*e^(-1) - 1/24*(x^2*e
+ d)^(3/2)*b*d*x*e^(-1) - 1/16*sqrt(x^2*e + d)*b*d^2*x*e^(-1) + 3/8*a*d^2*a
rcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + 1/4*(x^2*e + d)^(3/2)*a*x + 3/8*sqrt(x
^2*e + d)*a*d*x
```

Fricas [A]
time = 0.38, size = 148, normalized size = 0.85

$$\frac{1}{768}\left(3\left(3\,cd^4-8\,bd^3e+48\,ad^2e^2\right)e^{\frac{1}{2}}\log\left(-2\,x^2e-2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)-2\left(9\,cd^3xe-16\left(3\,cx^7+4\,bx^5+6\,ax^3\right)e^4-8\left(9\,cdx^5+14\,bdx^3+30\,adx\right)e^3-6\left(cd^2x^3+4\,bd^2x\right)e^2\right)\sqrt{x^2e+d}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/768*(3*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*e^(1/2)*log(-2*x^2*e - 2*sqrt
(x^2*e + d)*x*e^(1/2) - d) - 2*(9*c*d^3*x*e - 16*(3*c*x^7 + 4*b*x^5 + 6*a*x
^3)*e^4 - 8*(9*c*d*x^5 + 14*b*d*x^3 + 30*a*d*x)*e^3 - 6*(c*d^2*x^3 + 4*b*d^
2*x)*e^2)*sqrt(x^2*e + d))*e^(-3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(167) = 334$.
time = 34.02, size = 413, normalized size = 2.36

$$\frac{ad^{\frac{3}{2}}x\sqrt{1+\frac{ex^2}{d}}}{2}+\frac{ad^{\frac{3}{2}}x}{8\sqrt{1+\frac{ex^2}{d}}}+\frac{3a\sqrt{d}\,ex^3}{8\sqrt{1+\frac{ex^2}{d}}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8\sqrt{e}}+\frac{ae^2x^5}{4\sqrt{d}\,\sqrt{1+\frac{ex^2}{d}}}+\frac{bd^{\frac{5}{2}}x}{16e\sqrt{1+\frac{ex^2}{d}}}+\frac{17bd^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{ex^2}{d}}}+\frac{11b\sqrt{d}\,ex^5}{24\sqrt{1+\frac{ex^2}{d}}}-\frac{bd^3\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16e^{\frac{3}{2}}}+\frac{be^2x^7}{6\sqrt{d}\,\sqrt{1+\frac{ex^2}{d}}}-\frac{3cd^{\frac{7}{2}}x}{128e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{cd^{\frac{5}{2}}x^3}{128e\sqrt{1+\frac{ex^2}{d}}}+\frac{13cd^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{ex^2}{d}}}+\frac{5c\sqrt{d}\,ex^7}{16\sqrt{1+\frac{ex^2}{d}}}+\frac{3cd^4\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{128e^{\frac{5}{2}}}+\frac{ce^2x^9}{8\sqrt{d}\,\sqrt{1+\frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**(3/2)*x*sqrt(1 + e*x**2/d)/2 + a*d**(3/2)*x/(8*sqrt(1 + e*x**2/d)) + 3
*a*sqrt(d)*e*x**3/(8*sqrt(1 + e*x**2/d)) + 3*a*d**2*asinh(sqrt(e)*x/sqrt(d)
)/(8*sqrt(e)) + a*e**2*x**5/(4*sqrt(d)*sqrt(1 + e*x**2/d)) + b*d**(5/2)*x/(
16*e*sqrt(1 + e*x**2/d)) + 17*b*d**(3/2)*x**3/(48*sqrt(1 + e*x**2/d)) + 11*
b*sqrt(d)*e*x**5/(24*sqrt(1 + e*x**2/d)) - b*d**3*asinh(sqrt(e)*x/sqrt(d))/
(16*e**(3/2)) + b*e**2*x**7/(6*sqrt(d)*sqrt(1 + e*x**2/d)) - 3*c*d**(7/2)*x
/(128*e**2*sqrt(1 + e*x**2/d)) - c*d**(5/2)*x**3/(128*e*sqrt(1 + e*x**2/d))
 + 13*c*d**(3/2)*x**5/(64*sqrt(1 + e*x**2/d)) + 5*c*sqrt(d)*e*x**7/(16*sqrt
```

```
(1 + e*x**2/d)) + 3*c*d**4*asinh(sqrt(e)*x/sqrt(d))/(128*e**(5/2)) + c*e**2
*x**9/(8*sqrt(d)*sqrt(1 + e*x**2/d))
```

Giac [A]
time = 4.08, size = 145, normalized size = 0.83

$$-\frac{1}{128}\left(3\,cd^4-8\,bd^3e+48\,ad^2e^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{384}\left(2\left(4\left(6\,cx^2e+\left(9\,cde^6+8\,be^7\right)e^{(-6)}\right)x^2+\left(3\,cd^2e^5+56\,bde^6+48\,ae^7\right)e^{(-6)}\right)x^2-3\left(3\,cd^3e^4-8\,bd^2e^5-80\,ade^6\right)e^{(-6)}\right)\sqrt{x^2e+d}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/128*(3*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + s
qrt(x^2*e + d))) + 1/384*(2*(4*(6*c*x^2*e + (9*c*d*e^6 + 8*b*e^7)*e^(-6))*x
^2 + (3*c*d^2*e^5 + 56*b*d*e^6 + 48*a*e^7)*e^(-6))*x^2 - 3*(3*c*d^3*e^4 - 8
*b*d^2*e^5 - 80*a*d*e^6)*e^(-6))*sqrt(x^2*e + d)*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(e\,x^2+d\right)^{3/2}\,\left(c\,x^4+b\,x^2+a\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4), x)
```

3.278 $\int \sqrt{d+ex^2}\,(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=132

$$\frac{(cd^2-2bde+8ae^2)\,x\sqrt{d+ex^2}}{16e^2}-\frac{(cd-2be)x(d+ex^2)^{3/2}}{8e^2}+\frac{cx^3(d+ex^2)^{3/2}}{6e}+\frac{d(cd^2-2bde+8ae^2)\tanh^{-1}\left(\right.}{16e^{5/2}}$$

```
[Out] -1/8*(-2*b*e+c*d)*x*(e*x^2+d)^(3/2)/e^2+1/6*c*x^3*(e*x^2+d)^(3/2)/e+1/16*d*
(8*a*e^2-2*b*d*e+c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+1/16*(8*
a*e^2-2*b*d*e+c*d^2)*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1173, 396, 201, 223, 212}

$$\frac{x\sqrt{d+ex^2}\,(8ae^2-2bde+cd^2)}{16e^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)(8ae^2-2bde+cd^2)}{16e^{5/2}}-\frac{x(d+ex^2)^{3/2}(cd-2be)}{8e^2}+\frac{cx^3(d+ex^2)^{3/2}}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d^2 - 2*b*d*e + 8*a*e^2)*x*Sqrt[d + e*x^2])/(16*e^2) - ((c*d - 2*b*e)*x
*(d + e*x^2)^(3/2))/(8*e^2) + (c*x^3*(d + e*x^2)^(3/2))/(6*e) + (d*(c*d^2 -
 2*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(16*e^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+bx^2+cx^4\right)\,dx = \frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}+\frac{\int \sqrt{d+ex^2}\left(6ae-3(cd-2be)x^2\right)\,dx}{6e}$$

$$= -\frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2}+\frac{cx^3\left(d+ex^2\right)^{3/2}}{6e}+\frac{1}{8}\left(8a+\frac{d(cd-2be)}{e^2}\right)\int$$

$$= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2}-\frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2}+\frac{cx^3(d}{}$$

$$= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2}-\frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2}+\frac{cx^3(d}{}$$

$$= \frac{1}{16}\left(8a+\frac{d(cd-2be)}{e^2}\right)x\sqrt{d+ex^2}-\frac{(cd-2be)x\left(d+ex^2\right)^{3/2}}{8e^2}+\frac{cx^3(d}{}$$

Mathematica [A]
time = 0.15, size = 112, normalized size = 0.85

$$\frac{x\sqrt{d+ex^2}\left(-3cd^2+6bde+24ae^2+2cdex^2+12be^2x^2+8ce^2x^4\right)}{48e^2}-\frac{d\left(cd^2-2bde+8ae^2\right)\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{16e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2]*(-3*c*d^2 + 6*b*d*e + 24*a*e^2 + 2*c*d*e*x^2 + 12*b*e^2*
x^2 + 8*c*e^2*x^4))/(48*e^2) - (d*(c*d^2 - 2*b*d*e + 8*a*e^2)*Log[-(Sqrt[e]
*x) + Sqrt[d + e*x^2]])/(16*e^(5/2))
```

Maple [A]
time = 0.12, size = 181, normalized size = 1.37

method	result
risch	$\frac{x\left(8e^2x^4c+12b\,e^2x^2+2cde\,x^2+24a\,e^2+6deb-3c\,d^2\right)\sqrt{e\,x^2+d}}{48e^2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)a}{2\sqrt{e}}-\frac{d^2\ln\left(x\sqrt{e}+\sqrt{e\,x^2+\right.}{8e^{\frac{3}{2}}}$
default	$c\left(\frac{x^3\left(e\,x^2+d\right)^{\frac{3}{2}}}{6e}-\frac{d\left(\frac{x\left(e\,x^2+d\right)^{\frac{3}{2}}}{4e}-\frac{d\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{4e}\right)}{2e}\right)+b\left(\frac{x\left(e\,x^2+d\right)^{\frac{3}{2}}}{4e}-\frac{d\left(\frac{x\sqrt{e}}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/6*x^3*(e*x^2+d)^(3/2)/e-1/2*d/e*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*
x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))+b*(1/4*x*(
e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+
(e*x^2+d)^(1/2))))+a*(1/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x
^2+d)^(1/2)))
```

Maxima [A]
time = 0.32, size = 148, normalized size = 1.12

$$\frac{1}{6}\left(x^2e+d\right)^{\frac{3}{2}}cx^3e^{(-1)}+\frac{1}{16}\,cd^3\,\operatorname{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}-\frac{1}{8}\left(x^2e+d\right)^{\frac{3}{2}}cdxe^{(-2)}+\frac{1}{16}\sqrt{x^2e+d}\,cd^2xe^{(-2)}-\frac{1}{8}\,bd^2\,\operatorname{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}+\frac{1}{4}\left(x^2e+d\right)^{\frac{3}{2}}bxe^{(-1)}-\frac{1}{8}\sqrt{x^2e+d}\,bdxe^{(-1)}+\frac{1}{2}\,ad\,\operatorname{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}+\frac{1}{2}\sqrt{x^2e+d}\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*(x^2*e + d)^(3/2)*c*x^3*e^(-1) + 1/16*c*d^3*arcsinh(x*e^(1/2)/sqrt(d))*
e^(-5/2) - 1/8*(x^2*e + d)^(3/2)*c*d*x*e^(-2) + 1/16*sqrt(x^2*e + d)*c*d^2*
x*e^(-2) - 1/8*b*d^2*arcsinh(x*e^(1/2)/sqrt(d))*e^(-3/2) + 1/4*(x^2*e + d)^
(3/2)*b*x*e^(-1) - 1/8*sqrt(x^2*e + d)*b*d*x*e^(-1) + 1/2*a*d*arcsinh(x*e^(
1/2)/sqrt(d))*e^(-1/2) + 1/2*sqrt(x^2*e + d)*a*x
```

Fricas [A]
time = 0.50, size = 115, normalized size = 0.87

$$\frac{1}{96}\left(3\left(cd^3-2\,bd^2e+8\,ade^2\right)e^{\frac{1}{2}}\log\left(-2\,x^2e-2\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)-2\left(3\,cd^2xe-4\left(2\,cx^5+3\,bx^3+6\,ax\right)e^3-2\left(cdx^3+3\,bdx\right)e^2\right)\sqrt{x^2e+d}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/96*(3*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*e^(1/2)*log(-2*x^2*e - 2*sqrt(x^2*e
 + d)*x*e^(1/2) - d) - 2*(3*c*d^2*x*e - 4*(2*c*x^5 + 3*b*x^3 + 6*a*x)*e^3 -
 2*(c*d*x^3 + 3*b*d*x)*e^2)*sqrt(x^2*e + d))*e^(-3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(124) = 248$.
time = 8.21, size = 272, normalized size = 2.06

$$\frac{a\sqrt{d}\,x\sqrt{1+\frac{ex^2}{d}}}{2}+\frac{ad\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{e}}+\frac{bd^{\frac{3}{2}}x}{8e\sqrt{1+\frac{ex^2}{d}}}+\frac{3b\sqrt{d}\,x^3}{8\sqrt{1+\frac{ex^2}{d}}}-\frac{bd^2\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8e^{\frac{3}{2}}}+\frac{bex^5}{4\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}-\frac{cd^{\frac{5}{2}}x}{16e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{cd^{\frac{3}{2}}x^3}{48e\sqrt{1+\frac{ex^2}{d}}}+\frac{5c\sqrt{d}\,x^5}{24\sqrt{1+\frac{ex^2}{d}}}+\frac{cd^3\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{16e^{\frac{5}{2}}}+\frac{cex^7}{6\sqrt{d}\sqrt{1+\frac{ex^2}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] a*sqrt(d)*x*sqrt(1 + e*x**2/d)/2 + a*d*asinh(sqrt(e)*x/sqrt(d))/(2*sqrt(e))
 + b*d**(3/2)*x/(8*e*sqrt(1 + e*x**2/d)) + 3*b*sqrt(d)*x**3/(8*sqrt(1 + e*x
**2/d)) - b*d**2*asinh(sqrt(e)*x/sqrt(d))/(8*e**(3/2)) + b*e*x**5/(4*sqrt(d
)*sqrt(1 + e*x**2/d)) - c*d**(5/2)*x/(16*e**2*sqrt(1 + e*x**2/d)) - c*d**(3
/2)*x**3/(48*e*sqrt(1 + e*x**2/d)) + 5*c*sqrt(d)*x**5/(24*sqrt(1 + e*x**2/d
)) + c*d**3*asinh(sqrt(e)*x/sqrt(d))/(16*e**(5/2)) + c*e*x**7/(6*sqrt(d)*sq
rt(1 + e*x**2/d))
```

Giac [A]
time = 2.99, size = 106, normalized size = 0.80

$$-\frac{1}{16}\left(cd^3-2\,bd^2e+8\,ade^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{48}\left(2\left(4\,cx^2+\left(cde^3+6\,be^4\right)e^{(-4)}\right)x^2-3\left(cd^2e^2-2\,bde^3-8\,ae^4\right)e^{(-4)}\right)\sqrt{x^2e+d}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/16*(c*d^3 - 2*b*d^2*e + 8*a*d*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^
2*e + d))) + 1/48*(2*(4*c*x^2 + (c*d*e^3 + 6*b*e^4)*e^(-4))*x^2 - 3*(c*d^2*
e^2 - 2*b*d*e^3 - 8*a*e^4)*e^(-4))*sqrt(x^2*e + d)*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4), x)
```

3.279 $\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=97

$$-\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}+\frac{(3cd^2-4bde+8ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{8e^{5/2}}$$

```
[Out] 1/8*(8*a*e^2-4*b*d*e+3*c*d^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)-1/
8*(-4*b*e+3*c*d)*x*(e*x^2+d)^(1/2)/e^2+1/4*c*x^3*(e*x^2+d)^(1/2)/e
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1173, 396, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)(8ae^2-4bde+3cd^2)}{8e^{5/2}}-\frac{x\sqrt{d+ex^2}\,(3cd-4be)}{8e^2}+\frac{cx^3\sqrt{d+ex^2}}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]
```

```
[Out] -1/8*((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/e^2 + (c*x^3*Sqrt[d + e*x^2])/(4*e
) + ((3*c*d^2 - 4*b*d*e + 8*a*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8
*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\sqrt{d+ex^2}}\,dx = \frac{cx^3\sqrt{d+ex^2}}{4e} + \frac{\int \frac{4ae-(3cd-4be)x^2}{\sqrt{d+ex^2}}\,dx}{4e}$$

$$= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2} + \frac{cx^3\sqrt{d+ex^2}}{4e} - \frac{1}{8}\left(-8a-\frac{d(3cd-4be)}{e^2}\right)\int \frac{1}{\sqrt{d+ex^2}}$$

$$= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2} + \frac{cx^3\sqrt{d+ex^2}}{4e} - \frac{1}{8}\left(-8a-\frac{d(3cd-4be)}{e^2}\right)\text{Subst}\left(\int \frac{}{1}\right.$$

$$= -\frac{(3cd-4be)x\sqrt{d+ex^2}}{8e^2} + \frac{cx^3\sqrt{d+ex^2}}{4e} + \frac{(3cd^2-4bde+8ae^2)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+e}}\right.}{8e^{5/2}}$$

Mathematica [A]
time = 0.09, size = 85, normalized size = 0.88

$$\frac{\sqrt{d+ex^2}\,(-3cdx+4bex+2cex^3)}{8e^2} + \frac{(-3cd^2+4bde-8ae^2)\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{8e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(-3*c*d*x + 4*b*e*x + 2*c*e*x^3))/(8*e^2) + ((-3*c*d^2 + 4
*b*d*e - 8*a*e^2)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(8*e^(5/2))
```

Maple [A]
time = 0.12, size = 127, normalized size = 1.31

method	result
risch	$\frac{x(2ce\,x^2+4eb-3cd)\sqrt{e\,x^2+d}}{8e^2} + \frac{a\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{\sqrt{e}} - \frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)db}{2e^{\frac{3}{2}}} + \frac{3\ln\left(x\sqrt{e}+\sqrt{e\,x\right.}}{8e^{\frac{5}{2}}}$

default	$c\left(\frac{x^{3}\sqrt{e\,x^{2}+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)+b\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x\right.}}{2e^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
[Out] c*(1/4*x^3/e*(e*x^2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)
*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))+b*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*l
n(x*e^(1/2)+(e*x^2+d)^(1/2)))+a*ln(x*e^(1/2)+(e*x^2+d)^(1/2))/e^(1/2)
```

Maxima [A]
time = 0.29, size = 94, normalized size = 0.97

$$\frac{1}{4}\sqrt{x^{2}e+d}\,cx^{3}e^{(-1)}+\frac{3}{8}cd^{2}\,\mathrm{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}-\frac{3}{8}\sqrt{x^{2}e+d}\,cdxe^{(-2)}-\frac{1}{2}bd\,\mathrm{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}+\frac{1}{2}\sqrt{x^{2}e+d}\,bxe^{(-1)}+a\,\mathrm{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/4*sqrt(x^2*e + d)*c*x^3*e^(-1) + 3/8*c*d^2*arcsinh(x*e^(1/2)/sqrt(d))*e^(
-5/2) - 3/8*sqrt(x^2*e + d)*c*d*x*e^(-2) - 1/2*b*d*arcsinh(x*e^(1/2)/sqrt(d
))*e^(-3/2) + 1/2*sqrt(x^2*e + d)*b*x*e^(-1) + a*arcsinh(x*e^(1/2)/sqrt(d))
*e^(-1/2)
```

Fricas [A]
time = 0.37, size = 87, normalized size = 0.90

$$\frac{1}{16}\left(\left(3\,cd^{2}-4\,bde+8\,ae^{2}\right)e^{\frac{1}{2}}\log\left(-2\,x^{2}e-2\,\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}-d\right)-2\left(3\,cdxe-2\left(cx^{3}+2\,bx\right)e^{2}\right)\sqrt{x^{2}e+d}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] 1/16*((3*c*d^2 - 4*b*d*e + 8*a*e^2)*e^(1/2)*log(-2*x^2*e - 2*sqrt(x^2*e + d
)*x*e^(1/2) - d) - 2*(3*c*d*x*e - 2*(c*x^3 + 2*b*x)*e^2)*sqrt(x^2*e + d))*e
^(-3)
```

Sympy [A]
time = 3.97, size = 230, normalized size = 2.37

$$a\left(\begin{cases}\frac{\sqrt{-\frac{d}{e}}\operatorname{asin}\left(x\sqrt{-\frac{e}{d}}\right)}{\sqrt{d}} & \text{for}\: d>0\wedge e<0\\ \frac{\sqrt{\frac{d}{e}}\operatorname{asinh}\left(x\sqrt{\frac{e}{d}}\right)}{\sqrt{d}} & \text{for}\: d>0\wedge e>0\\ \frac{\sqrt{-\frac{d}{e}}\operatorname{acosh}\left(x\sqrt{-\frac{e}{d}}\right)}{\sqrt{-d}} & \text{for}\: e>0\wedge d<0\end{cases}\right)+\frac{b\sqrt{d}\,x\sqrt{1+\frac{ex^{2}}{d}}}{2e}-\frac{bd\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2e^{\frac{3}{2}}}-\frac{3cd^{\frac{3}{2}}x}{8e^{2}\sqrt{1+\frac{ex^{2}}{d}}}-\frac{c\sqrt{d}\,x^{3}}{8e\sqrt{1+\frac{ex^{2}}{d}}}+\frac{3cd^{2}\operatorname{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8e^{\frac{5}{2}}}+\frac{cx^{5}}{4\sqrt{d}\sqrt{1+\frac{ex^{2}}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] a*Piecewise((sqrt(-d/e)*asin(x*sqrt(-e/d))/sqrt(d), (d > 0) & (e < 0)), (sq
rt(d/e)*asinh(x*sqrt(e/d))/sqrt(d), (d > 0) & (e > 0)), (sqrt(-d/e)*acosh(x
*sqrt(-e/d))/sqrt(-d), (e > 0) & (d < 0))) + b*sqrt(d)*x*sqrt(1 + e*x**2/d)
/(2*e) - b*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(3/2)) - 3*c*d**(3/2)*x/(8*e**2
*sqrt(1 + e*x**2/d)) - c*sqrt(d)*x**3/(8*e*sqrt(1 + e*x**2/d)) + 3*c*d**2*a
sinh(sqrt(e)*x/sqrt(d))/(8*e**(5/2)) + c*x**5/(4*sqrt(d)*sqrt(1 + e*x**2/d)
)
```

Giac [A]
time = 3.66, size = 79, normalized size = 0.81

$$-\frac{1}{8}\left(3\,cd^2-4\,bde+8\,ae^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)+\frac{1}{8}\left(2\,cx^2e^{(-1)}-\left(3\,cde-4\,be^2\right)e^{(-3)}\right)\sqrt{x^2e+d}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*c*d^2 - 4*b*d*e + 8*a*e^2)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e
 + d))) + 1/8*(2*c*x^2*e^(-1) - (3*c*d*e - 4*b*e^2)*e^(-3))*sqrt(x^2*e + d)
*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^4+b\,x^2+a}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(1/2), x)
```

3.280 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}-\frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}$$

```
[Out] -1/2*(-2*b*e+3*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/e^(5/2)+(a+d*(-b*e+c
*d)/e^2)*x/d/(e*x^2+d)^(1/2)+1/2*c*x*(e*x^2+d)^(1/2)/e^2
```

Rubi [A]
time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1171, 396, 223, 212}

$$\frac{x(ae^2-bde+cd^2)}{de^2\sqrt{d+ex^2}}-\frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(d*e^2*Sqrt[d + e*x^2]) + (c*x*Sqrt[d + e*x^2])
/(2*e^2) - ((3*c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(5/2
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{(d+ex^2)^{3/2}}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}}-\frac{\int \frac{\frac{d(cd-be)}{e^2}-\frac{cdx^2}{e}}{\sqrt{d+ex^2}}\,dx}{d}\\ &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}-\frac{(3cd-2be)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2e^2}\\ &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}-\frac{(3cd-2be)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{2e^2}\\ &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{d\sqrt{d+ex^2}}+\frac{cx\sqrt{d+ex^2}}{2e^2}-\frac{(3cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2e^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 87, normalized size = 0.98

$$\frac{x(3cd^2-2bde+2ae^2+cdex^2)}{2de^2\sqrt{d+ex^2}}+\frac{(3cd-2be)\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{2e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(3*c*d^2 - 2*b*d*e + 2*a*e^2 + c*d*e*x^2))/(2*d*e^2*Sqrt[d + e*x^2]) + (
(3*c*d - 2*b*e)*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/(2*e^(5/2))
```

Maple [A]
time = 0.13, size = 117, normalized size = 1.31

method	result

risch	$\frac{cx\sqrt{e\,x^2+d}}{2e^2}-\frac{xb}{e\sqrt{e\,x^2+d}}+\frac{xcd}{e^2\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)b}{e^{\frac{3}{2}}}-\frac{3\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)cd}{2e^{\frac{5}{2}}}$
default	$c\left(\frac{x^3}{2e\sqrt{e\,x^2+d}}-\frac{3d\left(-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}\right)}{2e}\right)+b\left(-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\right.}}{e^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(1/2*x^3/e/(e*x^2+d)^(1/2)-3/2*d/e*(-x/e/(e*x^2+d)^(1/2)+1/e^(3/2)*ln(x*e
^(1/2)+(e*x^2+d)^(1/2))))+b*(-x/e/(e*x^2+d)^(1/2)+1/e^(3/2)*ln(x*e^(1/2)+(e
*x^2+d)^(1/2)))+a*x/d/(e*x^2+d)^(1/2)
```

Maxima [A]
time = 0.30, size = 94, normalized size = 1.06

$$\frac{cx^3e^{(-1)}}{2\sqrt{x^2e+d}}-\frac{3}{2}\,cd\,\mathrm{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}+\frac{3\,cdxe^{(-2)}}{2\sqrt{x^2e+d}}+b\,\mathrm{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}-\frac{bxe^{(-1)}}{\sqrt{x^2e+d}}+\frac{ax}{\sqrt{x^2e+d}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*x^3*e^(-1)/sqrt(x^2*e + d) - 3/2*c*d*arcsinh(x*e^(1/2)/sqrt(d))*e^(-5
/2) + 3/2*c*d*x*e^(-2)/sqrt(x^2*e + d) + b*arcsinh(x*e^(1/2)/sqrt(d))*e^(-3
/2) - b*x*e^(-1)/sqrt(x^2*e + d) + a*x/(sqrt(x^2*e + d)*d)
```

Fricas [A]
time = 0.43, size = 128, normalized size = 1.44

$$\frac{(2\,bdx^2e^2-3\,cd^3-(3\,cd^2x^2-2\,bd^2)e)e^{\frac{1}{2}}\log\left(-2\,x^2e-2\,\sqrt{x^2e+d}\,xe^{\frac{1}{2}}-d\right)+2\,(3\,cd^2xe+2\,axe^3+(cdx^3-2\,bdx)e^2)\sqrt{x^2e+d}}{4\,(dx^2e^4+d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*d*x^2*e^2 - 3*c*d^3 - (3*c*d^2*x^2 - 2*b*d^2)*e)*e^(1/2)*log(-2*x
^2*e - 2*sqrt(x^2*e + d)*x*e^(1/2) - d) + 2*(3*c*d^2*x*e + 2*a*x*e^3 + (c*d
*x^3 - 2*b*d*x)*e^2)*sqrt(x^2*e + d))/(d*x^2*e^4 + d^2*e^3)
```

Sympy [A]
time = 4.62, size = 134, normalized size = 1.51

$$\frac{ax}{d^{\frac{3}{2}}\sqrt{1+\frac{ex^2}{d}}}+b\left(\frac{\mathrm{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{\frac{3}{2}}}-\frac{x}{\sqrt{d}\,e\sqrt{1+\frac{ex^2}{d}}}\right)+c\left(\frac{3\sqrt{d}\,x}{2e^2\sqrt{1+\frac{ex^2}{d}}}-\frac{3d\,\mathrm{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2e^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{d}\,e\sqrt{1+\frac{ex^2}{d}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(3/2),x)
```

```
[Out] a*x/(d**(3/2)*sqrt(1 + e*x**2/d)) + b*(asinh(sqrt(e)*x/sqrt(d))/e**(3/2) -
x/(sqrt(d)*e*sqrt(1 + e*x**2/d))) + c*(3*sqrt(d)*x/(2*e**2*sqrt(1 + e*x**2/
d)) - 3*d*asinh(sqrt(e)*x/sqrt(d))/(2*e**(5/2)) + x**3/(2*sqrt(d)*e*sqrt(1
+ e*x**2/d)))
```

Giac [A]
time = 3.40, size = 80, normalized size = 0.90

$$\frac{1}{2}\,(3\,cd - 2\,be)e^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}} + \sqrt{x^2e + d}\right|\right) + \frac{\left(cx^2e^{(-1)} + \frac{(3\,cd^2e - 2\,bde^2 + 2\,ae^3)e^{(-3)}}{d}\right)x}{2\,\sqrt{x^2e + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(3*c*d - 2*b*e)*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) + 1/2*(
c*x^2*e^(-1) + (3*c*d^2*e - 2*b*d*e^2 + 2*a*e^3)*e^(-3)/d)*x/sqrt(x^2*e + d
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^4 + b\,x^2 + a}{\left(e\,x^2 + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(3/2), x)
```

3.281 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=101

$$\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\left(d+ex^2\right)^{3/2}}-\frac{\left(4cd^2-e(bd+2ae)\right)x}{3d^2e^2\sqrt{d+ex^2}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

```
[Out] 1/3*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^(3/2)+c*arctanh(x*e^(1/2)/(e*x^2+d)^
(1/2))/e^(5/2)-1/3*(4*c*d^2-e*(2*a*e+b*d))*x/d^2/e^2/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1171, 393, 223, 212}

$$-\frac{x\left(4cd^2-e(2ae+bd)\right)}{3d^2e^2\sqrt{d+ex^2}}+\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^2\right)^{3/2}}+\frac{c\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^2)^(3/2)) - ((4*c*d^2 - e*(b*
d + 2*a*e))*x)/(3*d^2*e^2*Sqrt[d + e*x^2]) + (c*ArcTanh[(Sqrt[e]*x)/Sqrt[d
+ e*x^2]])/e^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{(d+ex^2)^{5/2}}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\,(d+ex^2)^{3/2}} - \frac{\int \frac{-2a+\frac{d(cd-be)}{e^2}-\frac{3cdx^2}{e}}{(d+ex^2)^{3/2}}\,dx}{3d}\\
&= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\,(d+ex^2)^{3/2}} - \frac{(4cd^2-e(bd+2ae))\,x}{3d^2e^2\sqrt{d+ex^2}} + \frac{c\int \frac{1}{\sqrt{d+ex^2}}\,dx}{e^2}\\
&= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\,(d+ex^2)^{3/2}} - \frac{(4cd^2-e(bd+2ae))\,x}{3d^2e^2\sqrt{d+ex^2}} + \frac{c\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{e^2}\\
&= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{3d\,(d+ex^2)^{3/2}} - \frac{(4cd^2-e(bd+2ae))\,x}{3d^2e^2\sqrt{d+ex^2}} + \frac{c\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 92, normalized size = 0.91

$$\frac{-cd^2x(3d+4ex^2)+e^2x(3ad+bdx^2+2aex^2)}{3d^2e^2\,(d+ex^2)^{3/2}} - \frac{c\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}\right)}{e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x]
```

```
[Out] (-(c*d^2*x*(3*d + 4*e*x^2)) + e^2*x*(3*a*d + b*d*x^2 + 2*a*e*x^2))/(3*d^2*e
^2*(d + e*x^2)^(3/2)) - (c*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2]])/e^(5/2)
```

Maple [A]
time = 0.12, size = 150, normalized size = 1.49

method	result

default	$c\left(-\frac{x^{3}}{3e\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{-\frac{x}{e\sqrt{e\,x^{2}+d}}+\frac{\ln\left(x\sqrt{e}+\sqrt{e\,x^{2}+d}\right)}{e^{\frac{3}{2}}}}{e}\right)+b\left(-\frac{x}{2e\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{d\left(\frac{x}{3d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{2x}{3d^{2}\sqrt{e\,x\cdots}}\right.}{2e}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/3*x^3/e/(e*x^2+d)^(3/2)+1/e*(-x/e/(e*x^2+d)^(1/2)+1/e^(3/2)*ln(x*e^(1
/2)+(e*x^2+d)^(1/2))))+b*(-1/2*x/e/(e*x^2+d)^(3/2)+1/2*d/e*(1/3*x/d/(e*x^2+
d)^(3/2)+2/3*x/d^2/(e*x^2+d)^(1/2)))+a*(1/3*x/d/(e*x^2+d)^(3/2)+2/3*x/d^2/(
e*x^2+d)^(1/2))
```

Maxima [A]
time = 0.31, size = 135, normalized size = 1.34

$$-\frac{1}{3}\left(\frac{3\,x^{2}e^{(-1)}}{(x^{2}e+d)^{\frac{3}{2}}}+\frac{2\,de^{(-2)}}{(x^{2}e+d)^{\frac{3}{2}}}\right)cx+c\,\mathrm{arsinh}\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}-\frac{cxe^{(-2)}}{3\sqrt{x^{2}e+d}}-\frac{bxe^{(-1)}}{3\,(x^{2}e+d)^{\frac{3}{2}}}+\frac{bxe^{(-1)}}{3\sqrt{x^{2}e+d}\,d}+\frac{2\,ax}{3\sqrt{x^{2}e+d}\,d^{2}}+\frac{ax}{3\,(x^{2}e+d)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^2*e^(-1)/(x^2*e + d)^(3/2) + 2*d*e^(-2)/(x^2*e + d)^(3/2))*c*x +
c*arcsinh(x*e^(1/2)/sqrt(d))*e^(-5/2) - 1/3*c*x*e^(-2)/sqrt(x^2*e + d) - 1/
3*b*x*e^(-1)/(x^2*e + d)^(3/2) + 1/3*b*x*e^(-1)/(sqrt(x^2*e + d)*d) + 2/3*a
*x/(sqrt(x^2*e + d)*d^2) + 1/3*a*x/((x^2*e + d)^(3/2)*d)
```

Fricas [A]
time = 0.45, size = 146, normalized size = 1.45

$$\frac{3\,(cd^{2}x^{4}e^{2}+2\,cd^{3}x^{2}e+cd^{4})e^{\frac{1}{2}}\log\left(-2\,x^{2}e-2\sqrt{x^{2}e+d}\,xe^{\frac{1}{2}}-d\right)-2\,(4\,cd^{2}x^{3}e^{2}+3\,cd^{3}xe-2\,ax^{3}e^{4}-(bdx^{3}+3\,adx)e^{3})\sqrt{x^{2}e+d}}{6\,(d^{2}x^{4}e^{5}+2\,d^{3}x^{2}e^{4}+d^{4}e^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(c*d^2*x^4*e^2 + 2*c*d^3*x^2*e + c*d^4)*e^(1/2)*log(-2*x^2*e - 2*sqr
t(x^2*e + d)*x*e^(1/2) - d) - 2*(4*c*d^2*x^3*e^2 + 3*c*d^3*x*e - 2*a*x^3*e^
4 - (b*d*x^3 + 3*a*d*x)*e^3)*sqrt(x^2*e + d))/(d^2*x^4*e^5 + 2*d^3*x^2*e^4
+ d^4*e^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(94) = 188$.
time = 7.08, size = 450, normalized size = 4.46

$$a\left(\frac{3dx}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^{2}}{d}}+3d^{\frac{5}{2}}ex^{2}\sqrt{1+\frac{ex^{2}}{d}}}+\frac{2ex^{3}}{3d^{\frac{7}{2}}\sqrt{1+\frac{ex^{2}}{d}}+3d^{\frac{5}{2}}ex^{2}\sqrt{1+\frac{ex^{2}}{d}}}\right)+\frac{bx^{3}}{3d^{\frac{5}{2}}\sqrt{1+\frac{ex^{2}}{d}}+3d^{\frac{3}{2}}ex^{2}\sqrt{1+\frac{ex^{2}}{d}}}+c\left(\frac{3d^{\frac{39}{2}}e^{11}\sqrt{1+\frac{ex^{2}}{d}}\,\mathrm{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^{2}}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^{2}\sqrt{1+\frac{ex^{2}}{d}}}+\frac{3d^{\frac{37}{2}}e^{12}x^{2}\sqrt{1+\frac{ex^{2}}{d}}\,\mathrm{asinh}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^{2}}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^{2}\sqrt{1+\frac{ex^{2}}{d}}}-\frac{3d^{19}e^{\frac{23}{2}}x}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^{2}}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^{2}\sqrt{1+\frac{ex^{2}}{d}}}-\frac{4d^{18}e^{\frac{25}{2}}x^{3}}{3d^{\frac{39}{2}}e^{\frac{27}{2}}\sqrt{1+\frac{ex^{2}}{d}}+3d^{\frac{37}{2}}e^{\frac{29}{2}}x^{2}\sqrt{1+\frac{ex^{2}}{d}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(5/2),x)
```

```
[Out] a*(3*d*x/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 + e*x**2
/d)) + 2*e*x**3/(3*d**(7/2)*sqrt(1 + e*x**2/d) + 3*d**(5/2)*e*x**2*sqrt(1 +
 e*x**2/d))) + b*x**3/(3*d**(5/2)*sqrt(1 + e*x**2/d) + 3*d**(3/2)*e*x**2*sq
rt(1 + e*x**2/d)) + c*(3*d**(39/2)*e**11*sqrt(1 + e*x**2/d)*asinh(sqrt(e)*x
/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)
*x**2*sqrt(1 + e*x**2/d)) + 3*d**(37/2)*e**12*x**2*sqrt(1 + e*x**2/d)*asinh
(sqrt(e)*x/sqrt(d))/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)
*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) - 3*d**19*e**(23/2)*x/(3*d**(39/2)*e**(
27/2)*sqrt(1 + e*x**2/d) + 3*d**(37/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)) -
 4*d**18*e**(25/2)*x**3/(3*d**(39/2)*e**(27/2)*sqrt(1 + e*x**2/d) + 3*d**(3
7/2)*e**(29/2)*x**2*sqrt(1 + e*x**2/d)))
```

Giac [A]
time = 3.49, size = 88, normalized size = 0.87

$$-ce^{\left(-\frac{5}{2}\right)}\log\left(\left|-xe^{\frac{1}{2}}+\sqrt{x^2e+d}\right|\right)-\frac{\left(\frac{\left(4\,cd^2e^2-bde^3-2\,ae^4\right)x^2e^{(-3)}}{d^2}+\frac{3\,\left(cd^3e-ade^3\right)e^{(-3)}}{d^2}\right)x}{3\,\left(x^2e+d\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] -c*e^(-5/2)*log(abs(-x*e^(1/2) + sqrt(x^2*e + d))) - 1/3*((4*c*d^2*e^2 - b*
d*e^3 - 2*a*e^4)*x^2*e^(-3)/d^2 + 3*(c*d^3*e - a*d*e^3)*e^(-3)/d^2)*x/(x^2*
e + d)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^4+b\,x^2+a}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(5/2), x)
```

3.282 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^{7/2}}\,dx$

Optimal. Leaf size=86

$$\frac{ax}{d\left(d+ex^2\right)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{\left(3cd^2+2e(bd+4ae)\right)x^5}{15d^3\left(d+ex^2\right)^{5/2}}$$

```
[Out] a*x/d/(e*x^2+d)^(5/2)+1/3*(4*a*e+b*d)*x^3/d^2/(e*x^2+d)^(5/2)+1/15*(3*c*d^2
+2*e*(4*a*e+b*d))*x^5/d^3/(e*x^2+d)^(5/2)
```

Rubi [A]

time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1169, 1817, 12, 270}

$$\frac{x^5\left(2e(4ae+bd)+3cd^2\right)}{15d^3\left(d+ex^2\right)^{5/2}}+\frac{x^3(4ae+bd)}{3d^2\left(d+ex^2\right)^{5/2}}+\frac{ax}{d\left(d+ex^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(5/2)) + ((b*d + 4*a*e)*x^3)/(3*d^2*(d + e*x^2)^(5/2))
 + ((3*c*d^2 + 2*e*(b*d + 4*a*e))*x^5)/(15*d^3*(d + e*x^2)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{(d+ex^2)^{7/2}}\,dx &= \frac{ax}{d(d+ex^2)^{5/2}}+\frac{\int \frac{x^2(4ae+d(b+cx^2))}{(d+ex^2)^{7/2}}\,dx}{d}\\
&= \frac{ax}{d(d+ex^2)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2(d+ex^2)^{5/2}}+\frac{\int \frac{(3cd^2+2e(bd+4ae))x^4}{(d+ex^2)^{7/2}}\,dx}{3d^2}\\
&= \frac{ax}{d(d+ex^2)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2(d+ex^2)^{5/2}}+\frac{1}{3}\left(3c+\frac{2e(bd+4ae)}{d^2}\right)\int \frac{x^4}{(d+ex^2)^{7/2}}\,dx\\
&= \frac{ax}{d(d+ex^2)^{5/2}}+\frac{(bd+4ae)x^3}{3d^2(d+ex^2)^{5/2}}+\frac{(3cd^2+2e(bd+4ae))x^5}{15d^3(d+ex^2)^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 69, normalized size = 0.80

$$\frac{15ad^2x+5bd^2x^3+20adex^3+3cd^2x^5+2bdex^5+8ae^2x^5}{15d^3(d+ex^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x]
```

```
[Out] (15*a*d^2*x + 5*b*d^2*x^3 + 20*a*d*e*x^3 + 3*c*d^2*x^5 + 2*b*d*e*x^5 + 8*a*
e^2*x^5)/(15*d^3*(d + e*x^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(76) = 152$.
time = 0.13, size = 232, normalized size = 2.70

method	result
gosper	$\frac{x(8a\,e^2x^4+2bde\,x^4+3c\,d^2x^4+20ade\,x^2+5b\,d^2x^2+15a\,d^2)}{15(e\,x^2+d)^{\frac{5}{2}}\,d^3}$
trager	$\frac{x(8a\,e^2x^4+2bde\,x^4+3c\,d^2x^4+20ade\,x^2+5b\,d^2x^2+15a\,d^2)}{15(e\,x^2+d)^{\frac{5}{2}}\,d^3}$

default	$c\left(-\frac{x^{3}}{2e\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{3d\left(-\frac{x}{4e\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{d\left(\frac{x}{5d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^{2}\sqrt{e\,x^{2}+d}}}{d}\right)}{4e}\right)}{2e}\right)+b\left(-\frac{x}{4e\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/2*x^3/e/(e*x^2+d)^(5/2)+3/2*d/e*(-1/4*x/e/(e*x^2+d)^(5/2)+1/4*d/e*(1/
5*x/d/(e*x^2+d)^(5/2)+4/5/d*(1/3*x/d/(e*x^2+d)^(3/2)+2/3*x/d^2/(e*x^2+d)^(1
/2)))))+b*(-1/4*x/e/(e*x^2+d)^(5/2)+1/4*d/e*(1/5*x/d/(e*x^2+d)^(5/2)+4/5/d*
(1/3*x/d/(e*x^2+d)^(3/2)+2/3*x/d^2/(e*x^2+d)^(1/2))))+a*(1/5*x/d/(e*x^2+d)^
(5/2)+4/5/d*(1/3*x/d/(e*x^2+d)^(3/2)+2/3*x/d^2/(e*x^2+d)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(82) = 164$.
time = 0.30, size = 176, normalized size = 2.05

$$-\frac{cx^{3}e^{(-1)}}{2\,(x^{2}e+d)^{\frac{5}{2}}}+\frac{cxe^{(-2)}}{10\,(x^{2}e+d)^{\frac{3}{2}}}+\frac{cxe^{(-2)}}{5\,\sqrt{x^{2}e+d}\,d}-\frac{3\,cdxe^{(-2)}}{10\,(x^{2}e+d)^{\frac{5}{2}}}-\frac{bxe^{(-1)}}{5\,(x^{2}e+d)^{\frac{5}{2}}}+\frac{2\,bxe^{(-1)}}{15\,\sqrt{x^{2}e+d}\,d^{2}}+\frac{bxe^{(-1)}}{15\,(x^{2}e+d)^{\frac{3}{2}}d}+\frac{8\,ax}{15\,\sqrt{x^{2}e+d}\,d^{3}}+\frac{4\,ax}{15\,(x^{2}e+d)^{\frac{3}{2}}d^{2}}+\frac{ax}{5\,(x^{2}e+d)^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/2*c*x^3*e^(-1)/(x^2*e + d)^(5/2) + 1/10*c*x*e^(-2)/(x^2*e + d)^(3/2) + 1
/5*c*x*e^(-2)/(sqrt(x^2*e + d)*d) - 3/10*c*d*x*e^(-2)/(x^2*e + d)^(5/2) - 1
/5*b*x*e^(-1)/(x^2*e + d)^(5/2) + 2/15*b*x*e^(-1)/(sqrt(x^2*e + d)*d^2) + 1
/15*b*x*e^(-1)/((x^2*e + d)^(3/2)*d) + 8/15*a*x/(sqrt(x^2*e + d)*d^3) + 4/1
5*a*x/((x^2*e + d)^(3/2)*d^2) + 1/5*a*x/((x^2*e + d)^(5/2)*d)
```

Fricas [A]
time = 0.39, size = 99, normalized size = 1.15

$$\frac{(3\,cd^{2}x^{5}+8\,ax^{5}e^{2}+5\,bd^{2}x^{3}+15\,ad^{2}x+2\,(bdx^{5}+10\,adx^{3})e)\sqrt{x^{2}e+d}}{15\,(d^{3}x^{6}e^{3}+3\,d^{4}x^{4}e^{2}+3\,d^{5}x^{2}e+d^{6})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*c*d^2*x^5 + 8*a*x^5*e^2 + 5*b*d^2*x^3 + 15*a*d^2*x + 2*(b*d*x^5 + 1
0*a*d*x^3)*e)*sqrt(x^2*e + d)/(d^3*x^6*e^3 + 3*d^4*x^4*e^2 + 3*d^5*x^2*e +
d^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(80) = 160$.
time = 16.19, size = 639, normalized size = 7.43

$$a\left(\frac{15d^{5}x}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^{2}}{d}}+45d^{\frac{15}{2}}ex^{2}\sqrt{1+\frac{ex^{2}}{d}}+45d^{\frac{13}{2}}e^{2}x^{4}\sqrt{1+\frac{ex^{2}}{d}}+15d^{\frac{11}{2}}e^{3}x^{6}\sqrt{1+\frac{ex^{2}}{d}}}+\frac{35d^{4}ex^{3}}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^{2}}{d}}+45d^{\frac{15}{2}}ex^{2}\sqrt{1+\frac{ex^{2}}{d}}+45d^{\frac{13}{2}}e^{2}x^{4}\sqrt{1+\frac{ex^{2}}{d}}+15d^{\frac{11}{2}}e^{3}x^{6}\sqrt{1+\frac{ex^{2}}{d}}}+\frac{28d^{3}e^{2}x^{5}}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^{2}}{d}}+45d^{\frac{15}{2}}ex^{2}\sqrt{1+\frac{ex^{2}}{d}}+45d^{\frac{13}{2}}e^{2}x^{4}\sqrt{1+\frac{ex^{2}}{d}}+15d^{\frac{11}{2}}e^{3}x^{6}\sqrt{1+\frac{ex^{2}}{d}}}+\frac{8d^{2}e^{3}x^{7}}{15d^{\frac{17}{2}}\sqrt{1+\frac{ex^{2}}{d}}+45d^{\frac{15}{2}}ex^{2}\sqrt{1+\frac{ex^{2}}{d}}+45d^{\frac{13}{2}}e^{2}x^{4}\sqrt{1+\frac{ex^{2}}{d}}+15d^{\frac{11}{2}}e^{3}x^{6}\sqrt{1+\frac{ex^{2}}{d}}}\right)+b\left(\frac{5dx^{3}}{15d^{\frac{9}{2}}\sqrt{1+\frac{ex^{2}}{d}}+30d^{\frac{7}{2}}ex^{2}\sqrt{1+\frac{ex^{2}}{d}}+15d^{\frac{5}{2}}e^{2}x^{4}\sqrt{1+\frac{ex^{2}}{d}}}+\frac{2ex^{5}}{15d^{\frac{9}{2}}\sqrt{1+\frac{ex^{2}}{d}}+30d^{\frac{7}{2}}ex^{2}\sqrt{1+\frac{ex^{2}}{d}}+15d^{\frac{5}{2}}e^{2}x^{4}\sqrt{1+\frac{ex^{2}}{d}}}\right)+\frac{cx^{5}}{5d^{\frac{7}{2}}\sqrt{1+\frac{ex^{2}}{d}}+10d^{\frac{5}{2}}ex^{2}\sqrt{1+\frac{ex^{2}}{d}}+5d^{\frac{3}{2}}e^{2}x^{4}\sqrt{1+\frac{ex^{2}}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(7/2),x)
```

```
[Out] a*(15*d**5*x/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1
+ e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3
*x**6*sqrt(1 + e*x**2/d)) + 35*d**4*e*x**3/(15*d**(17/2)*sqrt(1 + e*x**2/d)
 + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 +
 e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 28*d**3*e**2*x**5
/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(15/2)*e*x**2*sqrt(1 + e*x**2/d)
+ 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 15*d**(11/2)*e**3*x**6*sqrt(1
 + e*x**2/d)) + 8*d**2*e**3*x**7/(15*d**(17/2)*sqrt(1 + e*x**2/d) + 45*d**(
15/2)*e*x**2*sqrt(1 + e*x**2/d) + 45*d**(13/2)*e**2*x**4*sqrt(1 + e*x**2/d)
 + 15*d**(11/2)*e**3*x**6*sqrt(1 + e*x**2/d))) + b*(5*d*x**3/(15*d**(9/2)*s
qrt(1 + e*x**2/d) + 30*d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**
2*x**4*sqrt(1 + e*x**2/d)) + 2*e*x**5/(15*d**(9/2)*sqrt(1 + e*x**2/d) + 30*
d**(7/2)*e*x**2*sqrt(1 + e*x**2/d) + 15*d**(5/2)*e**2*x**4*sqrt(1 + e*x**2/
d))) + c*x**5/(5*d**(7/2)*sqrt(1 + e*x**2/d) + 10*d**(5/2)*e*x**2*sqrt(1 +
e*x**2/d) + 5*d**(3/2)*e**2*x**4*sqrt(1 + e*x**2/d))
```

Giac [A]
time = 4.07, size = 75, normalized size = 0.87

$$\frac{\left(x^2\left(\frac{(3\,cd^2e^2+2\,bde^3+8\,ae^4)x^2e^{(-2)}}{d^3}+\frac{5\,(bd^2e^2+4\,ade^3)e^{(-2)}}{d^3}\right)+\frac{15\,a}{d}\right)x}{15\,(x^2e+d)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^2*((3*c*d^2*e^2 + 2*b*d*e^3 + 8*a*e^4)*x^2*e^(-2)/d^3 + 5*(b*d^2*e^
2 + 4*a*d*e^3)*e^(-2)/d^3) + 15*a/d)*x/(x^2*e + d)^(5/2)
```

Mupad [B]
time = 4.70, size = 133, normalized size = 1.55

$$\frac{3\,c\,d^4\,x-6\,c\,d^3\,x\,\left(e\,x^2+d\right)-3\,b\,d^3\,e\,x+8\,a\,e^2\,x\,{\left(e\,x^2+d\right)}^2+3\,c\,d^2\,x\,{\left(e\,x^2+d\right)}^2+3\,a\,d^2\,e^2\,x+4\,a\,d\,e^2\,x\,\left(e\,x^2+d\right)+2\,b\,d\,e\,x\,{\left(e\,x^2+d\right)}^2+b\,d^2\,e\,x\,\left(e\,x^2+d\right)}{15\,d^3\,e^2\,{\left(e\,x^2+d\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(7/2),x)
```

```
[Out] (3*c*d^4*x - 6*c*d^3*x*(d + e*x^2) - 3*b*d^3*e*x + 8*a*e^2*x*(d + e*x^2)^2
+ 3*c*d^2*x*(d + e*x^2)^2 + 3*a*d^2*e^2*x + 4*a*d*e^2*x*(d + e*x^2) + 2*b*d
*e*x*(d + e*x^2)^2 + b*d^2*e*x*(d + e*x^2))/(15*d^3*e^2*(d + e*x^2)^(5/2))
```

3.283 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx$

Optimal. Leaf size=126

$$\frac{ax}{d\,(d+ex^2)^{7/2}}+\frac{(bd+6ae)x^3}{3d^2\,(d+ex^2)^{7/2}}+\frac{(3cd^2+4e(bd+6ae))\,x^5}{15d^3\,(d+ex^2)^{7/2}}+\frac{2e(3cd^2+4e(bd+6ae))\,x^7}{105d^4\,(d+ex^2)^{7/2}}$$

```
[Out] a*x/d/(e*x^2+d)^(7/2)+1/3*(6*a*e+b*d)*x^3/d^2/(e*x^2+d)^(7/2)+1/15*(3*c*d^2
+4*e*(6*a*e+b*d))*x^5/d^3/(e*x^2+d)^(7/2)+2/105*e*(3*c*d^2+4*e*(6*a*e+b*d))
*x^7/d^4/(e*x^2+d)^(7/2)
```

Rubi [A]

time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1169, 1817, 12, 277, 270}

$$\frac{2ex^7(4e(6ae+bd)+3cd^2)}{105d^4\,(d+ex^2)^{7/2}}+\frac{x^5(4e(6ae+bd)+3cd^2)}{15d^3\,(d+ex^2)^{7/2}}+\frac{x^3(6ae+bd)}{3d^2\,(d+ex^2)^{7/2}}+\frac{ax}{d\,(d+ex^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(7/2)) + ((b*d + 6*a*e)*x^3)/(3*d^2*(d + e*x^2)^(7/2))
 + ((3*c*d^2 + 4*e*(b*d + 6*a*e))*x^5)/(15*d^3*(d + e*x^2)^(7/2)) + (2*e*(3
*c*d^2 + 4*e*(b*d + 6*a*e))*x^7)/(105*d^4*(d + e*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{(d+ex^2)^{9/2}}\,dx &= \frac{ax}{d(d+ex^2)^{7/2}} + \frac{\int \frac{x^2(6ae+d(b+cx^2))}{(d+ex^2)^{9/2}}\,dx}{d} \\
&= \frac{ax}{d(d+ex^2)^{7/2}} + \frac{(bd+6ae)x^3}{3d^2(d+ex^2)^{7/2}} + \frac{\int \frac{(3cd^2+4e(bd+6ae))x^4}{(d+ex^2)^{9/2}}\,dx}{3d^2} \\
&= \frac{ax}{d(d+ex^2)^{7/2}} + \frac{(bd+6ae)x^3}{3d^2(d+ex^2)^{7/2}} + \frac{1}{3}\left(3c+\frac{4e(bd+6ae)}{d^2}\right)\int \frac{x^4}{(d+ex^2)^{9/2}}\,dx \\
&= \frac{ax}{d(d+ex^2)^{7/2}} + \frac{(bd+6ae)x^3}{3d^2(d+ex^2)^{7/2}} + \frac{(3cd^2+4e(bd+6ae))x^5}{15d^3(d+ex^2)^{7/2}} + \frac{(2e(3cd^2+4e(bd+6a}{15d} \\
&= \frac{ax}{d(d+ex^2)^{7/2}} + \frac{(bd+6ae)x^3}{3d^2(d+ex^2)^{7/2}} + \frac{(3cd^2+4e(bd+6ae))x^5}{15d^3(d+ex^2)^{7/2}} + \frac{2e(3cd^2+4e(bd+6a}{105d^4(d+ex^2)^{7/}}
\end{aligned}$$

Mathematica [A]
time = 0.20, size = 103, normalized size = 0.82

$$\frac{105ad^3x+35bd^3x^3+210ad^2ex^3+21cd^3x^5+28bd^2ex^5+168ade^2x^5+6cd^2ex^7+8bde^2x^7+48ae^3x^7}{105d^4(d+ex^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x]
```

```
[Out] (105*a*d^3*x + 35*b*d^3*x^3 + 210*a*d^2*e*x^3 + 21*c*d^3*x^5 + 28*b*d^2*e*x
^5 + 168*a*d*e^2*x^5 + 6*c*d^2*e*x^7 + 8*b*d*e^2*x^7 + 48*a*e^3*x^7)/(105*d
^4*(d + e*x^2)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(112) = 224$.
time = 0.13, size = 295, normalized size = 2.34

method	result
gosper	$\frac{x\left(48a\,e^{3}x^{6}+8bd\,e^{2}x^{6}+6c\,d^{2}e\,x^{6}+168ad\,e^{2}x^{4}+28b\,d^{2}e\,x^{4}+21c\,d^{3}x^{4}+210a\,d^{2}e\,x^{2}+35b\,d^{3}x^{2}+105d^{3}a\right)}{105\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
trager	$\frac{x\left(48a\,e^{3}x^{6}+8bd\,e^{2}x^{6}+6c\,d^{2}e\,x^{6}+168ad\,e^{2}x^{4}+28b\,d^{2}e\,x^{4}+21c\,d^{3}x^{4}+210a\,d^{2}e\,x^{2}+35b\,d^{3}x^{2}+105d^{3}a\right)}{105\left(e\,x^{2}+d\right)^{\frac{7}{2}}d^{4}}$
default	$c\left(-\frac{x^{3}}{4e\left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{3d\left(-\frac{x}{6e\left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{d\left(\frac{x}{7d\left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^{2}\sqrt{e\,x^{2}+d}}\right)}{7d}}{d}\right)}{6e}\right)}{4e}\right)+ \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/4*x^3/e/(e*x^2+d)^(7/2)+3/4*d/e*(-1/6*x/e/(e*x^2+d)^(7/2)+1/6*d/e*(1/
7*x/d/(e*x^2+d)^(7/2)+6/7/d*(1/5*x/d/(e*x^2+d)^(5/2)+4/5/d*(1/3*x/d/(e*x^2+
d)^(3/2)+2/3*x/d^2/(e*x^2+d)^(1/2))))))+b*(-1/6*x/e/(e*x^2+d)^(7/2)+1/6*d/e
```

```
*(1/7*x/d/(e*x^2+d)^(7/2)+6/7/d*(1/5*x/d/(e*x^2+d)^(5/2)+4/5/d*(1/3*x/d/(e*
x^2+d)^(3/2)+2/3*x/d^2/(e*x^2+d)^(1/2)))))+a*(1/7*x/d/(e*x^2+d)^(7/2)+6/7/d
*(1/5*x/d/(e*x^2+d)^(5/2)+4/5/d*(1/3*x/d/(e*x^2+d)^(3/2)+2/3*x/d^2/(e*x^2+d
)^(1/2))))
```

Maxima [A]
time = 0.29, size = 231, normalized size = 1.83

$$-\frac{cx^3e^{(-1)}}{4\,(x^2e+d)^{\frac{7}{2}}}+\frac{3\,cxe^{(-2)}}{140\,(x^2e+d)^{\frac{5}{2}}}+\frac{2\,cxe^{(-2)}}{35\,\sqrt{x^2e+d}\,d^2}+\frac{cxe^{(-2)}}{35\,(x^2e+d)^{\frac{3}{2}}d}-\frac{3\,cdxe^{(-2)}}{28\,(x^2e+d)^{\frac{7}{2}}}-\frac{bxe^{(-1)}}{7\,(x^2e+d)^{\frac{7}{2}}}+\frac{8\,bxe^{(-1)}}{105\,\sqrt{x^2e+d}\,d^3}+\frac{4\,bxe^{(-1)}}{105\,(x^2e+d)^{\frac{3}{2}}d^2}+\frac{bxe^{(-1)}}{35\,(x^2e+d)^{\frac{5}{2}}d}+\frac{16\,ax}{35\,\sqrt{x^2e+d}\,d^4}+\frac{8\,ax}{35\,(x^2e+d)^{\frac{3}{2}}d^3}+\frac{6\,ax}{35\,(x^2e+d)^{\frac{5}{2}}d^2}+\frac{ax}{7\,(x^2e+d)^{\frac{7}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*c*x^3*e^(-1)/(x^2*e + d)^(7/2) + 3/140*c*x*e^(-2)/(x^2*e + d)^(5/2) +
2/35*c*x*e^(-2)/(sqrt(x^2*e + d)*d^2) + 1/35*c*x*e^(-2)/((x^2*e + d)^(3/2)*
d) - 3/28*c*d*x*e^(-2)/(x^2*e + d)^(7/2) - 1/7*b*x*e^(-1)/(x^2*e + d)^(7/2)
 + 8/105*b*x*e^(-1)/(sqrt(x^2*e + d)*d^3) + 4/105*b*x*e^(-1)/((x^2*e + d)^(
3/2)*d^2) + 1/35*b*x*e^(-1)/((x^2*e + d)^(5/2)*d) + 16/35*a*x/(sqrt(x^2*e +
 d)*d^4) + 8/35*a*x/((x^2*e + d)^(3/2)*d^3) + 6/35*a*x/((x^2*e + d)^(5/2)*d
^2) + 1/7*a*x/((x^2*e + d)^(7/2)*d)
```

Fricas [A]
time = 0.49, size = 141, normalized size = 1.12

$$\frac{(21\,cd^3x^5+48\,ax^7e^3+35\,bd^3x^3+105\,ad^3x+8\,(bdx^7+21\,adx^5)e^2+2\,(3\,cd^2x^7+14\,bd^2x^5+105\,ad^2x^3)e)\sqrt{x^2e+d}}{105\,(d^4x^8e^4+4\,d^5x^6e^3+6\,d^6x^4e^2+4\,d^7x^2e+d^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(21*c*d^3*x^5 + 48*a*x^7*e^3 + 35*b*d^3*x^3 + 105*a*d^3*x + 8*(b*d*x^
7 + 21*a*d*x^5)*e^2 + 2*(3*c*d^2*x^7 + 14*b*d^2*x^5 + 105*a*d^2*x^3)*e)*sqr
t(x^2*e + d)/(d^4*x^8*e^4 + 4*d^5*x^6*e^3 + 6*d^6*x^4*e^2 + 4*d^7*x^2*e + d
^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1989 vs. 2(119) = 238.
time = 41.06, size = 1989, normalized size = 15.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(9/2),x)
```

```
[Out] a*(35*d**14*x/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(
1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*
```

```
e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) +
 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt
(1 + e*x**2/d)) + 175*d**13*e*x**3/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d
**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**
2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8
*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(
25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 371*d**12*e**2*x**5/(35*d**(37/2)*sq
rt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*
e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) +
 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt
(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 429*d**11*e*
*3*x**7/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*
x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x
**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d
**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e
*x**2/d)) + 286*d**10*e**4*x**9/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(
35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d
) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sq
rt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/
2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 104*d**9*e**5*x**11/(35*d**(37/2)*sqrt(
1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2/d) + 525*d**(33/2)*e**
2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 52
5*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(27/2)*e**5*x**10*sqrt(1
+ e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**2/d)) + 16*d**8*e**6*x*
*13/(35*d**(37/2)*sqrt(1 + e*x**2/d) + 210*d**(35/2)*e*x**2*sqrt(1 + e*x**2
/d) + 525*d**(33/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 700*d**(31/2)*e**3*x**6*
sqrt(1 + e*x**2/d) + 525*d**(29/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 210*d**(2
7/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 35*d**(25/2)*e**6*x**12*sqrt(1 + e*x**
2/d))) + b*(35*d**5*x**3/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420*d**(17/2)*
e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 42
0*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x**8*sqrt(1 +
 e*x**2/d)) + 63*d**4*e*x**5/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420*d**(17
/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x**2/d)
+ 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x**8*sqrt
(1 + e*x**2/d)) + 36*d**3*e**2*x**7/(105*d**(19/2)*sqrt(1 + e*x**2/d) + 420
*d**(17/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1 + e*x
**2/d) + 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e**4*x*
*8*sqrt(1 + e*x**2/d)) + 8*d**2*e**3*x**9/(105*d**(19/2)*sqrt(1 + e*x**2/d)
 + 420*d**(17/2)*e*x**2*sqrt(1 + e*x**2/d) + 630*d**(15/2)*e**2*x**4*sqrt(1
 + e*x**2/d) + 420*d**(13/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 105*d**(11/2)*e
**4*x**8*sqrt(1 + e*x**2/d))) + c*(7*d*x**5/(35*d**(11/2)*sqrt(1 + e*x**2/d
) + 105*d**(9/2)*e*x**2*sqrt(1 + e*x**2/d) + 105*d**(7/2)*e**2*x**4*sqrt(1
+ e*x**2/d) + 35*d**(5/2)*e**3*x**6*sqrt(1 + e*x**2/d)) + 2*e*x**7/(35*d**(
11/2)*sqrt(1 + e*x**2/d) + 105*d**(9/2)*e*x**2*sqrt(1 + e*x**2/d) + 105*d**
(7/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 35*d**(5/2)*e**3*x**6*sqrt(1 + e*x**2/
```

```
d)))
```

Giac [A]
time = 3.38, size = 113, normalized size = 0.90

$$\frac{\left(\left(x^2\left(\frac{2\left(3\,cd^2e^4+4\,bde^5+24\,ae^6\right)x^2e^{(-3)}}{d^4}+\frac{7\left(3\,cd^3e^3+4\,bd^2e^4+24\,ade^5\right)e^{(-3)}}{d^4}\right)+\frac{35\left(bd^3e^3+6\,ad^2e^4\right)e^{(-3)}}{d^4}\right)x^2+\frac{105\,a}{d}\right)x}{105\left(x^2e+d\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*(2*(3*c*d^2*e^4 + 4*b*d*e^5 + 24*a*e^6)*x^2*e^(-3)/d^4 + 7*(3*c
*d^3*e^3 + 4*b*d^2*e^4 + 24*a*d*e^5)*e^(-3)/d^4) + 35*(b*d^3*e^3 + 6*a*d^2*
e^4)*e^(-3)/d^4)*x^2 + 105*a/d)*x/(x^2*e + d)^(7/2)
```

Mupad [B]
time = 4.67, size = 154, normalized size = 1.22

$$\frac{x\left(\frac{a}{7\,d}-\frac{d\left(\frac{b}{7\,d}-\frac{c}{7\,e}\right)}{e}\right)}{\left(e\,x^2+d\right)^{7/2}}-\frac{x\left(\frac{c}{5\,e^2}-\frac{-c\,d^2+b\,d\,e+6\,a\,e^2}{35\,d^2\,e^2}\right)}{\left(e\,x^2+d\right)^{5/2}}+\frac{x\left(3\,c\,d^2+4\,b\,d\,e+24\,a\,e^2\right)}{105\,d^3\,e^2\left(e\,x^2+d\right)^{3/2}}+\frac{x\left(6\,c\,d^2+8\,b\,d\,e+48\,a\,e^2\right)}{105\,d^4\,e^2\,\sqrt{e\,x^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(9/2),x)
```

```
[Out] (x*(a/(7*d) - (d*(b/(7*d) - c/(7*e)))/e))/(d + e*x^2)^(7/2) - (x*(c/(5*e^2)
 - (6*a*e^2 - c*d^2 + b*d*e)/(35*d^2*e^2)))/(d + e*x^2)^(5/2) + (x*(24*a*e^
2 + 3*c*d^2 + 4*b*d*e))/(105*d^3*e^2*(d + e*x^2)^(3/2)) + (x*(48*a*e^2 + 6*
c*d^2 + 8*b*d*e))/(105*d^4*e^2*(d + e*x^2)^(1/2))
```

3.284 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx$

Optimal. Leaf size=165

$$\frac{ax}{d\,(d+ex^2)^{9/2}}+\frac{(bd+8ae)x^3}{3d^2\,(d+ex^2)^{9/2}}+\frac{(cd^2+2e(bd+8ae))\,x^5}{5d^3\,(d+ex^2)^{9/2}}+\frac{4e(cd^2+2e(bd+8ae))\,x^7}{35d^4\,(d+ex^2)^{9/2}}+\frac{8e^2(cd^2+2e(bd+8ae))\,x^9}{315d^5\,(d+ex^2)^{9/2}}$$

```
[Out] a*x/d/(e*x^2+d)^(9/2)+1/3*(8*a*e+b*d)*x^3/d^2/(e*x^2+d)^(9/2)+1/5*(c*d^2+2*
e*(8*a*e+b*d))*x^5/d^3/(e*x^2+d)^(9/2)+4/35*e*(c*d^2+2*e*(8*a*e+b*d))*x^7/d
^4/(e*x^2+d)^(9/2)+8/315*e^2*(c*d^2+2*e*(8*a*e+b*d))*x^9/d^5/(e*x^2+d)^(9/2
)
```

Rubi [A]
time = 0.14, antiderivative size = 164, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1169, 1817, 12, 277, 270}

$$\frac{x^5\left(\frac{2e(8ae+bd)}{d^2}+c\right)}{5d\,(d+ex^2)^{9/2}}+\frac{8e^2x^9(2e(8ae+bd)+cd^2)}{315d^5\,(d+ex^2)^{9/2}}+\frac{4ex^7(2e(8ae+bd)+cd^2)}{35d^4\,(d+ex^2)^{9/2}}+\frac{x^3(8ae+bd)}{3d^2\,(d+ex^2)^{9/2}}+\frac{ax}{d\,(d+ex^2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(9/2)) + ((b*d + 8*a*e)*x^3)/(3*d^2*(d + e*x^2)^(9/2))
 + ((c + (2*e*(b*d + 8*a*e))/d^2)*x^5)/(5*d*(d + e*x^2)^(9/2)) + (4*e*(c*d^
2 + 2*e*(b*d + 8*a*e))*x^7)/(35*d^4*(d + e*x^2)^(9/2)) + (8*e^2*(c*d^2 + 2*
e*(b*d + 8*a*e))*x^9)/(315*d^5*(d + e*x^2)^(9/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{11/2}}\,dx = \frac{ax}{d\,(d+ex^2)^{9/2}} + \frac{\int \frac{x^2(8ae+d(b+cx^2))}{(d+ex^2)^{11/2}}\,dx}{d}$$

$$= \frac{ax}{d\,(d+ex^2)^{9/2}} + \frac{(bd+8ae)x^3}{3d^2\,(d+ex^2)^{9/2}} + \frac{\int \frac{(3cd^2+6e(bd+8ae))x^4}{(d+ex^2)^{11/2}}\,dx}{3d^2}$$

$$= \frac{ax}{d\,(d+ex^2)^{9/2}} + \frac{(bd+8ae)x^3}{3d^2\,(d+ex^2)^{9/2}} + \left(c+\frac{2e(bd+8ae)}{d^2}\right)\int \frac{x^4}{(d+ex^2)^{11/2}}\,dx$$

$$= \frac{ax}{d\,(d+ex^2)^{9/2}} + \frac{(bd+8ae)x^3}{3d^2\,(d+ex^2)^{9/2}} + \frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\,(d+ex^2)^{9/2}} + \frac{\left(4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)\right)\int \frac{x^{\dots}}{(d+e\dots}}{5d}$$

$$= \frac{ax}{d\,(d+ex^2)^{9/2}} + \frac{(bd+8ae)x^3}{3d^2\,(d+ex^2)^{9/2}} + \frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\,(d+ex^2)^{9/2}} + \frac{4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^7}{35d^2\,(d+ex^2)^{9/2}} + \frac{(8e\dots}{\dots}$$

$$= \frac{ax}{d\,(d+ex^2)^{9/2}} + \frac{(bd+8ae)x^3}{3d^2\,(d+ex^2)^{9/2}} + \frac{\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^5}{5d\,(d+ex^2)^{9/2}} + \frac{4e\left(c+\frac{2e(bd+8ae)}{d^2}\right)x^7}{35d^2\,(d+ex^2)^{9/2}} + \frac{8e^2\dots}{31\dots}$$

Mathematica [A]
time = 0.28, size = 132, normalized size = 0.80

$$\frac{a(315d^4x+840d^3ex^3+1008d^2e^2x^5+576de^3x^7+128e^4x^9)+dx^3(cdx^2(63d^2+36dex^2+8e^2x^4)+b(105d^3+126d^2ex^2+72de^2x^4+16e^3x^6))}{315d^5\,(d+ex^2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x]
```

```
[Out] (a*(315*d^4*x + 840*d^3*e*x^3 + 1008*d^2*e^2*x^5 + 576*d*e^3*x^7 + 128*e^4*
x^9) + d*x^3*(c*d*x^2*(63*d^2 + 36*d*e*x^2 + 8*e^2*x^4) + b*(105*d^3 + 126*
d^2*e*x^2 + 72*d*e^2*x^4 + 16*e^3*x^6)))/(315*d^5*(d + e*x^2)^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(147) = 294$.
time = 0.13, size = 358, normalized size = 2.17

method	result
gosper	$\frac{x\left(128a\,e^4x^8+16bd\,e^3x^8+8c\,d^2e^2x^8+576ad\,e^3x^6+72b\,d^2e^2x^6+36c\,d^3e\,x^6+1008a\,d^2e^2x^4+126b\,d^3e\,x^4+63c\,d^4x^4+840a\,d^3e\,x^2+105b\right.}{315(e\,x^2+d)^{\frac{9}{2}}d^5}$
trager	$\frac{x\left(128a\,e^4x^8+16bd\,e^3x^8+8c\,d^2e^2x^8+576ad\,e^3x^6+72b\,d^2e^2x^6+36c\,d^3e\,x^6+1008a\,d^2e^2x^4+126b\,d^3e\,x^4+63c\,d^4x^4+840a\,d^3e\,x^2+105b\right.}{315(e\,x^2+d)^{\frac{9}{2}}d^5}$

default	$c\left(-\frac{x^{3}}{6e\left(e\,x^{2}+d\right)^{\frac{9}{2}}}+\frac{d\left(-\frac{x}{8e\left(e\,x^{2}+d\right)^{\frac{9}{2}}}+\frac{d\left(\frac{x}{9d\left(e\,x^{2}+d\right)^{\frac{9}{2}}}+\frac{\frac{8x}{63d\left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{8x}{15d^{2}\sqrt{e\,x^{2}+d}}\right.}{7d}\right.}{9d}}{d}\right.}{8e}\right.}{2e}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/6*x^3/e/(e*x^2+d)^(9/2)+1/2*d/e*(-1/8*x/e/(e*x^2+d)^(9/2)+1/8*d/e*(1/
9*x/d/(e*x^2+d)^(9/2)+8/9/d*(1/7*x/d/(e*x^2+d)^(7/2)+6/7/d*(1/5*x/d/(e*x^2+
d)^(5/2)+4/5/d*(1/3*x/d/(e*x^2+d)^(3/2)+2/3*x/d^2/(e*x^2+d)^(1/2)))))))+b*(
-1/8*x/e/(e*x^2+d)^(9/2)+1/8*d/e*(1/9*x/d/(e*x^2+d)^(9/2)+8/9/d*(1/7*x/d/(e
*x^2+d)^(7/2)+6/7/d*(1/5*x/d/(e*x^2+d)^(5/2)+4/5/d*(1/3*x/d/(e*x^2+d)^(3/2)
+2/3*x/d^2/(e*x^2+d)^(1/2))))))+a*(1/9*x/d/(e*x^2+d)^(9/2)+8/9/d*(1/7*x/d/(
e*x^2+d)^(7/2)+6/7/d*(1/5*x/d/(e*x^2+d)^(5/2)+4/5/d*(1/3*x/d/(e*x^2+d)^(3/2
)+2/3*x/d^2/(e*x^2+d)^(1/2)))))
```

Maxima [A]
time = 0.29, size = 286, normalized size = 1.73

$$-\frac{cx^3e^{(-1)}}{6\,(x^2e+d)^{\frac{9}{2}}}+\frac{cxe^{(-2)}}{126\,(x^2e+d)^{\frac{7}{2}}}+\frac{8\,cxe^{(-2)}}{315\,\sqrt{x^2e+d}\,d^3}+\frac{4\,cxe^{(-2)}}{315\,(x^2e+d)^{\frac{3}{2}}d^2}+\frac{cxe^{(-2)}}{105\,(x^2e+d)^{\frac{5}{2}}d}-\frac{cdxe^{(-2)}}{18\,(x^2e+d)^{\frac{9}{2}}}-\frac{bxe^{(-1)}}{9\,(x^2e+d)^{\frac{9}{2}}}+\frac{16\,bxe^{(-1)}}{315\,\sqrt{x^2e+d}\,d^4}+\frac{8\,bxe^{(-1)}}{315\,(x^2e+d)^{\frac{3}{2}}d^3}+\frac{2\,bxe^{(-1)}}{105\,(x^2e+d)^{\frac{5}{2}}d^2}+\frac{bxe^{(-1)}}{63\,(x^2e+d)^{\frac{7}{2}}d}+\frac{128\,ax}{315\,\sqrt{x^2e+d}\,d^5}+\frac{64\,ax}{315\,(x^2e+d)^{\frac{3}{2}}d^4}+\frac{16\,ax}{105\,(x^2e+d)^{\frac{5}{2}}d^3}+\frac{8\,ax}{63\,(x^2e+d)^{\frac{7}{2}}d^2}+\frac{ax}{9\,(x^2e+d)^{\frac{9}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/6*c*x^3*e^(-1)/(x^2*e + d)^(9/2) + 1/126*c*x*e^(-2)/(x^2*e + d)^(7/2) +
8/315*c*x*e^(-2)/(sqrt(x^2*e + d)*d^3) + 4/315*c*x*e^(-2)/((x^2*e + d)^(3/2
)*d^2) + 1/105*c*x*e^(-2)/((x^2*e + d)^(5/2)*d) - 1/18*c*d*x*e^(-2)/(x^2*e
+ d)^(9/2) - 1/9*b*x*e^(-1)/(x^2*e + d)^(9/2) + 16/315*b*x*e^(-1)/(sqrt(x^2
*e + d)*d^4) + 8/315*b*x*e^(-1)/((x^2*e + d)^(3/2)*d^3) + 2/105*b*x*e^(-1)/
((x^2*e + d)^(5/2)*d^2) + 1/63*b*x*e^(-1)/((x^2*e + d)^(7/2)*d) + 128/315*a
*x/(sqrt(x^2*e + d)*d^5) + 64/315*a*x/((x^2*e + d)^(3/2)*d^4) + 16/105*a*x/
((x^2*e + d)^(5/2)*d^3) + 8/63*a*x/((x^2*e + d)^(7/2)*d^2) + 1/9*a*x/((x^2*
e + d)^(9/2)*d)
```

Fricas [A]
time = 0.62, size = 182, normalized size = 1.10

$$\frac{(128\,ax^9e^4+63\,cd^4x^5+105\,bd^4x^3+315\,ad^4x+16\,(bdx^9+36\,adx^7)e^3+8\,(cd^2x^9+9\,bd^2x^7+126\,ad^2x^5)e^2+6\,(6\,cd^3x^7+21\,bd^3x^5+140\,ad^3x^3)e)\sqrt{x^2e+d}}{315\,(d^5x^{10}e^5+5\,d^6x^8e^4+10\,d^7x^6e^3+10\,d^8x^4e^2+5\,d^9x^2e+d^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(128*a*x^9*e^4 + 63*c*d^4*x^5 + 105*b*d^4*x^3 + 315*a*d^4*x + 16*(b*d
*x^9 + 36*a*d*x^7)*e^3 + 8*(c*d^2*x^9 + 9*b*d^2*x^7 + 126*a*d^2*x^5)*e^2 +
6*(6*c*d^3*x^7 + 21*b*d^3*x^5 + 140*a*d^3*x^3)*e)*sqrt(x^2*e + d)/(d^5*x^10
*e^5 + 5*d^6*x^8*e^4 + 10*d^7*x^6*e^3 + 10*d^8*x^4*e^2 + 5*d^9*x^2*e + d^10
)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5187 vs. $2(160) = 320$.
time = 91.42, size = 5187, normalized size = 31.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(11/2),x)

[Out] a*(315*d**30*x/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e*x**2*sq
rt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 37800*d**
(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*sqrt(1 + e*
x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*d**(59/2)*e
6*x12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(1 + e*x**2/d
) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53/2)*e**9*x**
18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x**2/d)) + 273
0*d**29*e*x**3/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e*x**2*sq
rt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 37800*d**
(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*sqrt(1 + e*
x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*d**(59/2)*e
6*x12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(1 + e*x**2/d
) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53/2)*e**9*x**
18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x**2/d)) + 107
73*d**28*e**2*x**5/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e*x**
2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 37800
*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*sqrt(1
+ e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*d**(59/
2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(1 + e*x*
*2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53/2)*e**9
*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x**2/d)) +
 25524*d**27*e**3*x**7/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/2)*e
*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d) + 3
7800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8*sqr
t(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150*d**
(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(1 +
e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53/2)*
e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x**2/d
)) + 40229*d**26*e**4*x**9/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d**(69/
2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x**2/d)
 + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4*x**8
*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) + 66150
*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*sqrt(
1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d**(53
/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 + e*x*
*2/d)) + 44058*d**25*e**5*x**11/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 3150*d*
*(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 + e*x*
*2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)*e**4
*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/d) +
66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x**14*
sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3150*d

```
**(53/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqrt(1 +
 e*x**2/d)) + 33915*d**24*e**6*x**13/(315*d**(71/2)*sqrt(1 + e*x**2/d) + 31
50*d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqrt(1 +
 e*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(63/2)
*e**4*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*x**2/
d) + 66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e**7*x
**14*sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d) + 3
150*d**(53/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**20*sqr
t(1 + e*x**2/d)) + 18088*d**23*e**7*x**15/(315*d**(71/2)*sqrt(1 + e*x**2/d)
 + 3150*d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4*sqr
t(1 + e*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*d**(
63/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1 + e*
x**2/d) + 66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/2)*e
**7*x**14*sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x**2/d
) + 3150*d**(53/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*x**2
0*sqrt(1 + e*x**2/d)) + 6384*d**22*e**8*x**17/(315*d**(71/2)*sqrt(1 + e*x**
2/d) + 3150*d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*x**4
*sqrt(1 + e*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66150*
d**(63/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 79380*d**(61/2)*e**5*x**10*sqrt(1
+ e*x**2/d) + 66150*d**(59/2)*e**6*x**12*sqrt(1 + e*x**2/d) + 37800*d**(57/
2)*e**7*x**14*sqrt(1 + e*x**2/d) + 14175*d**(55/2)*e**8*x**16*sqrt(1 + e*x*
*2/d) + 3150*d**(53/2)*e**9*x**18*sqrt(1 + e*x**2/d) + 315*d**(51/2)*e**10*
x**20*sqrt(1 + e*x**2/d)) + 1344*d**21*e**9*x**19/(315*d**(71/2)*sqrt(1 + e
*x**2/d) + 3150*d**(69/2)*e*x**2*sqrt(1 + e*x**2/d) + 14175*d**(67/2)*e**2*
x**4*sqrt(1 + e*x**2/d) + 37800*d**(65/2)*e**3*x**6*sqrt(1 + e*x**2/d) + 66
150*d**(63/2)*e**4*x**8*sqrt(1 + e*x**2/d) + 79...
```

Giac [A]

time = 3.45, size = 148, normalized size = 0.90

$$\frac{\left(\left(\left(4\,x^2\left(\frac{2\,(cd^2e^6+2\,bde^7+16\,ae^8)x^2e^{(-4)}}{d^5}+\frac{9\,(cd^3e^5+2\,bd^2e^6+16\,ade^7)e^{(-4)}}{d^5}\right)+\frac{63\,(cd^4e^4+2\,bd^3e^5+16\,ad^2e^6)e^{(-4)}}{d^5}\right)x^2+\frac{105\,(bd^4e^4+8\,ad^3e^5)e^{(-4)}}{d^5}\right)x^2+\frac{315\,a}{d}\right)x}{315\,(x^2e+d)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((4*x^2*(2*(c*d^2*e^6 + 2*b*d*e^7 + 16*a*e^8)*x^2*e^(-4)/d^5 + 9*(c*
d^3*e^5 + 2*b*d^2*e^6 + 16*a*d*e^7)*e^(-4)/d^5) + 63*(c*d^4*e^4 + 2*b*d^3*e
^5 + 16*a*d^2*e^6)*e^(-4)/d^5)*x^2 + 105*(b*d^4*e^4 + 8*a*d^3*e^5)*e^(-4)/d
^5)*x^2 + 315*a/d)*x/(x^2*e + d)^(9/2)
```

Mupad [B]

time = 4.75, size = 189, normalized size = 1.15

$$\frac{x\left(\frac{a}{9\,d}-\frac{d\left(\frac{b}{9\,d}-\frac{c}{9\,e}\right)}{e}\right)}{{\left(e\,x^2+d\right)}^{9/2}}-\frac{x\left(\frac{c}{7\,e^2}-\frac{-c\,d^2+b\,d\,e+8\,a\,e^2}{63\,d^2\,e^2}\right)}{{\left(e\,x^2+d\right)}^{7/2}}+\frac{x\,\left(c\,d^2+2\,b\,d\,e+16\,a\,e^2\right)}{105\,d^3\,e^2\,{\left(e\,x^2+d\right)}^{5/2}}+\frac{x\,\left(4\,c\,d^2+8\,b\,d\,e+64\,a\,e^2\right)}{315\,d^4\,e^2\,{\left(e\,x^2+d\right)}^{3/2}}+\frac{x\,\left(8\,c\,d^2+16\,b\,d\,e+128\,a\,e^2\right)}{315\,d^5\,e^2\,\sqrt{e\,x^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(11/2),x)
```

```
[Out] (x*(a/(9*d) - (d*(b/(9*d) - c/(9*e)))/e))/(d + e*x^2)^(9/2) - (x*(c/(7*e^2)
 - (8*a*e^2 - c*d^2 + b*d*e)/(63*d^2*e^2)))/(d + e*x^2)^(7/2) + (x*(16*a*e^
2 + c*d^2 + 2*b*d*e))/(105*d^3*e^2*(d + e*x^2)^(5/2)) + (x*(64*a*e^2 + 4*c*
d^2 + 8*b*d*e))/(315*d^4*e^2*(d + e*x^2)^(3/2)) + (x*(128*a*e^2 + 8*c*d^2 +
 16*b*d*e))/(315*d^5*e^2*(d + e*x^2)^(1/2))
```

3.285 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}} \, dx$

Optimal. Leaf size=210

$$\frac{ax}{d(d+ex^2)^{11/2}}+\frac{(bd+10ae)x^3}{3d^2(d+ex^2)^{11/2}}+\frac{(3cd^2+8e(bd+10ae))x^5}{15d^3(d+ex^2)^{11/2}}+\frac{2e(3cd^2+8e(bd+10ae))x^7}{35d^4(d+ex^2)^{11/2}}+\frac{8e^2(3cd^2+8e(bd+10ae))x^9}{315d^5(d+ex^2)^{11/2}}$$

```
[Out] a*x/d/(e*x^2+d)^(11/2)+1/3*(10*a*e+b*d)*x^3/d^2/(e*x^2+d)^(11/2)+1/15*(3*c*
d^2+8*e*(10*a*e+b*d))*x^5/d^3/(e*x^2+d)^(11/2)+2/35*e*(3*c*d^2+8*e*(10*a*e+
b*d))*x^7/d^4/(e*x^2+d)^(11/2)+8/315*e^2*(3*c*d^2+8*e*(10*a*e+b*d))*x^9/d^5
/(e*x^2+d)^(11/2)+16/3465*e^3*(3*c*d^2+8*e*(10*a*e+b*d))*x^11/d^6/(e*x^2+d)
^(11/2)
```

Rubi [A]
time = 0.14, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1169, 1817, 12, 277, 270}

$$\frac{16e^3x^{11}(8e(10ae+bd)+3cd^2)}{3465d^6(d+ex^2)^{11/2}}+\frac{8e^2x^9(8e(10ae+bd)+3cd^2)}{315d^5(d+ex^2)^{11/2}}+\frac{2ex^7(8e(10ae+bd)+3cd^2)}{35d^4(d+ex^2)^{11/2}}+\frac{x^5(8e(10ae+bd)+3cd^2)}{15d^3(d+ex^2)^{11/2}}+\frac{x^3(10ae+bd)}{3d^2(d+ex^2)^{11/2}}+\frac{ax}{d(d+ex^2)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

```
[Out] (a*x)/(d*(d + e*x^2)^(11/2)) + ((b*d + 10*a*e)*x^3)/(3*d^2*(d + e*x^2)^(11/
2)) + ((3*c*d^2 + 8*e*(b*d + 10*a*e))*x^5)/(15*d^3*(d + e*x^2)^(11/2)) + (2
*e*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^7)/(35*d^4*(d + e*x^2)^(11/2)) + (8*e^2
*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^9)/(315*d^5*(d + e*x^2)^(11/2)) + (16*e^3
*(3*c*d^2 + 8*e*(b*d + 10*a*e))*x^11)/(3465*d^6*(d + e*x^2)^(11/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Dist[1/d, Int[x^2*(d
+ e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*a
^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4*
p + 2*q + 1, 0]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{(d+ex^2)^{13/2}}\,dx = \frac{ax}{d\,(d+ex^2)^{11/2}} + \frac{\int \frac{x^2(10ae+d(b+cx^2))}{(d+ex^2)^{13/2}}\,dx}{d}$$

$$= \frac{ax}{d\,(d+ex^2)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\,(d+ex^2)^{11/2}} + \frac{\int \frac{(3cd^2+8e(bd+10ae))x^4}{(d+ex^2)^{13/2}}\,dx}{3d^2}$$

$$= \frac{ax}{d\,(d+ex^2)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\,(d+ex^2)^{11/2}} + \frac{1}{3}\left(3c+\frac{8e(bd+10ae)}{d^2}\right)\int \frac{x^4}{(d+ex^2)^{13/2}}\,dx$$

$$= \frac{ax}{d\,(d+ex^2)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\,(d+ex^2)^{11/2}} + \frac{(3cd^2+8e(bd+10ae))\,x^5}{15d^3\,(d+ex^2)^{11/2}} + \frac{(2e(3cd^2+8e(bd+}{}$$

$$= \frac{ax}{d\,(d+ex^2)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\,(d+ex^2)^{11/2}} + \frac{(3cd^2+8e(bd+10ae))\,x^5}{15d^3\,(d+ex^2)^{11/2}} + \frac{2e(3cd^2+8e(bd+}{35d^4\,(d+ex^2}$$

$$= \frac{ax}{d\,(d+ex^2)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\,(d+ex^2)^{11/2}} + \frac{(3cd^2+8e(bd+10ae))\,x^5}{15d^3\,(d+ex^2)^{11/2}} + \frac{2e(3cd^2+8e(bd+}{35d^4\,(d+ex^2}$$

$$= \frac{ax}{d\,(d+ex^2)^{11/2}} + \frac{(bd+10ae)x^3}{3d^2\,(d+ex^2)^{11/2}} + \frac{(3cd^2+8e(bd+10ae))\,x^5}{15d^3\,(d+ex^2)^{11/2}} + \frac{2e(3cd^2+8e(bd+}{35d^4\,(d+ex^2}$$

Mathematica [**A**]

time = 0.45, size = 167, normalized size = 0.80

$$\frac{5a(693d^5x + 2310d^4ex^3 + 3696d^3e^2x^5 + 3168d^2e^3x^7 + 1408de^4x^9 + 256e^5x^{11}) + dx^3(3cdx^2(231d^3 + 198d^2ex^2 + 88de^2x^4 + 16e^3x^6) + b(1155d^4 + 1848d^3ex^2 + 1584d^2e^2x^4 + 704de^3x^6 + 128e^4x^8))}{3465d^6\left(d + ex^2\right)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x]
```

```
[Out] (5*a*(693*d^5*x + 2310*d^4*e*x^3 + 3696*d^3*e^2*x^5 + 3168*d^2*e^3*x^7 + 14
08*d*e^4*x^9 + 256*e^5*x^11) + d*x^3*(3*c*d*x^2*(231*d^3 + 198*d^2*e*x^2 +
88*d*e^2*x^4 + 16*e^3*x^6) + b*(1155*d^4 + 1848*d^3*e*x^2 + 1584*d^2*e^2*x^
4 + 704*d*e^3*x^6 + 128*e^4*x^8)))/(3465*d^6*(d + e*x^2)^(11/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(188) = 376$.
time = 0.11, size = 421, normalized size = 2.00

method	result
gosper	$\frac{x(1280a\,e^5x^{10}+128bd\,e^4x^{10}+48c\,d^2e^3x^{10}+7040ad\,e^4x^8+704b\,d^2e^3x^8+264c\,d^3e^2x^8+15840a\,d^2e^3x^6+1584b\,d^3e^2x^6+594c\,d^4e\,x^6+18}{3465(e\,x^2+d)^{\frac{11}{2}}d^6}$
trager	$\frac{x(1280a\,e^5x^{10}+128bd\,e^4x^{10}+48c\,d^2e^3x^{10}+7040ad\,e^4x^8+704b\,d^2e^3x^8+264c\,d^3e^2x^8+15840a\,d^2e^3x^6+1584b\,d^3e^2x^6+594c\,d^4e\,x^6+18}{3465(e\,x^2+d)^{\frac{11}{2}}d^6}$

$$10\left(\frac{8x}{63d\left(e\,x^{2}+d\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35d\left(e\,x^{2}+d\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15d\left(e\,x^{2}\right.}\right.}{}\right)}{}\right)$$

$$d\left(\frac{x}{11d\left(e\,x^{2}+d\right)^{\frac{11}{2}}}+\frac{\frac{10x}{99d\left(e\,x^{2}+d\right)^{\frac{9}{2}}}+\frac{\cdots}{11d}}{d}\right)$$

$$3d\left(-\frac{x}{10e\left(e\,x^{2}+d\right)^{\frac{11}{2}}}+\frac{\cdots}{10e}\right)$$

default	c	$-\frac{x^{3}}{8e(e\,x^{2}+d)^{\frac{11}{2}}}+\frac{\cdots}{8e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(-1/8*x^3/e/(e*x^2+d)^(11/2)+3/8*d/e*(-1/10*x/e/(e*x^2+d)^(11/2)+1/10*d/e
*(1/11*x/d/(e*x^2+d)^(11/2)+10/11/d*(1/9*x/d/(e*x^2+d)^(9/2)+8/9/d*(1/7*x/d
/(e*x^2+d)^(7/2)+6/7/d*(1/5*x/d/(e*x^2+d)^(5/2)+4/5/d*(1/3*x/d/(e*x^2+d)^(3
/2)+2/3*x/d^2/(e*x^2+d)^(1/2))))))))+b*(-1/10*x/e/(e*x^2+d)^(11/2)+1/10*d/e
*(1/11*x/d/(e*x^2+d)^(11/2)+10/11/d*(1/9*x/d/(e*x^2+d)^(9/2)+8/9/d*(1/7*x/d
/(e*x^2+d)^(7/2)+6/7/d*(1/5*x/d/(e*x^2+d)^(5/2)+4/5/d*(1/3*x/d/(e*x^2+d)^(3
/2)+2/3*x/d^2/(e*x^2+d)^(1/2)))))))+a*(1/11*x/d/(e*x^2+d)^(11/2)+10/11/d*(1
/9*x/d/(e*x^2+d)^(9/2)+8/9/d*(1/7*x/d/(e*x^2+d)^(7/2)+6/7/d*(1/5*x/d/(e*x^2
+d)^(5/2)+4/5/d*(1/3*x/d/(e*x^2+d)^(3/2)+2/3*x/d^2/(e*x^2+d)^(1/2))))))
```

Maxima [A]
time = 0.30, size = 341, normalized size = 1.62

$$-\frac{cx^3e^{(-1)}}{8\,(x^2e+d)^{\frac{11}{2}}}+\frac{cxe^{(-2)}}{264\,(x^2e+d)^{\frac{9}{2}}}+\frac{16\,cxe^{(-2)}}{1155\,\sqrt{x^2e+d}\,d^4}+\frac{8\,cxe^{(-2)}}{1155\,(x^2e+d)^{\frac{3}{2}}d^3}+\frac{2\,cxe^{(-2)}}{385\,(x^2e+d)^{\frac{5}{2}}d^2}+\frac{cxe^{(-2)}}{231\,(x^2e+d)^{\frac{7}{2}}d}-\frac{3\,cdxe^{(-2)}}{88\,(x^2e+d)^{\frac{11}{2}}}-\frac{bxe^{(-1)}}{11\,(x^2e+d)^{\frac{11}{2}}}+\frac{128\,bxe^{(-1)}}{3465\,\sqrt{x^2e+d}\,d^5}+\frac{64\,bxe^{(-1)}}{3465\,(x^2e+d)^{\frac{3}{2}}d^4}+\frac{16\,bxe^{(-1)}}{1155\,(x^2e+d)^{\frac{5}{2}}d^3}+\frac{8\,bxe^{(-1)}}{693\,(x^2e+d)^{\frac{7}{2}}d^2}+\frac{bxe^{(-1)}}{99\,(x^2e+d)^{\frac{9}{2}}d}+\frac{256\,ax}{693\,\sqrt{x^2e+d}\,d^6}+\frac{128\,ax}{693\,(x^2e+d)^{\frac{3}{2}}d^5}+\frac{32\,ax}{231\,(x^2e+d)^{\frac{5}{2}}d^4}+\frac{80\,ax}{693\,(x^2e+d)^{\frac{7}{2}}d^3}+\frac{10\,ax}{99\,(x^2e+d)^{\frac{9}{2}}d^2}+\frac{ax}{11\,(x^2e+d)^{\frac{11}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/8*c*x^3*e^(-1)/(x^2*e + d)^(11/2) + 1/264*c*x*e^(-2)/(x^2*e + d)^(9/2) +
 16/1155*c*x*e^(-2)/(sqrt(x^2*e + d)*d^4) + 8/1155*c*x*e^(-2)/((x^2*e + d)^
(3/2)*d^3) + 2/385*c*x*e^(-2)/((x^2*e + d)^(5/2)*d^2) + 1/231*c*x*e^(-2)/((
x^2*e + d)^(7/2)*d) - 3/88*c*d*x*e^(-2)/(x^2*e + d)^(11/2) - 1/11*b*x*e^(-1
)/(x^2*e + d)^(11/2) + 128/3465*b*x*e^(-1)/(sqrt(x^2*e + d)*d^5) + 64/3465*
b*x*e^(-1)/((x^2*e + d)^(3/2)*d^4) + 16/1155*b*x*e^(-1)/((x^2*e + d)^(5/2)*
d^3) + 8/693*b*x*e^(-1)/((x^2*e + d)^(7/2)*d^2) + 1/99*b*x*e^(-1)/((x^2*e +
 d)^(9/2)*d) + 256/693*a*x/(sqrt(x^2*e + d)*d^6) + 128/693*a*x/((x^2*e + d)
^(3/2)*d^5) + 32/231*a*x/((x^2*e + d)^(5/2)*d^4) + 80/693*a*x/((x^2*e + d)^
(7/2)*d^3) + 10/99*a*x/((x^2*e + d)^(9/2)*d^2) + 1/11*a*x/((x^2*e + d)^(11/
2)*d)
```

Fricas [A]
time = 0.73, size = 224, normalized size = 1.07

$$\frac{(1280\,ax^{11}e^5+693\,cd^5x^5+1155\,bd^5x^3+3465\,ad^5x+128\,(bdx^{11}+55\,adx^9)e^4+16\,(3\,cd^2x^{11}+44\,bd^2x^9+990\,ad^2x^7)e^3+264\,(cd^3x^9+6\,bd^3x^7+70\,ad^3x^5)e^2+66\,(9\,cd^4x^7+28\,bd^4x^5+175\,ad^4x^3)e)\sqrt{x^2e+d}}{3465\,(d^6x^{12}e^6+6\,d^7x^{10}e^5+15\,d^8x^8e^4+20\,d^9x^6e^3+15\,d^{10}x^4e^2+6\,d^{11}x^2e+d^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(1280*a*x^11*e^5 + 693*c*d^5*x^5 + 1155*b*d^5*x^3 + 3465*a*d^5*x + 1
28*(b*d*x^11 + 55*a*d*x^9)*e^4 + 16*(3*c*d^2*x^11 + 44*b*d^2*x^9 + 990*a*d^
```

```
2*x^7)*e^3 + 264*(c*d^3*x^9 + 6*b*d^3*x^7 + 70*a*d^3*x^5)*e^2 + 66*(9*c*d^4
*x^7 + 28*b*d^4*x^5 + 175*a*d^4*x^3)*e)*sqrt(x^2*e + d)/(d^6*x^12*e^6 + 6*d
^7*x^10*e^5 + 15*d^8*x^8*e^4 + 20*d^9*x^6*e^3 + 15*d^10*x^4*e^2 + 6*d^11*x^
2*e + d^12)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**(13/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.68, size = 189, normalized size = 0.90

$$\frac{\left(\left(\left(2\left(4x^2\left(\frac{2(3cd^2e^8+8bde^9+80ae^{10})x^2e^{(-5)}}{d^6}+\frac{11(3cd^3e^7+8bd^2e^8+80ade^9)e^{(-5)}}{d^6}\right)+\frac{99(3cd^4e^6+8bd^3e^7+80ad^2e^8)e^{(-5)}}{d^6}\right)x^2+\frac{231(3cd^5e^5+8bd^4e^6+80ad^3e^7)e^{(-5)}}{d^6}\right)x^2+\frac{1155(bd^5e^5+10ad^4e^6)e^{(-5)}}{d^6}\right)x^2+\frac{3465a}{d}\right)x}{3465(x^2e+d)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^(13/2),x, algorithm="giac")
```

```
[Out] 1/3465*(((2*(4*x^2*(2*(3*c*d^2*e^8 + 8*b*d*e^9 + 80*a*e^10)*x^2*e^(-5)/d^6
+ 11*(3*c*d^3*e^7 + 8*b*d^2*e^8 + 80*a*d*e^9)*e^(-5)/d^6) + 99*(3*c*d^4*e^6
 + 8*b*d^3*e^7 + 80*a*d^2*e^8)*e^(-5)/d^6)*x^2 + 231*(3*c*d^5*e^5 + 8*b*d^4
*e^6 + 80*a*d^3*e^7)*e^(-5)/d^6)*x^2 + 1155*(b*d^5*e^5 + 10*a*d^4*e^6)*e^(-
5)/d^6)*x^2 + 3465*a/d)*x/(x^2*e + d)^(11/2)
```

Mupad [B]
time = 4.76, size = 226, normalized size = 1.08

$$\frac{x\left(\frac{a}{11\,d}-\frac{d\left(\frac{b}{11\,d}-\frac{c}{11\,e}\right)}{e}\right)}{\left(e\,x^2+d\right)^{11/2}}-\frac{x\left(\frac{c}{9\,e^2}-\frac{-c\,d^2+b\,d\,e+10\,a\,e^2}{99\,d^2\,e^2}\right)}{\left(e\,x^2+d\right)^{9/2}}+\frac{x\left(3\,c\,d^2+8\,b\,d\,e+80\,a\,e^2\right)}{693\,d^3\,e^2\,\left(e\,x^2+d\right)^{7/2}}+\frac{x\left(6\,c\,d^2+16\,b\,d\,e+160\,a\,e^2\right)}{1155\,d^4\,e^2\,\left(e\,x^2+d\right)^{5/2}}+\frac{x\left(24\,c\,d^2+64\,b\,d\,e+640\,a\,e^2\right)}{3465\,d^5\,e^2\,\left(e\,x^2+d\right)^{3/2}}+\frac{x\left(48\,c\,d^2+128\,b\,d\,e+1280\,a\,e^2\right)}{3465\,d^6\,e^2\,\sqrt{e\,x^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^(13/2),x)
```

```
[Out] (x*(a/(11*d) - (d*(b/(11*d) - c/(11*e)))/e))/(d + e*x^2)^(11/2) - (x*(c/(9*
e^2) - (10*a*e^2 - c*d^2 + b*d*e)/(99*d^2*e^2)))/(d + e*x^2)^(9/2) + (x*(80
*a*e^2 + 3*c*d^2 + 8*b*d*e))/(693*d^3*e^2*(d + e*x^2)^(7/2)) + (x*(160*a*e^
2 + 6*c*d^2 + 16*b*d*e))/(1155*d^4*e^2*(d + e*x^2)^(5/2)) + (x*(640*a*e^2 +
 24*c*d^2 + 64*b*d*e))/(3465*d^5*e^2*(d + e*x^2)^(3/2)) + (x*(1280*a*e^2 +
48*c*d^2 + 128*b*d*e))/(3465*d^6*e^2*(d + e*x^2)^(1/2))
```

3.286 $\int (7+5x^2)^3 \sqrt{2+3x^2+x^4}\, dx$

Optimal. Leaf size=193

$$\frac{577x(2+x^2)}{3\sqrt{2+3x^2+x^4}}+\frac{1}{21}x(2608+757x^2)\sqrt{2+3x^2+x^4}+\frac{275}{7}x(2+3x^2+x^4)^{3/2}+\frac{125}{9}x^3(2+3x^2+x^4)^{3/2}-$$

```
[Out] 275/7*x*(x^4+3*x^2+2)^(3/2)+125/9*x^3*(x^4+3*x^2+2)^(3/2)+577/3*x*(x^2+2)/(
x^4+3*x^2+2)^(1/2)-577/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1
)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+29
45/21*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2)
)*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/21*x*(757*x^2+2608)
*(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1693, 1190, 1203, 1113, 1149}

$$\frac{2945\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}}-\frac{577\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{275}{7}(x^4+3x^2+2)^{3/2}x+\frac{1}{21}(757x^2+2608)\sqrt{x^4+3x^2+2}\,x+\frac{577(x^2+2)x}{3\sqrt{x^4+3x^2+2}}+\frac{125}{9}(x^4+3x^2+2)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (577*x*(2 + x^2))/(3*Sqrt[2 + 3*x^2 + x^4]) + (x*(2608 + 757*x^2)*Sqrt[2 +
3*x^2 + x^4])/21 + (275*x*(2 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(2 + 3*x^2
+ x^4)^(3/2))/9 - (577*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
E[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) + (2945*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
```

```
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int (7+5x^2)^3 \sqrt{2+3x^2+x^4}\, dx = \frac{125}{9}x^3(2+3x^2+x^4)^{3/2} + \frac{1}{9}\int \sqrt{2+3x^2+x^4}\,(3087+5865x^2+2475x$$
$$= \frac{275}{7}x(2+3x^2+x^4)^{3/2} + \frac{125}{9}x^3(2+3x^2+x^4)^{3/2} + \frac{1}{63}\int (16659+113$$
$$= \frac{1}{21}x(2608+757x^2)\sqrt{2+3x^2+x^4} + \frac{275}{7}x(2+3x^2+x^4)^{3/2} + \frac{125}{9}x^3$$
$$= \frac{1}{21}x(2608+757x^2)\sqrt{2+3x^2+x^4} + \frac{275}{7}x(2+3x^2+x^4)^{3/2} + \frac{125}{9}x^3$$
$$= \frac{577x(2+x^2)}{3\sqrt{2+3x^2+x^4}} + \frac{1}{21}x(2608+757x^2)\sqrt{2+3x^2+x^4} + \frac{275}{7}x(2+3x$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.61, size = 119, normalized size = 0.62

$$\frac{25548x+61214x^3+57312x^5+28496x^7+7725x^9+875x^{11}-12117i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-5553i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{63\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25548*x + 61214*x^3 + 57312*x^5 + 28496*x^7 + 7725*x^9 + 875*x^11 - (12117
*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (5553*
I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(63*Sqrt
[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 172, normalized size = 0.89

method	result
risch	$\frac{x(875x^6+5100x^4+11446x^2+12774)\sqrt{x^4+3x^2+2}}{63} + \frac{577i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\right.}{6\sqrt{x^4+3x^2+2}}$
default	$\frac{125x^7\sqrt{x^4+3x^2+2}}{9} + \frac{1700x^5\sqrt{x^4+3x^2+2}}{21} + \frac{11446x^3\sqrt{x^4+3x^2+2}}{63} + \frac{4258x\sqrt{x^4+3x^2+2}}{21} +$
elliptic	$\frac{125x^7\sqrt{x^4+3x^2+2}}{9} + \frac{1700x^5\sqrt{x^4+3x^2+2}}{21} + \frac{11446x^3\sqrt{x^4+3x^2+2}}{63} + \frac{4258x\sqrt{x^4+3x^2+2}}{21} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 125/9*x^7*(x^4+3*x^2+2)^(1/2)+1700/21*x^5*(x^4+3*x^2+2)^(1/2)+11446/63*x^3*
(x^4+3*x^2+2)^(1/2)+4258/21*x*(x^4+3*x^2+2)^(1/2)+577/6*I*2^(1/2)*(2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2)
)-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-2945/21*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^
2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(x^2+1)(x^2+2)}\left(5x^2+7\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)^3\,\sqrt{x^4+3\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2), x)
```

3.287 $\int \left(7+5x^2\right)^2 \sqrt{2+3x^2+x^4}\, dx$

Optimal. Leaf size=168

$$\frac{31x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{21}x\left(407+114x^2\right)\sqrt{2+3x^2+x^4}+\frac{25}{7}x\left(2+3x^2+x^4\right)^{3/2}-\frac{31\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E(\text{ta}}{\sqrt{2+3x^2+x^4}}$$

```
[Out] 25/7*x*(x^4+3*x^2+2)^(3/2)+31*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-31*(x^2+1)^(3/2
)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)
/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+472/21*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*
EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4
+3*x^2+2)^(1/2)+1/21*x*(114*x^2+407)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1190, 1203, 1113, 1149}

$$\frac{472\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{21\sqrt{x^4+3x^2+2}}-\frac{31\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{25}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{21}x\left(114x^2+407\right)\sqrt{x^4+3x^2+2}+\frac{31x(x^2+2)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (31*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(407 + 114*x^2)*Sqrt[2 + 3*x^2
+ x^4])/21 + (25*x*(2 + 3*x^2 + x^4)^(3/2))/7 - (31*Sqrt[2]*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (47
2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2
1*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int (7+5x^2)^2\sqrt{2+3x^2+x^4}\,dx &= \frac{25}{7}x(2+3x^2+x^4)^{3/2}+\frac{1}{7}\int (293+190x^2)\sqrt{2+3x^2+x^4}\,dx\\ &= \frac{1}{21}x(407+114x^2)\sqrt{2+3x^2+x^4}+\frac{25}{7}x(2+3x^2+x^4)^{3/2}+\frac{1}{105}\int \frac{4}{\sqrt{}}\\ &= \frac{1}{21}x(407+114x^2)\sqrt{2+3x^2+x^4}+\frac{25}{7}x(2+3x^2+x^4)^{3/2}+31\int \frac{}{\sqrt{2}}\\ &= \frac{31x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{21}x(407+114x^2)\sqrt{2+3x^2+x^4}+\frac{25}{7}x(2+3x^2+\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.49, size = 114, normalized size = 0.68

$$\frac{1114x+2349x^3+1724x^5+564x^7+75x^9-651i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-293i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{21\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (1114*x + 2349*x^3 + 1724*x^5 + 564*x^7 + 75*x^9 - (651*I)*Sqrt[1 + x^2]*Sq
rt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (293*I)*Sqrt[1 + x^2]*Sqrt
[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(21*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 155, normalized size = 0.92

method	result
risch	$\frac{x(75x^4+339x^2+557)\sqrt{x^4+3x^2+2}}{21}+\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}}{2}\right.\right.}{2\sqrt{x^4+3x^2+2}}$
default	$\frac{25x^5\sqrt{x^4+3x^2+2}}{7}+\frac{113x^3\sqrt{x^4+3x^2+2}}{7}+\frac{557x\sqrt{x^4+3x^2+2}}{21}-\frac{472i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}}{21\sqrt{x^4+3x^2}}$
elliptic	$\frac{25x^5\sqrt{x^4+3x^2+2}}{7}+\frac{113x^3\sqrt{x^4+3x^2+2}}{7}+\frac{557x\sqrt{x^4+3x^2+2}}{21}-\frac{472i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}}{21\sqrt{x^4+3x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/7*x^5*(x^4+3*x^2+2)^(1/2)+113/7*x^3*(x^4+3*x^2+2)^(1/2)+557/21*x*(x^4+3*
x^2+2)^(1/2)-472/21*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(
1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+31/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE
(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(x^2+1)(x^2+2)}\,(5x^2+7)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(5\,x^2+7\right)}^2\,\sqrt{x^4+3\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2), x)
```

3.288 $\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx$

Optimal. Leaf size=149

$$\frac{5x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x(10+3x^2)\sqrt{2+3x^2+x^4}-\frac{5\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{11\sqrt{2}(1+x^2)\sqrt{}}{3\sqrt{2}}$$

```
[Out] 5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE
(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+11/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2
^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/3*x*(3*x^2+10
)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1190, 1203, 1113, 1149}

$$\frac{11\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\text{ArcTan}(x)\left|\frac{1}{2}\right.\right)}{3\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\text{ArcTan}(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}+\frac{5x(x^2+2)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}x(3x^2+10)\sqrt{x^4+3x^2+2}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(10 + 3*x^2)*Sqrt[2 + 3*x^2 + x^
4])/3 - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
 1/2])/Sqrt[2 + 3*x^2 + x^4] + (11*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^
2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int (7+5x^2)\sqrt{2+3x^2+x^4}\,dx &= \frac{1}{3}x(10+3x^2)\sqrt{2+3x^2+x^4}+\frac{1}{15}\int \frac{110+75x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{1}{3}x(10+3x^2)\sqrt{2+3x^2+x^4}+5\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{22}{3}\int \frac{}{\sqrt{2+3}}\\ &= \frac{5x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x(10+3x^2)\sqrt{2+3x^2+x^4}-\frac{5\sqrt{2}\,(1+x^2)\sqrt{\frac{2+}{1+}}}{\sqrt{2+3x}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.56, size = 109, normalized size = 0.73

$$\frac{20x+36x^3+19x^5+3x^7-15i\sqrt{1+x^2}\,\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-7i\sqrt{1+x^2}\,\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{3\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (20*x + 36*x^3 + 19*x^5 + 3*x^7 - (15*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipt
icE[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[
I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 137, normalized size = 0.92

method	result
risch	$\frac{x(3x^2+10)\sqrt{x^4+3x^2+2}}{3}+\frac{5i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}}$
default	$x^3\sqrt{x^4+3x^2+2}+\frac{10x\sqrt{x^4+3x^2+2}}{3}-\frac{11i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{}}{}$
elliptic	$x^3\sqrt{x^4+3x^2+2}+\frac{10x\sqrt{x^4+3x^2+2}}{3}-\frac{11i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{5i\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^3*(x^4+3*x^2+2)^(1/2)+10/3*x*(x^4+3*x^2+2)^(1/2)-11/3*I*2^(1/2)*(2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))
+5/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF
(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{(x^2+1)(x^2+2)}\,(5x^2+7)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)\,\sqrt{x^4+3\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2), x)
```

3.289 $\int \sqrt{2+3x^2+x^4}\, dx$

Optimal. Leaf size=141

$$\frac{x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\sqrt{2+3x^2+x^4}-\frac{\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{2\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}$$

```
[Out] x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(
x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/
2)+2/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2
))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/3*x*(x^4+3*x^2+2)^
(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1105, 1203, 1113, 1149}

$$\frac{2\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}\sqrt{x^4+3x^2+2}\,x+\frac{(x^2+2)\,x}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*Sqrt[2 + 3*x^2 + x^4])/3 - (Sqrt[2
]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3
*x^2 + x^4] + (2*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcT
an[x], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{2+3x^2+x^4}\,dx &= \frac{1}{3}x\sqrt{2+3x^2+x^4}+\frac{1}{3}\int \frac{4+3x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{2+3x^2+x^4}+\frac{4}{3}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx+\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}x\sqrt{2+3x^2+x^4}-\frac{\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.68, size = 102, normalized size = 0.72

$$\frac{2x+3x^3+x^5-3i\sqrt{1+x^2}\,\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-i\sqrt{1+x^2}\,\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{3\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (2*x + 3*x^3 + x^5 - (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[
x/Sqrt[2]], 2] - I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2
]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 121, normalized size = 0.86

method	result
default	$\frac{x\sqrt{x^4+3x^2+2}}{3}-\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{E}\right.}{2}$
risch	$\frac{x\sqrt{x^4+3x^2+2}}{3}-\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{E}\right.}{2}$
elliptic	$\frac{x\sqrt{x^4+3x^2+2}}{3}-\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{E}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(x^4+3*x^2+2)^(1/2)-2/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+
3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/2*I*2^(1/2)*(2*x^2+4)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-
EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**4 + 3*x**2 + 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^4+3\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(1/2), x)
```

3.290 $\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx$

Optimal. Leaf size=178

$$\frac{x(2+x^2)}{5\sqrt{2+3x^2+x^4}}-\frac{\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}}+\frac{(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}}+\frac{3(1+x^2)\,\sqrt{\ }}{35\sqrt{\ }}$$

```
[Out] 1/5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+3/70*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Elli
pticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^
4+3*x^2+2)^(1/2)-1/5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1
/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/5*(x
^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+
2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 232, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1222, 1203, 1113, 1149, 1228, 1470, 553}

$$\frac{4\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{25\sqrt{x^4+3x^2+2}}-\frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{25\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}+\frac{3(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{35\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}+\frac{x(x^2+2)}{5\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] (x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) - (3*(1 +
x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(25*Sqrt[2]*Sqrt[
2 + 3*x^2 + x^4]) + (4*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
F[ArcTan[x], 1/2])/(25*Sqrt[2 + 3*x^2 + x^4]) + (3*(2 + x^2)*EllipticPi[2/7
, ArcTan[x], 1/2])/(35*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x
^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
```

```
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
```

```
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int \frac{-8-5x^2}{\sqrt{2+3x^2+x^4}}\,dx\right) - \frac{6}{25}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$$
$$= -\left(\frac{3}{25}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\right) + \frac{1}{5}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{3}{10}\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3}}$$
$$= \frac{x(2+x^2)}{5\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} - \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F}{25\sqrt{2}\sqrt{2+3x}}$$
$$= \frac{x(2+x^2)}{5\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{5\sqrt{2+3x^2+x^4}} - \frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F}{25\sqrt{2}\sqrt{2+3x}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.67, size = 90, normalized size = 0.51

$$-\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\left(35E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+21F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-6\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{175\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2),x]
[Out] ((-1/175*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(35*EllipticE[I*ArcSinh[x/Sqrt[2]],
 2] + 21*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - 6*EllipticPi[10/7, I*ArcSinh[
x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 138, normalized size = 0.78

method	result
default	$-\frac{3i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{50\sqrt{x^4+3x^2+2}} - \frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{10\sqrt{x^4+3x^2+2}}$

elliptic	$-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{50\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right.}{10\sqrt{x^4+3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] -3/50*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF
(1/2*I*2^(1/2)*x,2^(1/2))-1/10*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4
+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+6/175*I*2^(1/2)*(1+1/2*x
^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7
,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x^2+1)(x^2+2)}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7),x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7), x)
```

3.291 $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=209

$$-\frac{x(2+x^2)}{70\sqrt{2+3x^2+x^4}}+\frac{x\sqrt{2+3x^2+x^4}}{14(7+5x^2)}+\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(\right.}{140\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] -1/70*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/1960*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1
)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1
))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/70*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE
(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+3/280*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*
2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/14*x*(x^4+3*
x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A]
time = 0.08, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1240, 1203, 1113, 1149, 1228, 1470, 553}

$$\frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{140\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{980\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}+\frac{\sqrt{x^4+3x^2+2}\,x}{14(5x^2+7)}-\frac{(x^2+2)\,x}{70\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] -1/70*(x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*Sqrt[2 + 3*x^2 + x^4])/(14*(
7 + 5*x^2)) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2
])/(35*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^
2)]*EllipticF[ArcTan[x], 1/2])/(140*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - ((2 +
x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(980*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)
]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1240

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(d + e*x^2))), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
```

```
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^2}\,dx = \frac{x\sqrt{2+3x^2+x^4}}{14(7+5x^2)} + \frac{1}{350}\int \frac{7-5x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{1}{350}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{14(7+5x^2)} + \frac{1}{700}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{1}{280}\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}$$

$$= -\frac{x(2+x^2)}{70\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{14(7+5x^2)} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}} +$$

$$= -\frac{x(2+x^2)}{70\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{14(7+5x^2)} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{35\sqrt{2}\sqrt{2+3x^2+x^4}} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 208, normalized size = 1.00

$$\frac{350x+525x^3+175x^5+35i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-84i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-7i\sqrt{1+x^2}\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-5ix^2\sqrt{1+x^2}\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{2450(7+5x^2)\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (350*x + 525*x^3 + 175*x^5 + (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)
*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (84*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7
 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (5*I)*x^2*Sqrt[1 + x^2]*S
qrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(2450*(7 + 5*x^2)*S
qrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 162, normalized size = 0.78

method	result

default	$\frac{x\sqrt{x^4+3x^2+2}}{70x^2+98}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{175\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{El}}{140\sqrt{x^4+3x^2}}$
elliptic	$\frac{x\sqrt{x^4+3x^2+2}}{70x^2+98}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{175\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{El}}{140\sqrt{x^4+3x^2}}$
risch	$\frac{x\sqrt{x^4+3x^2+2}}{70x^2+98}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{140\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-3/175*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/140*I*2^(1
/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/
2)*x,2^(1/2))-1/2450*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x^2+1)\,(x^2+2)}}{(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{{\left(5\,x^2+7\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^2,x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^2, x)
```

3.292 $\int \frac{\sqrt{2+3x^2+x^4}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=237

$$-\frac{11x(2+x^2)}{11760\sqrt{2+3x^2+x^4}}+\frac{x\sqrt{2+3x^2+x^4}}{28\,(7+5x^2)^2}+\frac{11x\sqrt{2+3x^2+x^4}}{2352\,(7+5x^2)}+\frac{11(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{5880\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{81(}{}$$

```
[Out] -11/11760*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1201/329280*(x^2+2)*(1/(x^2+1))^(1/
2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+
2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+11/11760*(x^2+1)^(3/2)*(1/(x^2+1))^(1
/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/
(x^4+3*x^2+2)^(1/2)+81/15680*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x
^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2
)+1/28*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+11/2352*x*(x^4+3*x^2+2)^(1/2)/(5*x
^2+7)
```

Rubi [A]
time = 0.38, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1242, 1237, 1710, 1730, 1203, 1113, 1149, 1228, 1470, 553}

$$\frac{81(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{7840\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{11(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{5880\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{1201(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{164640\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}+\frac{11\sqrt{x^4+3x^2+2}\,x}{2352\,(5x^2+7)}+\frac{\sqrt{x^4+3x^2+2}\,x}{28\,(5x^2+7)^2}-\frac{11(x^2+2)\,x}{11760\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (-11*x*(2 + x^2))/(11760*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])
/(28*(7 + 5*x^2)^2) + (11*x*Sqrt[2 + 3*x^2 + x^4])/(2352*(7 + 5*x^2)) + (11
*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5880*Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellipti
cF[ArcTan[x], 1/2])/(7840*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (1201*(2 + x^2)*
EllipticPi[2/7, ArcTan[x], 1/2])/(164640*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*
Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{2+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx = \int \left(-\frac{6}{25\left(7+5x^2\right)^3\sqrt{2+3x^2+x^4}} + \frac{1}{25\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}} + \frac{}{25\,(7+5x}\right.$$

$$= \frac{1}{25}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{2+3x^2+x^4}}\,dx + \frac{1}{25}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx - \frac{6}{25}\int$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} - \frac{x\sqrt{2+3x^2+x^4}}{84\left(7+5x^2\right)} + \frac{\int \frac{62+70x^2+25x^4}{\left(7+5x^2\right)\sqrt{2+3x^2+x^4}}\,dx}{2100} - \frac{1}{700}\int \frac{}{(7+}$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{50\sqrt{2}\sqrt{2+3x^2+x^4}} - \int$$

$$= \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} + \frac{\left(1+x^2\right)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{50\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{}{7}$$

$$= \frac{x\left(2+x^2\right)}{420\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} - \frac{\left(1+x^2\right)\sqrt{\frac{2+}{1+}}}{210\sqrt{2}\sqrt{2}}$$

$$= -\frac{11x\left(2+x^2\right)}{11760\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} + \frac{11\left(1+x^2\right)}{5880\sqrt{}}$$

$$= -\frac{11x\left(2+x^2\right)}{11760\sqrt{2+3x^2+x^4}} + \frac{x\sqrt{2+3x^2+x^4}}{28\left(7+5x^2\right)^2} + \frac{11x\sqrt{2+3x^2+x^4}}{2352\left(7+5x^2\right)} + \frac{11\left(1+x^2\right)}{5880\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 174, normalized size = 0.73

$$\frac{\frac{14700x\left(2+3x^2+x^4\right)}{\left(7+5x^2\right)^2} + \frac{1925x\left(2+3x^2+x^4\right)}{7+5x^2} + 385i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 434i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 1201i\sqrt{1+x^2}\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{411600\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] ((14700*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (1925*x*(2 + 3*x^2 + x^4))/(7
+ 5*x^2) + (385*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2
]], 2] - (434*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]]
, 2] - (1201*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sq
rt[2]], 2])/(411600*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 186, normalized size = 0.78

method	result
risch	$\frac{\sqrt{x^4+3x^2+2}\,x(55x^2+161)}{2352(5x^2+7)^2} - \frac{11i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right.\right.}{23520\sqrt{x^4+3x^2+2}}$
default	$\frac{x\sqrt{x^4+3x^2+2}}{28(5x^2+7)^2} + \frac{11x\sqrt{x^4+3x^2+2}}{2352(5x^2+7)} - \frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{58800\sqrt{x^4+3x^2+2}} + \frac{11i\sqrt{}}{}$
elliptic	$\frac{x\sqrt{x^4+3x^2+2}}{28(5x^2+7)^2} + \frac{11x\sqrt{x^4+3x^2+2}}{2352(5x^2+7)} - \frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{58800\sqrt{x^4+3x^2+2}} + \frac{11i\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+11/2352*x*(x^4+3*x^2+2)^(1/2)/(5*x^2
+7)-31/58800*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*El
lipticF(1/2*I*2^(1/2)*x,2^(1/2))+11/23520*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-1201/411600*I
*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2
*I*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x^2+1)\,(x^2+2)}}{(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(sqrt((x**2 + 1)*(x**2 + 2))/(5*x**2 + 7)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 2)/(5*x^2 + 7)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4+3\,x^2+2}}{{\left(5\,x^2+7\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^3,x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(1/2)/(5*x^2 + 7)^3, x)
```

3.293 $\int (7+5x^2)^3 (2+3x^2+x^4)^{3/2}\, dx$

Optimal. Leaf size=219

$$\frac{20884x(2+x^2)}{65\sqrt{2+3x^2+x^4}}+\frac{x(1032541+297911x^2)\sqrt{2+3x^2+x^4}}{5005}+\frac{x(208212+65345x^2)(2+3x^2+x^4)^{3/2}}{3003}+\frac{3825}{143}$$

```
[Out] 1/3003*x*(65345*x^2+208212)*(x^4+3*x^2+2)^(3/2)+3825/143*x*(x^4+3*x^2+2)^(5
/2)+125/13*x^3*(x^4+3*x^2+2)^(5/2)+20884/65*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-2
0884/65*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/
2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1171349/5005*(x^2+1
)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((
x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/5005*x*(297911*x^2+1032541)*(x^
4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1693, 1190, 1203, 1113, 1149}

$$\frac{1171349\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{5005\sqrt{x^4+3x^2+2}}-\frac{20884\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{65\sqrt{x^4+3x^2+2}}+\frac{3825}{143}(x^4+3x^2+2)^{5/2}x+\frac{(65345x^2+208212)(x^4+3x^2+2)^{3/2}x}{3003}+\frac{(297911x^2+1032541)\sqrt{x^4+3x^2+2}\,x}{5005}+\frac{20884(x^2+2)x}{65\sqrt{x^4+3x^2+2}}+\frac{125}{13}(x^4+3x^2+2)^{5/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (20884*x*(2 + x^2))/(65*Sqrt[2 + 3*x^2 + x^4]) + (x*(1032541 + 297911*x^2)*
Sqrt[2 + 3*x^2 + x^4])/5005 + (x*(208212 + 65345*x^2)*(2 + 3*x^2 + x^4)^(3/
2))/3003 + (3825*x*(2 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(2 + 3*x^2 + x^4
)^(5/2))/13 - (20884*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[
ArcTan[x], 1/2])/(65*Sqrt[2 + 3*x^2 + x^4]) + (1171349*Sqrt[2]*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5005*Sqrt[2 + 3*x^2 + x
^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
```

```
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(7+5x^2\right)^3 \left(2+3x^2+x^4\right)^{3/2}\,dx &= \frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2}+\frac{1}{13}\int \left(2+3x^2+x^4\right)^{3/2}\left(4459+8805x^2+3\right.\\ &= \frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2}+\frac{125}{13}x^3\left(2+3x^2+x^4\right)^{5/2}+\frac{1}{143}\int \left(41399+\right.\\ &= \frac{x(208212+65345x^2)\left(2+3x^2+x^4\right)^{3/2}}{3003}+\frac{3825}{143}x\left(2+3x^2+x^4\right)^{5/2}+\frac{1}{}\\ &= \frac{x(1032541+297911x^2)\sqrt{2+3x^2+x^4}}{5005}+\frac{x(208212+65345x^2)\left(2+3\right.}{3003}\\ &= \frac{x(1032541+297911x^2)\sqrt{2+3x^2+x^4}}{5005}+\frac{x(208212+65345x^2)\left(2+3\right.}{3003}\\ &= \frac{20884x(2+x^2)}{65\sqrt{2+3x^2+x^4}}+\frac{x(1032541+297911x^2)\sqrt{2+3x^2+x^4}}{5005}+\frac{x(208}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 129, normalized size = 0.59

$$\frac{13572486x+40493455x^3+54938052x^5+46218643x^7+25350660x^9+8705725x^{11}+1701000x^{13}+144375x^{15}-4824204i\sqrt{1+x^2}\,\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-2203890i\sqrt{1+x^2}\,\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{15015\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (13572486*x + 40493455*x^3 + 54938052*x^5 + 46218643*x^7 + 25350660*x^9 + 8
705725*x^11 + 1701000*x^13 + 144375*x^15 - (4824204*I)*Sqrt[1 + x^2]*Sqrt[2
 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (2203890*I)*Sqrt[1 + x^2]*Sqrt
[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(15015*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 206, normalized size = 0.94

method	result
risch	$\frac{x\left(144375x^{10}+1267875x^8+4613350x^6+8974860x^4+10067363x^2+6786243\right)\sqrt{x^4+3x^2+2}}{15015}+\frac{10442i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x}}{}$
default	$\frac{598324x^5\sqrt{x^4+3x^2+2}}{1001}+\frac{131810x^7\sqrt{x^4+3x^2+2}}{429}+\frac{12075x^9\sqrt{x^4+3x^2+2}}{143}+\frac{125x^{11}\sqrt{x^4+3x^2+2}}{13}$

elliptic	$\frac{598324x^5\sqrt{x^4+3x^2+2}}{1001}+\frac{131810x^7\sqrt{x^4+3x^2+2}}{429}+\frac{12075x^9\sqrt{x^4+3x^2+2}}{143}+\frac{125x^{11}\sqrt{x^4+3x^2+}}{13}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 598324/1001*x^5*(x^4+3*x^2+2)^(1/2)+131810/429*x^7*(x^4+3*x^2+2)^(1/2)+1207
5/143*x^9*(x^4+3*x^2+2)^(1/2)+125/13*x^11*(x^4+3*x^2+2)^(1/2)-1171349/5005*
I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I
*2^(1/2)*x,2^(1/2))+10442/65*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3
*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x
,2^(1/2)))+2262081/5005*x*(x^4+3*x^2+2)^(1/2)+10067363/15015*x^3*(x^4+3*x^2
+2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (5\,x^2+7)^3\,(x^4+3\,x^2+2)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2), x)
```

3.294 $\int (7+5x^2)^2 (2+3x^2+x^4)^{3/2}\, dx$

Optimal. Leaf size=198

$$\frac{742x(2+x^2)}{15\sqrt{2+3x^2+x^4}}+\frac{x(36783+10643x^2)\sqrt{2+3x^2+x^4}}{1155}+\frac{1}{693}x(7281+2240x^2)(2+3x^2+x^4)^{3/2}+\frac{25}{11}x(2$$

```
[Out] 1/693*x*(2240*x^2+7281)*(x^4+3*x^2+2)^(3/2)+25/11*x*(x^4+3*x^2+2)^(5/2)+742
/15*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-742/15*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*El
lipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3
*x^2+2)^(1/2)+13879/385*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)
^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/1
155*x*(10643*x^2+36783)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1190, 1203, 1113, 1149}

$$\frac{13879\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{385\sqrt{x^4+3x^2+2}}-\frac{742\sqrt{2}(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{15\sqrt{x^4+3x^2+2}}+\frac{25}{11}x(x^4+3x^2+2)^{5/2}+\frac{1}{693}x(2240x^2+7281)(x^4+3x^2+2)^{3/2}+\frac{x(10643x^2+36783)\sqrt{x^4+3x^2+2}}{1155}+\frac{742x(x^2+2)}{15\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (742*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(36783 + 10643*x^2)*Sqrt[
2 + 3*x^2 + x^4])/1155 + (x*(7281 + 2240*x^2)*(2 + 3*x^2 + x^4)^(3/2))/693
+ (25*x*(2 + 3*x^2 + x^4)^(5/2))/11 - (742*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sqrt[2 + 3*x^2 + x^4]) + (13879*
Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(385
*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
```

```
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int (7+5x^2)^2 \left(2+3x^2+x^4\right)^{3/2} dx = \frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2} + \frac{1}{11}\int (489+320x^2)\left(2+3x^2+x^4\right)^{3/2} dx$$

$$= \frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x^2+x^4\right)^{3/2} + \frac{25}{11}x\left(2+3x^2+x^4\right)^{5/2} + \frac{}{2}$$

$$= \frac{x(36783+10643x^2)\sqrt{2+3x^2+x^4}}{1155} + \frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x\right.$$

$$= \frac{x(36783+10643x^2)\sqrt{2+3x^2+x^4}}{1155} + \frac{1}{693}x\left(7281+2240x^2\right)\left(2+3x\right.$$

$$= \frac{742x(2+x^2)}{15\sqrt{2+3x^2+x^4}} + \frac{x(36783+10643x^2)\sqrt{2+3x^2+x^4}}{1155} + \frac{1}{693}x(72$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.07, size = 124, normalized size = 0.63

$$\frac{429318x+1160065x^3+1333551x^5+892084x^7+363480x^9+82075x^{11}+7875x^{13}-171402i\sqrt{1+x^2}\sqrt{2+x^2}E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-78420i\sqrt{1+x^2}\sqrt{2+x^2}F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{3465\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (429318*x + 1160065*x^3 + 1333551*x^5 + 892084*x^7 + 363480*x^9 + 82075*x^1
1 + 7875*x^13 - (171402*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[
x/Sqrt[2]], 2] - (78420*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[
x/Sqrt[2]], 2])/(3465*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 189, normalized size = 0.95

method	result
risch	$\frac{x(7875x^8+58450x^6+172380x^4+258044x^2+214659)\sqrt{x^4+3x^2+2}}{3465} + \frac{371i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i}{}\right.\right.}{15\sqrt{x^4+3x}}$
default	$\frac{25x^9\sqrt{x^4+3x^2+2}}{11} + \frac{1670x^7\sqrt{x^4+3x^2+2}}{99} + \frac{11492x^5\sqrt{x^4+3x^2+2}}{231} + \frac{258044x^3\sqrt{x^4+3x^2+2}}{3465}$
elliptic	$\frac{25x^9\sqrt{x^4+3x^2+2}}{11} + \frac{1670x^7\sqrt{x^4+3x^2+2}}{99} + \frac{11492x^5\sqrt{x^4+3x^2+2}}{231} + \frac{258044x^3\sqrt{x^4+3x^2+2}}{3465}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/11*x^9*(x^4+3*x^2+2)^(1/2)+1670/99*x^7*(x^4+3*x^2+2)^(1/2)+11492/231*x^5
*(x^4+3*x^2+2)^(1/2)+258044/3465*x^3*(x^4+3*x^2+2)^(1/2)+23851/385*x*(x^4+3
*x^2+2)^(1/2)+371/15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^
(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)
))-13879/385*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*El
lipticF(1/2*I*2^(1/2)*x,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)^2\,\left(x^4+3\,x^2+2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2), x)
```

3.295 $\int (7+5x^2)(2+3x^2+x^4)^{3/2}\,dx$

Optimal. Leaf size=179

$$\frac{116x(2+x^2)}{15\sqrt{2+3x^2+x^4}}+\frac{1}{105}x(519+149x^2)\sqrt{2+3x^2+x^4}+\frac{1}{63}x(108+35x^2)(2+3x^2+x^4)^{3/2}-\frac{116\sqrt{2}\,(1+}{1}$$

```
[Out] 1/63*x*(35*x^2+108)*(x^4+3*x^2+2)^(3/2)+116/15*x*(x^2+2)/(x^4+3*x^2+2)^(1/2
)-116/15*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1
/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+197/35*(x^2+1)^(3/
2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2
)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/105*x*(149*x^2+519)*(x^4+3*x^2+2)^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1190, 1203, 1113, 1149}

$$\frac{197\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}-\frac{116\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{15\sqrt{x^4+3x^2+2}}+\frac{1}{63}x(35x^2+108)(x^4+3x^2+2)^{3/2}+\frac{1}{105}x(149x^2+519)\sqrt{x^4+3x^2+2}+\frac{116x(x^2+2)}{15\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (116*x*(2 + x^2))/(15*Sqrt[2 + 3*x^2 + x^4]) + (x*(519 + 149*x^2)*Sqrt[2 +
3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(2 + 3*x^2 + x^4)^(3/2))/63 - (116*Sq
rt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(15*Sq
rt[2 + 3*x^2 + x^4]) + (197*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
```

```
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int (7+5x^2)\,(2+3x^2+x^4)^{3/2}\,dx &= \frac{1}{63}x(108+35x^2)\,(2+3x^2+x^4)^{3/2}+\frac{1}{21}\int (222+149x^2)\,\sqrt{2+3x^2-}\\
&= \frac{1}{105}x(519+149x^2)\,\sqrt{2+3x^2+x^4}+\frac{1}{63}x(108+35x^2)\,(2+3x^2+x^4\\
&= \frac{1}{105}x(519+149x^2)\,\sqrt{2+3x^2+x^4}+\frac{1}{63}x(108+35x^2)\,(2+3x^2+x^4\\
&= \frac{116x(2+x^2)}{15\sqrt{2+3x^2+x^4}}+\frac{1}{105}x(519+149x^2)\,\sqrt{2+3x^2+x^4}+\frac{1}{63}x(108\cdot
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.61, size = 119, normalized size = 0.66

$$
\frac{5274x+12745x^3+12018x^5+5962x^7+1590x^9+175x^{11}-2436i\sqrt{1+x^2}\,\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-1110i\sqrt{1+x^2}\,\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{315\sqrt{2+3x^2+x^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (5274*x + 12745*x^3 + 12018*x^5 + 5962*x^7 + 1590*x^9 + 175*x^11 - (2436*I)
*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (1110*I)*
Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(315*Sqrt[2
 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 172, normalized size = 0.96

method	result
risch	$\frac{x(175x^6+1065x^4+2417x^2+2637)\sqrt{x^4+3x^2+2}}{315}+\frac{58i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{Ellipt}\right.}{15\sqrt{x^4+3x^2+2}}$
default	$\frac{5x^7\sqrt{x^4+3x^2+2}}{9}+\frac{71x^5\sqrt{x^4+3x^2+2}}{21}+\frac{2417x^3\sqrt{x^4+3x^2+2}}{315}+\frac{293x\sqrt{x^4+3x^2+2}}{35}-\frac{197i\sqrt{2}}{}$
elliptic	$\frac{5x^7\sqrt{x^4+3x^2+2}}{9}+\frac{71x^5\sqrt{x^4+3x^2+2}}{21}+\frac{2417x^3\sqrt{x^4+3x^2+2}}{315}+\frac{293x\sqrt{x^4+3x^2+2}}{35}-\frac{197i\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/9*x^7*(x^4+3*x^2+2)^(1/2)+71/21*x^5*(x^4+3*x^2+2)^(1/2)+2417/315*x^3*(x^4
+3*x^2+2)^(1/2)+293/35*x*(x^4+3*x^2+2)^(1/2)-197/35*I*2^(1/2)*(2*x^2+4)^(1/
2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+58/
15*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1
/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}} \cdot \left(5x^{2}+7\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+2)**(3/2),x)
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)\,{\left(x^4+3\,x^2+2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2), x)
```

3.296 $\int \left(2+3x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=172

$$\frac{6x(2+x^2)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}-\frac{6\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(\right.}{5\sqrt{2+3x^2+x^4}}$$

```
[Out] 1/7*x*(x^4+3*x^2+2)^(3/2)+6/5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-6/5*(x^2+1)^(3/
2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2
)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+31/35*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*
EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4
+3*x^2+2)^(1/2)+1/35*x*(9*x^2+29)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1105, 1190, 1203, 1113, 1149}

$$\frac{31\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{35\sqrt{x^4+3x^2+2}}-\frac{6\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}+\frac{1}{7}x\left(x^4+3x^2+2\right)^{3/2}+\frac{1}{35}x\left(9x^2+29\right)\sqrt{x^4+3x^2+2}+\frac{6x(x^2+2)}{5\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (6*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + (x*(29 + 9*x^2)*Sqrt[2 + 3*x^2
+ x^4])/35 + (x*(2 + 3*x^2 + x^4)^(3/2))/7 - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 +
 x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + (31
*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(35
*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(2+3x^2+x^4\right)^{3/2}\,dx &= \frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{3}{7}\int \left(4+3x^2\right)\sqrt{2+3x^2+x^4}\,dx\\ &= \frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{1}{35}\int \frac{62+42x^2}{\sqrt{2+3x^2+x^4}}\\ &= \frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}+\frac{6}{5}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,d\\ &= \frac{6x(2+x^2)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{35}x\left(29+9x^2\right)\sqrt{2+3x^2+x^4}+\frac{1}{7}x\left(2+3x^2+x^4\right)^{3/2}-\frac{6\sqrt{}}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.92, size = 114, normalized size = 0.66

$$\frac{78x+165x^3+121x^5+39x^7+5x^9-42i\sqrt{1+x^2}\,\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-20i\sqrt{1+x^2}\,\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{35\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (78*x + 165*x^3 + 121*x^5 + 39*x^7 + 5*x^9 - (42*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (20*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(35*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 155, normalized size = 0.90

method	result
risch	$\frac{x(5x^4+24x^2+39)\sqrt{x^4+3x^2+2}}{35} - \frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{35\sqrt{x^4+3x^2+2}} + \frac{3i\sqrt{2}\,\sqrt{2x^2+}}{}$
default	$\frac{x^5\sqrt{x^4+3x^2+2}}{7} + \frac{24x^3\sqrt{x^4+3x^2+2}}{35} + \frac{39x\sqrt{x^4+3x^2+2}}{35} - \frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{Ellipti}}{35\sqrt{x^4+3x^2+2}}$
elliptic	$\frac{x^5\sqrt{x^4+3x^2+2}}{7} + \frac{24x^3\sqrt{x^4+3x^2+2}}{35} + \frac{39x\sqrt{x^4+3x^2+2}}{35} - \frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{Ellipti}}{35\sqrt{x^4+3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x^5*(x^4+3*x^2+2)^(1/2)+24/35*x^3*(x^4+3*x^2+2)^(1/2)+39/35*x*(x^4+3*x^2+2)^(1/2)-31/35*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+3/5*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4 + 3x^2 + 2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x^4+3\,x^2+2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(3/2), x)
```

3.297 $\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2} \, dx$

Optimal. Leaf size=207

$$\frac{24x(2+x^2)}{125\sqrt{2+3x^2+x^4}}+\frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4}-\frac{24\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{125\sqrt{2+3x^2+x^4}}+\frac{56\sqrt{2}\,(1+}{3}$$

```
[Out] 24/125*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-9/875*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1
)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1
))^(1/2)/(x^4+3*x^2+2)^(1/2)-24/125*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellipti
cE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+
2)^(1/2)+56/375*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1
/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/75*x*(3*x
^2+11)*(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1222, 1190, 1203, 1113, 1149, 1228, 1470, 553}

$$\frac{56\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{375\sqrt{x^4+3x^2+2}}-\frac{24\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{125\sqrt{x^4+3x^2+2}}-\frac{9\sqrt{2}\,(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}+\frac{24x(x^2+2)}{125\sqrt{x^4+3x^2+2}}+\frac{1}{75}x\left(3x^2+11\right)\sqrt{x^4+3x^2+2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (24*x*(2 + x^2))/(125*Sqrt[2 + 3*x^2 + x^4]) + (x*(11 + 3*x^2)*Sqrt[2 + 3*x
^2 + x^4])/75 - (24*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[A
rcTan[x], 1/2])/(125*Sqrt[2 + 3*x^2 + x^4]) + (56*Sqrt[2]*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(375*Sqrt[2 + 3*x^2 + x^4]) -
 (9*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(875*Sqrt[(2 + x^2)/
(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
```

```
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(2+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int \left(-8-5x^2\right)\sqrt{2+3x^2+x^4}\,dx\right) - \frac{6}{25}\int \frac{\sqrt{2+3x^2+x^4}}{7+5x^2}\,dx$$

$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} - \frac{1}{375}\int \frac{-130-90x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{6}{625}\int \frac{-8-5x}{\sqrt{2+3x^2-}}$$

$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} + \frac{18}{625}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{6}{125}\int \frac{x^2}{\sqrt{2+3x^2-}}$$

$$= \frac{24x(2+x^2)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} - \frac{24\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}}{125\sqrt{2+3x^2-}}$$

$$= \frac{24x(2+x^2)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\left(11+3x^2\right)\sqrt{2+3x^2+x^4} - \frac{24\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}}{125\sqrt{2+3x^2-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.12, size = 148, normalized size = 0.71

$$\frac{3850x+6825x^3+3500x^5+525x^7-2520i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-1022i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-108i\sqrt{1+x^2}\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{13125\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (3850*x + 6825*x^3 + 3500*x^5 + 525*x^7 - (2520*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (1022*I)*Sqrt[1 + x^2]*Sqrt[2 + x^
```

```
2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (108*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]
*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(13125*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 170, normalized size = 0.82

method	result
default	$\frac{x^3\sqrt{x^4+3x^2+2}}{25}+\frac{11x\sqrt{x^4+3x^2+2}}{75}-\frac{73i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{1875\sqrt{x^4+3x^2+2}}-\frac{12i}{}$
elliptic	$\frac{x^3\sqrt{x^4+3x^2+2}}{25}+\frac{11x\sqrt{x^4+3x^2+2}}{75}-\frac{73i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{1875\sqrt{x^4+3x^2+2}}-\frac{12i}{}$
risch	$\frac{x(3x^2+11)\sqrt{x^4+3x^2+2}}{75}+\frac{12i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right.\right.}{125\sqrt{x^4+3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] 1/25*x^3*(x^4+3*x^2+2)^(1/2)+11/75*x*(x^4+3*x^2+2)^(1/2)-73/1875*I*2^(1/2)*
(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x
,2^(1/2))-12/125*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2
)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-36/4375*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}}{5x^{2}+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^4+3\,x^2+2\right)}^{3/2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7),x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.298 $\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=222

$$\frac{9x(2+x^2)}{175\sqrt{2+3x^2+x^4}}+\frac{1}{75}x\sqrt{2+3x^2+x^4}-\frac{3x\sqrt{2+3x^2+x^4}}{175\,(7+5x^2)}-\frac{9\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{175\sqrt{2+3x^2+x^4}}+ 59$$

```
[Out] 9/175*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-9/175*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*E
llipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+
3*x^2+2)^(1/2)+59/1050*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^
(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)+9/2450*(x^
2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*((
x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/75*x*(x^4+3*x^2+2)^(1/2)-3/17
5*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A]
time = 0.28, antiderivative size = 333, normalized size of antiderivative = 1.50, number of steps used = 21, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1242, 1113, 1149, 1136, 1203, 1237, 1730, 1228, 1470, 553}

$$\frac{44\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{1875\sqrt{x^4+3x^2+2}}+\frac{81(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{8750\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{9\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{175\sqrt{x^4+3x^2+2}}+\frac{3\sqrt{2}\,(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{875\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}-\frac{39(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{12250\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}-\frac{3\sqrt{x^4+3x^2+2}\,x}{175\,(5x^2+7)}+\frac{1}{75}\sqrt{x^4+3x^2+2}\,x+\frac{9(x^2+2)\,x}{175\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (9*x*(2 + x^2))/(175*Sqrt[2 + 3*x^2 + x^4]) + (x*Sqrt[2 + 3*x^2 + x^4])/75
- (3*x*Sqrt[2 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) - (9*Sqrt[2]*(1 + x^2)*Sqrt
[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(175*Sqrt[2 + 3*x^2 + x^4]
) + (81*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(875
0*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (44*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
+ x^2)]*EllipticF[ArcTan[x], 1/2])/(1875*Sqrt[2 + 3*x^2 + x^4]) - (39*(2 +
x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(12250*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^
2)]*Sqrt[2 + 3*x^2 + x^4]) + (3*Sqrt[2]*(2 + x^2)*EllipticPi[2/7, ArcTan[x]
, 1/2])/(875*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
```

```
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \left(\frac{52}{625\sqrt{2+3x^2+x^4}} + \frac{16x^2}{125\sqrt{2+3x^2+x^4}} + \frac{x^4}{25\sqrt{2+3x^2+x^4}} + \frac{\cdots}{625\,(7+5\cdots}\right.$$

$$= -\left(\frac{12}{625}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx\right) + \frac{1}{25}\int \frac{x^4}{\sqrt{2+3x^2+x^4}}\,dx + \frac{36}{625}\int \frac{\cdots}{(7\cdots}$$

$$= \frac{16x(2+x^2)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\,(7+5x^2)} - \frac{16\sqrt{2}\,(1+x^2)\cdots}{125\cdots}$$

$$= \frac{16x(2+x^2)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\,(7+5x^2)} - \frac{16\sqrt{2}\,(1+x^2)\cdots}{125\cdots}$$

$$= \frac{6x(2+x^2)}{125\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\,(7+5x^2)} - \frac{6\sqrt{2}\,(1+x^2)\cdots}{125\sqrt{\cdots}}$$

$$= \frac{9x(2+x^2)}{175\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\,(7+5x^2)} - \frac{9\sqrt{2}\,(1+x^2)\cdots}{175\sqrt{\cdots}}$$

$$= \frac{9x(2+x^2)}{175\sqrt{2+3x^2+x^4}} + \frac{1}{75}x\sqrt{2+3x^2+x^4} - \frac{3x\sqrt{2+3x^2+x^4}}{175\,(7+5x^2)} - \frac{9\sqrt{2}\,(1+x^2)\cdots}{175\sqrt{\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.11, size = 213, normalized size = 0.96

$$\frac{2800x + 6650x^3 + 5075x^5 + 1225x^7 - 945i\sqrt{1+x^2}\sqrt{2+x^2}\,(7+5x^2)\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 182i\sqrt{1+x^2}\sqrt{2+x^2}\,(7+5x^2)\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 189i\sqrt{1+x^2}\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 135ix^2\sqrt{1+x^2}\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{18375\,(7+5x^2)\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (2800*x + 6650*x^3 + 5075*x^5 + 1225*x^7 - (945*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*(7 + 5*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (182*I)*Sqrt[1 + x^2]*
Sqrt[2 + x^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (189*I)*Sqrt
[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (135*I)
```

```
*x^2*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])
/(18375*(7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 177, normalized size = 0.80

method	result
default	$-\frac{3x\sqrt{x^4+3x^2+2}}{175(5x^2+7)}+\frac{x\sqrt{x^4+3x^2+2}}{75}-\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2625\sqrt{x^4+3x^2+2}}-\frac{9i}{}$
elliptic	$-\frac{3x\sqrt{x^4+3x^2+2}}{175(5x^2+7)}+\frac{x\sqrt{x^4+3x^2+2}}{75}-\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2625\sqrt{x^4+3x^2+2}}-\frac{9i}{}$
risch	$\frac{\sqrt{x^4+3x^2+2}\,x(7x^2+8)}{525x^2+735}+\frac{9i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right.}{350\sqrt{x^4+3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/175*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)+1/75*x*(x^4+3*x^2+2)^(1/2)-13/2625*I
*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*
2^(1/2)*x,2^(1/2))-9/350*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2
+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+9/6125*I*2^(1/2)*(1+1/2*x^2)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1
/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}}{\left(5x^{2}+7\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^4+3\,x^2+2\right)}^{3/2}}{{\left(5\,x^2+7\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^2,x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.299 $\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3} \, dx$

Optimal. Leaf size=231

$$\frac{3x(2+x^2)}{392\sqrt{2+3x^2+x^4}}-\frac{3x\sqrt{2+3x^2+x^4}}{350\,(7+5x^2)^2}+\frac{17x\sqrt{2+3x^2+x^4}}{9800\,(7+5x^2)}-\frac{3(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{196\sqrt{2+3x^2+x^4}}+\frac{5(1-}{}$$

```
[Out] 3/392*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+141/54880*(x^2+2)*(1/(x^2+1))^(1/2)*(x^
2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^
2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-3/196*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellip
ticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(
1/2)+5/784*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^
(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)-3/350*x*(x^4+3*x^2+2)^
(1/2)/(5*x^2+7)^2+17/9800*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A]
time = 0.42, antiderivative size = 288, normalized size of antiderivative = 1.25, number of steps used = 27, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1242, 1113, 1149, 1237, 1710, 1730, 1203, 1228, 1470, 553}

$$\frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{784\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{6\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{875\sqrt{x^4+3x^2+2}}-\frac{39(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{24500\sqrt{2}\,\sqrt{x^4+3x^2+2}}+\frac{141(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{27440\sqrt{2}\,\sqrt{\frac{x^2+2}{x^2+1}}\,\sqrt{x^4+3x^2+2}}+\frac{17\sqrt{x^4+3x^2+2}\,x}{9800\,(5x^2+7)}-\frac{3\sqrt{x^4+3x^2+2}\,x}{350\,(5x^2+7)^2}+\frac{3(x^2+2)\,x}{392\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (3*x*(2 + x^2))/(392*Sqrt[2 + 3*x^2 + x^4]) - (3*x*Sqrt[2 + 3*x^2 + x^4])/(
350*(7 + 5*x^2)^2) + (17*x*Sqrt[2 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) - (39*
(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(24500*Sqrt[
2]*Sqrt[2 + 3*x^2 + x^4]) - (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*
EllipticE[ArcTan[x], 1/2])/(875*Sqrt[2 + 3*x^2 + x^4]) + (5*(1 + x^2)*Sqrt[
(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[2 + 3*x^2
 + x^4]) + (141*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(27440*Sqrt[2]*S
qrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{(2+3x^2+x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \left(\frac{9}{625\sqrt{2+3x^2+x^4}} + \frac{x^2}{125\sqrt{2+3x^2+x^4}} + \frac{36}{625\,(7+5x^2)^3\,\sqrt{2+3x^2+x^4}}\right.$$

$$= \frac{1}{125}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{9}{625}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{11}{625}\int \frac{1}{(7+5x^2)\,\sqrt{2+}}$$

$$= \frac{x(2+x^2)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\,(7+5x^2)^2} + \frac{x\sqrt{2+3x^2+x^4}}{175\,(7+5x^2)} - \frac{\sqrt{2}\,(1+x^2)\sqrt{\frac{2}{1}}}{125\sqrt{2}}$$

$$= \frac{x(2+x^2)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\,(7+5x^2)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\,(7+5x^2)} - \frac{\sqrt{2}\,(1+x^2)}{125\sqrt{}}$$

$$= \frac{x(2+x^2)}{125\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\,(7+5x^2)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\,(7+5x^2)} - \frac{\sqrt{2}\,(1+x^2)}{125\sqrt{}}$$

$$= \frac{6x(2+x^2)}{875\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\,(7+5x^2)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\,(7+5x^2)} - \frac{6\sqrt{2}\,(1+x^2)}{875\sqrt{}}$$

$$= \frac{3x(2+x^2)}{392\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\,(7+5x^2)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\,(7+5x^2)} - \frac{39(1+x^2)\sqrt{\frac{2}{1}}}{24500\sqrt{2}}$$

$$= \frac{3x(2+x^2)}{392\sqrt{2+3x^2+x^4}} - \frac{3x\sqrt{2+3x^2+x^4}}{350\,(7+5x^2)^2} + \frac{17x\sqrt{2+3x^2+x^4}}{9800\,(7+5x^2)} - \frac{39(1+x^2)\sqrt{\frac{2}{1}}}{24500\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.15, size = 174, normalized size = 0.75

$$\frac{-\frac{588x(2+3x^2+x^4)}{(7+5x^2)^2} + \frac{119x(2+3x^2+x^4)}{7+5x^2} - 525i\sqrt{1+x^2}\,\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 406i\sqrt{1+x^2}\,\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 141i\sqrt{1+x^2}\,\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{68600\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] ((-588*x*(2 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + (119*x*(2 + 3*x^2 + x^4))/(7 +
5*x^2) - (525*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]]
, 2] - (406*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]],
2] + (141*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[
2]], 2])/(68600*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 186, normalized size = 0.81

method	result
risch	$\frac{\sqrt{x^4+3x^2+2}\,x(17x^2+7)}{1960(5x^2+7)^2}+\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right.}{784\sqrt{x^4+3x^2+2}}$
default	$-\frac{3x\sqrt{x^4+3x^2+2}}{350(5x^2+7)^2}+\frac{17x\sqrt{x^4+3x^2+2}}{9800(5x^2+7)}-\frac{29i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9800\sqrt{x^4+3x^2+2}}-$
elliptic	$-\frac{3x\sqrt{x^4+3x^2+2}}{350(5x^2+7)^2}+\frac{17x\sqrt{x^4+3x^2+2}}{9800(5x^2+7)}-\frac{29i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{9800\sqrt{x^4+3x^2+2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/350*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+17/9800*x*(x^4+3*x^2+2)^(1/2)/(5*x
^2+7)-29/9800*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*E
llipticF(1/2*I*2^(1/2)*x,2^(1/2))-3/784*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+141/68600*I*2^(
1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2
^(1/2)*x,10/7,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 3*x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+2)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(((x**2 + 1)*(x**2 + 2))**(3/2)/(5*x**2 + 7)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+3\,x^2+2\right)^{3/2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^3,x)
```

```
[Out] int((3*x^2 + x^4 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.300 $\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=157

$$\frac{135x(2+x^2)}{\sqrt{2+3x^2+x^4}}+75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}-\frac{135\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{19}{}$$

```
[Out] 135*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+193/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ell
ipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*
x^2+2)^(1/2)-135*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),
1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+75*x*(x^4+
3*x^2+2)^(1/2)+25*x^3*(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1693, 1203, 1113, 1149}

$$\frac{193(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{135\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+75\sqrt{x^4+3x^2+2}\,x+\frac{135(x^2+2)\,x}{\sqrt{x^4+3x^2+2}}+25\sqrt{x^4+3x^2+2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (135*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + 75*x*Sqrt[2 + 3*x^2 + x^4] + 25*x
^3*Sqrt[2 + 3*x^2 + x^4] - (135*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]
*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (193*(1 + x^2)*Sqrt[(2
+ x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]
)
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
```

```
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \frac{(7+5x^2)^3}{\sqrt{2+3x^2+x^4}}\,dx = 25x^3\sqrt{2+3x^2+x^4}+\frac{1}{5}\int \frac{1715+2925x^2+1125x^4}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= 75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}+\frac{1}{15}\int \frac{2895+2025x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= 75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}+135\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+193\int \frac{}{\sqrt{}}$$

$$= \frac{135x(2+x^2)}{\sqrt{2+3x^2+x^4}}+75x\sqrt{2+3x^2+x^4}+25x^3\sqrt{2+3x^2+x^4}-\frac{135\sqrt{2}\,(1+x^2)\sqrt{}}{\sqrt{2+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 106, normalized size = 0.68

$$\frac{25x(6+11x^2+6x^4+x^6)-135i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-58i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*(6 + 11*x^2 + 6*x^4 + x^6) - (135*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Elli
pticE[I*ArcSinh[x/Sqrt[2]], 2] - (58*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipti
cF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 138, normalized size = 0.88

method	result
risch	$25x(x^2+3)\sqrt{x^4+3x^2+2}+\frac{135i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2}\right.\right.}{2\sqrt{x^4+3x^2+2}}$
default	$25x^3\sqrt{x^4+3x^2+2}+75x\sqrt{x^4+3x^2+2}-\frac{193i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+$
elliptic	$25x^3\sqrt{x^4+3x^2+2}+75x\sqrt{x^4+3x^2+2}-\frac{193i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25*x^3*(x^4+3*x^2+2)^(1/2)+75*x*(x^4+3*x^2+2)^(1/2)-193/2*I*2^(1/2)*(2*x^2+
4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2
))+135/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Ellip
ticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^3}{\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
[Out] Integral((5*x**2 + 7)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+7\right)}^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.301 $\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=142

$$\frac{20x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{25}{3}x\sqrt{2+3x^2+x^4}-\frac{20\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{97(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F(\cdots}{3\sqrt{2}\,\sqrt{2+3x^2+\cdots}}$$

```
[Out] 20*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+97/6*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellip
ticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^
2+2)^(1/2)-20*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2
*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+25/3*x*(x^4+3
*x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1220, 1203, 1113, 1149}

$$\frac{97(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{20\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{25}{3}\sqrt{x^4+3x^2+2}\,x+\frac{20(x^2+2)\,x}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (20*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (25*x*Sqrt[2 + 3*x^2 + x^4])/3 - (
20*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/S
qrt[2 + 3*x^2 + x^4] + (97*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[Ar
cTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
```

```
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(7+5x^2)^2}{\sqrt{2+3x^2+x^4}}\,dx &= \frac{25}{3}x\sqrt{2+3x^2+x^4}+\frac{1}{3}\int \frac{97+60x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{25}{3}x\sqrt{2+3x^2+x^4}+20\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+\frac{97}{3}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{20x(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{25}{3}x\sqrt{2+3x^2+x^4}-\frac{20\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 104, normalized size = 0.73

$$\frac{25x(2+3x^2+x^4)-60i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-37i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{3\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*(2 + 3*x^2 + x^4) - (60*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*Ar
cSinh[x/Sqrt[2]], 2] - (37*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSi
nh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 121, normalized size = 0.85

method	result
default	$\frac{25x\sqrt{x^4+3x^2+2}}{3}-\frac{97i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}}+\frac{10i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+}$
risch	$\frac{25x\sqrt{x^4+3x^2+2}}{3}-\frac{97i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}}+\frac{10i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+}$
elliptic	$\frac{25x\sqrt{x^4+3x^2+2}}{3}-\frac{97i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}}+\frac{10i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/3*x*(x^4+3*x^2+2)^(1/2)-97/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^
4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+10*I*2^(1/2)*(2*x^2+4)^
(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))
-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^2}{\sqrt{(x^2+1)\,(x^2+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+7\right)}^2}{\sqrt{x^4+3\,x^2+2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.302 $\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=121

$$\frac{5x(2+x^2)}{\sqrt{2+3x^2+x^4}}-\frac{5\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{7(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{2+3x^2+x^4}}$$

```
[Out] 5*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+7/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellipti
cF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+
2)^(1/2)-5*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^
(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1203, 1113, 1149}

$$\frac{7(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{5\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{5x(x^2+2)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (5*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (7*(1 + x^2)
*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x
^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{7+5x^2}{\sqrt{2+3x^2+x^4}}\,dx = 5\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + 7\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{5x(2+x^2)}{\sqrt{2+3x^2+x^4}} - \frac{5\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{7(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F}{\sqrt{2}\,\sqrt{2+3x^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 69, normalized size = 0.57

$$-\frac{i\sqrt{1+x^2}\,\sqrt{2+x^2}\left(5E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+2F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(5*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + 2
*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.02, size = 106, normalized size = 0.88

method	result
default	$\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}} - \frac{7i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+}}{2\sqrt{x^4+3}}$
elliptic	$\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}} - \frac{7i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+}}{2\sqrt{x^4+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(
1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-7/2*I*2^(1/2)*
(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x
,2^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2 + 7}{\sqrt{(x^2 + 1)(x^2 + 2)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.303 $\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=48

$$\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{2+3x^2+x^4}}$$

```
[Out] 1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1113}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sq
rt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx = \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{2+3x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 50, normalized size = 1.04

$$-\frac{i\sqrt{1+x^2}\,\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/Sqrt[
2 + 3*x^2 + x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.02, size = 46, normalized size = 0.96

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^{4}+3x^{2}+2}}$	46
elliptic	$-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^{4}+3x^{2}+2}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(
1/2*I*2^(1/2)*x,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.08, size = 10, normalized size = 0.21

$$-i\,\mathrm{ellipticF}\left(\frac{1}{2}i\,\sqrt{2}\,x,2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -I*ellipticF(1/2*I*sqrt(2)*x, 2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 3*x**2 + 2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.304 $\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=106

$$\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{5(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}$$

```
[Out] -5/28*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/
7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/4*(x^2
+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*
((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1228, 1113, 1470, 553}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{5(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*
Sqrt[2 + 3*x^2 + x^4]) - (5*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(14*
Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
```

```
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx - \frac{5}{4}\int \frac{2+2x^2}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{\left(5\sqrt{1+\frac{x^2}{2}}\sqrt{2+2x^2}\right)\int \frac{\sqrt{2}}{\sqrt{1+}}}{4\sqrt{2+3x^2+x^4}}\\ &= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{2\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{5(2+x^2)\,\Pi\left(\frac{2}{7};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{14\sqrt{2}\sqrt{\frac{2+x^2}{1+x^2}}\sqrt{2+3x^2+x^4}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.08, size = 55, normalized size = 0.52

$$-\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{7\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((-1/7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]]
, 2])/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 47, normalized size = 0.44

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^{2}}{2}}\,\sqrt{x^{2}+1}\,\mathrm{EllipticPi}\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{7\sqrt{x^{4}+3x^{2}+2}}$	47
elliptic	$-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^{2}}{2}}\,\sqrt{x^{2}+1}\,\mathrm{EllipticPi}\left(\frac{i\sqrt{2}\,x}{2},\frac{10}{7},\sqrt{2}\right)}{7\sqrt{x^{4}+3x^{2}+2}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
Pi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^6 + 22*x^4 + 31*x^2 + 14), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^{2}+1)\,(x^{2}+2)}\,(5x^{2}+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(5\,x^2+7)\,\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.305 $\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=209

$$\frac{5x(2+x^2)}{84\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)}-\frac{5(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{42\sqrt{2}\,\sqrt{2+3x^2+x^4}}+\frac{9(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}\right.}{56\sqrt{2}\,\sqrt{2+3x^2+x^4}}$$

```
[Out] 5/84*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-65/2352*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1
)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1
))^(1/2)/(x^4+3*x^2+2)^(1/2)-5/84*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE
(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+9/112*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*
2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-25/84*x*(x^4+3
*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A]
time = 0.12, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1237, 1730, 1203, 1113, 1149, 1228, 1470, 553}

$$\frac{9(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{56\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{5(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{42\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{65(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{1176\sqrt{2}\,\sqrt{\frac{x^2+2}{x^2+1}}\,\sqrt{x^4+3x^2+2}}-\frac{25\sqrt{x^4+3x^2+2}\,x}{84\,(5x^2+7)}+\frac{5(x^2+2)\,x}{84\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (5*x*(2 + x^2))/(84*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4])/(
84*(7 + 5*x^2)) - (5*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(42*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (9*(1 + x^2)*Sqrt[(2 + x^2)/(
1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(56*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (
65*(2 + x^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(1176*Sqrt[2]*Sqrt[(2 + x^2)/
(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
```

```
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx = -\frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)} + \frac{1}{84}\int \frac{62+70x^2+25x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)} - \frac{\int \frac{-175-125x^2}{\sqrt{2+3x^2+x^4}}\,dx}{2100} + \frac{13}{84}\int \frac{1}{(7+5x^2)\sqrt{2+3}}$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)} + \frac{5}{84}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{13}{168}\int \frac{1}{\sqrt{2+3x^2+}}$$

$$= \frac{5x(2+x^2)}{84\sqrt{2+3x^2+x^4}} - \frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)} - \frac{5(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\big(\tan^{-}}{42\sqrt{2}\,\sqrt{2+3x^2+x^4}}$$

$$= \frac{5x(2+x^2)}{84\sqrt{2+3x^2+x^4}} - \frac{25x\sqrt{2+3x^2+x^4}}{84\,(7+5x^2)} - \frac{5(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\big(\tan^{-}}{42\sqrt{2}\,\sqrt{2+3x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 208, normalized size = 1.00

$$\frac{-350x-525x^3-175x^5-35i\sqrt{1+x^2}\,\sqrt{2+x^2}\,(7+5x^2)\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-14i\sqrt{1+x^2}\,\sqrt{2+x^2}\,(7+5x^2)\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-91i\sqrt{1+x^2}\,\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-65ix^2\sqrt{1+x^2}\,\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{588\,(7+5x^2)\,\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (-350*x - 525*x^3 - 175*x^5 - (35*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2
)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(
7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (91*I)*Sqrt[1 + x^2]*Sqrt[2
 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] - (65*I)*x^2*Sqrt[1 + x^2
]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(588*(7 + 5*x^2)
*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 162, normalized size = 0.78

method	result
default	$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{84\sqrt{x^4+3x^2+2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+}}{168\sqrt{x^4+}}$
elliptic	$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{84\sqrt{x^4+3x^2+2}}-\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+}}{168\sqrt{x^4+}}$
risch	$-\frac{25x\sqrt{x^4+3x^2+2}}{84(5x^2+7)}+\frac{5i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{168\sqrt{x^4+3x^2+2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -25/84*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-1/84*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-5/168*I*2^(
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1
/2)*x,2^(1/2))-13/588*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^8 + 145*x^6 + 309*x^4 + 287*x^2 + 98),
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2+1)(x^2+2)}\,(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(5\,x^2+7\right)}^2\,\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.306 $\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=237

$$\frac{65x(2+x^2)}{4704\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{168\,(7+5x^2)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704\,(7+5x^2)}-\frac{65(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{2352\sqrt{2}\,\sqrt{2+3x^2+x^4}}+\cdots$$

```
[Out] 65/4704*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-2525/131712*(x^2+2)*(1/(x^2+1))^(1/2)
*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)
/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-65/4704*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)
*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^
4+3*x^2+2)^(1/2)+631/18816*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2
+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-
25/168*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2-325/4704*x*(x^4+3*x^2+2)^(1/2)/(5*
x^2+7)
```

Rubi [A]

time = 0.16, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1237, 1710, 1730, 1203, 1113, 1149, 1228, 1470, 553}

$$\frac{631(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{9408\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{65(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{2352\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{2525(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{65856\sqrt{2}\,\sqrt{\frac{x^2+2}{x^2+1}}\,\sqrt{x^4+3x^2+2}}-\frac{325\sqrt{x^4+3x^2+2}\,x}{4704\,(5x^2+7)}-\frac{25\sqrt{x^4+3x^2+2}\,x}{168\,(5x^2+7)^2}+\frac{65(x^2+2)\,x}{4704\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (65*x*(2 + x^2))/(4704*Sqrt[2 + 3*x^2 + x^4]) - (25*x*Sqrt[2 + 3*x^2 + x^4]
)/(168*(7 + 5*x^2)^2) - (325*x*Sqrt[2 + 3*x^2 + x^4])/(4704*(7 + 5*x^2)) -
(65*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(2352*Sq
rt[2]*Sqrt[2 + 3*x^2 + x^4]) + (631*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ell
ipticF[ArcTan[x], 1/2])/(9408*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (2525*(2 + x
^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(65856*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2
)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+3x^2+x^4}}\,dx = -\frac{25x\sqrt{2+3x^2+x^4}}{168\,(7+5x^2)^2}+\frac{1}{168}\int \frac{74-10x^2-25x^4}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{168\,(7+5x^2)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704\,(7+5x^2)}+\frac{\int \frac{2838+2310x^2+975x^4}{(7+5x^2)\sqrt{2+3x^2+x^4}}}{14112}$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{168\,(7+5x^2)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704\,(7+5x^2)}-\frac{\int \frac{-4725-4875x^2}{\sqrt{2+3x^2+x^4}}\,dx}{352800}+$$

$$= -\frac{25x\sqrt{2+3x^2+x^4}}{168\,(7+5x^2)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704\,(7+5x^2)}+\frac{3}{224}\int \frac{1}{\sqrt{2+3x^2+x^4}}$$

$$= \frac{65x(2+x^2)}{4704\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{168\,(7+5x^2)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704\,(7+5x^2)}-\frac{65}{}$$

$$= \frac{65x(2+x^2)}{4704\sqrt{2+3x^2+x^4}}-\frac{25x\sqrt{2+3x^2+x^4}}{168\,(7+5x^2)^2}-\frac{325x\sqrt{2+3x^2+x^4}}{4704\,(7+5x^2)}-\frac{65}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.15, size = 186, normalized size = 0.78

$$\frac{-175x(238+487x^2+314x^4+65x^6)-455i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)^2E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+14i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)^2F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-505i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)^2\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{32928(7+5x^2)^2\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] (-175*x*(238 + 487*x^2 + 314*x^4 + 65*x^6) - (455*I)*Sqrt[1 + x^2]*Sqrt[2 +
 x^2]*(7 + 5*x^2)^2*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (14*I)*Sqrt[1 + x^
2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (505*I)
*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5*x^2)^2*EllipticPi[10/7, I*ArcSinh[x/Sqr
t[2]], 2])/(32928*(7 + 5*x^2)^2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 186, normalized size = 0.78

method	result

risch	$-\frac{25\sqrt{x^4+3x^2+2}\,x(65x^2+119)}{4704(5x^2+7)^2}+\frac{65i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}}{2}\right.\right.}{9408\sqrt{x^4+3x^2+2}}$
default	$-\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}-\frac{325x\sqrt{x^4+3x^2+2}}{4704(5x^2+7)}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4704\sqrt{x^4+3x^2+2}}-$
elliptic	$-\frac{25x\sqrt{x^4+3x^2+2}}{168(5x^2+7)^2}-\frac{325x\sqrt{x^4+3x^2+2}}{4704(5x^2+7)}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4704\sqrt{x^4+3x^2+2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -25/168*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2-325/4704*x*(x^4+3*x^2+2)^(1/2)/(5
*x^2+7)+1/4704*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*
EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-65/9408*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-505/32928*I*
2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*
I*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^10 + 900*x^8 + 2560*x^6 + 3598*x^4 +
2499*x^2 + 686), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2+1)(x^2+2)}\,(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(5*x**2 + 7)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(5*x^2 + 7)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(5\,x^2+7\right)}^3\,\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.307 $\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=189

$$\frac{7679x(2+x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{5000}{3}x\sqrt{2+3x^2+x^4}+625x^3\sqrt{2+3x^2+x^4}-\frac{7679(1+x^2)\sqrt{\frac{2+x}{1+x}}}{\sqrt{2}\sqrt{2+3}}$$

```
[Out] 7679/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-1/2*x*(179*x^2+115)/(x^4+3*x^2+2)^(1/2
)-7679/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1
/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+15383/6*(x^2+1)^(3
/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+
2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+5000/3*x*(x^4+3*x^2+2)^(1/2)+625*x^3*
(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1219, 1693, 1203, 1113, 1149}

$$\frac{15383(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}F\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{7679(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}E\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{5000}{3}\sqrt{x^4+3x^2+2}\,x+\frac{7679(x^2+2)x}{2\sqrt{x^4+3x^2+2}}-\frac{(179x^2+115)x}{2\sqrt{x^4+3x^2+2}}+625\sqrt{x^4+3x^2+2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (7679*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) - (x*(115 + 179*x^2))/(2*Sqrt[
2 + 3*x^2 + x^4]) + (5000*x*Sqrt[2 + 3*x^2 + x^4])/3 + 625*x^3*Sqrt[2 + 3*x
^2 + x^4] - (7679*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x],
1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (15383*(1 + x^2)*Sqrt[(2 + x^2)/(1
+ x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
```

```
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \frac{(7+5x^2)^5}{(2+3x^2+x^4)^{3/2}}\,dx = -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{-16922-35179x^2-25000x^4-6250x^6}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + 625x^3\sqrt{2+3x^2+x^4} - \frac{1}{10}\int \frac{-84610-138395x^2-5000}{\sqrt{2+3x^2+x^4}}$$
$$= -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{5000}{3}x\sqrt{2+3x^2+x^4} + 625x^3\sqrt{2+3x^2+x^4} - \frac{1}{30}\int \frac{-1}{}$$
$$= -\frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{5000}{3}x\sqrt{2+3x^2+x^4} + 625x^3\sqrt{2+3x^2+x^4} + \frac{7679}{2}\int$$
$$= \frac{7679x(2+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{x(115+179x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{5000}{3}x\sqrt{2+3x^2+x^4} + 625x^3\sqrt{2+3x^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 109, normalized size = 0.58

$$\frac{19655x+36963x^3+21250x^5+3750x^7-23037i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-7729i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{6\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^5/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (19655*x + 36963*x^3 + 21250*x^5 + 3750*x^7 - (23037*I)*Sqrt[1 + x^2]*Sqrt[
2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (7729*I)*Sqrt[1 + x^2]*Sqrt[2
 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(6*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 274, normalized size = 1.45

method	result
risch	$\frac{x(3750x^6+21250x^4+36963x^2+19655)}{6\sqrt{x^4+3x^2+2}} + \frac{7679i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}}{2}\right.\right.}{4\sqrt{x^4+3x^2+2}}$
elliptic	$-\frac{2\left(\frac{179}{4}x^3+\frac{115}{4}x\right)}{\sqrt{x^4+3x^2+2}} + 625x^3\sqrt{x^4+3x^2+2} + \frac{5000x\sqrt{x^4+3x^2+2}}{3} + \frac{7679i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+}}{}$
default	$-\frac{6250\left(\frac{17}{2}x^3+9x\right)}{\sqrt{x^4+3x^2+2}} + 625x^3\sqrt{x^4+3x^2+2} + \frac{5000x\sqrt{x^4+3x^2+2}}{3} - \frac{15383i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+}}{6\sqrt{x^4+3x}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -6250*(17/2*x^3+9*x)/(x^4+3*x^2+2)^(1/2)+625*x^3*(x^4+3*x^2+2)^(1/2)+5000/3*x*(x^4+3*x^2+2)^(1/2)-15383/6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+7679/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-43750*(-9/2*x^3-5*x)/(x^4+3*x^2+2)^(1/2)-122500*(5/2*x^3+3*x)/(x^4+3*x^2+2)^(1/2)-171500*(-3/2*x^3-2*x)/(x^4+3*x^2+2)^(1/2)-120050*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-33614*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^5}{\left(\left(x^2+1\right)\left(x^2+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**5/(x**4+3*x**2+2)**(3/2),x)

[Out] Integral((5*x**2 + 7)**5/((x**2 + 1)*(x**2 + 2))**(3/2), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+7\right)}^5}{{\left(x^4+3\,x^2+2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.308 $\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=170

$$\frac{637x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(145+113x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{625}{3}x\sqrt{2+3x^2+x^4}-\frac{637(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{1067\sqrt{2}}{}$$

```
[Out] 637/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(113*x^2+145)/(x^4+3*x^2+2)^(1/2)
-637/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2
))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1067/3*(x^2+1)^(3/2)
*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/
(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/3*x*(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1219, 1693, 1203, 1113, 1149}

$$\frac{1067\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{637(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{625}{3}\sqrt{x^4+3x^2+2}\,x+\frac{637(x^2+2)\,x}{2\sqrt{x^4+3x^2+2}}+\frac{(113x^2+145)\,x}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (637*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(145 + 113*x^2))/(2*Sqrt[2
 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/3 - (637*(1 + x^2)*Sqrt[(2
 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4
]) + (1067*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x],
 1/2])/(3*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
```

```
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \frac{(7+5x^2)^4}{(2+3x^2+x^4)^{3/2}}\,dx = \frac{x(145+113x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{-2256-3137x^2-1250x^4}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{x(145+113x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{625}{3}x\sqrt{2+3x^2+x^4} - \frac{1}{6}\int \frac{-4268-1911x^2}{\sqrt{2+3x^2+x^4}}\,dx$$
$$= \frac{x(145+113x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{625}{3}x\sqrt{2+3x^2+x^4} + \frac{637}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{2134}{3}\int$$
$$= \frac{637x(2+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{x(145+113x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{625}{3}x\sqrt{2+3x^2+x^4} - \frac{637(1+x^2)\sqrt{\frac{2}{1}}}{\sqrt{2}\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 104, normalized size = 0.61

$$\frac{2935x+4089x^3+1250x^5-1911i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-2357i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{6\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^4/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2935*x + 4089*x^3 + 1250*x^5 - (1911*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipt
icE[I*ArcSinh[x/Sqrt[2]], 2] - (2357*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*Ellipti
cF[I*ArcSinh[x/Sqrt[2]], 2])/(6*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 234, normalized size = 1.38

method	result
risch	$\frac{x(1250x^4+4089x^2+2935)}{6\sqrt{x^4+3x^2+2}} + \frac{637i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}} -$
elliptic	$-\frac{2\left(-\frac{113}{4}x^3-\frac{145}{4}x\right)}{\sqrt{x^4+3x^2+2}} + \frac{625x\sqrt{x^4+3x^2+2}}{3} + \frac{637i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{Ellip}\right.}{4\sqrt{x^4+3x^2+2}}$
default	$-\frac{1250\left(-\frac{9}{2}x^3-5x\right)}{\sqrt{x^4+3x^2+2}} + \frac{625x\sqrt{x^4+3x^2+2}}{3} + \frac{637i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{Ellip}\right.}{4\sqrt{x^4+3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

[Out] -1250*(-9/2*x^3-5*x)/(x^4+3*x^2+2)^(1/2)+625/3*x*(x^4+3*x^2+2)^(1/2)+637/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-1067/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-7000*(5/2*x^3+3*x)/(x^4+3*x^2+2)^(1/2)-14700*(-3/2*x^3-2*x)/(x^4+3*x^2+2)^(1/2)-13720*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-4802*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^4}{((x^2+1)(x^2+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**4/(x**4+3*x**2+2)**(3/2),x)

[Out] Integral((5*x**2 + 7)**4/((x**2 + 1)*(x**2 + 2))**(3/2), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5x^2+7)^4}{(x^4+3x^2+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.309 $\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=149

$$\frac{x(5-11x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{261x(2+x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{261(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{169(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan\right.}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] 1/2*x*(-11*x^2+5)/(x^4+3*x^2+2)^(1/2)+261/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)-2
61/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))
*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+169/2*(x^2+1)^(3/2)*(1
/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^
2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1219, 1203, 1113, 1149}

$$\frac{169(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{261(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{x(5-11x^2)}{2\sqrt{x^4+3x^2+2}}+\frac{261x(x^2+2)}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(5 - 11*x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (261*x*(2 + x^2))/(2*Sqrt[2 +
3*x^2 + x^4]) - (261*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (169*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
```

```
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(7+5x^2)^3}{(2+3x^2+x^4)^{3/2}}\,dx &= \frac{x(5-11x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{-338-261x^2}{\sqrt{2+3x^2+x^4}}\,dx \\ &= \frac{x(5-11x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{261}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+169\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\ &= \frac{x(5-11x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{261x(2+x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{261(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\cdots\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 99, normalized size = 0.66

$$-\frac{-5x+11x^3+261i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+77i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{2\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/2*(-5*x + 11*x^3 + (261*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSi
nh[x/Sqrt[2]], 2] + (77*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[
x/Sqrt[2]], 2])/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 196, normalized size = 1.32

method	result
risch	$-\frac{x(11x^2-5)}{2\sqrt{x^4+3x^2+2}}+\frac{261i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}-$
elliptic	$-\frac{2\left(\frac{11}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{261i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}}-$
default	$-\frac{250\left(\frac{5}{2}x^3+3x\right)}{\sqrt{x^4+3x^2+2}}-\frac{169i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{261i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -250*(5/2*x^3+3*x)/(x^4+3*x^2+2)^(1/2)-169/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2
+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+261/4*I*2^
(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^
(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-1050*(-3/2*x^3-2*x)/(x
^4+3*x^2+2)^(1/2)-1470*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-686*(-3/4*x^3-5/4*x)
/(x^4+3*x^2+2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^3}{((x^2+1)(x^2+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**3/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.310 $\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{17x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{17(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{6\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F(\mathrm{t}}{\sqrt{2+3x^2+x^4}}$$

```
[Out] -17/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(17*x^2+25)/(x^4+3*x^2+2)^(1/2)+1
7/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*
2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+6*(x^2+1)^(3/2)*(1/(x^2
+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))
^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1219, 1203, 1113, 1149}

$$\frac{6\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\mathrm{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{17(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\mathrm{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{17x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{x(17x^2+25)}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (-17*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(25 + 17*x^2))/(2*Sqrt[2 +
 3*x^2 + x^4]) + (17*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (6*Sqrt[2]*(1 + x^2)*Sqrt[(2 + x
^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
```

```
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(7+5x^2)^2}{(2+3x^2+x^4)^{3/2}}\,dx &= \frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{-24+17x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{17}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx+12\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\ &= -\frac{17x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(25+17x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{17(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 99, normalized size = 0.66

$$\frac{25x+17x^3+17i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-41i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{2\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(7 + 5*x^2)^2/(2 + 3*x^2 + x^4)^(3/2),x]

[Out] (25*x + 17*x^3 + (17*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (41*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(2*Sqrt[2 + 3*x^2 + x^4])

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 173, normalized size = 1.16

method	result
risch	$\frac{x(17x^2+25)}{2\sqrt{x^4+3x^2+2}} - \frac{17i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}} - \frac{6i}{}$
elliptic	$-\frac{2\left(-\frac{17}{4}x^3-\frac{25}{4}x\right)}{\sqrt{x^4+3x^2+2}} - \frac{17i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}} -$
default	$-\frac{50\left(-\frac{3}{2}x^3-2x\right)}{\sqrt{x^4+3x^2+2}} - \frac{6i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}} - \frac{17i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)

[Out] -50*(-3/2*x^3-2*x)/(x^4+3*x^2+2)^(1/2)-6*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-17/4*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-140*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-98*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")

[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^2}{((x^2+1)(x^2+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
[Out] Integral((5*x**2 + 7)**2/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5\,x^2+7)^2}{(x^4+3\,x^2+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.311 $\int \frac{7+5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(5+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}+\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\right.}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] -1/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(x^2+5)/(x^4+3*x^2+2)^(1/2)+1/2*(x
^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2
)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/2*(x^2+1)^(3/2)*(1/(x^2+1))
^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/
2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1192, 1203, 1113, 1149}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/2*(x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(5 + x^2))/(2*Sqrt[2 + 3*x^2
+ x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(
Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Ellip
ticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
```

```
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{7+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx &= \frac{x(5+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{-2+x^2}{\sqrt{2+3x^2+x^4}}\,dx \\ &= \frac{x(5+x^2)}{2\sqrt{2+3x^2+x^4}} - \frac{1}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx \\ &= -\frac{x(2+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{x(5+x^2)}{2\sqrt{2+3x^2+x^4}} + \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}} + \frac{(1}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 97, normalized size = 0.67

$$\frac{5x+x^3+i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-3i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{2\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/(2 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (5*x + x^3 + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]],
2] - (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/
(2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 150, normalized size = 1.03

method	result
risch	$\frac{x(x^2+5)}{2\sqrt{x^4+3x^2+2}} - \frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}} - \frac{i\sqrt{}}{}$
elliptic	$-\frac{2\left(-\frac{1}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}} - \frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{4\sqrt{x^4+3x^2+2}} - \frac{i\sqrt{}}{}$
default	$-\frac{10\left(x^3+\frac{3}{2}x\right)}{\sqrt{x^4+3x^2+2}} - \frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}} - \frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -10*(x^3+3/2*x)/(x^4+3*x^2+2)^(1/2)-1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-1/4*I*2^(1/2)*(
2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x
,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-14*(-3/4*x^3-5/4*x)/(x^4+3*x^
2+2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+7}{\left((x^2+1)\,(x^2+2)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/((x**2 + 1)*(x**2 + 2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^2+7}{{\left(x^4+3\,x^2+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.312 $\int \frac{1}{(2+3x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=149

$$-\frac{3x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(5+3x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan\right.}{\sqrt{2+3x^2+x^4}}$$

```
[Out] -3/2*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*x*(3*x^2+5)/(x^4+3*x^2+2)^(1/2)+3/2*
(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1
/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-(x^2+1)^(3/2)*(1/(x^2+1))^(
1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)
/(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1106, 1203, 1113, 1149}

$$-\frac{\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{3(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{3x(x^2+2)}{2\sqrt{x^4+3x^2+2}}+\frac{x(3x^2+5)}{2\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (-3*x*(2 + x^2))/(2*Sqrt[2 + 3*x^2 + x^4]) + (x*(5 + 3*x^2))/(2*Sqrt[2 + 3*
x^2 + x^4]) + (3*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1
/2])/(Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) - (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1
 + x^2)]*EllipticF[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
```

```
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+3x^2+x^4)^{3/2}}\,dx &= \frac{x(5+3x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{1}{2}\int \frac{4+3x^2}{\sqrt{2+3x^2+x^4}}\,dx\\ &= \frac{x(5+3x^2)}{2\sqrt{2+3x^2+x^4}}-\frac{3}{2}\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx-2\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx\\ &= -\frac{3x(2+x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{x(5+3x^2)}{2\sqrt{2+3x^2+x^4}}+\frac{3(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}-\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.96, size = 99, normalized size = 0.66

$$\frac{5x+3x^3+3i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{2\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (5*x + 3*x^3 + (3*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt
[2]], 2] + I*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]
)/(2*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 129, normalized size = 0.87

method	result
risch	$\frac{x(3x^2+5)}{2\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,(}{}$
default	$-\frac{2\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}}{}$
elliptic	$-\frac{2\left(-\frac{3}{4}x^3-\frac{5}{4}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{\sqrt{x^4+3x^2+2}}-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*(-3/4*x^3-5/4*x)/(x^4+3*x^2+2)^(1/2)+I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-3/4*I*2^(1/2)*(
2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x
,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4+3x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 2)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 2)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^4+3\,x^2+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + x^4 + 2)^(3/2),x)
```

```
[Out] int(1/(3*x^2 + x^4 + 2)^(3/2), x)
```

3.313 $\int \frac{1}{(7+5x^2)\,(2+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{x}{6\sqrt{2+3x^2+x^4}}+\frac{\sqrt{2}\,(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{3\sqrt{2+3x^2+x^4}}-\frac{9(1+x^2)\sqrt{\frac{2+x^2}{2+2x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{2+3x^2+x^4}}+\frac{125(1+}{8}$$

```
[Out] 1/6*x/(x^4+3*x^2+2)^(1/2)+125/168*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticP
i(x/(x^2+1)^(1/2),2/7,1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*x
^2+2)^(1/2)+1/3*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1
/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-9/4*(x^2+1)
^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(2
*x^2+2))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 207, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1235, 1192, 1203, 1113, 1149, 1228, 1470, 553}

$$-\frac{9(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{4\sqrt{2}\,\sqrt{x^4+3x^2+2}}+\frac{\sqrt{2}\,(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{3\sqrt{x^4+3x^2+2}}+\frac{125(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{84\sqrt{2}\,\sqrt{\frac{x^2+2}{x^2+1}}\,\sqrt{x^4+3x^2+2}}-\frac{x(x^2+2)}{3\sqrt{x^4+3x^2+2}}+\frac{x(2x^2+5)}{6\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/3*(x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (x*(5 + 2*x^2))/(6*Sqrt[2 + 3*x^
2 + x^4]) + (Sqrt[2]*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x
], 1/2])/(3*Sqrt[2 + 3*x^2 + x^4]) - (9*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]
*EllipticF[ArcTan[x], 1/2])/(4*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (125*(2 + x
^2)*EllipticPi[2/7, ArcTan[x], 1/2])/(84*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*
Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
```

```
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)\left(2+3x^2+x^4\right)^{3/2}}\,dx = -\left(\frac{1}{6}\int \frac{-8-5x^2}{\left(2+3x^2+x^4\right)^{3/2}}\,dx\right) - \frac{25}{6}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} + \frac{1}{12}\int \frac{-2-4x^2}{\sqrt{2+3x^2+x^4}}\,dx - \frac{25}{12}\int \frac{1}{\sqrt{2+3x^2+x^4}}$$

$$= \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} - \frac{25(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{12\sqrt{2}\sqrt{2+3x^2+x^4}} - \frac{1}{6}\int \frac{}{\sqrt{2+}}$$

$$= -\frac{x(2+x^2)}{3\sqrt{2+3x^2+x^4}} + \frac{x(5+2x^2)}{6\sqrt{2+3x^2+x^4}} + \frac{\sqrt{2}(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E(\text{ta}}{3\sqrt{2+3x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.13, size = 138, normalized size = 0.80

$$\frac{35x+14x^3+14i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-7i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+25i\sqrt{1+x^2}\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{42\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (35*x + 14*x^3 + (14*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/S
qrt[2]], 2] - (7*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[
2]], 2] + (25*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/S
qrt[2]], 2])/(42*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 161, normalized size = 0.93

method	result
default	$-\frac{2\left(-\frac{1}{6}x^{3}-\frac{5}{12}x\right)}{\sqrt{x^{4}+3x^{2}+2}}-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{12\sqrt{x^{4}+3x^{2}+2}}+\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\mathrm{El}}{6\sqrt{x^{4}+3x^{2}}}$
elliptic	$-\frac{2\left(-\frac{1}{6}x^{3}-\frac{5}{12}x\right)}{\sqrt{x^{4}+3x^{2}+2}}-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{12\sqrt{x^{4}+3x^{2}+2}}+\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\mathrm{El}}{6\sqrt{x^{4}+3x^{2}}}$
risch	$\frac{x\left(2x^{2}+5\right)}{6\sqrt{x^{4}+3x^{2}+2}}-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{6\sqrt{x^{4}+3x^{2}+2}}+\frac{i\sqrt{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/6*x^3-5/12*x)/(x^4+3*x^2+2)^(1/2)-1/12*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^
2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/6*I*2^(
1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1
/2)*x,2^(1/2))+25/42*I*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2
)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(5*x^10 + 37*x^8 + 107*x^6 + 151*x^4 + 104*x
^2 + 28), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}\cdot\left(5x^{2}+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+2)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(5\,x^2+7\right)\,{\left(x^4+3\,x^2+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 2)^(3/2)), x)
```

3.314 $\int \frac{1}{\left(7+5x^2\right)^2\left(2+3x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=235

$$-\frac{31x(2+x^2)}{56\sqrt{2+3x^2+x^4}}+\frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}}+\frac{625x\sqrt{2+3x^2+x^4}}{504\,(7+5x^2)}+\frac{31(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{28\sqrt{2}\sqrt{2+3x^2+x^4}}-\frac{463(}{}$$

```
[Out] -31/56*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/36*x*(11*x^2+20)/(x^4+3*x^2+2)^(1/2)
+375/1568*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2
),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+31/5
6*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^
(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-463/672*(x^2+1)^(3/2)*(1/
(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2
+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/504*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A]

time = 0.27, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1242, 1192, 1203, 1113, 1149, 1237, 1730, 1228, 1470, 553}

$$-\frac{463(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{336\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{31(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{28\sqrt{2}\sqrt{x^4+3x^2+2}}+\frac{375(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{784\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\sqrt{x^4+3x^2+2}}+\frac{625\sqrt{x^4+3x^2+2}\,x}{504\,(5x^2+7)}-\frac{31(x^2+2)\,x}{56\sqrt{x^4+3x^2+2}}+\frac{(11x^2+20)\,x}{36\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-31*x*(2 + x^2))/(56*Sqrt[2 + 3*x^2 + x^4]) + (x*(20 + 11*x^2))/(36*Sqrt[2
 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(504*(7 + 5*x^2)) + (31*(1
 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(28*Sqrt[2]*Sq
rt[2 + 3*x^2 + x^4]) - (463*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[A
rcTan[x], 1/2])/(336*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (375*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(784*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 +
3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
```

```
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^2 (2+3x^2+x^4)^{3/2}}\,dx = \int \left(\frac{14+5x^2}{36(2+3x^2+x^4)^{3/2}} - \frac{25}{6(7+5x^2)^2\sqrt{2+3x^2+x^4}} - \frac{}{36(7+5x\right.$$

$$= \frac{1}{36}\int \frac{14+5x^2}{(2+3x^2+x^4)^{3/2}}\,dx - \frac{25}{36}\int \frac{1}{(7+5x^2)\sqrt{2+3x^2+x^4}}\,dx - \frac{25}{6}$$

$$= \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504(7+5x^2)} - \frac{1}{72}\int \frac{26+22x^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504(7+5x^2)} - \frac{25(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F(}{72\sqrt{2}\sqrt{2+3x^2}}$$

$$= -\frac{11x(2+x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504(7+5x^2)} + \frac{1}{}$$

$$= -\frac{31x(2+x^2)}{56\sqrt{2+3x^2+x^4}} + \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504(7+5x^2)} + \frac{3}{}$$

$$= -\frac{31x(2+x^2)}{56\sqrt{2+3x^2+x^4}} + \frac{x(20+11x^2)}{36\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{504(7+5x^2)} + \frac{3}{}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 10.11, size = 208, normalized size = 0.89

$$\frac{7490x+10157x^3+3255x^5+651i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+182i\sqrt{1+x^2}\sqrt{2+x^2}(7+5x^2)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+1575i\sqrt{1+x^2}\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+1125ix^2\sqrt{1+x^2}\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{1176(7+5x^2)\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (7490*x + 10157*x^3 + 3255*x^5 + (651*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(7 + 5
*x^2)*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (182*I)*Sqrt[1 + x^2]*Sqrt[2 + x
^2]*(7 + 5*x^2)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (1575*I)*Sqrt[1 + x^2]
*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2] + (1125*I)*x^2*Sqr
t[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])/(1176*(
7 + 5*x^2)*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 185, normalized size = 0.79

method	result
default	$\frac{625x\sqrt{x^4+3x^2+2}}{504(5x^2+7)}-\frac{2\left(-\frac{11}{72}x^3-\frac{5}{18}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{168\sqrt{x^4+3x^2+2}}+\frac{31i\sqrt{2}}{}$
elliptic	$\frac{625x\sqrt{x^4+3x^2+2}}{504(5x^2+7)}-\frac{2\left(-\frac{11}{72}x^3-\frac{5}{18}x\right)}{\sqrt{x^4+3x^2+2}}+\frac{13i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{168\sqrt{x^4+3x^2+2}}+\frac{31i\sqrt{2}}{}$
risch	$\frac{x(465x^4+1451x^2+1070)}{168(5x^2+7)\sqrt{x^4+3x^2+2}}-\frac{31i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right.}{112\sqrt{x^4+3x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 625/504*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)-2*(-11/72*x^3-5/18*x)/(x^4+3*x^2+2)
^(1/2)+13/168*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*E
llipticF(1/2*I*2^(1/2)*x,2^(1/2))+31/112*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+75/392*I*2^(1/
2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(
1/2)*x,10/7,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(25*x^12 + 220*x^10 + 794*x^8 + 1504*x^6 + 1
577*x^4 + 868*x^2 + 196), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+2)**(3/2),x)
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(5\,x^2+7\right)}^2\,{\left(x^4+3\,x^2+2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 2)^(3/2)), x)
```

3.315 $\int \frac{1}{(7+5x^2)^3 (2+3x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=263

$$-\frac{5797x(2+x^2)}{28224\sqrt{2+3x^2+x^4}}+\frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}}+\frac{625x\sqrt{2+3x^2+x^4}}{1008\,(7+5x^2)^2}+\frac{41875x\sqrt{2+3x^2+x^4}}{84672\,(7+5x^2)}+\frac{5797(1+x^2)}{14112\cdots}$$

```
[Out] -5797/28224*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/216*x*(23*x^2+50)/(x^4+3*x^2+2)
^(1/2)+192625/790272*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(
x^2+1)^(1/2),2/7,1/2*2^(1/2))*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)
^(1/2)+5797/28224*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2)
,1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)-49907/112
896*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+625/1008*x*(x^4+3*x^2+2
)^(1/2)/(5*x^2+7)^2+41875/84672*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A]
time = 0.50, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1242, 1192, 1203, 1113, 1149, 1237, 1710, 1730, 1228, 1470, 553}

$$-\frac{49907(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{56448\sqrt{2}\,\sqrt{x^4+3x^2+2}}+\frac{5797(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{14112\sqrt{2}\,\sqrt{x^4+3x^2+2}}+\frac{192625(x^2+2)\,\Pi\left(\frac{2}{7};\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{395136\sqrt{2}\sqrt{\frac{x^2+2}{x^2+1}}\,\sqrt{x^4+3x^2+2}}+\frac{41875\sqrt{x^4+3x^2+2}\,x}{84672\,(5x^2+7)}+\frac{625\sqrt{x^4+3x^2+2}\,x}{1008\,(5x^2+7)^2}-\frac{5797(x^2+2)\,x}{28224\sqrt{x^4+3x^2+2}}+\frac{(23x^2+50)\,x}{216\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-5797*x*(2 + x^2))/(28224*Sqrt[2 + 3*x^2 + x^4]) + (x*(50 + 23*x^2))/(216*
Sqrt[2 + 3*x^2 + x^4]) + (625*x*Sqrt[2 + 3*x^2 + x^4])/(1008*(7 + 5*x^2)^2)
 + (41875*x*Sqrt[2 + 3*x^2 + x^4])/(84672*(7 + 5*x^2)) + (5797*(1 + x^2)*Sq
rt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(14112*Sqrt[2]*Sqrt[2 +
3*x^2 + x^4]) - (49907*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan
[x], 1/2])/(56448*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4]) + (192625*(2 + x^2)*Ellipt
icPi[2/7, ArcTan[x], 1/2])/(395136*Sqrt[2]*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2
 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
```

```
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^3 (2+3x^2+x^4)^{3/2}}\,dx = \int \left(-\frac{-62-35x^2}{216\,(2+3x^2+x^4)^{3/2}} - \frac{25}{6\,(7+5x^2)^3\sqrt{2+3x^2+x^4}} - \frac{}{36\,(7+\cdots}\right.$$

$$= -\left(\frac{1}{216}\int \frac{-62-35x^2}{(2+3x^2+x^4)^{3/2}}\,dx\right) - \frac{25}{36}\int \frac{1}{(7+5x^2)^2\sqrt{2+3x^2+x^4}}\cdots$$

$$= \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\,(7+5x^2)^2} + \frac{625x\sqrt{2+3x^2+x^4}}{3024\,(7+5x^2)} + \cdots$$

$$= \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\,(7+5x^2)^2} + \frac{41875x\sqrt{2+3x^2+x^4}}{84672\,(7+5x^2)}\cdots$$

$$= -\frac{23x(2+x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\,(7+5x^2)^2} + \cdots$$

$$= -\frac{149x(2+x^2)}{1008\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\,(7+5x^2)^2}\cdots$$

$$= -\frac{5797x(2+x^2)}{28224\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\,(7+5x^2)^2}\cdots$$

$$= -\frac{5797x(2+x^2)}{28224\sqrt{2+3x^2+x^4}} + \frac{x(50+23x^2)}{216\sqrt{2+3x^2+x^4}} + \frac{625x\sqrt{2+3x^2+x^4}}{1008\,(7+5x^2)^2}\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.23, size = 159, normalized size = 0.60

$$\frac{\frac{7x(550550+1089803x^2+698290x^4+144925x^6)}{(7+5x^2)^2} + 40579i\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - 742i\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) + 38525i\sqrt{1+x^2}\sqrt{2+x^2}\,\Pi\left(\frac{10}{7};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{197568\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*(2 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] ((7*x*(550550 + 1089803*x^2 + 698290*x^4 + 144925*x^6))/(7 + 5*x^2)^2 + (40
579*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (74
2*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (3852
5*I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[10/7, I*ArcSinh[x/Sqrt[2]], 2])
/(197568*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 209, normalized size = 0.79

method	result
risch	$\frac{x\left(144925x^6+698290x^4+1089803x^2+550550\right)}{28224(5x^2+7)^2\sqrt{x^4+3x^2+2}} - \frac{5797i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i}{}\right.\right.}{56448\sqrt{x^4+3x^2+2}}$
default	$\frac{625x\sqrt{x^4+3x^2+2}}{1008(5x^2+7)^2}+\frac{41875x\sqrt{x^4+3x^2+2}}{84672(5x^2+7)}-\frac{2\left(-\frac{23}{432}x^3-\frac{25}{216}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{53i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{Ellipt}}{28224\sqrt{x^4+3x^2+}}$
elliptic	$\frac{625x\sqrt{x^4+3x^2+2}}{1008(5x^2+7)^2}+\frac{41875x\sqrt{x^4+3x^2+2}}{84672(5x^2+7)}-\frac{2\left(-\frac{23}{432}x^3-\frac{25}{216}x\right)}{\sqrt{x^4+3x^2+2}}-\frac{53i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{Ellipt}}{28224\sqrt{x^4+3x^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 625/1008*x*(x^4+3*x^2+2)^(1/2)/(5*x^2+7)^2+41875/84672*x*(x^4+3*x^2+2)^(1/2
)/(5*x^2+7)-2*(-23/432*x^3-25/216*x)/(x^4+3*x^2+2)^(1/2)-53/28224*I*2^(1/2)
*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*
x,2^(1/2))+5797/56448*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)
^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))+38525/197568*I*2^(1/2)*(1+1/2*x^2
)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,10/7,2
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(125*x^14 + 1275*x^12 + 5510*x^10 + 13078*x^
8 + 18413*x^6 + 15379*x^4 + 7056*x^2 + 1372), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^{2}+1\right)\left(x^{2}+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+2)**(3/2),x)
[Out] Integral(1/(((x**2 + 1)*(x**2 + 2))**(3/2)*(5*x**2 + 7)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 3*x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(5\,x^2+7\right)}^3\,{\left(x^4+3\,x^2+2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 2)^(3/2)), x)
```

3.316 $\int (7+5x^2)^4 \sqrt{2+x^2-x^4}\, dx$

Optimal. Leaf size=116

$$\frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4}-\frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2}-\frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2}-\frac{625}{11}x^5\left(2+\right.$$

```
[Out] -116100/77*x*(-x^4+x^2+2)^(3/2)-14500/33*x^3*(-x^4+x^2+2)^(3/2)-625/11*x^5*
(-x^4+x^2+2)^(3/2)+3764813/231*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-539419/77
*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/231*x*(717372*x^2+177953)*(-x^4+x^2+2
)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1220, 1693, 1190, 1194, 538, 435, 430}

$$-\frac{539419}{77}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{3764813}{231}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{116100}{77}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{231}\left(717372x^2+177953\right)\sqrt{-x^4+x^2+2}\,x-\frac{625}{11}\left(-x^4+x^2+2\right)^{3/2}x^5-\frac{14500}{33}\left(-x^4+x^2+2\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(177953 + 717372*x^2)*Sqrt[2 + x^2 - x^4])/231 - (116100*x*(2 + x^2 - x^
4)^(3/2))/77 - (14500*x^3*(2 + x^2 - x^4)^(3/2))/33 - (625*x^5*(2 + x^2 - x
^4)^(3/2))/11 + (3764813*EllipticE[ArcSin[x/Sqrt[2]], -2])/231 - (539419*El
lipticF[ArcSin[x/Sqrt[2]], -2])/77
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c])))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(7+5x^2\right)^4 \sqrt{2+x^2-x^4}\, dx = -\frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2} - \frac{1}{11}\int \sqrt{2+x^2-x^4}\,\left(-26411-75460x^2-871\right.$$

$$= -\frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2} - \frac{625}{11}x^5\left(2+x^2-x^4\right)^{3/2} + \frac{1}{99}\int \sqrt{2+x^2-x}$$

$$= -\frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} - \frac{14500}{33}x^3\left(2+x^2-x^4\right)^{3/2} - \frac{625}{11}x^5\left(2+x^2-\right.$$

$$= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} - \frac{1}{}$$

$$= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} - \frac{1}{}$$

$$= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} - \frac{1}{}$$

$$= \frac{1}{231}x\left(177953+717372x^2\right)\sqrt{2+x^2-x^4} - \frac{116100}{77}x\left(2+x^2-x^4\right)^{3/2} - \frac{1}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.08, size = 112, normalized size = 0.97

$$\frac{-1037294x-186503x^3+1125819x^5+231228x^7-105925x^9-75250x^{11}-13125x^{13}+3764813i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-4838091i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{231\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^4*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-1037294*x - 186503*x^3 + 1125819*x^5 + 231228*x^7 - 105925*x^9 - 75250*x^
11 - 13125*x^13 + (3764813*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x
], -1/2] - (4838091*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2
])/(231*Sqrt[2 + x^2 - x^4])
```

Maple [A]
time = 0.12, size = 193, normalized size = 1.66

method	result
risch	$-\frac{x\left(13125x^8+88375x^6+220550x^4+166072x^2-518647\right)\left(x^4-x^2-2\right)}{231\sqrt{-x^4+x^2+2}} - \frac{3764813\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\sqrt{\right.\right.}{462\sqrt{-x^4+x^2-}}$
default	$\frac{20050x^5\sqrt{-x^4+x^2+2}}{21}+\frac{12625x^7\sqrt{-x^4+x^2+2}}{33}+\frac{625x^9\sqrt{-x^4+x^2+2}}{11}-\frac{518647x\sqrt{-x^4+x^2+2}}{231}$

elliptic	$\frac{20050x^5\sqrt{-x^4+x^2+2}}{21}+\frac{12625x^7\sqrt{-x^4+x^2+2}}{33}+\frac{625x^9\sqrt{-x^4+x^2+2}}{11}-\frac{518647x\sqrt{-x^4+x^2+}}{231}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 20050/21*x^5*(-x^4+x^2+2)^(1/2)+12625/33*x^7*(-x^4+x^2+2)^(1/2)+625/11*x^9*(-x^4+x^2+2)^(1/2)-518647/231*x*(-x^4+x^2+2)^(1/2)+166072/231*x^3*(-x^4+x^2+2)^(1/2)+1073278/231*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-3764813/462*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)

Fricas [A]
time = 0.10, size = 44, normalized size = 0.38

$$\frac{(13125\,x^{10}+88375\,x^{8}+220550\,x^{6}+166072\,x^{4}-518647\,x^{2}-3764813)\sqrt{-x^4+x^2+2}}{231\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")

[Out] 1/231*(13125*x^10 + 88375*x^8 + 220550*x^6 + 166072*x^4 - 518647*x^2 - 3764813)*sqrt(-x^4 + x^2 + 2)/x

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**2+7)**4*(-x**4+x**2+2)**(1/2),x)

[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**4, x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)^4\,\sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(1/2), x)
```

3.317 $\int (7+5x^2)^3 \sqrt{2+x^2-x^4}\, dx$

Optimal. Leaf size=95

$$\frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4}-\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2}-\frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2}+\frac{79411}{63}E\left(\sin^{-1}\left(\right.\right.$$

```
[Out] -1825/21*x*(-x^4+x^2+2)^(3/2)-125/9*x^3*(-x^4+x^2+2)^(3/2)+79411/63*Ellipti
cE(1/2*x*2^(1/2),I*2^(1/2))-8735/21*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/63
*x*(14691*x^2+5956)*(-x^4+x^2+2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1220, 1693, 1190, 1194, 538, 435, 430}

$$-\frac{8735}{21}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{79411}{63}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1825}{21}\left(-x^4+x^2+2\right)^{3/2}x+\frac{1}{63}\left(14691x^2+5956\right)\sqrt{-x^4+x^2+2}\,x-\frac{125}{9}\left(-x^4+x^2+2\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(5956 + 14691*x^2)*Sqrt[2 + x^2 - x^4])/63 - (1825*x*(2 + x^2 - x^4)^(3/
2))/21 - (125*x^3*(2 + x^2 - x^4)^(3/2))/9 + (79411*EllipticE[ArcSin[x/Sqrt
[2]], -2])/63 - (8735*EllipticF[ArcSin[x/Sqrt[2]], -2])/21
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(7+5x^2\right)^3 \sqrt{2+x^2-x^4}\, dx = -\frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2} - \frac{1}{9}\int \left(-3087-7365x^2-5475x^4\right)\sqrt{2+x^2-$$
$$= -\frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2} - \frac{125}{9}x^3\left(2+x^2-x^4\right)^{3/2} + \frac{1}{63}\int \left(32559+734\right.$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4} - \frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2} - \frac{125}{9}x^3$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4} - \frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2} - \frac{125}{9}x^3$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4} - \frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2} - \frac{125}{9}x^3$$
$$= \frac{1}{63}x\left(5956+14691x^2\right)\sqrt{2+x^2-x^4} - \frac{1825}{21}x\left(2+x^2-x^4\right)^{3/2} - \frac{125}{9}x^3$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.01, size = 107, normalized size = 1.13

$$\frac{-9988x+9938x^3+21660x^5-1116x^7-3725x^9-875x^{11}+79411i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-106014i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{63\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-9988*x + 9938*x^3 + 21660*x^5 - 1116*x^7 - 3725*x^9 - 875*x^11 + (79411*I
)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (106014*I)*Sqrt[4
 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(63*Sqrt[2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(85) = 170.
time = 0.12, size = 176, normalized size = 1.85

method	result
risch	$-\frac{x\left(875x^6+4600x^4+7466x^2-4994\right)\left(x^4-x^2-2\right)}{63\sqrt{-x^4+x^2+2}} - \frac{79411\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)-\mathrm{Ellip}\right.}{126\sqrt{-x^4+x^2+2}}$
default	$\frac{125x^7\sqrt{-x^4+x^2+2}}{9}+\frac{4600x^5\sqrt{-x^4+x^2+2}}{63}+\frac{7466x^3\sqrt{-x^4+x^2+2}}{63}-\frac{4994x\sqrt{-x^4+x^2+2}}{63}$
elliptic	$\frac{125x^7\sqrt{-x^4+x^2+2}}{9}+\frac{4600x^5\sqrt{-x^4+x^2+2}}{63}+\frac{7466x^3\sqrt{-x^4+x^2+2}}{63}-\frac{4994x\sqrt{-x^4+x^2+2}}{63}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 125/9*x^7*(-x^4+x^2+2)^(1/2)+4600/63*x^5*(-x^4+x^2+2)^(1/2)+7466/63*x^3*(-x
^4+x^2+2)^(1/2)-4994/63*x*(-x^4+x^2+2)^(1/2)+26603/63*2^(1/2)*(-2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-794
11/126*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF
(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)
```

Fricas [A]
time = 0.08, size = 39, normalized size = 0.41

$$\frac{(875\,x^8+4600\,x^6+7466\,x^4-4994\,x^2-79411)\sqrt{-x^4+x^2+2}}{63\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/63*(875*x^8 + 4600*x^6 + 7466*x^4 - 4994*x^2 - 79411)*sqrt(-x^4 + x^2 + 2
)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)^3\,\sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2),x)

[Out] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2), x)

3.318 $\int (7+5x^2)^2 \sqrt{2+x^2-x^4}\, dx$

Optimal. Leaf size=74

$$\frac{1}{21}x(275+354x^2)\sqrt{2+x^2-x^4}-\frac{25}{7}x(2+x^2-x^4)^{3/2}+\frac{2045}{21}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{79}{7}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.$$

```
[Out] -25/7*x*(-x^4+x^2+2)^(3/2)+2045/21*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-79/7*
EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/21*x*(354*x^2+275)*(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1190, 1194, 538, 435, 430}

$$-\frac{79}{7}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{2045}{21}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{7}x(-x^4+x^2+2)^{3/2}+\frac{1}{21}x(354x^2+275)\sqrt{-x^4+x^2+2}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*(275 + 354*x^2)*Sqrt[2 + x^2 - x^4])/21 - (25*x*(2 + x^2 - x^4)^(3/2))/7
 + (2045*EllipticE[ArcSin[x/Sqrt[2]], -2])/21 - (79*EllipticF[ArcSin[x/Sqrt
[2]], -2])/7
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int \left(7+5x^2\right)^2\sqrt{2+x^2-x^4}\,dx &= -\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}-\frac{1}{7}\int\left(-393-590x^2\right)\sqrt{2+x^2-x^4}\,dx\\ &= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{1}{105}\int\frac{9040}{\sqrt{2-}}\\ &= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{2}{105}\int\frac{90}{\sqrt{4-}}\\ &= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}-\frac{158}{7}\int\frac{}{\sqrt{4-}}\\ &= \frac{1}{21}x\left(275+354x^2\right)\sqrt{2+x^2-x^4}-\frac{25}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{2045}{21}E\Big(\sin\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 6.24, size = 102, normalized size = 1.38

$$\frac{250x+683x^3+304x^5-204x^7-75x^9+2045i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)-2949i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)}{21\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (250*x + 683*x^3 + 304*x^5 - 204*x^7 - 75*x^9 + (2045*I)*Sqrt[4 + 2*x^2 - 2
*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2949*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Elli
pticF[I*ArcSinh[x], -1/2])/(21*Sqrt[2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(68) = 136.
time = 0.12, size = 159, normalized size = 2.15

method	result
risch	$-\frac{x(75x^4+279x^2+125)(x^4-x^2-2)}{21\sqrt{-x^4+x^2+2}}-\frac{2045\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\right.\right.}{42\sqrt{-x^4+x^2+2}}$
default	$\frac{25x^5\sqrt{-x^4+x^2+2}}{7}+\frac{93x^3\sqrt{-x^4+x^2+2}}{7}+\frac{125x\sqrt{-x^4+x^2+2}}{21}+\frac{904\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}}{21\sqrt{-x^4+x}}$
elliptic	$\frac{25x^5\sqrt{-x^4+x^2+2}}{7}+\frac{93x^3\sqrt{-x^4+x^2+2}}{7}+\frac{125x\sqrt{-x^4+x^2+2}}{21}+\frac{904\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}}{21\sqrt{-x^4+x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/7*x^5*(-x^4+x^2+2)^(1/2)+93/7*x^3*(-x^4+x^2+2)^(1/2)+125/21*x*(-x^4+x^2+
2)^(1/2)+904/21*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*E
llipticF(1/2*2^(1/2)*x,I*2^(1/2))-2045/42*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*
2^(1/2)*x,I*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

Fricas [A]
time = 0.09, size = 34, normalized size = 0.46

$$\frac{(75\,x^6+279\,x^4+125\,x^2-2045)\sqrt{-x^4+x^2+2}}{21\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/21*(75*x^6 + 279*x^4 + 125*x^2 - 2045)*sqrt(-x^4 + x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-(x^2-2)\,(x^2+1)}\,\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(-x**4+x**2+2)**(1/2),x)
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\left(5\,x^2+7\right)^2\,\sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2), x)
```

3.319 $\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=46

$$x(2+x^2)\sqrt{2+x^2-x^4}+7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 7*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+3*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+x
*(x^2+2)*(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1190, 1194, 538, 435, 430}

$$3F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+7E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+x\sqrt{-x^4+x^2+2}\,(x^2+2)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] x*(2 + x^2)*Sqrt[2 + x^2 - x^4] + 7*EllipticE[ArcSin[x/Sqrt[2]], -2] + 3*El
lipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c])))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int (7+5x^2)\sqrt{2+x^2-x^4}\,dx &= x(2+x^2)\sqrt{2+x^2-x^4}-\frac{1}{15}\int \frac{-150-105x^2}{\sqrt{2+x^2-x^4}}\,dx\\ &= x(2+x^2)\sqrt{2+x^2-x^4}-\frac{2}{15}\int \frac{-150-105x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\ &= x(2+x^2)\sqrt{2+x^2-x^4}+6\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+7\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\\ &= x(2+x^2)\sqrt{2+x^2-x^4}+7E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+3F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.98, size = 94, normalized size = 2.04

$$\frac{4x+4x^3-x^5-x^7+7i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-12i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (4*x + 4*x^3 - x^5 - x^7 + (7*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSin
h[x], -1/2] - (12*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])
/Sqrt[2 + x^2 - x^4]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(50) = 100$.
time = 0.03, size = 141, normalized size = 3.07

method	result
risch	$-\frac{x(x^2+2)(x^4-x^2-2)}{\sqrt{-x^4+x^2+2}}-\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^4+x^2+2}}+$
default	$x^3\sqrt{-x^4+x^2+2}+2x\sqrt{-x^4+x^2+2}+\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{\sqrt{-x^4+x^2+2}}-$
elliptic	$x^3\sqrt{-x^4+x^2+2}+2x\sqrt{-x^4+x^2+2}+\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{\sqrt{-x^4+x^2+2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^3*(-x^4+x^2+2)^(1/2)+2*x*(-x^4+x^2+2)^(1/2)+5*2^(1/2)*(-2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-7/2*2^(1/
2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)
*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

Fricas [A]
time = 0.10, size = 26, normalized size = 0.57

$$\frac{(x^4+2\,x^2-7)\sqrt{-x^4+x^2+2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] (x^4 + 2*x^2 - 7)*sqrt(-x^4 + x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-(x^2-2)(x^2+1)}\,(5x^2+7)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(-x**4+x**2+2)**(1/2),x)
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(5\,x^2+7\right)\,\sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2), x)
```

3.320 $\int \sqrt{2+x^2-x^4}\,dx$

Optimal. Leaf size=44

$$\frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 1/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1
/3*x*(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1105, 1194, 538, 435, 430}

$$F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{3}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{3}\sqrt{-x^4+x^2+2}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/3 + EllipticE[ArcSin[x/Sqrt[2]], -2]/3 + EllipticF[
ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{2+x^2-x^4}\,dx &= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}\int \frac{4+x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{2}{3}\int \frac{4+x^2}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\
&= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+2\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\
&= \frac{1}{3}x\sqrt{2+x^2-x^4}+\frac{1}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.88, size = 90, normalized size = 2.05

$$\frac{2x+x^3-x^5+i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-3i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{3\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (2*x + x^3 - x^5 + I*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2]
- (3*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(3*Sqrt[2 +
x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(44) = 88.
time = 0.03, size = 125, normalized size = 2.84

method	result
default	$\frac{x\sqrt{-x^4+x^2+2}}{3}+\frac{2\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{3\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}}{}$
elliptic	$\frac{x\sqrt{-x^4+x^2+2}}{3}+\frac{2\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{3\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}}{}$
risch	$-\frac{x(x^4-x^2-2)}{3\sqrt{-x^4+x^2+2}}+\frac{2\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{3\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(-x^4+x^2+2)^(1/2)+2/3*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-1/6*2^(1/2)*(-2*x^2+4)^(1/2)
*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-Ellip
ticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 22, normalized size = 0.50

$$\frac{\sqrt{-x^4+x^2+2}\,(x^2-1)}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-x^4 + x^2 + 2)*(x^2 - 1)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 + x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-x^4+x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(1/2),x)
```

```
[Out] int((x^2 - x^4 + 2)^(1/2), x)
```

3.321 $\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{5}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{17}{25}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -1/5*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+17/25*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))-34/175*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Rubi [A]

time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1222, 1194, 538, 435, 430, 1226, 551}

$$\frac{17}{25}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{5}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
```

```
[Out] -1/5*EllipticE[ArcSin[x/Sqrt[2]], -2] + (17*EllipticF[ArcSin[x/Sqrt[2]], -2
])/25 - (34*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/175
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx &= -\left(\frac{1}{25}\int \frac{-12+5x^2}{\sqrt{2+x^2-x^4}}\,dx\right)-\frac{34}{25}\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx\\ &= -\left(\frac{2}{25}\int \frac{-12+5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\right)-\frac{68}{25}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\,(7+5x^2)}\,dx\\ &= -\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{5}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{34}{25}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+\cdots}}\\ &= -\frac{1}{5}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{17}{25}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{34}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\cdots\right.\right.\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.10, size = 51, normalized size = 1.11

$$-\frac{1}{175} i \sqrt{2} \left(35 E\left(i \sinh^{-1}(x)|-\frac{1}{2}\right)+7 F\left(i \sinh^{-1}(x)|-\frac{1}{2}\right)-17 \Pi\left(\frac{5}{7}; i \sinh^{-1}(x)|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2),x]
```

```
[Out] (-1/175*I)*Sqrt[2]*(35*EllipticE[I*ArcSinh[x], -1/2] + 7*EllipticF[I*ArcSin
h[x], -1/2] - 17*EllipticPi[5/7, I*ArcSinh[x], -1/2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(47) = 94.
time = 0.10, size = 141, normalized size = 3.07

method	result
default	$\frac{17\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)}{50\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right.}{10\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{17\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)}{50\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right.}{10\sqrt{-x^4+x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] 17/50*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1
/2*2^(1/2)*x,I*2^(1/2))-1/10*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))-34/175*2^(1/2)*(1-1/2*x^2)^(
1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1
/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x^2-2)(x^2+1)}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7),x)
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{-x^4+x^2+2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7),x)
[Out] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7), x)
```

3.322 $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=74

$$\frac{x\sqrt{2+x^2-x^4}}{14(7+5x^2)}+\frac{1}{70}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{6}{175}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{99\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450}$$

```
[Out] 1/70*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-6/175*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))+99/2450*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/14*x*(-x^4+x^2+2)
^(1/2)/(5*x^2+7)
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1240, 1194, 538, 435, 430, 1226, 551}

$$-\frac{6}{175}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{70}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{99\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450}+\frac{\sqrt{-x^4+x^2+2}\,x}{14(5x^2+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(14*(7 + 5*x^2)) + EllipticE[ArcSin[x/Sqrt[2]], -2]
/70 - (6*EllipticF[ArcSin[x/Sqrt[2]], -2])/175 + (99*EllipticPi[-10/7, ArcS
in[x/Sqrt[2]], -2])/2450
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1240

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(d + e*x^2))), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^2}\,dx &= \frac{x\sqrt{2+x^2-x^4}}{14(7+5x^2)} - \frac{1}{350}\int \frac{7-5x^2}{\sqrt{2+x^2-x^4}}\,dx + \frac{99}{350}\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx \\ &= \frac{x\sqrt{2+x^2-x^4}}{14(7+5x^2)} - \frac{1}{175}\int \frac{7-5x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx + \frac{99}{175}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\,(7 \\ &= \frac{x\sqrt{2+x^2-x^4}}{14(7+5x^2)} + \frac{99\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2450} + \frac{1}{70}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx - \frac{12}{175}\int \\ &= \frac{x\sqrt{2+x^2-x^4}}{14(7+5x^2)} + \frac{1}{70}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{6}{175}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{99\Pi}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.13, size = 196, normalized size = 2.65

$$\frac{700x+350x^3-350x^5+70i\sqrt{2}(7+5x^2)\sqrt{2+x^2-x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-21i\sqrt{2}(7+5x^2)\sqrt{2+x^2-x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-693i\sqrt{2}\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-495i\sqrt{2}x^2\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{4900(7+5x^2)\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (700*x + 350*x^3 - 350*x^5 + (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (21*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 -
x^4]*EllipticF[I*ArcSinh[x], -1/2] - (693*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*El
lipticPi[5/7, I*ArcSinh[x], -1/2] - (495*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]
*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(4900*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4]
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(71) = 142.
time = 0.12, size = 165, normalized size = 2.23

method	result
default	$\frac{x\sqrt{-x^4+x^2+2}}{70x^2+98} - \frac{3\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)}{175\sqrt{-x^4+x^2+2}} + \frac{\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}}{140\sqrt{-x^4+}}$
elliptic	$\frac{x\sqrt{-x^4+x^2+2}}{70x^2+98} - \frac{3\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)}{175\sqrt{-x^4+x^2+2}} + \frac{\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}}{140\sqrt{-x^4+}}$
risch	$-\frac{(x^4-x^2-2)x}{14(5x^2+7)\sqrt{-x^4+x^2+2}} - \frac{\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right.\right.}{140\sqrt{-x^4+x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-3/175*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+1/140*2^(1/2)*(
-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*
2^(1/2))+99/2450*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)
*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x^{2}-2\right)\left(x^{2}+1\right)}}{\left(5x^{2}+7\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^4+x^2+2}}{\left(5\,x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^2,x)
```

```
[Out] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^2, x)
```

3.323 $\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx$

Optimal. Leaf size=102

$$\frac{x\sqrt{2+x^2-x^4}}{28\,(7+5x^2)^2}-\frac{31x\sqrt{2+x^2-x^4}}{13328\,(7+5x^2)}-\frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640}-\frac{269F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{166600}+\frac{16601\Pi\left(\right.}{}$$

```
[Out] -31/66640*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-269/166600*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))+16601/2332400*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/2
8*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-31/13328*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A]
time = 0.25, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1242, 1237, 1710, 1730, 1194, 538, 435, 430, 1226, 551}

$$-\frac{269F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{166600}-\frac{31E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640}+\frac{16601\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2332400}-\frac{31\sqrt{-x^4+x^2+2}\,x}{13328\,(5x^2+7)}+\frac{\sqrt{-x^4+x^2+2}\,x}{28\,(5x^2+7)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (x*Sqrt[2 + x^2 - x^4])/(28*(7 + 5*x^2)^2) - (31*x*Sqrt[2 + x^2 - x^4])/(13
328*(7 + 5*x^2)) - (31*EllipticE[ArcSin[x/Sqrt[2]], -2])/66640 - (269*Ellip
ticF[ArcSin[x/Sqrt[2]], -2])/166600 + (16601*EllipticPi[-10/7, ArcSin[x/Sqr
t[2]], -2])/2332400
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{2+x^2-x^4}}{(7+5x^2)^3}\,dx = \int \left(-\frac{34}{25\,(7+5x^2)^3\sqrt{2+x^2-x^4}} + \frac{19}{25\,(7+5x^2)^2\sqrt{2+x^2-x^4}} - \frac{1}{25\,(7+5x^2)\sqrt{\cdots}}\right.$$

$$= -\left(\frac{1}{25}\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx\right) + \frac{19}{25}\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx - \frac{34}{25}\int$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\,(7+5x^2)^2} - \frac{19x\sqrt{2+x^2-x^4}}{476\,(7+5x^2)} - \frac{1}{700}\int \frac{186-190x^2+25x^4}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx + \frac{19\int}{}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\,(7+5x^2)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\,(7+5x^2)} - \frac{1}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{\int \frac{3769}{(7+5x}}{}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\,(7+5x^2)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\,(7+5x^2)} - \frac{1}{175}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{\int \frac{757}{\sqrt{2}}}{8}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\,(7+5x^2)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\,(7+5x^2)} + \frac{2697\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{83300} + \frac{\int}{\sqrt{4}}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\,(7+5x^2)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\,(7+5x^2)} - \frac{19E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{2380} - \frac{19F\left(\sin^{-1}\left(\right.\right.}{59}$$

$$= \frac{x\sqrt{2+x^2-x^4}}{28\,(7+5x^2)^2} - \frac{31x\sqrt{2+x^2-x^4}}{13328\,(7+5x^2)} - \frac{31E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{66640} - \frac{269F\left(\sin^{-1}\right.}{166}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.15, size = 244, normalized size = 2.39

$$\frac{181300x-17850x^3-144900x^5+54250x^7-2170i\sqrt{2}\,(7+5x^2)^2\sqrt{2+x^2-x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+7021i\sqrt{2}\,(7+5x^2)^2\sqrt{2+x^2-x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-813449i\sqrt{2}\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-1162070i\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-415025i\sqrt{2}\,x^4\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{4664800\,(7+5x^2)^2\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^2 - x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (181300*x - 17850*x^3 - 144900*x^5 + 54250*x^7 - (2170*I)*Sqrt[2]*(7 + 5*x^
2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (7021*I)*Sqrt[2]*(
7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (813449*I)
*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (1162070
*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (
415025*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/
2])/(4664800*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

Maple [A]
time = 0.13, size = 189, normalized size = 1.85

method	result
default	$\frac{x\sqrt{-x^4+x^2+2}}{28(5x^2+7)^2}-\frac{31x\sqrt{-x^4+x^2+2}}{13328(5x^2+7)}-\frac{269\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{333200\sqrt{-x^4+x^2+2}}-$
elliptic	$\frac{x\sqrt{-x^4+x^2+2}}{28(5x^2+7)^2}-\frac{31x\sqrt{-x^4+x^2+2}}{13328(5x^2+7)}-\frac{269\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{333200\sqrt{-x^4+x^2+2}}-$
risch	$\frac{(x^4-x^2-2)x(155x^2-259)}{13328(5x^2+7)^2\sqrt{-x^4+x^2+2}}+\frac{31\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\operatorname{EllipticE}\left(\frac{\sqrt{2}\,x}{2}\right.\right.}{133280\sqrt{-x^4+x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-31/13328*x*(-x^4+x^2+2)^(1/2)/(5*x^2+
7)-269/333200*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ell
ipticF(1/2*2^(1/2)*x,I*2^(1/2))-31/133280*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))+16601/2332400*2
^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^
(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}}{\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(1/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral(sqrt(-(x**2 - 2)*(x**2 + 1))/(5*x**2 + 7)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + x^2 + 2)/(5*x^2 + 7)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-x^4+x^2+2}}{{\left(5\,x^2+7\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^3,x)
```

```
[Out] int((x^2 - x^4 + 2)^(1/2)/(5*x^2 + 7)^3, x)
```

3.324 $\int (7+5x^2)^4 (2+x^2-x^4)^{3/2} \, dx$

Optimal. Leaf size=142

$$\frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005}-\frac{x(69817-1581440x^2)(2+x^2-x^4)^{3/2}}{1001}-\frac{132300}{143}x(2+x^2-x^4)$$

```
[Out] -1/1001*x*(-1581440*x^2+69817)*(-x^4+x^2+2)^(3/2)-132300/143*x*(-x^4+x^2+2)
^(5/2)-11750/39*x^3*(-x^4+x^2+2)^(5/2)-125/3*x^5*(-x^4+x^2+2)^(5/2)+1241414
22/5005*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-50794416/5005*EllipticF(1/2*x*2^
(1/2),I*2^(1/2))+3/5005*x*(7837383*x^2+2193559)*(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1220, 1693, 1190, 1194, 538, 435, 430}

$$-\frac{50794416F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}+\frac{124141422E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}-\frac{132300}{143}(-x^4+x^2+2)^{5/2}x-\frac{(69817-1581440x^2)(-x^4+x^2+2)^{3/2}x}{1001}+\frac{3(7837383x^2+2193559)\sqrt{-x^4+x^2+2}\,x}{5005}-\frac{125}{3}(-x^4+x^2+2)^{5/2}x^5-\frac{11750}{39}(-x^4+x^2+2)^{5/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (3*x*(2193559 + 7837383*x^2)*Sqrt[2 + x^2 - x^4])/5005 - (x*(69817 - 158144
0*x^2)*(2 + x^2 - x^4)^(3/2))/1001 - (132300*x*(2 + x^2 - x^4)^(5/2))/143 -
 (11750*x^3*(2 + x^2 - x^4)^(5/2))/39 - (125*x^5*(2 + x^2 - x^4)^(5/2))/3 +
 (124141422*EllipticE[ArcSin[x/Sqrt[2]], -2])/5005 - (50794416*EllipticF[Ar
cSin[x/Sqrt[2]], -2])/5005
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(7+5x^2\right)^4 \left(2+x^2-x^4\right)^{3/2} dx = -\frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2} - \frac{1}{15}\int \left(2+x^2-x^4\right)^{3/2} (-36015-102900x$$
$$= -\frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2} - \frac{125}{3}x^5\left(2+x^2-x^4\right)^{5/2} + \frac{1}{195}\int \left(2+x^2\right.$$
$$= -\frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2} - \frac{11750}{39}x^3\left(2+x^2-x^4\right)^{5/2} - \frac{125}{3}x^5\left(2+x\right.$$
$$= -\frac{x(69817-1581440x^2)\left(2+x^2-x^4\right)^{3/2}}{1001} - \frac{132300}{143}x\left(2+x^2-x^4\right)^{5/2}$$
$$= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005} - \frac{x(69817-1581440x^2)\,(2}{1001}$$
$$= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005} - \frac{x(69817-1581440x^2)\,(2}{1001}$$
$$= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005} - \frac{x(69817-1581440x^2)\,(2}{1001}$$
$$= \frac{3x(2193559+7837383x^2)\sqrt{2+x^2-x^4}}{5005} - \frac{x(69817-1581440x^2)\,(2}{1001}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 122, normalized size = 0.86

$$\frac{-75836958x+48624305x^3+172881581x^5+32834763x^7-36649955x^9-24642275x^{11}-1556625x^{13}+2646875x^{15}+625625x^{17}+372424266i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-482444775i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{15015\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^4*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (-75836958*x + 48624305*x^3 + 172881581*x^5 + 32834763*x^7 - 36649955*x^9 -
 24642275*x^11 - 1556625*x^13 + 2646875*x^15 + 625625*x^17 + (372424266*I)*
Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (482444775*I)*Sqrt[
4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(15015*Sqrt[2 + x^2 - x^4
])
```

Maple [A]
time = 0.11, size = 227, normalized size = 1.60

method	result
risch	$\frac{x\left(625625x^{12}+3272500x^{10}+2967125x^8-15130150x^6-45845855x^4-43271392x^2+37918479\right)\left(x^4-x^2-2\right)}{15015\sqrt{-x^4+x^2+2}} - \frac{62070711\sqrt{2}\,\sqrt{-2}}{}$

default	$\frac{833561x^5\sqrt{-x^4+x^2+2}}{273}+\frac{432290x^7\sqrt{-x^4+x^2+2}}{429}-\frac{84775x^9\sqrt{-x^4+x^2+2}}{429}-\frac{12639493x\sqrt{-x^4+x}}{5005}$
elliptic	$\frac{833561x^5\sqrt{-x^4+x^2+2}}{273}+\frac{432290x^7\sqrt{-x^4+x^2+2}}{429}-\frac{84775x^9\sqrt{-x^4+x^2+2}}{429}-\frac{12639493x\sqrt{-x^4+x}}{5005}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 833561/273*x^5*(-x^4+x^2+2)^(1/2)+432290/429*x^7*(-x^4+x^2+2)^(1/2)-84775/4
29*x^9*(-x^4+x^2+2)^(1/2)-12639493/5005*x*(-x^4+x^2+2)^(1/2)-8500/39*x^11*(
-x^4+x^2+2)^(1/2)-125/3*x^13*(-x^4+x^2+2)^(1/2)+43271392/15015*x^3*(-x^4+x^
2+2)^(1/2)+36673503/5005*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2
)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-62070711/5005*2^(1/2)*(-2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))
-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

Fricas [A]
time = 0.09, size = 54, normalized size = 0.38

$$-\frac{(625625\,x^{14}+3272500\,x^{12}+2967125\,x^{10}-15130150\,x^{8}-45845855\,x^{6}-43271392\,x^{4}+37918479\,x^{2}+372424266)\sqrt{-x^4+x^2+2}}{15015\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15015*(625625*x^14 + 3272500*x^12 + 2967125*x^10 - 15130150*x^8 - 458458
55*x^6 - 43271392*x^4 + 37918479*x^2 + 372424266)*sqrt(-x^4 + x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(x^{2}-2\right)\left(x^{2}+1\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(5\,x^2+7\right)}^4\,{\left(-x^4+x^2+2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^4*(x^2 - x^4 + 2)^(3/2), x)
```

3.325 $\int \left(7+5x^2\right)^3 \left(2+x^2-x^4\right)^{3/2}\, dx$

Optimal. Leaf size=121

$$\frac{x(2512273+5712051x^2)\sqrt{2+x^2-x^4}}{15015}+\frac{x(33792+374045x^2)\left(2+x^2-x^4\right)^{3/2}}{3003}-\frac{7825}{143}x\left(2+x^2-x^4\right)^{5/2}-\frac{1}{1}$$

```
[Out] 1/3003*x*(374045*x^2+33792)*(-x^4+x^2+2)^(3/2)-7825/143*x*(-x^4+x^2+2)^(5/2
)-125/13*x^3*(-x^4+x^2+2)^(5/2)+31072528/15015*EllipticE(1/2*x*2^(1/2),I*2^
(1/2))-3199778/5005*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/15015*x*(5712051*x
^2+2512273)*(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1220, 1693, 1190, 1194, 538, 435, 430}

$$-\frac{3199778F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{5005}+\frac{31072528E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{15015}-\frac{7825}{143}\left(-x^4+x^2+2\right)^{5/2}x+\frac{(374045x^2+33792)\left(-x^4+x^2+2\right)^{3/2}x}{3003}+\frac{(5712051x^2+2512273)\sqrt{-x^4+x^2+2}\,x}{15015}-\frac{125}{13}\left(-x^4+x^2+2\right)^{5/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(2512273 + 5712051*x^2)*Sqrt[2 + x^2 - x^4])/15015 + (x*(33792 + 374045*
x^2)*(2 + x^2 - x^4)^(3/2))/3003 - (7825*x*(2 + x^2 - x^4)^(5/2))/143 - (12
5*x^3*(2 + x^2 - x^4)^(5/2))/13 + (31072528*EllipticE[ArcSin[x/Sqrt[2]], -2
])/15015 - (3199778*EllipticF[ArcSin[x/Sqrt[2]], -2])/5005
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int (7+5x^2)^3 (2+x^2-x^4)^{3/2} \, dx = -\frac{125}{13} x^3 (2+x^2-x^4)^{5/2} - \frac{1}{13} \int (-4459-10305x^2-7825x^4) (2+x^2$$
$$= -\frac{7825}{143} x (2+x^2-x^4)^{5/2} - \frac{125}{13} x^3 (2+x^2-x^4)^{5/2} + \frac{1}{143} \int (64699+1$$
$$= \frac{x(33792+374045x^2)(2+x^2-x^4)^{3/2}}{3003} - \frac{7825}{143} x (2+x^2-x^4)^{5/2} - \frac{125}{13} x$$
$$= \frac{x(2512273+5712051x^2)\sqrt{2+x^2-x^4}}{15015} + \frac{x(33792+374045x^2)(2+x}{3003}$$
$$= \frac{x(2512273+5712051x^2)\sqrt{2+x^2-x^4}}{15015} + \frac{x(33792+374045x^2)(2+x}{3003}$$
$$= \frac{x(2512273+5712051x^2)\sqrt{2+x^2-x^4}}{15015} + \frac{x(33792+374045x^2)(2+x}{3003}$$
$$= \frac{x(2512273+5712051x^2)\sqrt{2+x^2-x^4}}{15015} + \frac{x(33792+374045x^2)(2+x}{3003}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 117, normalized size = 0.97

$$\frac{-872614x+11078615x^3+13371048x^5-1756521x^7-4448240x^9-1027775x^{11}+388500x^{13}+144375x^{15}+31072528i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-41809125i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{15015\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (-872614*x + 11078615*x^3 + 13371048*x^5 - 1756521*x^7 - 4448240*x^9 - 1027
775*x^11 + 388500*x^13 + 144375*x^15 + (31072528*I)*Sqrt[4 + 2*x^2 - 2*x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (41809125*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Ellip
ticF[I*ArcSinh[x], -1/2])/(15015*Sqrt[2 + x^2 - x^4])
```

Maple [A]
time = 0.12, size = 210, normalized size = 1.74

method	result
risch	$\frac{x(144375x^{10}+532875x^8-206150x^6-3588640x^4-5757461x^2+436307)(x^4-x^2-2)}{15015\sqrt{-x^4+x^2+2}} - \frac{15536264\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}}{15015\sqrt{}}$
default	$\frac{65248x^5\sqrt{-x^4+x^2+2}}{273} + \frac{5890x^7\sqrt{-x^4+x^2+2}}{429} - \frac{5075x^9\sqrt{-x^4+x^2+2}}{143} - \frac{436307x\sqrt{-x^4+x^2+2}}{15015}$

elliptic	$\frac{65248x^5\sqrt{-x^4+x^2+2}}{273}+\frac{5890x^7\sqrt{-x^4+x^2+2}}{429}-\frac{5075x^9\sqrt{-x^4+x^2+2}}{143}-\frac{436307x\sqrt{-x^4+x^2+}}{15015}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 65248/273*x^5*(-x^4+x^2+2)^(1/2)+5890/429*x^7*(-x^4+x^2+2)^(1/2)-5075/143*x
^9*(-x^4+x^2+2)^(1/2)-436307/15015*x*(-x^4+x^2+2)^(1/2)-125/13*x^11*(-x^4+x
^2+2)^(1/2)+5757461/15015*x^3*(-x^4+x^2+2)^(1/2)+10736597/15015*2^(1/2)*(-2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^
(1/2))-15536264/15015*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(
1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Fricas [A]
time = 0.08, size = 49, normalized size = 0.40

$$-\frac{(144375\,x^{12}+532875\,x^{10}-206150\,x^{8}-3588640\,x^{6}-5757461\,x^{4}+436307\,x^{2}+31072528)\sqrt{-x^4+x^2+2}}{15015\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15015*(144375*x^12 + 532875*x^10 - 206150*x^8 - 3588640*x^6 - 5757461*x^
4 + 436307*x^2 + 31072528)*sqrt(-x^4 + x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)^3\,{\left(-x^4+x^2+2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2), x)
```

3.326 $\int (7+5x^2)^2 (2+x^2-x^4)^{3/2} \, dx$

Optimal. Leaf size=100

$$\frac{1}{495}x(11497+14889x^2)\sqrt{2+x^2-x^4}+\frac{1}{99}x(363+920x^2)(2+x^2-x^4)^{3/2}-\frac{25}{11}x(2+x^2-x^4)^{5/2}+\frac{85942}{495}$$

```
[Out] 1/99*x*(920*x^2+363)*(-x^4+x^2+2)^(3/2)-25/11*x*(-x^4+x^2+2)^(5/2)+85942/49
5*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-3392/165*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))+1/495*x*(14889*x^2+11497)*(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1190, 1194, 538, 435, 430}

$$-\frac{3392}{165}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{85942}{495}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{11}x\left(-x^4+x^2+2\right)^{5/2}+\frac{1}{99}x\left(920x^2+363\right)\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{495}x\left(14889x^2+11497\right)\sqrt{-x^4+x^2+2}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(11497 + 14889*x^2)*Sqrt[2 + x^2 - x^4])/495 + (x*(363 + 920*x^2)*(2 + x
^2 - x^4)^(3/2))/99 - (25*x*(2 + x^2 - x^4)^(5/2))/11 + (85942*EllipticE[Ar
cSin[x/Sqrt[2]], -2])/495 - (3392*EllipticF[ArcSin[x/Sqrt[2]], -2])/165
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int (7+5x^2)^2 (2+x^2-x^4)^{3/2}\,dx = -\frac{25}{11}x(2+x^2-x^4)^{5/2} - \frac{1}{11}\int (-589-920x^2)(2+x^2-x^4)^{3/2}\,dx$$

$$= \frac{1}{99}x(363+920x^2)(2+x^2-x^4)^{3/2} - \frac{25}{11}x(2+x^2-x^4)^{5/2} + \frac{1}{231}\int (23$$

$$= \frac{1}{495}x(11497+14889x^2)\sqrt{2+x^2-x^4} + \frac{1}{99}x(363+920x^2)(2+x^2-$$

$$= \frac{1}{495}x(11497+14889x^2)\sqrt{2+x^2-x^4} + \frac{1}{99}x(363+920x^2)(2+x^2-$$

$$= \frac{1}{495}x(11497+14889x^2)\sqrt{2+x^2-x^4} + \frac{1}{99}x(363+920x^2)(2+x^2-$$

$$= \frac{1}{495}x(11497+14889x^2)\sqrt{2+x^2-x^4} + \frac{1}{99}x(363+920x^2)(2+x^2-$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.34, size = 112, normalized size = 1.12

$$\frac{21254x+53435x^3+23097x^5-19944x^7-10760x^9+1225x^{11}+1125x^{13}+85942i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)-123825i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)|-\frac{1}{2}\right)}{495\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (21254*x + 53435*x^3 + 23097*x^5 - 19944*x^7 - 10760*x^9 + 1225*x^11 + 1125
*x^13 + (85942*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (
123825*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(495*Sqrt[
2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(90) = 180.
time = 0.12, size = 193, normalized size = 1.93

method	result
risch	$\frac{x(1125x^8+2350x^6-6160x^4-21404x^2-10627)(x^4-x^2-2)}{495\sqrt{-x^4+x^2+2}}-\frac{42971\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right.\right.}{495\sqrt{-x^4+x^2+2}}$
default	$-\frac{25x^9\sqrt{-x^4+x^2+2}}{11}-\frac{470x^7\sqrt{-x^4+x^2+2}}{99}+\frac{112x^5\sqrt{-x^4+x^2+2}}{9}+\frac{21404x^3\sqrt{-x^4+x^2+2}}{495}$
elliptic	$-\frac{25x^9\sqrt{-x^4+x^2+2}}{11}-\frac{470x^7\sqrt{-x^4+x^2+2}}{99}+\frac{112x^5\sqrt{-x^4+x^2+2}}{9}+\frac{21404x^3\sqrt{-x^4+x^2+2}}{495}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -25/11*x^9*(-x^4+x^2+2)^(1/2)-470/99*x^7*(-x^4+x^2+2)^(1/2)+112/9*x^5*(-x^4
+x^2+2)^(1/2)+21404/495*x^3*(-x^4+x^2+2)^(1/2)+10627/495*x*(-x^4+x^2+2)^(1/
2)+37883/495*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elli
pticF(1/2*2^(1/2)*x,I*2^(1/2))-42971/495*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(
1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2
^(1/2)*x,I*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Fricas [A]
time = 0.09, size = 44, normalized size = 0.44

$$-\frac{(1125\,x^{10}+2350\,x^{8}-6160\,x^{6}-21404\,x^{4}-10627\,x^{2}+85942)\sqrt{-x^{4}+x^{2}+2}}{495\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/495*(1125*x^10 + 2350*x^8 - 6160*x^6 - 21404*x^4 - 10627*x^2 + 85942)*sq
rt(-x^4 + x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-\left(x^{2}-2\right)\left(x^{2}+1\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(5\,x^2+7\right)}^2\,{\left(-x^4+x^2+2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2), x)
```

3.327 $\int (7+5x^2)(2+x^2-x^4)^{3/2}\,dx$

Optimal. Leaf size=81

$$\frac{1}{315}x(1087+669x^2)\sqrt{2+x^2-x^4}+\frac{1}{63}x(48+35x^2)(2+x^2-x^4)^{3/2}+\frac{4432}{315}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{418}{105}$$

```
[Out] 1/63*x*(35*x^2+48)*(-x^4+x^2+2)^(3/2)+4432/315*EllipticE(1/2*x*2^(1/2),I*2^
(1/2))+418/105*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+1/315*x*(669*x^2+1087)*(-
x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1190, 1194, 538, 435, 430}

$$\frac{418}{105}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{4432}{315}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{63}x(35x^2+48)(-x^4+x^2+2)^{3/2}+\frac{1}{315}x(669x^2+1087)\sqrt{-x^4+x^2+2}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(1087 + 669*x^2)*Sqrt[2 + x^2 - x^4])/315 + (x*(48 + 35*x^2)*(2 + x^2 -
x^4)^(3/2))/63 + (4432*EllipticE[ArcSin[x/Sqrt[2]], -2])/315 + (418*Ellipti
cF[ArcSin[x/Sqrt[2]], -2])/105
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \left(7+5x^2\right)\left(2+x^2-x^4\right)^{3/2}\,dx = \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2} - \frac{1}{21}\int \left(-262-223x^2\right)\sqrt{2+x^2-x^4}$$

$$= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}$$

$$= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}$$

$$= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}$$

$$= \frac{1}{315}x\left(1087+669x^2\right)\sqrt{2+x^2-x^4} + \frac{1}{63}x\left(48+35x^2\right)\left(2+x^2-x^4\right)^{3/2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.35, size = 107, normalized size = 1.32

$$\frac{3134x+4085x^3-438x^5-1674x^7-110x^9+175x^{11}+4432i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-7275i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{315\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (3134*x + 4085*x^3 - 438*x^5 - 1674*x^7 - 110*x^9 + 175*x^11 + (4432*I)*Sqr
t[4 + 2*x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] - (7275*I)*Sqrt[4 + 2*x^
2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(315*Sqrt[2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(75) = 150.
time = 0.04, size = 176, normalized size = 2.17

method	result
risch	$\frac{x(175x^6+65x^4-1259x^2-1567)(x^4-x^2-2)}{315\sqrt{-x^4+x^2+2}} - \frac{2216\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)-\text{EllipticE}\left(\right.\right.}{315\sqrt{-x^4+x^2+2}}$
default	$-\frac{5x^7\sqrt{-x^4+x^2+2}}{9} - \frac{13x^5\sqrt{-x^4+x^2+2}}{63} + \frac{1259x^3\sqrt{-x^4+x^2+2}}{315} + \frac{1567x\sqrt{-x^4+x^2+2}}{315} +$
elliptic	$-\frac{5x^7\sqrt{-x^4+x^2+2}}{9} - \frac{13x^5\sqrt{-x^4+x^2+2}}{63} + \frac{1259x^3\sqrt{-x^4+x^2+2}}{315} + \frac{1567x\sqrt{-x^4+x^2+2}}{315} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/9*x^7*(-x^4+x^2+2)^(1/2)-13/63*x^5*(-x^4+x^2+2)^(1/2)+1259/315*x^3*(-x^4
+x^2+2)^(1/2)+1567/315*x*(-x^4+x^2+2)^(1/2)+2843/315*2^(1/2)*(-2*x^2+4)^(1/
2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-2216
/315*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1
/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Fricas [A]
time = 0.08, size = 39, normalized size = 0.48

$$-\frac{(175\,x^8+65\,x^6-1259\,x^4-1567\,x^2+4432)\sqrt{-x^4+x^2+2}}{315\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/315*(175*x^8 + 65*x^6 - 1259*x^4 - 1567*x^2 + 4432)*sqrt(-x^4 + x^2 + 2)
/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(x^{2}-2\right)\left(x^{2}+1\right)\right)^{\frac{3}{2}}\cdot\left(5x^{2}+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(-x**4+x**2+2)**(3/2),x)
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^{2}+7\right)\,\left(-x^{4}+x^{2}+2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2), x)
```

3.328 $\int \left(2+x^2-x^4\right)^{3/2} \, dx$

Optimal. Leaf size=74

$$\frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{48}{35}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|-$$

```
[Out] 1/7*x*(-x^4+x^2+2)^(3/2)+34/35*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+48/35*Ell
ipticF(1/2*x*2^(1/2),I*2^(1/2))+1/35*x*(3*x^2+19)*(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1105, 1190, 1194, 538, 435, 430}

$$\frac{48}{35}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{34}{35}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{7}x\left(-x^4+x^2+2\right)^{3/2}+\frac{1}{35}x\left(3x^2+19\right)\sqrt{-x^4+x^2+2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(19 + 3*x^2)*Sqrt[2 + x^2 - x^4])/35 + (x*(2 + x^2 - x^4)^(3/2))/7 + (34
*EllipticE[ArcSin[x/Sqrt[2]], -2])/35 + (48*EllipticF[ArcSin[x/Sqrt[2]], -2
])/35
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(2+x^2-x^4\right)^{3/2}\,dx &= \frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{3}{7}\int \left(4+x^2\right)\sqrt{2+x^2-x^4}\,dx\\ &= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}-\frac{1}{35}\int \frac{-82-34x^2}{\sqrt{2+x^2-x^4}}\,dx\\ &= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}-\frac{2}{35}\int \frac{-82-34x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\\ &= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{34}{35}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{96}{35}\int\\ &= \frac{1}{35}x\left(19+3x^2\right)\sqrt{2+x^2-x^4}+\frac{1}{7}x\left(2+x^2-x^4\right)^{3/2}+\frac{34}{35}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.20, size = 102, normalized size = 1.38

$$\frac{58x+45x^3-31x^5-13x^7+5x^9+34i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-75i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{35\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (58*x + 45*x^3 - 31*x^5 - 13*x^7 + 5*x^9 + (34*I)*Sqrt[4 + 2*x^2 - 2*x^4]*E
llipticE[I*ArcSinh[x], -1/2] - (75*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*A
rcSinh[x], -1/2])/(35*Sqrt[2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(68) = 136$.
time = 0.04, size = 159, normalized size = 2.15

method	result
risch	$\frac{x(5x^4-8x^2-29)(x^4-x^2-2)}{35\sqrt{-x^4+x^2+2}}+\frac{41\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\operatorname{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{35\sqrt{-x^4+x^2+2}}-\frac{17\sqrt{2}\,\sqrt{-2x^2+4}}{}$
default	$-\frac{x^5\sqrt{-x^4+x^2+2}}{7}+\frac{8x^3\sqrt{-x^4+x^2+2}}{35}+\frac{29x\sqrt{-x^4+x^2+2}}{35}+\frac{41\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}}{35\sqrt{-x^4+x}}$
elliptic	$-\frac{x^5\sqrt{-x^4+x^2+2}}{7}+\frac{8x^3\sqrt{-x^4+x^2+2}}{35}+\frac{29x\sqrt{-x^4+x^2+2}}{35}+\frac{41\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}}{35\sqrt{-x^4+x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*x^5*(-x^4+x^2+2)^(1/2)+8/35*x^3*(-x^4+x^2+2)^(1/2)+29/35*x*(-x^4+x^2+2
)^(1/2)+41/35*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ell
ipticF(1/2*2^(1/2)*x,I*2^(1/2))-17/35*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2
)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1
/2)*x,I*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [A]
time = 0.08, size = 34, normalized size = 0.46

$$-\frac{(5\,x^6-8\,x^4-29\,x^2+34)\sqrt{-x^4+x^2+2}}{35\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/35*(5*x^6 - 8*x^4 - 29*x^2 + 34)*sqrt(-x^4 + x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-x^4 + x^2 + 2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**4 + x**2 + 2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(-x^4+x^2+2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((x^2 - x^4 + 2)^(3/2), x)
```

3.329 $\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2} dx$

Optimal. Leaf size=72

$$\frac{1}{75}x(13-3x^2)\sqrt{2+x^2-x^4}+\frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{178}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1156\Pi\left(-\frac{10}{7};\text{s}\right.}{}$$

```
[Out] 92/375*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-178/625*EllipticF(1/2*x*2^(1/2),I
*2^(1/2))+1156/4375*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/75*x*(-3*x^
2+13)*(-x^4+x^2+2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1222, 1190, 1194, 538, 435, 430, 1226, 551}

$$-\frac{178}{625}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{92}{375}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1156\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375}+\frac{1}{75}x\sqrt{-x^4+x^2+2}\,(13-3x^2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (x*(13 - 3*x^2)*Sqrt[2 + x^2 - x^4])/75 + (92*EllipticE[ArcSin[x/Sqrt[2]],
-2])/375 - (178*EllipticF[ArcSin[x/Sqrt[2]], -2])/625 + (1156*EllipticPi[-1
0/7, ArcSin[x/Sqrt[2]], -2])/4375
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{(2+x^2-x^4)^{3/2}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int (-12+5x^2)\sqrt{2+x^2-x^4}\,dx\right) - \frac{34}{25}\int \frac{\sqrt{2+x^2-x^4}}{7+5x^2}\,dx$$

$$= \frac{1}{75}x(13-3x^2)\sqrt{2+x^2-x^4} + \frac{1}{375}\int \frac{230-10x^2}{\sqrt{2+x^2-x^4}}\,dx + \frac{34}{625}\int \frac{-12+5x^2}{\sqrt{2+x^2-x^{\cdots}}}$$

$$= \frac{1}{75}x(13-3x^2)\sqrt{2+x^2-x^4} + \frac{2}{375}\int \frac{230-10x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx + \frac{68}{625}\int \frac{-\cdots}{\sqrt{4-\cdots}}$$

$$= \frac{1}{75}x(13-3x^2)\sqrt{2+x^2-x^4} + \frac{1156\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{4375} - \frac{2}{75}\int \frac{\sqrt{2+\cdots}}{\sqrt{4-\cdots}}$$

$$= \frac{1}{75}x(13-3x^2)\sqrt{2+x^2-x^4} + \frac{92}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{178}{625}F\left(\sin^{-1}\left(\frac{\cdots}{\sqrt{\cdots}}\right.\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.14, size = 130, normalized size = 1.81

$$\frac{4550x+1225x^3-2800x^5+525x^7+3220i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-2961i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-1734i\sqrt{4+2x^2-2x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{13125\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (4550*x + 1225*x^3 - 2800*x^5 + 525*x^7 + (3220*I)*Sqrt[4 + 2*x^2 - 2*x^4]*
EllipticE[I*ArcSinh[x], -1/2] - (2961*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[
I*ArcSinh[x], -1/2] - (1734*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticPi[5/7, I*Ar
cSinh[x], -1/2])/(13125*Sqrt[2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(69) = 138.
time = 0.13, size = 173, normalized size = 2.40

method	result
default	$-\frac{x^3\sqrt{-x^4+x^2+2}}{25}+\frac{13x\sqrt{-x^4+x^2+2}}{75}-\frac{89\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{625\sqrt{-x^4+x^2+2}}$
elliptic	$-\frac{x^3\sqrt{-x^4+x^2+2}}{25}+\frac{13x\sqrt{-x^4+x^2+2}}{75}-\frac{89\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{625\sqrt{-x^4+x^2+2}}$

risch	$\frac{x(3x^2-13)(x^4-x^2-2)}{75\sqrt{-x^4+x^2+2}} - \frac{46\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)\right)}{375\sqrt{-x^4+x^2+2}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] -1/25*x^3*(-x^4+x^2+2)^(1/2)+13/75*x*(-x^4+x^2+2)^(1/2)-89/625*2^(1/2)*(-2*
x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(
1/2))+46/375*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elli
pticE(1/2*2^(1/2)*x,I*2^(1/2))+1156/4375*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^
(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7),x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7),x, algorithm="giac")
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(-x^4+x^2+2\right)^{3/2}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7),x)
[Out] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7), x)
```

3.330 $\int \frac{\left(2+x^2-x^4\right)^{3/2}}{\left(7+5x^2\right)^2} dx$

Optimal. Leaf size=93

$$-\frac{1}{75}x\sqrt{2+x^2-x^4}-\frac{17x\sqrt{2+x^2-x^4}}{175\left(7+5x^2\right)}-\frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{458}{875}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1241}{}$$

[Out] -97/525*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+458/875*EllipticF(1/2*x*2^(1/2), I*2^(1/2))-1241/6125*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-1/75*x*(-x^4 +x^2+2)^(1/2)-17/175*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)

Rubi [A]
time = 0.20, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {1242, 1109, 430, 1146, 507, 435, 1136, 1194, 1237, 1730, 538, 1226, 551}

$$\frac{458}{875}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{97}{525}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1241\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{6125}-\frac{17\sqrt{-x^4+x^2+2}\,x}{175\left(5x^2+7\right)}-\frac{1}{75}\sqrt{-x^4+x^2+2}\,x$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] -1/75*(x*Sqrt[2 + x^2 - x^4]) - (17*x*Sqrt[2 + x^2 - x^4])/(175*(7 + 5*x^2)
) - (97*EllipticE[ArcSin[x/Sqrt[2]], -2])/525 + (458*EllipticF[ArcSin[x/Sqr
t[2]], -2])/875 - (1241*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/6125
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
```

```
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1146

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt[
-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] &
& LtQ[c, 0]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
```

```
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^2}\,dx = \int \left(\frac{212}{625\sqrt{2+x^2-x^4}} - \frac{24x^2}{125\sqrt{2+x^2-x^4}} + \frac{x^4}{25\sqrt{2+x^2-x^4}} + \frac{1}{625\,(7+5x^2)}\right.$$

$$= \frac{1}{25}\int \frac{x^4}{\sqrt{2+x^2-x^4}}\,dx - \frac{24}{125}\int \frac{x^2}{\sqrt{2+x^2-x^4}}\,dx + \frac{212}{625}\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx -$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\,(7+5x^2)} + \frac{17\int \frac{118-70x^2-25x^4}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx}{4375} + \frac{1}{75}\int$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\,(7+5x^2)} + \frac{212}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{1292}{}$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\,(7+5x^2)} - \frac{62}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{332}{625}F$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\,(7+5x^2)} - \frac{62}{375}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{332}{625}F$$

$$= -\frac{1}{75}x\sqrt{2+x^2-x^4} - \frac{17x\sqrt{2+x^2-x^4}}{175\,(7+5x^2)} - \frac{97}{525}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{458}{875}F$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.12, size = 201, normalized size = 2.16

$$\frac{-14000x-11900x^3+4550x^5+2450x^7-6790i\sqrt{2}\,(7+5x^2)\sqrt{2+x^2-x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+567i\sqrt{2}\,(7+5x^2)\sqrt{2+x^2-x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+26061i\sqrt{2}\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+18615i\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{36750\,(7+5x^2)\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (-14000*x - 11900*x^3 + 4550*x^5 + 2450*x^7 - (6790*I)*Sqrt[2]*(7 + 5*x^2)*
Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] + (567*I)*Sqrt[2]*(7 + 5*
x^2)*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (26061*I)*Sqrt[2]*
Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (18615*I)*Sqrt[2]
*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(36750*(7 + 5
*x^2)*Sqrt[2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(86) = 172.
time = 0.13, size = 180, normalized size = 1.94

method	result
default	$-\frac{17x\sqrt{-x^4+x^2+2}}{175(5x^2+7)}-\frac{x\sqrt{-x^4+x^2+2}}{75}+\frac{229\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{875\sqrt{-x^4+x^2+2}}-$
elliptic	$-\frac{17x\sqrt{-x^4+x^2+2}}{175(5x^2+7)}-\frac{x\sqrt{-x^4+x^2+2}}{75}+\frac{229\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{875\sqrt{-x^4+x^2+2}}-$
risch	$\frac{(x^4-x^2-2)x(7x^2+20)}{105(5x^2+7)\sqrt{-x^4+x^2+2}}+\frac{97\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right.\right.}{1050\sqrt{-x^4+x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -17/175*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/75*x*(-x^4+x^2+2)^(1/2)+229/875*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/
2)*x,I*2^(1/2))-97/1050*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))-1241/6125*2^(1/2)*(1-1/2*x^2)^(1/
2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}{\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(-x^4+x^2+2\right)}^{3/2}}{{\left(5\,x^2+7\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^2,x)
```

```
[Out] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^2, x)
```

3.331 $\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3} dx$

Optimal. Leaf size=102

$$-\frac{17x\sqrt{2+x^2-x^4}}{350(7+5x^2)^2}+\frac{563x\sqrt{2+x^2-x^4}}{9800(7+5x^2)}+\frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}-\frac{1251F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24500}+\frac{987}{}$$

```
[Out] 191/9800*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1251/24500*EllipticF(1/2*x*2^(1
/2),I*2^(1/2))+9879/343000*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-17/350
*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+563/9800*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A]

time = 0.31, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {1242, 1109, 430, 1146, 507, 435, 1237, 1710, 1730, 1194, 538, 1226, 551}

$$-\frac{1251F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24500}+\frac{191E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800}+\frac{9879\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{343000}+\frac{563\sqrt{-x^4+x^2+2}\,x}{9800(5x^2+7)}-\frac{17\sqrt{-x^4+x^2+2}\,x}{350(5x^2+7)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (-17*x*Sqrt[2 + x^2 - x^4])/(350*(7 + 5*x^2)^2) + (563*x*Sqrt[2 + x^2 - x^4
])/(9800*(7 + 5*x^2)) + (191*EllipticE[ArcSin[x/Sqrt[2]], -2])/9800 - (1251
*EllipticF[ArcSin[x/Sqrt[2]], -2])/24500 + (9879*EllipticPi[-10/7, ArcSin[x
/Sqrt[2]], -2])/343000
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 507

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
```

```
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-b/a, -d/c])
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1146

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[x^2/(Sqrt[b + q + 2*c*x^2]*Sqrt[
-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] &
& LtQ[c, 0]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
```

```
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{(2+x^2-x^4)^{3/2}}{(7+5x^2)^3}\,dx = \int \left(-\frac{31}{625\sqrt{2+x^2-x^4}} + \frac{x^2}{125\sqrt{2+x^2-x^4}} + \frac{1156}{625\,(7+5x^2)^3\,\sqrt{2+x^2-x^4}} -$$

$$= \frac{1}{125}\int \frac{x^2}{\sqrt{2+x^2-x^4}}\,dx - \frac{31}{625}\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx + \frac{429}{625}\int \frac{1}{(7+5x^2)\,\sqrt{2+x^2}}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\,(7+5x^2)^2} + \frac{19x\sqrt{2+x^2-x^4}}{175\,(7+5x^2)} + \frac{17\int \frac{186-190x^2+25x^4}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx}{8750} - \frac{19}{}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\,(7+5x^2)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\,(7+5x^2)} - \frac{31}{625}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{429}{}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\,(7+5x^2)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\,(7+5x^2)} + \frac{1}{125}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{36}{625}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\,(7+5x^2)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\,(7+5x^2)} + \frac{1}{125}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{36}{625}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\,(7+5x^2)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\,(7+5x^2)} + \frac{26}{875}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{214}{}$$

$$= -\frac{17x\sqrt{2+x^2-x^4}}{350\,(7+5x^2)^2} + \frac{563x\sqrt{2+x^2-x^4}}{9800\,(7+5x^2)} + \frac{191E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{9800} - \frac{125}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 244, normalized size = 2.39

$$\frac{485100x + 636650x^3 - 45500x^5 - 197050x^7 + 13370i\sqrt{2}\,(7+5x^2)^2\sqrt{2+x^2-x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) - 2541i\sqrt{2}\,(7+5x^2)^2\sqrt{2+x^2-x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) - 484071i\sqrt{2}\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) - 691530i\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) - 246975i\sqrt{2}\,x^4\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{686000\,(7+5x^2)^2\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (485100*x + 636650*x^3 - 45500*x^5 - 197050*x^7 + (13370*I)*Sqrt[2]*(7 + 5*
x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (2541*I)*Sqrt[2]
*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] - (484071*
I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (69153
0*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] -
(246975*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1
/2])/(686000*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4])
```

Maple [A]
time = 0.13, size = 189, normalized size = 1.85

method	result
default	$-\frac{17x\sqrt{-x^4+x^2+2}}{350(5x^2+7)^2}+\frac{563x\sqrt{-x^4+x^2+2}}{9800(5x^2+7)}-\frac{1251\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{49000\sqrt{-x^4+x^2+2}}$
elliptic	$-\frac{17x\sqrt{-x^4+x^2+2}}{350(5x^2+7)^2}+\frac{563x\sqrt{-x^4+x^2+2}}{9800(5x^2+7)}-\frac{1251\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{49000\sqrt{-x^4+x^2+2}}$
risch	$-\frac{(x^4-x^2-2)x(563x^2+693)}{1960(5x^2+7)^2\sqrt{-x^4+x^2+2}}-\frac{191\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\text{EllipticE}\left(\frac{\sqrt{2}\,x}{2}\right.\right.}{19600\sqrt{-x^4+x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -17/350*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+563/9800*x*(-x^4+x^2+2)^(1/2)/(5*x
^2+7)-1251/49000*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*
EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+191/19600*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))+9879/343000*
2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2
^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
```

```
[Out] integral((-x^4 + x^2 + 2)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(x^{2}-2\right)\left(x^{2}+1\right)\right)^{\frac{3}{2}}}{\left(5x^{2}+7\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**2+2)**(3/2)/(5*x**2+7)**3,x)
```

```
[Out] Integral((-(x**2 - 2)*(x**2 + 1))**(3/2)/(5*x**2 + 7)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^2+2)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(-x^4+x^2+2\right)}^{3/2}}{{\left(5\,x^2+7\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^3,x)
```

```
[Out] int((x^2 - x^4 + 2)^(3/2)/(5*x^2 + 7)^3, x)
```

3.332 $\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=65

$$-\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-542F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 3905/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-542*EllipticF(1/2*x*2^(1/2),I*2^(
1/2))-625/3*x*(-x^4+x^2+2)^(1/2)-25*x^3*(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1693, 1194, 538, 435, 430}

$$-542F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{3905}{3}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{625}{3}\sqrt{-x^4+x^2+2}\,x-25\sqrt{-x^4+x^2+2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-625*x*Sqrt[2 + x^2 - x^4])/3 - 25*x^3*Sqrt[2 + x^2 - x^4] + (3905*Ellipti
cE[ArcSin[x/Sqrt[2]], -2])/3 - 542*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(7+5x^2)^3}{\sqrt{2+x^2-x^4}}\,dx &= -25x^3\sqrt{2+x^2-x^4}-\frac{1}{5}\int \frac{-1715-4425x^2-3125x^4}{\sqrt{2+x^2-x^4}}\,dx\\ &= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{1}{15}\int \frac{11395+19525x^2}{\sqrt{2+x^2-x^4}}\,dx\\ &= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{2}{15}\int \frac{11395+19525x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx\\ &= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}-1084\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx+\\ &= -\frac{625}{3}x\sqrt{2+x^2-x^4}-25x^3\sqrt{2+x^2-x^4}+\frac{3905}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-542\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.07, size = 97, normalized size = 1.49

$$\frac{-2500x-1550x^3+1100x^5+150x^7+7810i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-10089i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{6\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-2500*x - 1550*x^3 + 1100*x^5 + 150*x^7 + (7810*I)*Sqrt[4 + 2*x^2 - 2*x^4]
*EllipticE[I*ArcSinh[x], -1/2] - (10089*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Elliptic
F[I*ArcSinh[x], -1/2])/(6*Sqrt[2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(63) = 126.
time = 0.12, size = 142, normalized size = 2.18

method	result
default	$-25x^3\sqrt{-x^4+x^2+2}-\frac{625x\sqrt{-x^4+x^2+2}}{3}+\frac{2279\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}$
risch	$\frac{25x(3x^2+25)(x^4-x^2-2)}{3\sqrt{-x^4+x^2+2}}-\frac{3905\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)\right)}{6\sqrt{-x^4+x^2+2}}$
elliptic	$-25x^3\sqrt{-x^4+x^2+2}-\frac{625x\sqrt{-x^4+x^2+2}}{3}+\frac{2279\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -25*x^3*(-x^4+x^2+2)^(1/2)-625/3*x*(-x^4+x^2+2)^(1/2)+2279/6*2^(1/2)*(-2*x^
2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/
2))-3905/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(Ellip
ticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [A]
time = 0.07, size = 29, normalized size = 0.45

$$-\frac{5\left(15\,x^{4}+125\,x^{2}+781\right)\sqrt{-x^{4}+x^{2}+2}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
[Out] -5/3*(15*x^4 + 125*x^2 + 781)*sqrt(-x^4 + x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^{2}+7\right)^{3}}{\sqrt{-\left(x^{2}-2\right)\left(x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
[Out] Integral((5*x**2 + 7)**3/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/sqrt(-x^4 + x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(5\,x^{2}+7\right)^{3}}{\sqrt{-x^{4}+x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(1/2), x)
```

3.333 $\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=46

$$-\frac{25}{3}x\sqrt{2+x^2-x^4}+\frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 260/3*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-21*EllipticF(1/2*x*2^(1/2),I*2^(1/
2))-25/3*x*(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1194, 538, 435, 430}

$$-21F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{260}{3}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{3}\sqrt{-x^4+x^2+2}\,x$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/3 + (260*EllipticE[ArcSin[x/Sqrt[2]], -2])/3 -
21*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(7+5x^2)^2}{\sqrt{2+x^2-x^4}}\,dx &= -\frac{25}{3}x\sqrt{2+x^2-x^4}-\frac{1}{3}\int \frac{-197-260x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= -\frac{25}{3}x\sqrt{2+x^2-x^4}-\frac{2}{3}\int \frac{-197-260x^2}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\
&= -\frac{25}{3}x\sqrt{2+x^2-x^4}-42\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx+\frac{260}{3}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\
&= -\frac{25}{3}x\sqrt{2+x^2-x^4}+\frac{260}{3}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-21F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 92, normalized size = 2.00

$$\frac{-100x-50x^3+50x^5+520i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-717i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{6\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (-100*x - 50*x^3 + 50*x^5 + (520*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticE[I*Arc
Sinh[x], -1/2] - (717*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[x], -1
/2])/(6*Sqrt[2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(46) = 92.
time = 0.14, size = 125, normalized size = 2.72

method	result
default	$-\frac{25x\sqrt{-x^4+x^2+2}}{3}+\frac{197\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{130\sqrt{2}\,\sqrt{-2x^2+4}}{}$
elliptic	$-\frac{25x\sqrt{-x^4+x^2+2}}{3}+\frac{197\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{6\sqrt{-x^4+x^2+2}}-\frac{130\sqrt{2}\,\sqrt{-2x^2+4}}{}$
risch	$\frac{25x(x^4-x^2-2)}{3\sqrt{-x^4+x^2+2}}-\frac{130\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)\right)}{3\sqrt{-x^4+x^2+2}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -25/3*x*(-x^4+x^2+2)^(1/2)+197/6*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x
^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-130/3*2^(1/2)*(-2*x^2+4)
^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))
-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [A]
time = 0.08, size = 24, normalized size = 0.52

$$-\frac{5\sqrt{-x^4+x^2+2}\,(5x^2+52)}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -5/3*sqrt(-x^4 + x^2 + 2)*(5*x^2 + 52)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(5x^2+7)^2}{\sqrt{-(x^2-2)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(-x^4 + x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(5\,x^2+7\right)^2}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(1/2), x)
```

3.334 $\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=25

$$5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 5*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+2*EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1194, 538, 435, 430}

$$2F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+5E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
[Out] 5*EllipticE[ArcSin[x/Sqrt[2]], -2] + 2*EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{7+5x^2}{\sqrt{2+x^2-x^4}}\,dx &= 2\int \frac{7+5x^2}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\ &= 4\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx+5\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx\\ &= 5E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+2F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 34, normalized size = 1.36

$$\frac{i\left(10E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-17F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] (I*(10*EllipticE[I*ArcSinh[x], -1/2] - 17*EllipticF[I*ArcSinh[x], -1/2]))/S
qrt[2]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(31) = 62.
time = 0.02, size = 110, normalized size = 4.40

method	result
default	$-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^4+x^2+2}}+\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{}}{2\sqrt{-}}$
elliptic	$-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)\right)}{2\sqrt{-x^4+x^2+2}}+\frac{7\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{}}{2\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1
/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+7/2*2^(1/2)*(-2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^
(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [A]
time = 0.10, size = 17, normalized size = 0.68

$$-\frac{5\sqrt{-x^4+x^2+2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -5*sqrt(-x^4 + x^2 + 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{5x^2+7}{\sqrt{-(x^2-2)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt(-(x**2 - 2)*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(-x^4 + x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{5\,x^2+7}{\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(1/2),x)
[Out] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(1/2), x)
```

3.335 $\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1109, 430}

$$F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+x^2-x^4}}\,dx &= 2\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\ &= F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.03, size = 19, normalized size = 1.90

$$-\frac{iF\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x^2 - x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[x], -1/2])/Sqrt[2]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(13) = 26.
time = 0.02, size = 47, normalized size = 4.70

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{2\sqrt{-x^4+x^2+2}}$	47
elliptic	$\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{2\sqrt{-x^4+x^2+2}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2
*2^(1/2)*x,I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

Fricas [A]
time = 0.09, size = 8, normalized size = 0.80

$$\mathrm{ellipticF}\left(\frac{1}{2}\,\sqrt{2}\,x,-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] ellipticF(1/2*sqrt(2)*x, -2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + x**2 + 2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.10

$$\int \frac{1}{\sqrt{-x^4 + x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - x^4 + 2)^(1/2),x)
```

```
[Out] int(1/(x^2 - x^4 + 2)^(1/2), x)
```

3.336 $\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=17

$$\frac{1}{7}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 1/7*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1226, 551}

$$\frac{1}{7}\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2]/7
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx &= 2\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}\,(7+5x^2)}\,dx\\ &= \frac{1}{7}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 24, normalized size = 1.41

$$-\frac{i\Pi\left(\frac{5}{7};i\sinh^{-1}(x)|-\frac{1}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] ((-1/7*I)*EllipticPi[5/7, I*ArcSinh[x], -1/2])/Sqrt[2]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(16) = 32.
time = 0.15, size = 48, normalized size = 2.82

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\mathrm{EllipticPi}\left(\frac{\sqrt{2}\,x}{2},-\frac{10}{7},i\sqrt{2}\right)}{7\sqrt{-x^4+x^2+2}}$	48
elliptic	$\frac{\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}\,\sqrt{x^2+1}\,\mathrm{EllipticPi}\left(\frac{\sqrt{2}\,x}{2},-\frac{10}{7},i\sqrt{2}\right)}{7\sqrt{-x^4+x^2+2}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1
/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(5*x^6 + 2*x^4 - 17*x^2 - 14), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x^{2}-2\right)\left(x^{2}+1\right)}\,\left(5x^{2}+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\left(5\,x^2+7\right)\,\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(1/2)), x)
```

3.337 $\int \frac{1}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=74

$$-\frac{25x\sqrt{2+x^2-x^4}}{476(7+5x^2)}-\frac{5}{476}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{1}{238}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{167\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right.}{3332}$$

```
[Out] -5/476*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-1/238*EllipticF(1/2*x*2^(1/2),I*2
^(1/2))+167/3332*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-25/476*x*(-x^4+x
^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1237, 1730, 1194, 538, 435, 430, 1226, 551}

$$-\frac{1}{238}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{5}{476}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{167\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332}-\frac{25\sqrt{-x^4+x^2+2}\,x}{476(5x^2+7)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(476*(7 + 5*x^2)) - (5*EllipticE[ArcSin[x/Sqrt[
2]], -2])/476 - EllipticF[ArcSin[x/Sqrt[2]], -2]/238 + (167*EllipticPi[-10/
7, ArcSin[x/Sqrt[2]], -2])/3332
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
```

```
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(7+5x^2\right)^2 \sqrt{2+x^2-x^4}}\,dx &= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)} + \frac{1}{476}\int \frac{118-70x^2-25x^4}{\left(7+5x^2\right)\sqrt{2+x^2-x^4}}\,dx \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)} - \frac{\int \frac{175+125x^2}{\sqrt{2+x^2-x^4}}\,dx}{11900} + \frac{167}{476}\int \frac{1}{\left(7+5x^2\right)\sqrt{2+x^2-}} \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)} - \frac{\int \frac{175+125x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx}{5950} + \frac{167}{238}\int \frac{}{\sqrt{4-2x^2}\sqrt{}} \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)} + \frac{167\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3332} - \frac{1}{119}\int \frac{}{\sqrt{4-}} \\
&= -\frac{25x\sqrt{2+x^2-x^4}}{476\left(7+5x^2\right)} - \frac{5}{476}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) - \frac{1}{238}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.11, size = 196, normalized size = 2.65

$$
\frac{-700x-350x^3+350x^5-70i\sqrt{2}\left(7+5x^2\right)\sqrt{2+x^2-x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+119i\sqrt{2}\left(7+5x^2\right)\sqrt{2+x^2-x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-1169i\sqrt{2}\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-835i\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{6664\left(7+5x^2\right)\sqrt{2+x^2-x^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]),x]
```

```
[Out] (-700*x - 350*x^3 + 350*x^5 - (70*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2 - x^4
]*EllipticE[I*ArcSinh[x], -1/2] + (119*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2 + x^2
- x^4]*EllipticF[I*ArcSinh[x], -1/2] - (1169*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]
*EllipticPi[5/7, I*ArcSinh[x], -1/2] - (835*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x
^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(6664*(7 + 5*x^2)*Sqrt[2 + x^2 - x
^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(71) = 142.
time = 0.13, size = 165, normalized size = 2.23

method	result
default	$-\frac{25x\sqrt{-x^4+x^2+2}}{476(5x^2+7)} - \frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)}{476\sqrt{-x^4+x^2+2}} - \frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2}}{952\sqrt{-x}}$

elliptic	$-\frac{25x\sqrt{-x^4+x^2+2}}{476(5x^2+7)}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{476\sqrt{-x^4+x^2+2}}-\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x}}{952\sqrt{-}}$
risch	$\frac{25(x^4-x^2-2)x}{476(5x^2+7)\sqrt{-x^4+x^2+2}}+\frac{5\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{}\right.\right.}{952\sqrt{-x^4+x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -25/476*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-1/476*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-5/952*2^(1/2
)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x
,I*2^(1/2))+167/3332*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(
1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(25*x^8 + 45*x^6 - 71*x^4 - 189*x^2 - 98), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x^2-2\right)\left(x^2+1\right)}\,\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(5\,x^2+7\right)^2\,\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(1/2)), x)
```

3.338 $\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx$

Optimal. Leaf size=102

$$-\frac{25x\sqrt{2+x^2-x^4}}{952\,(7+5x^2)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152\,(7+5x^2)}-\frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152}-\frac{263F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{226576}+$$

[Out] -2505/453152*EllipticE(1/2*x*2^(1/2),I*2^(1/2))-263/226576*EllipticF(1/2*x*2^(1/2),I*2^(1/2))+58915/3172064*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))-25/952*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-12525/453152*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)

Rubi [A]
time = 0.12, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1237, 1710, 1730, 1194, 538, 435, 430, 1226, 551}

$$-\frac{263F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{226576}-\frac{2505E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{453152}+\frac{58915\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{3172064}-\frac{12525\sqrt{-x^4+x^2+2}\,x}{453152\,(5x^2+7)}-\frac{25\sqrt{-x^4+x^2+2}\,x}{952\,(5x^2+7)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]

[Out] (-25*x*Sqrt[2 + x^2 - x^4])/(952*(7 + 5*x^2)^2) - (12525*x*Sqrt[2 + x^2 - x^4])/(453152*(7 + 5*x^2)) - (2505*EllipticE[ArcSin[x/Sqrt[2]], -2])/453152 - (263*EllipticF[ArcSin[x/Sqrt[2]], -2])/226576 + (58915*EllipticPi[-10/7, ArcSin[x/Sqrt[2]], -2])/3172064

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
```

```
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^3\sqrt{2+x^2-x^4}}\,dx = -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}+\frac{1}{952}\int \frac{186-190x^2+25x^4}{(7+5x^2)^2\sqrt{2+x^2-x^4}}\,dx$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)}+\frac{\int \frac{37698-32690x^2-12525x^4}{(7+5x^2)\sqrt{2+x^2-x^4}}}{453152}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)}-\frac{\int \frac{75775+62625x^2}{\sqrt{2+x^2-x^4}}\,dx}{11328800}+$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)}-\frac{\int \frac{75775+62625x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}}{5664400}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)}+\frac{58915\Pi\left(-\frac{10}{7};\sin^{-1}\left(\right.\right.}{3172064}$$

$$= -\frac{25x\sqrt{2+x^2-x^4}}{952(7+5x^2)^2}-\frac{12525x\sqrt{2+x^2-x^4}}{453152(7+5x^2)}-\frac{2505E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|}{453152}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.18, size = 108, normalized size = 1.06

$$\frac{\frac{350x(-7966-8993x^2+1478x^4+2505x^6)}{(7+5x^2)^2\sqrt{2+x^2-x^4}}-35070i\sqrt{2}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+56287i\sqrt{2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-58915i\sqrt{2}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{6344128}$$

Antiderivative was successfully verified.

[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[2 + x^2 - x^4]),x]

[Out] ((350*x*(-7966 - 8993*x^2 + 1478*x^4 + 2505*x^6))/((7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]) - (35070*I)*Sqrt[2]*EllipticE[I*ArcSinh[x], -1/2] + (56287*I)*Sqrt[2]*EllipticF[I*ArcSinh[x], -1/2] - (58915*I)*Sqrt[2]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/6344128

Maple [A]
time = 0.12, size = 189, normalized size = 1.85

method	result
default	$-\frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}-\frac{12525x\sqrt{-x^4+x^2+2}}{453152(5x^2+7)}-\frac{263\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right.}{453152\sqrt{-x^4+x^2+2}}$
elliptic	$-\frac{25x\sqrt{-x^4+x^2+2}}{952(5x^2+7)^2}-\frac{12525x\sqrt{-x^4+x^2+2}}{453152(5x^2+7)}-\frac{263\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right.}{453152\sqrt{-x^4+x^2+2}}$
risch	$\frac{25(x^4-x^2-2)x(2505x^2+3983)}{453152(5x^2+7)^2\sqrt{-x^4+x^2+2}}+\frac{2505\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}}{2}\right.\right.}{906304\sqrt{-x^4+x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -25/952*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2-12525/453152*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)-263/453152*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-2505/906304*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))+58915/3172064*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + x^2 + 2)/(125*x^10 + 400*x^8 - 40*x^6 - 1442*x^4 - 18
13*x^2 - 686), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x^{2}-2\right)\left(x^{2}+1\right)}\,\left(5x^{2}+7\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x**2 - 2)*(x**2 + 1))*(5*x**2 + 7)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^4 + x^2 + 2)*(5*x^2 + 7)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(5\,x^2+7\right)^3\,\sqrt{-x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(1/2)), x)
```

3.339 $\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=93

$$\frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}}+\frac{27500}{3}x\sqrt{2+x^2-x^4}+625x^3\sqrt{2+x^2-x^4}-\frac{3482293}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+$$

```
[Out] -3482293/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+627857/6*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))+1/18*x*(1419793*x^2+1419985)/(-x^4+x^2+2)^(1/2)+27500/3*x*(
-x^4+x^2+2)^(1/2)+625*x^3*(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1219, 1693, 1194, 538, 435, 430}

$$\frac{627857}{6}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{3482293}{18}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{27500}{3}\sqrt{-x^4+x^2+2}\,x+\frac{(1419793x^2+1419985)\,x}{18\sqrt{-x^4+x^2+2}}+625\sqrt{-x^4+x^2+2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(1419985 + 1419793*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (27500*x*Sqrt[2 + x^
2 - x^4])/3 + 625*x^3*Sqrt[2 + x^2 - x^4] - (3482293*EllipticE[ArcSin[x/Sqr
t[2]], -2])/18 + (627857*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \frac{(7+5x^2)^5}{(2+x^2-x^4)^{3/2}}\,dx = \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{1268722+3084793x^2+450000x^4+56250x^6}{\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} + 625x^3\sqrt{2+x^2-x^4} + \frac{1}{90}\int \frac{-6343610-15761465x^2}{\sqrt{2+x^2-x^4}}$$
$$= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} + \frac{27500}{3}x\sqrt{2+x^2-x^4} + 625x^3\sqrt{2+x^2-x^4} - \frac{1}{270}\int$$
$$= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} + \frac{27500}{3}x\sqrt{2+x^2-x^4} + 625x^3\sqrt{2+x^2-x^4} - \frac{1}{135}\int$$
$$= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} + \frac{27500}{3}x\sqrt{2+x^2-x^4} + 625x^3\sqrt{2+x^2-x^4} - \frac{348229}{18}$$
$$= \frac{x(1419985+1419793x^2)}{18\sqrt{2+x^2-x^4}} + \frac{27500}{3}x\sqrt{2+x^2-x^4} + 625x^3\sqrt{2+x^2-x^4} - \frac{348229}{18}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.08, size = 97, normalized size = 1.04

$$\frac{1749985x+1607293x^3-153750x^5-11250x^7-3482293i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+4281654i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{18\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^5/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (1749985*x + 1607293*x^3 - 153750*x^5 - 11250*x^7 - (3482293*I)*Sqrt[4 + 2*
x^2 - 2*x^4]*EllipticE[I*ArcSinh[x], -1/2] + (4281654*I)*Sqrt[4 + 2*x^2 - 2
*x^4]*EllipticF[I*ArcSinh[x], -1/2])/(18*Sqrt[2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(85) = 170.
time = 0.13, size = 280, normalized size = 3.01

method	result
risch	$-\frac{x(11250x^6+153750x^4-1607293x^2-1749985)}{18\sqrt{-x^4+x^2+2}} + \frac{3482293\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{2}\right)-\mathrm{Ellip}\right.}{36\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{\frac{1419793}{18}x^3+\frac{1419985}{18}x}{\sqrt{-x^4+x^2+2}} + 625x^3\sqrt{-x^4+x^2+2} + \frac{27500x\sqrt{-x^4+x^2+2}}{3} - \frac{799361\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2}}{18\sqrt{-x^4+}}$
default	$\frac{\frac{53125}{9}x^3+\frac{43750}{9}x}{\sqrt{-x^4+x^2+2}} + 625x^3\sqrt{-x^4+x^2+2} + \frac{27500x\sqrt{-x^4+x^2+2}}{3} - \frac{799361\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2}}{18\sqrt{-x^4+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6250*(17/18*x^3+7/9*x)/(-x^4+x^2+2)^(1/2)+625*x^3*(-x^4+x^2+2)^(1/2)+27500/
3*x*(-x^4+x^2+2)^(1/2)-799361/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x
^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+3482293/36*2^(1/2)*(-2*x
^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(
1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+43750*(7/18*x^3+5/9*x)/(-x^4+x^2+
2)^(1/2)+122500*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^(1/2)+171500*(1/18*x^3+2/9*x)
/(-x^4+x^2+2)^(1/2)+120050*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+33614*(5/36*
x-1/36*x^3)/(-x^4+x^2+2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [A]
time = 0.12, size = 50, normalized size = 0.54

$$\frac{(5625\,x^8+76875\,x^6+937500\,x^4-2616139\,x^2-3482293)\sqrt{-x^4+x^2+2}}{9\,(x^5-x^3-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(5625*x^8 + 76875*x^6 + 937500*x^4 - 2616139*x^2 - 3482293)*sqrt(-x^4 +
 x^2 + 2)/(x^5 - x^3 - 2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(5x^2+7)^5}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**5/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^5/(-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(5\,x^2+7\right)^5}{{\left(-x^4+x^2+2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^5/(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^5/(x^2 - x^4 + 2)^(3/2), x)
```

3.340 $\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}}+\frac{625}{3}x\sqrt{2+x^2-x^4}-\frac{165239}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{31921}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -165239/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+31921/6*EllipticF(1/2*x*2^(1/
2),I*2^(1/2))+1/18*x*(83489*x^2+83585)/(-x^4+x^2+2)^(1/2)+625/3*x*(-x^4+x^2
+2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1219, 1693, 1194, 538, 435, 430}

$$\frac{31921}{6}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{165239}{18}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{625}{3}\sqrt{-x^4+x^2+2}\,x+\frac{(83489x^2+83585)\,x}{18\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(83585 + 83489*x^2))/(18*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^
4])/3 - (165239*EllipticE[ArcSin[x/Sqrt[2]], -2])/18 + (31921*EllipticF[Arc
Sin[x/Sqrt[2]], -2])/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-b/a, -d/c]))))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \frac{(7+5x^2)^4}{(2+x^2-x^4)^{3/2}}\,dx = \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{61976+157739x^2+11250x^4}{\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} + \frac{1}{54}\int \frac{-208428-495717x^2}{\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} + \frac{1}{27}\int \frac{-208428-495717x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx$$
$$= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} - \frac{165239}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{31921}{3}$$
$$= \frac{x(83585+83489x^2)}{18\sqrt{2+x^2-x^4}} + \frac{625}{3}x\sqrt{2+x^2-x^4} - \frac{165239}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) +$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 92, normalized size = 1.24

$$\frac{91085x+87239x^3-3750x^5-165239i\sqrt{4+2x^2-2x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+199977i\sqrt{4+2x^2-2x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{18\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^4/(2 + x^2 - x^4)^(3/2),x]
[Out] (91085*x + 87239*x^3 - 3750*x^5 - (165239*I)*Sqrt[4 + 2*x^2 - 2*x^4]*Ellipt
icE[I*ArcSinh[x], -1/2] + (199977*I)*Sqrt[4 + 2*x^2 - 2*x^4]*EllipticF[I*Ar
cSinh[x], -1/2])/(18*Sqrt[2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(68) = 136.
time = 0.14, size = 240, normalized size = 3.24

method	result
risch	$-\frac{x(3750x^4-87239x^2-91085)}{18\sqrt{-x^4+x^2+2}}+\frac{165239\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\text{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\right.\right.}{36\sqrt{-x^4+x^2+2}}$
elliptic	$\frac{\frac{83489}{18}x^3+\frac{83585}{18}x}{\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{3}+\frac{165239\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)\right.}{36\sqrt{-x^4+x^2+2}}$
default	$\frac{\frac{4375}{9}x^3+\frac{6250}{9}x}{\sqrt{-x^4+x^2+2}}+\frac{625x\sqrt{-x^4+x^2+2}}{3}-\frac{17369\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1250*(7/18*x^3+5/9*x)/(-x^4+x^2+2)^(1/2)+625/3*x*(-x^4+x^2+2)^(1/2)-17369/9
*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^
(1/2)*x,I*2^(1/2))+165239/36*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x
^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2
^(1/2)))+7000*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^(1/2)+14700*(1/18*x^3+2/9*x)/(-
x^4+x^2+2)^(1/2)+13720*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+4802*(5/36*x-1/3
6*x^3)/(-x^4+x^2+2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [A]
time = 0.09, size = 45, normalized size = 0.61

$$\frac{(1875\,x^6+39000\,x^4-128162\,x^2-165239)\sqrt{-x^4+x^2+2}}{9\,(x^5-x^3-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(1875*x^6 + 39000*x^4 - 128162*x^2 - 165239)*sqrt(-x^4 + x^2 + 2)/(x^5
- x^3 - 2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(5x^2+7)^4}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**4/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^4/(-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5\,x^2+7)^4}{(-x^4+x^2+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4/(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^4/(x^2 - x^4 + 2)^(3/2), x)
```

3.341 $\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}}-\frac{7147}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1763}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -7147/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1763/6*EllipticF(1/2*x*2^(1/2),
I*2^(1/2))+1/18*x*(4897*x^2+4945)/(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.208$, Rules used = {1219, 1194, 538, 435, 430}

$$\frac{1763}{6}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{7147}{18}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(4945 + 4897*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (7147*EllipticE[ArcSin[x/S
qrt[2]], -2])/18 + (1763*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c])))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(7+5x^2)^3}{(2+x^2-x^4)^{3/2}}\,dx &= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{1858+7147x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9}\int \frac{1858+7147x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{7147}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{1763}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(4945+4897x^2)}{18\sqrt{2+x^2-x^4}} - \frac{7147}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1763}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 79, normalized size = 1.44

$$\frac{1}{18}\left(\frac{4945x}{\sqrt{2+x^2-x^4}} + \frac{4897x^3}{\sqrt{2+x^2-x^4}} - 7147i\sqrt{2}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) + 8076i\sqrt{2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] ((4945*x)/Sqrt[2 + x^2 - x^4] + (4897*x^3)/Sqrt[2 + x^2 - x^4] - (7147*I)*S
qrt[2]*EllipticE[I*ArcSinh[x], -1/2] + (8076*I)*Sqrt[2]*EllipticF[I*ArcSinh
[x], -1/2])/18
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(53) = 106.
time = 0.12, size = 202, normalized size = 3.67

method	result
risch	$\frac{x(4897x^2+4945)}{18\sqrt{-x^4+x^2+2}}+\frac{7147\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\text{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}$ -
elliptic	$\frac{\frac{4897}{18}x^3+\frac{4945}{18}x}{\sqrt{-x^4+x^2+2}}-\frac{929\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{7147\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2}}{}$
default	$\frac{\frac{625}{9}x^3+\frac{250}{9}x}{\sqrt{-x^4+x^2+2}}-\frac{929\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{7147\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 250*(5/18*x^3+1/9*x)/(-x^4+x^2+2)^(1/2)-929/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^
2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+7147/36*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1
/2)*x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+1050*(1/18*x^3+2/9*x)/
(-x^4+x^2+2)^(1/2)+1470*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+686*(5/36*x-1/3
6*x^3)/(-x^4+x^2+2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [A]
time = 0.08, size = 40, normalized size = 0.73

$$\frac{(1125\,x^4-6046\,x^2-7147)\sqrt{-x^4+x^2+2}}{9\,(x^5-x^3-2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/9*(1125*x^4 - 6046*x^2 - 7147)*sqrt(-x^4 + x^2 + 2)/(x^5 - x^3 - 2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^3}{\left(-(x^2-2)(x^2+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
[Out] Integral((5*x**2 + 7)**3/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/(-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(5\,x^2+7)^3}{(-x^4+x^2+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(3/2),x)
[Out] int((5*x^2 + 7)^3/(x^2 - x^4 + 2)^(3/2), x)
```

3.342 $\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}}-\frac{281}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{139}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -281/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+139/6*EllipticF(1/2*x*2^(1/2),I*
2^(1/2))+1/18*x*(281*x^2+305)/(-x^4+x^2+2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1219, 1194, 538, 435, 430}

$$\frac{139}{6}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{281}{18}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(305 + 281*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (281*EllipticE[ArcSin[x/Sqrt
[2]], -2])/18 + (139*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c])))))
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(7+5x^2)^2}{(2+x^2-x^4)^{3/2}}\,dx &= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{-136+281x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9}\int \frac{-136+281x^2}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{281}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{139}{3}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2x^2}}\,dx \\
&= \frac{x(305+281x^2)}{18\sqrt{2+x^2-x^4}} - \frac{281}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{139}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.06, size = 79, normalized size = 1.44

$$
\frac{1}{18}\left(\frac{305x}{\sqrt{2+x^2-x^4}} + \frac{281x^3}{\sqrt{2+x^2-x^4}} - 281i\sqrt{2}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) + 213i\sqrt{2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] ((305*x)/Sqrt[2 + x^2 - x^4] + (281*x^3)/Sqrt[2 + x^2 - x^4] - (281*I)*Sqrt
[2]*EllipticE[I*ArcSinh[x], -1/2] + (213*I)*Sqrt[2]*EllipticF[I*ArcSinh[x],
 -1/2])/18
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(53) = 106.
time = 0.12, size = 179, normalized size = 3.25

method	result
risch	$\frac{x(281x^2+305)}{18\sqrt{-x^4+x^2+2}}+\frac{281\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\text{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}+$
elliptic	$\frac{\frac{281}{18}x^3+\frac{305}{18}x}{\sqrt{-x^4+x^2+2}}+\frac{281\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\text{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}+$
default	$\frac{\frac{25}{9}x^3+\frac{100}{9}x}{\sqrt{-x^4+x^2+2}}+\frac{34\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}+\frac{281\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 50*(1/18*x^3+2/9*x)/(-x^4+x^2+2)^(1/2)+34/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1
)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+281/36*2^(1/2
)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*
x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+140*(1/9*x^3-1/18*x)/(-x^4
+x^2+2)^(1/2)+98*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^2}{(-(x^2-2)(x^2+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/(-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(5\,x^2+7)^2}{(-x^4+x^2+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(x^2 - x^4 + 2)^(3/2), x)
```

3.343 $\int \frac{7+5x^2}{\left(2+x^2-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}}-\frac{13}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{17}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] -13/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+17/6*EllipticF(1/2*x*2^(1/2),I*2^
(1/2))+1/18*x*(13*x^2+25)/(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1192, 1194, 538, 435, 430}

$$\frac{17}{6}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{13}{18}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] (x*(25 + 13*x^2))/(18*Sqrt[2 + x^2 - x^4]) - (13*EllipticE[ArcSin[x/Sqrt[2]
], -2])/18 + (17*EllipticF[ArcSin[x/Sqrt[2]], -2])/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{7+5x^2}{(2+x^2-x^4)^{3/2}}\,dx &= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}}-\frac{1}{18}\int \frac{-38+13x^2}{\sqrt{2+x^2-x^4}}\,dx\\
&= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}}-\frac{1}{9}\int \frac{-38+13x^2}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\
&= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}}-\frac{13}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx+\frac{17}{3}\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx\\
&= \frac{x(25+13x^2)}{18\sqrt{2+x^2-x^4}}-\frac{13}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{17}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 79, normalized size = 1.44

$$\frac{1}{18}\left(\frac{25x}{\sqrt{2+x^2-x^4}}+\frac{13x^3}{\sqrt{2+x^2-x^4}}-13i\sqrt{2}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-6i\sqrt{2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/(2 + x^2 - x^4)^(3/2),x]
```

```
[Out] ((25*x)/Sqrt[2 + x^2 - x^4] + (13*x^3)/Sqrt[2 + x^2 - x^4] - (13*I)*Sqrt[2]
*EllipticE[I*ArcSinh[x], -1/2] - (6*I)*Sqrt[2]*EllipticF[I*ArcSinh[x], -1/2
])/18
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(53) = 106.
time = 0.04, size = 156, normalized size = 2.84

method	result
risch	$\frac{x(13x^2+25)}{18\sqrt{-x^4+x^2+2}}+\frac{13\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)\right)}{36\sqrt{-x^4+x^2+2}}+$
elliptic	$\frac{\frac{13}{18}x^3+\frac{25}{18}x}{\sqrt{-x^4+x^2+2}}+\frac{19\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{13\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+}}{}$
default	$\frac{\frac{10}{9}x^3-\frac{5}{9}x}{\sqrt{-x^4+x^2+2}}+\frac{19\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{18\sqrt{-x^4+x^2+2}}+\frac{13\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 10*(1/9*x^3-1/18*x)/(-x^4+x^2+2)^(1/2)+19/18*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+13/36*2^(1/2
)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*
x,I*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))+14*(5/36*x-1/36*x^3)/(-x^4
+x^2+2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+7}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/(-(x**2 - 2)*(x**2 + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/(-x^4 + x^2 + 2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{5\,x^2+7}{{\left(-x^4+x^2+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)/(x^2 - x^4 + 2)^(3/2), x)
```

3.344 $\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}}+\frac{1}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

```
[Out] 1/18*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1/6*EllipticF(1/2*x*2^(1/2),I*2^(1/
2))+1/18*x*(-x^2+5)/(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1106, 1194, 538, 435, 430}

$$\frac{1}{6}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{18}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{x(5-x^2)}{18\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 - x^4)^(-3/2),x]
```

```
[Out] (x*(5 - x^2))/(18*Sqrt[2 + x^2 - x^4]) + EllipticE[ArcSin[x/Sqrt[2]], -2]/1
8 + EllipticF[ArcSin[x/Sqrt[2]], -2]/6
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(2+x^2-x^4)^{3/2}}\,dx &= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{18}\int \frac{-4-x^2}{\sqrt{2+x^2-x^4}}\,dx \\
&= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} - \frac{1}{9}\int \frac{-4-x^2}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx \\
&= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} + \frac{1}{18}\int \frac{\sqrt{2+2x^2}}{\sqrt{4-2x^2}}\,dx + \frac{1}{3}\int \frac{1}{\sqrt{4-2x^2}\,\sqrt{2+2x^2}}\,dx \\
&= \frac{x(5-x^2)}{18\sqrt{2+x^2-x^4}} + \frac{1}{18}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1}{6}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.89, size = 79, normalized size = 1.44

$$\frac{1}{18}\left(\frac{5x}{\sqrt{2+x^2-x^4}} - \frac{x^3}{\sqrt{2+x^2-x^4}} + i\sqrt{2}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) - 3i\sqrt{2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 - x^4)^(-3/2),x]
```

```
[Out] ((5*x)/Sqrt[2 + x^2 - x^4] - x^3/Sqrt[2 + x^2 - x^4] + I*Sqrt[2]*EllipticE[
I*ArcSinh[x], -1/2] - (3*I)*Sqrt[2]*EllipticF[I*ArcSinh[x], -1/2])/18
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(53) = 106.
time = 0.04, size = 133, normalized size = 2.42

method	result
risch	$-\frac{x(x^2-5)}{18\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+}}{}$
default	$\frac{\frac{5}{18}x-\frac{1}{18}x^3}{\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,(}{3}$
elliptic	$\frac{\frac{5}{18}x-\frac{1}{18}x^3}{\sqrt{-x^4+x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{9\sqrt{-x^4+x^2+2}}-\frac{\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,(}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(5/36*x-1/36*x^3)/(-x^4+x^2+2)^(1/2)+1/9*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)
^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))-1/36*2^(1/2)*(
-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*(EllipticF(1/2*2^(1/2)*x,I
*2^(1/2))-EllipticE(1/2*2^(1/2)*x,I*2^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^4+x^2+2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral((-x**4 + x**2 + 2)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + x^2 + 2)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 - x^4 + 2)^(3/2),x)
```

```
[Out] int(1/(x^2 - x^4 + 2)^(3/2), x)
```

3.345 $\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}}+\frac{8}{153}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{1}{102}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|$$

```
[Out] 8/153*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+1/102*EllipticF(1/2*x*2^(1/2),I*2^
(1/2))-25/238*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/306*x*(-16*x^2+35
)/(-x^4+x^2+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1235, 1192, 1194, 538, 435, 430, 1226, 551}

$$\frac{1}{102}F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{8}{153}E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)-\frac{25}{238}\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)+\frac{x(35-16x^2)}{306\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(35 - 16*x^2))/(306*Sqrt[2 + x^2 - x^4]) + (8*EllipticE[ArcSin[x/Sqrt[2]
], -2])/153 + EllipticF[ArcSin[x/Sqrt[2]], -2]/102 - (25*EllipticPi[-10/7,
ArcSin[x/Sqrt[2]], -2])/238
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)(2+x^2-x^4)^{3/2}}\,dx = -\left(\frac{1}{34}\int \frac{-12+5x^2}{(2+x^2-x^4)^{3/2}}\,dx\right) - \frac{25}{34}\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx$$
$$= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} + \frac{1}{612}\int \frac{38+32x^2}{\sqrt{2+x^2-x^4}}\,dx - \frac{25}{17}\int \frac{1}{\sqrt{4-2x^2}\sqrt{2+2}}$$
$$= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} - \frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1}{306}\int \frac{38}{\sqrt{4-2}}$$
$$= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} - \frac{25}{238}\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1}{51}\int \frac{}{\sqrt{4-2x}}$$
$$= \frac{x(35-16x^2)}{306\sqrt{2+x^2-x^4}} + \frac{8}{153}E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right) + \frac{1}{102}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.15, size = 101, normalized size = 1.40

$$\frac{\frac{490x}{\sqrt{2+x^2-x^4}} - \frac{224x^3}{\sqrt{2+x^2-x^4}} + 224i\sqrt{2}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) - 357i\sqrt{2}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right) + 225i\sqrt{2}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{4284}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] ((490*x)/Sqrt[2 + x^2 - x^4] - (224*x^3)/Sqrt[2 + x^2 - x^4] + (224*I)*Sqrt
[2]*EllipticE[I*ArcSinh[x], -1/2] - (357*I)*Sqrt[2]*EllipticF[I*ArcSinh[x],
 -1/2] + (225*I)*Sqrt[2]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/4284
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(69) = 138.
time = 0.13, size = 164, normalized size = 2.28

method	result
default	$\frac{-\frac{8}{153}x^3+\frac{35}{306}x}{\sqrt{-x^4+x^2+2}} + \frac{\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{204\sqrt{-x^4+x^2+2}} + \frac{4\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}}{153\sqrt{-x^4+x}}$
elliptic	$\frac{-\frac{8}{153}x^3+\frac{35}{306}x}{\sqrt{-x^4+x^2+2}} + \frac{\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{204\sqrt{-x^4+x^2+2}} + \frac{4\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}}{153\sqrt{-x^4+x}}$
risch	$-\frac{x(16x^2-35)}{306\sqrt{-x^4+x^2+2}} - \frac{4\sqrt{2}\sqrt{-2x^2+4}\sqrt{x^2+1}\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)\right)}{153\sqrt{-x^4+x^2+2}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-4/153*x^3+35/612*x)/(-x^4+x^2+2)^(1/2)+1/204*2^(1/2)*(-2*x^2+4)^(1/2)*(
x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+4/153*2^
(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/
2)*x,I*2^(1/2))-25/238*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(5*x^10 - 3*x^8 - 29*x^6 - x^4 + 48*x^2 + 28)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x^{2}-2\right)\left(x^{2}+1\right)\right)^{\frac{3}{2}}\cdot\left(5x^{2}+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(-x**4+x**2+2)**(3/2),x)
```

```
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(5\,x^2+7)\,\left(-x^4+x^2+2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)*(x^2 - x^4 + 2)^(3/2)), x)
```

3.346 $\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)}+\frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656}+\frac{89F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24276}-\frac{10\cdots}{\cdots}$$

```
[Out] 5143/145656*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+89/24276*EllipticF(1/2*x*2^(
1/2),I*2^(1/2))-10825/113288*EllipticPi(1/2*x*2^(1/2),-10/7,I*2^(1/2))+1/10
404*x*(-287*x^2+580)/(-x^4+x^2+2)^(1/2)+625/16184*x*(-x^4+x^2+2)^(1/2)/(5*x
^2+7)
```

Rubi [A]
time = 0.19, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1242, 1192, 1194, 538, 435, 430, 1237, 1730, 1226, 551}

$$\frac{89F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{24276}+\frac{5143E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{145656}-\frac{10825\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{113288}+\frac{625\sqrt{-x^4+x^2+2}\,x}{16184(5x^2+7)}+\frac{(580-287x^2)\,x}{10404\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(580 - 287*x^2))/(10404*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 - x^4
])/(16184*(7 + 5*x^2)) + (5143*EllipticE[ArcSin[x/Sqrt[2]], -2])/145656 + (
89*EllipticF[ArcSin[x/Sqrt[2]], -2])/24276 - (10825*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/113288
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
```

```
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^2(2+x^2-x^4)^{3/2}}\,dx = \int \left(\frac{194-95x^2}{1156(2+x^2-x^4)^{3/2}} - \frac{25}{34(7+5x^2)^2\sqrt{2+x^2-x^4}} - \frac{}{1156(7+}\right.$$

$$= \frac{\int \frac{194-95x^2}{(2+x^2-x^4)^{3/2}}\,dx}{1156} - \frac{475\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx}{1156} - \frac{25}{34}\int \frac{}{(7+5x^2)^2}$$

$$= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} - \frac{\int \frac{-586-574x^2}{\sqrt{2+x^2-x^4}}\,dx}{20808} -$$

$$= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} - \frac{475\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.}{8092}$$

$$= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} - \frac{475\Pi\left(-\frac{10}{7};\sin^{-1}\left(\frac{x}{\sqrt{2}}\right.\right.}{8092}$$

$$= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} + \frac{287E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right| -}{10404}$$

$$= \frac{x(580-287x^2)}{10404\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{16184(7+5x^2)} + \frac{5143E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right|}{145656}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.13, size = 196, normalized size = 1.96

$$\frac{953260x+253386x^3-360010x^5+72002i\sqrt{2}\,(7+5x^2)\,\sqrt{2+x^2-x^4}\,E\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)-111741i\sqrt{2}\,(7+5x^2)\,\sqrt{2+x^2-x^4}\,F\left(i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+681975i\sqrt{2}\,\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)+487125i\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\,\Pi\left(\frac{5}{7};i\sinh^{-1}(x)\middle|-\frac{1}{2}\right)}{2039184\,(7+5x^2)\,\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (953260*x + 253386*x^3 - 360010*x^5 + (72002*I)*Sqrt[2]*(7 + 5*x^2)*Sqrt[2
+ x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2] - (111741*I)*Sqrt[2]*(7 + 5*x^2)
*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSinh[x], -1/2] + (681975*I)*Sqrt[2]*Sqr
t[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2] + (487125*I)*Sqrt[2]*x
^2*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(2039184*(7 + 5
*x^2)*Sqrt[2 + x^2 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(93) = 186.
time = 0.14, size = 188, normalized size = 1.88

method	result
default	$\frac{625x\sqrt{-x^4+x^2+2}}{16184(5x^2+7)}+\frac{-\frac{287}{10404}x^3+\frac{145}{2601}x}{\sqrt{-x^4+x^2+2}}+\frac{89\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{48552\sqrt{-x^4+x^2+2}}+\frac{514}{}$
elliptic	$\frac{625x\sqrt{-x^4+x^2+2}}{16184(5x^2+7)}+\frac{-\frac{287}{10404}x^3+\frac{145}{2601}x}{\sqrt{-x^4+x^2+2}}+\frac{89\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{48552\sqrt{-x^4+x^2+2}}+\frac{514}{}$
risch	$-\frac{x(25715x^4-18099x^2-68090)}{145656(5x^2+7)\sqrt{-x^4+x^2+2}}-\frac{5143\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{\sqrt{2}}{2}\right.\right.}{291312\sqrt{-x^4+x^2+2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 625/16184*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)+2*(-287/20808*x^3+145/5202*x)/(-x^
4+x^2+2)^(1/2)+89/48552*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)
^(1/2)*EllipticF(1/2*2^(1/2)*x,I*2^(1/2))+5143/291312*2^(1/2)*(-2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))-108
25/113288*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Ellipt
icPi(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + x^2 + 2)/(25*x^12 + 20*x^10 - 166*x^8 - 208*x^6 + 233*
x^4 + 476*x^2 + 196), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x^2-2\right)\left(x^2+1\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(-x**4+x**2+2)**(3/2),x)
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(5\,x^2+7\right)}^2\,{\left(-x^4+x^2+2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)^2*(x^2 - x^4 + 2)^(3/2)), x)
```

3.347 $\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{x(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}}+\frac{625x\sqrt{2+x^2-x^4}}{32368\,(7+5x^2)^2}+\frac{645625x\sqrt{2+x^2-x^4}}{15407168\,(7+5x^2)}+\frac{3086453E\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{138664512}+\frac{60\cdots}{}$$

```
[Out] 3086453/138664512*EllipticE(1/2*x*2^(1/2),I*2^(1/2))+60409/23110752*Ellipti
cF(1/2*x*2^(1/2),I*2^(1/2))-6898575/107850176*EllipticPi(1/2*x*2^(1/2),-10/
7,I*2^(1/2))+1/353736*x*(-4909*x^2+9830)/(-x^4+x^2+2)^(1/2)+625/32368*x*(-x
^4+x^2+2)^(1/2)/(5*x^2+7)^2+645625/15407168*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)
```

Rubi [A]

time = 0.36, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1242, 1192, 1194, 538, 435, 430, 1237, 1710, 1730, 1226, 551}

$$\frac{60409F\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{23110752}+\frac{3086453E\left(\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{138664512}-\frac{6898575\Pi\left(-\frac{10}{7};\text{ArcSin}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{107850176}+\frac{645625\sqrt{-x^4+x^2+2}\,x}{15407168\,(5x^2+7)}+\frac{625\sqrt{-x^4+x^2+2}\,x}{32368\,(5x^2+7)^2}+\frac{(9830-4909x^2)\,x}{353736\sqrt{-x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (x*(9830 - 4909*x^2))/(353736*Sqrt[2 + x^2 - x^4]) + (625*x*Sqrt[2 + x^2 -
x^4])/(32368*(7 + 5*x^2)^2) + (645625*x*Sqrt[2 + x^2 - x^4])/(15407168*(7 +
 5*x^2)) + (3086453*EllipticE[ArcSin[x/Sqrt[2]], -2])/138664512 + (60409*El
lipticF[ArcSin[x/Sqrt[2]], -2])/23110752 - (6898575*EllipticPi[-10/7, ArcSi
n[x/Sqrt[2]], -2])/107850176
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
```

```
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1226

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/((d + e*x^2
)*Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
```

```
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^3 (2+x^2-x^4)^{3/2}}\,dx = \int \left(-\frac{-3278+1635x^2}{39304\,(2+x^2-x^4)^{3/2}} - \frac{25}{34\,(7+5x^2)^3\sqrt{2+x^2-x^4}} - \frac{}{1156\,(7}\right.$$

$$= -\frac{\int \frac{-3278+1635x^2}{(2+x^2-x^4)^{3/2}}\,dx}{39304} - \frac{8175\int \frac{1}{(7+5x^2)\sqrt{2+x^2-x^4}}\,dx}{39304} - \frac{475\int \frac{}{(7+5x^2)^2\sqrt{}}}{1}$$

$$= \frac{x(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\,(7+5x^2)^2} + \frac{11875x\sqrt{2+x^2-x^4}}{550256\,(7+5x^2)}$$

$$= \frac{x(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\,(7+5x^2)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\,(7+5x^2)}$$

$$= \frac{x(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\,(7+5x^2)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\,(7+5x^2)}$$

$$= \frac{x(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\,(7+5x^2)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\,(7+5x^2)}$$

$$= \frac{x(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\,(7+5x^2)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\,(7+5x^2)}$$

$$= \frac{x(9830-4909x^2)}{353736\sqrt{2+x^2-x^4}} + \frac{625x\sqrt{2+x^2-x^4}}{32368\,(7+5x^2)^2} + \frac{645625x\sqrt{2+x^2-x^4}}{15407168\,(7+5x^2)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 244, normalized size = 1.91

$$\frac{3857257460x + 3876617542x^3 - 737347940x^5 - 1080258550x^7 + 43210342i\sqrt{2}\,(7+5x^2)^2\sqrt{2+x^2-x^4}\,E\left(i\sinh^{-1}(x)\middle|-\tfrac{1}{2}\right) - 67352691i\sqrt{2}\,(7+5x^2)^2\sqrt{2+x^2-x^4}\,F\left(i\sinh^{-1}(x)\middle|-\tfrac{1}{2}\right) + 3042271575i\sqrt{2}\sqrt{2+x^2-x^4}\,\Pi\left(\tfrac{5}{7};i\sinh^{-1}(x)\middle|-\tfrac{1}{2}\right) + 4346102250i\sqrt{2}\,x^2\sqrt{2+x^2-x^4}\,\Pi\left(\tfrac{5}{7};i\sinh^{-1}(x)\middle|-\tfrac{1}{2}\right) + 1552179375i\sqrt{2}\,x^4\sqrt{2+x^2-x^4}\,\Pi\left(\tfrac{5}{7};i\sinh^{-1}(x)\middle|-\tfrac{1}{2}\right)}{1941303168\,(7+5x^2)^2\sqrt{2+x^2-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*(2 + x^2 - x^4)^(3/2)),x]
```

```
[Out] (3857257460*x + 3876617542*x^3 - 737347940*x^5 - 1080258550*x^7 + (43210342
*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticE[I*ArcSinh[x], -1/2]
 - (67352691*I)*Sqrt[2]*(7 + 5*x^2)^2*Sqrt[2 + x^2 - x^4]*EllipticF[I*ArcSi
nh[x], -1/2] + (3042271575*I)*Sqrt[2]*Sqrt[2 + x^2 - x^4]*EllipticPi[5/7, I
*ArcSinh[x], -1/2] + (4346102250*I)*Sqrt[2]*x^2*Sqrt[2 + x^2 - x^4]*Ellipti
cPi[5/7, I*ArcSinh[x], -1/2] + (1552179375*I)*Sqrt[2]*x^4*Sqrt[2 + x^2 - x^
4]*EllipticPi[5/7, I*ArcSinh[x], -1/2])/(1941303168*(7 + 5*x^2)^2*Sqrt[2 +
x^2 - x^4])
```

Maple [A]
time = 0.13, size = 212, normalized size = 1.66

method	result
risch	$-\frac{x(77161325x^6+52667710x^4-276901253x^2-275518390)}{138664512(5x^2+7)^2\sqrt{-x^4+x^2+2}} - \frac{3086453\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{\sqrt{2}\,x}{2}, i\sqrt{\right.\right.}}{277329024\sqrt{-x^4+x^2+2}}$
default	$\frac{625x\sqrt{-x^4+x^2+2}}{32368(5x^2+7)^2} + \frac{645625x\sqrt{-x^4+x^2+2}}{15407168(5x^2+7)} + \frac{-\frac{4909}{353736}x^3+\frac{4915}{176868}x}{\sqrt{-x^4+x^2+2}} + \frac{60409\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+}}{46221504\sqrt{-x^4}}$
elliptic	$\frac{625x\sqrt{-x^4+x^2+2}}{32368(5x^2+7)^2} + \frac{645625x\sqrt{-x^4+x^2+2}}{15407168(5x^2+7)} + \frac{-\frac{4909}{353736}x^3+\frac{4915}{176868}x}{\sqrt{-x^4+x^2+2}} + \frac{60409\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{x^2+}}{46221504\sqrt{-x^4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 625/32368*x*(-x^4+x^2+2)^(1/2)/(5*x^2+7)^2+645625/15407168*x*(-x^4+x^2+2)^(
1/2)/(5*x^2+7)+2*(-4909/707472*x^3+4915/353736*x)/(-x^4+x^2+2)^(1/2)+60409/
46221504*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*Elliptic
F(1/2*2^(1/2)*x,I*2^(1/2))+3086453/277329024*2^(1/2)*(-2*x^2+4)^(1/2)*(x^2+
1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticE(1/2*2^(1/2)*x,I*2^(1/2))-6898575/1078
50176*2^(1/2)*(1-1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(-x^4+x^2+2)^(1/2)*EllipticPi
(1/2*2^(1/2)*x,-10/7,I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + x^2 + 2)/(125*x^14 + 275*x^12 - 690*x^10 - 2202*x^8 -
291*x^6 + 4011*x^4 + 4312*x^2 + 1372), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(x^{2}-2\right)\left(x^{2}+1\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(-x**4+x**2+2)**(3/2),x)
[Out] Integral(1/((-(x**2 - 2)*(x**2 + 1))**(3/2)*(5*x**2 + 7)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(-x^4+x^2+2)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-x^4 + x^2 + 2)^(3/2)*(5*x^2 + 7)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(5\,x^2+7\right)}^3\,{\left(-x^4+x^2+2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)^3*(x^2 - x^4 + 2)^(3/2)), x)
```

3.348 $\int (7+5x^2)^4 \sqrt{4+3x^2+x^4}\, dx$

Optimal. Leaf size=242

$$\frac{51665x\sqrt{4+3x^2+x^4}}{33\left(2+x^2\right)}+\frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4}+\frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2}+\frac{23500}{99}x^3\left(4+3x^2\right.$$

```
[Out] 3050/11*x*(x^4+3*x^2+4)^(3/2)+23500/99*x^3*(x^4+3*x^2+4)^(3/2)+625/11*x^5*(
x^4+3*x^2+4)^(3/2)+51665/33*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/33*x*(4516*x^2+
18727)*(x^4+3*x^2+4)^(1/2)+33159/22*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2
)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2)))
,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-5
1665/33*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2
^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1693, 1190, 1211, 1117, 1209}

$$\frac{33159(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{11\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{51665\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33\sqrt{x^4+3x^2+4}}+\frac{3050}{11}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{33}\left(4516x^2+18727\right)\sqrt{x^4+3x^2+4}\,x+\frac{51665\sqrt{x^4+3x^2+4}\,x}{33\left(x^2+2\right)}+\frac{625}{11}\left(x^4+3x^2+4\right)^{3/2}x^5+\frac{23500}{99}\left(x^4+3x^2+4\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (51665*x*Sqrt[4 + 3*x^2 + x^4])/(33*(2 + x^2)) + (x*(18727 + 4516*x^2)*Sqrt
[4 + 3*x^2 + x^4])/33 + (3050*x*(4 + 3*x^2 + x^4)^(3/2))/11 + (23500*x^3*(4
 + 3*x^2 + x^4)^(3/2))/99 + (625*x^5*(4 + 3*x^2 + x^4)^(3/2))/11 - (51665*S
qrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/S
qrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4]) + (33159*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(11*Sqrt[2]*Sqrt
[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
```

```
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \left(7+5x^2\right)^4 \sqrt{4+3x^2+x^4}\, dx &= \frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{11}\int \sqrt{4+3x^2+x^4}\, \left(26411+75460x^2+683\right. \\
&= \frac{23500}{99}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{625}{11}x^5\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{99}\int \sqrt{4+3x^2} \\
&= \frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2}+\frac{23500}{99}x^3\left(4+3x^2+x^4\right)^{3/2}+\frac{625}{11}x^5\left(4+3x^2+\right. \\
&= \frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4}+\frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2}+\frac{2350}{99} \\
&= \frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4}+\frac{3050}{11}x\left(4+3x^2+x^4\right)^{3/2}+\frac{2350}{99} \\
&= \frac{51665x\sqrt{4+3x^2+x^4}}{33\left(2+x^2\right)}+\frac{1}{33}x\left(18727+4516x^2\right)\sqrt{4+3x^2+x^4}+\frac{3050}{11}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.69, size = 354, normalized size = 1.46

$$
\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(663924+1257535x^2+1217475x^4+712748x^6+264075x^8+57250x^{10}+5625x^{12})-154995\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+3\sqrt{2}\left(-36253i+51665\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{396\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^4*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(663924 + 1257535*x^2 + 1217475*x^4 + 7127
48*x^6 + 264075*x^8 + 57250*x^10 + 5625*x^12) - 154995*Sqrt[2]*(3*I + Sqrt[
7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt
[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-36253*I + 51665*Sq
rt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt
[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(396*Sqrt[(-I)/(-3*I + Sqrt[
7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 292, normalized size = 1.21

method	result

risch	$\frac{x(5625x^8+40375x^6+120450x^4+189898x^2+165981)\sqrt{x^4+3x^2+4}}{99} - \frac{1653280\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-(}{}$
default	$\frac{625x^9\sqrt{x^4+3x^2+4}}{11} + \frac{40375x^7\sqrt{x^4+3x^2+4}}{99} + \frac{189898x^3\sqrt{x^4+3x^2+4}}{99} + \frac{55327x\sqrt{x^4+3x^2+4}}{33}$
elliptic	$\frac{625x^9\sqrt{x^4+3x^2+4}}{11} + \frac{40375x^7\sqrt{x^4+3x^2+4}}{99} + \frac{189898x^3\sqrt{x^4+3x^2+4}}{99} + \frac{55327x\sqrt{x^4+3x^2+4}}{33}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 625/11*x^9*(x^4+3*x^2+4)^(1/2)+40375/99*x^7*(x^4+3*x^2+4)^(1/2)+189898/99*x
^3*(x^4+3*x^2+4)^(1/2)+55327/33*x*(x^4+3*x^2+4)^(1/2)+382496/33/(-6+2*I*7^(
1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)
^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*
I*7^(1/2))^(1/2))-1653280/33/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))
*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(
1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-El
lipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+3650/3*x^5
*(x^4+3*x^2+4)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(x^2-x+2)\,(x^2+x+2)}\,\left(5x^2+7\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(5\,x^2+7\right)}^4\,\sqrt{x^4+3\,x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.349 $\int (7+5x^2)^3 \sqrt{4+3x^2+x^4}\, dx$

Optimal. Leaf size=221

$$\frac{4717x\sqrt{4+3x^2+x^4}}{21\,(2+x^2)}+\frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4}+\frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{125}{9}x^3\left(4+3x^2+x^4\right.$$

```
[Out] 275/7*x*(x^4+3*x^2+4)^(3/2)+125/9*x^3*(x^4+3*x^2+4)^(3/2)+4717/21*x*(x^4+3*
x^2+4)^(1/2)/(x^2+2)+1/21*x*(407*x^2+1708)*(x^4+3*x^2+4)^(1/2)+1301/6*(x^2+
2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Elli
pticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(
1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-4717/21*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2
)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1
/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1
/2)
```

Rubi [A]
time = 0.07, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1693, 1190, 1211, 1117, 1209}

$$\frac{1301(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{4717\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{21\sqrt{x^4+3x^2+4}}+\frac{275}{7}\left(x^4+3x^2+4\right)^{3/2}x+\frac{1}{21}\left(407x^2+1708\right)\sqrt{x^4+3x^2+4}\,x+\frac{4717\sqrt{x^4+3x^2+4}\,x}{21\,(x^2+2)}+\frac{125}{9}\left(x^4+3x^2+4\right)^{3/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4717*x*Sqrt[4 + 3*x^2 + x^4])/(21*(2 + x^2)) + (x*(1708 + 407*x^2)*Sqrt[4
+ 3*x^2 + x^4])/21 + (275*x*(4 + 3*x^2 + x^4)^(3/2))/7 + (125*x^3*(4 + 3*x^
2 + x^4)^(3/2))/9 - (4717*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(21*Sqrt[4 + 3*x^2 + x^4]) + (130
1*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2
]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
```

```
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int (7+5x^2)^3 \sqrt{4+3x^2+x^4}\, dx = \frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2} + \frac{1}{9}\int \sqrt{4+3x^2+x^4}\,\left(3087+5115x^2+2475x\right.$$

$$= \frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2} + \frac{125}{9}x^3\left(4+3x^2+x^4\right)^{3/2} + \frac{1}{63}\int (11709+610$$

$$= \frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4} + \frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2} + \frac{125}{9}x^3$$

$$= \frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4} + \frac{275}{7}x\left(4+3x^2+x^4\right)^{3/2} + \frac{125}{9}x^3$$

$$= \frac{4717x\sqrt{4+3x^2+x^4}}{21\left(2+x^2\right)} + \frac{1}{21}x\left(1708+407x^2\right)\sqrt{4+3x^2+x^4} + \frac{275}{7}x(4$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.24, size = 349, normalized size = 1.58

$$\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(60096+93656x^2+71862x^4+30946x^6+7725x^8+875x^{10}\right)-14151\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+3\sqrt{2}\left(-3409i+4717\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{252\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(60096 + 93656*x^2 + 71862*x^4 + 30946*x^6
 + 7725*x^8 + 875*x^10) - 14151*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7
] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sq
rt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7
])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-3409*I + 4717*Sqrt[7])*Sqrt[(-3*I + Sqrt[
7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + S
qrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[
7])/(3*I + Sqrt[7])])/(252*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4
])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 275, normalized size = 1.24

method	result
risch	$\frac{x\left(875x^6+5100x^4+12146x^2+15024\right)\sqrt{x^4+3x^2+4}}{63} - \frac{150944\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right.}}{2}$

default	$\frac{125x^7\sqrt{x^4+3x^2+4}}{9}+\frac{1700x^5\sqrt{x^4+3x^2+4}}{21}+\frac{12146x^3\sqrt{x^4+3x^2+4}}{63}+\frac{5008x\sqrt{x^4+3x^2+4}}{21}+$
elliptic	$\frac{125x^7\sqrt{x^4+3x^2+4}}{9}+\frac{1700x^5\sqrt{x^4+3x^2+4}}{21}+\frac{12146x^3\sqrt{x^4+3x^2+4}}{63}+\frac{5008x\sqrt{x^4+3x^2+4}}{21}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 125/9*x^7*(x^4+3*x^2+4)^(1/2)+1700/21*x^5*(x^4+3*x^2+4)^(1/2)+12146/63*x^3*
(x^4+3*x^2+4)^(1/2)+5008/21*x*(x^4+3*x^2+4)^(1/2)+35120/21/(-6+2*I*7^(1/2))
^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2
)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(
1/2))^(1/2))-150944/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^
(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*
(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-Elliptic
E(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{(x^2-x+2)(x^2+x+2)}\left(5x^2+7\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(5\,x^2+7\right)^3\,\sqrt{x^4+3\,x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.350 $\int (7+5x^2)^2 \sqrt{4+3x^2+x^4}\, dx$

Optimal. Leaf size=198

$$\frac{319x\sqrt{4+3x^2+x^4}}{7(2+x^2)}+\frac{1}{7}x(119+38x^2)\sqrt{4+3x^2+x^4}+\frac{25}{7}x(4+3x^2+x^4)^{3/2}-\frac{319\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2\cdots}{(2+x^{\cdots}}}}{7\sqrt{4+\cdots}}$$

```
[Out] 25/7*x*(x^4+3*x^2+4)^(3/2)+319/7*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/7*x*(38*x^
2+119)*(x^4+3*x^2+4)^(1/2)+81/2*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1
/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4
*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-319/7
*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2))
)*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4
)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1190, 1211, 1117, 1209}

$$\frac{81(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{319\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{7\sqrt{x^4+3x^2+4}}+\frac{25}{7}x(x^4+3x^2+4)^{3/2}+\frac{1}{7}x(38x^2+119)\sqrt{x^4+3x^2+4}+\frac{319x\sqrt{x^4+3x^2+4}}{7(x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (319*x*Sqrt[4 + 3*x^2 + x^4])/(7*(2 + x^2)) + (x*(119 + 38*x^2)*Sqrt[4 + 3*
x^2 + x^4])/7 + (25*x*(4 + 3*x^2 + x^4)^(3/2))/7 - (319*Sqrt[2]*(2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(7*
Sqrt[4 + 3*x^2 + x^4]) + (81*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*
EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
```

```
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\int (7+5x^2)^2 \sqrt{4+3x^2+x^4}\, dx &= \frac{25}{7}x(4+3x^2+x^4)^{3/2} + \frac{1}{7}\int (243+190x^2)\sqrt{4+3x^2+x^4}\, dx \\
&= \frac{1}{7}x(119+38x^2)\sqrt{4+3x^2+x^4} + \frac{25}{7}x(4+3x^2+x^4)^{3/2} + \frac{1}{105}\int \frac{744}{\sqrt{4}} \\
&= \frac{1}{7}x(119+38x^2)\sqrt{4+3x^2+x^4} + \frac{25}{7}x(4+3x^2+x^4)^{3/2} - \frac{638}{7}\int \frac{}{\sqrt{4}} \\
&= \frac{319x\sqrt{4+3x^2+x^4}}{7(2+x^2)} + \frac{1}{7}x(119+38x^2)\sqrt{4+3x^2+x^4} + \frac{25}{7}x(4+3x
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.86, size = 343, normalized size = 1.73

$$\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(876+1109x^2+658x^4+188x^6+25x^8)-319\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\sqrt{2}\left(-35i+319\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{28\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(876 + 1109*x^2 + 658*x^4 + 188*x^6 + 25*x
^8) - 319*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I +
 Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*Ar
cSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] +
Sqrt[2]*(-35*I + 319*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(28*S
qrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 258, normalized size = 1.30

method	result
risch	$\frac{x(25x^4+113x^2+219)\sqrt{x^4+3x^2+4}}{7} - \frac{10208\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\text{EllipticF}\right.}{7\sqrt{-6+2i\sqrt{}}}$
default	$\frac{25x^5\sqrt{x^4+3x^2+4}}{7}+\frac{113x^3\sqrt{x^4+3x^2+4}}{7}+\frac{219x\sqrt{x^4+3x^2+4}}{7}+\frac{1984\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}}{}$
elliptic	$\frac{25x^5\sqrt{x^4+3x^2+4}}{7}+\frac{113x^3\sqrt{x^4+3x^2+4}}{7}+\frac{219x\sqrt{x^4+3x^2+4}}{7}+\frac{1984\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/7*x^5*(x^4+3*x^2+4)^(1/2)+113/7*x^3*(x^4+3*x^2+4)^(1/2)+219/7*x*(x^4+3*x
^2+4)^(1/2)+1984/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2
)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-
6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-10208/7/(-6+2*I*7^(1/2))^(1
/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
```

```
4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*
7^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(x^2 - x + 2)(x^2 + x + 2)}\,(5x^2 + 7)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(1/2),x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(5\,x^2+7\right)}^2\,\sqrt{x^4+3\,x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2),x)
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.351 $\int (7+5x^2) \sqrt{4+3x^2+x^4}\, dx$

Optimal. Leaf size=177

$$\frac{9x\sqrt{4+3x^2+x^4}}{2+x^2}+\frac{1}{3}x\left(10+3x^2\right)\sqrt{4+3x^2+x^4}-\frac{9\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}$$

```
[Out] 9*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/3*x*(3*x^2+10)*(x^4+3*x^2+4)^(1/2)+49/6*(
x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*
EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^
2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-9*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2
))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2
)
```

Rubi [A]
time = 0.03, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1190, 1211, 1117, 1209}

$$\frac{49(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{9\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+\frac{1}{3}(3x^2+10)\sqrt{x^4+3x^2+4}\,x+\frac{9\sqrt{x^4+3x^2+4}\,x}{x^2+2}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (x*(10 + 3*x^2)*Sqrt[4 + 3*x^2 + x^
4])/3 - (9*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (49*(2 + x^2)*Sqrt[(4 +
3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
```

```
b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int (7+5x^2)\sqrt{4+3x^2+x^4}\,dx = \frac{1}{3}x(10+3x^2)\sqrt{4+3x^2+x^4} + \frac{1}{15}\int \frac{220+135x^2}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{1}{3}x(10+3x^2)\sqrt{4+3x^2+x^4} - 18\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx + \frac{98}{3}\int \frac{}{\sqrt{4+3}}$$

$$= \frac{9x\sqrt{4+3x^2+x^4}}{2+x^2} + \frac{1}{3}x(10+3x^2)\sqrt{4+3x^2+x^4} - \frac{9\sqrt{2}\,(2+x^2)\sqrt{\frac{4}{(}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.41, size = 338, normalized size = 1.91

$$\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(40+42x^2+19x^4+3x^6) - 27\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right) + \sqrt{2}\left(-7i+27\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{12\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(40 + 42*x^2 + 19*x^4 + 3*x^6) - 27*Sqrt[2
]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[
```

```
(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I
)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I +
27*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I +
 Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*
I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-3*I + S
qrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.06, size = 240, normalized size = 1.36

method	result
risch	$\frac{x(3x^2+10)\sqrt{x^4+3x^2+4}}{3} - \frac{288\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-6}}{}\right.\right.}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x}}$
default	$x^3\sqrt{x^4+3x^2+4} + \frac{10x\sqrt{x^4+3x^2+4}}{3} + \frac{176\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4}}$
elliptic	$x^3\sqrt{x^4+3x^2+4} + \frac{10x\sqrt{x^4+3x^2+4}}{3} + \frac{176\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^3*(x^4+3*x^2+4)^(1/2)+10/3*x*(x^4+3*x^2+4)^(1/2)+176/3/(-6+2*I*7^(1/2))^(
1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/
(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/
2))^(1/2))-288/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1
-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(Ellipti
cF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*
(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(x^2 - x + 2)(x^2 + x + 2)}\,(5x^2 + 7)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(5\,x^2+7\right)\,\sqrt{x^4+3\,x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2), x)
```

3.352 $\int \sqrt{4+3x^2+x^4}\, dx$

Optimal. Leaf size=169

$$\frac{1}{3}x\sqrt{4+3x^2+x^4}+\frac{x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}+\frac{7(2+x^2)\sqrt{\cdots}}{\cdots}$$

```
[Out] 1/3*x*(x^4+3*x^2+4)^(1/2)+x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+7/6*(x^2+2)*(cos(2*
arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(
2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/
2)/(x^4+3*x^2+4)^(1/2)-(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2
*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))
*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1105, 1211, 1117, 1209}

$$\frac{7(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+\frac{\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{1}{3}\sqrt{x^4+3x^2+4}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/3 + (x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (Sqrt[2
]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2
]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (7*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt[2]*Sqrt[4 + 3*x^2 + x^
4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \sqrt{4+3x^2+x^4}\,dx = \frac{1}{3}x\sqrt{4+3x^2+x^4} + \frac{1}{3}\int \frac{8+3x^2}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{1}{3}x\sqrt{4+3x^2+x^4} - 2\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx + \frac{14}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{1}{3}x\sqrt{4+3x^2+x^4} + \frac{x\sqrt{4+3x^2+x^4}}{2+x^2} - \frac{\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\Big(2\tan^{-1}\Big(}{\sqrt{4+3x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.68, size = 331, normalized size = 1.96

$$\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(4+3x^2+x^4) - 3\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right) + \sqrt{2}\left(-7i+3\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{12\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 3*Sqrt[2]*(3*I + Sqrt[
7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt
[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqr
t[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I
```

```
)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x
], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(12*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4
 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 224, normalized size = 1.33

method	result
default	$\frac{x\sqrt{x^4+3x^2+4}}{3}+\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\right.}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
risch	$\frac{x\sqrt{x^4+3x^2+4}}{3}+\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\right.}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{x\sqrt{x^4+3x^2+4}}{3}+\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\right.}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(x^4+3*x^2+4)^(1/2)+32/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2
))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipt
icF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-32/(-6+2*I*7^(1
/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^
(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*
(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^4+3x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(sqrt(x**4 + 3*x**2 + 4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^4+3\,x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int((3*x^2 + x^4 + 4)^(1/2), x)
```

3.353 $\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$

Optimal. Leaf size=322

$$\frac{x\sqrt{4+3x^2+x^4}}{5(2+x^2)}+\frac{1}{5}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)-\frac{\sqrt{2}(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5\sqrt{4+3x^2+x^4}}$$

```
[Out] 1/175*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/5*x*(x^4+3*x
^2+4)^(1/2)/(x^2+2)+1/30*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos
(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2
))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+187/1050*(x^
2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*El
lipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x
^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-1/5*(x^2+2)*(cos(2*arctan(1/2*x*
2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*
x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2
+4)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1222, 1211, 1117, 1209, 1230, 1720}

$$\frac{1}{5}\sqrt{\frac{11}{35}}\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)-\frac{11\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{75\sqrt{x^4+3x^2+4}}+\frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{25\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5\sqrt{x^4+3x^2+4}}+\frac{187(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{525\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{\sqrt{x^4+3x^2+4}\,x}{5(x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/(5*(2 + x^2)) + (Sqrt[11/35]*ArcTan[(2*Sqrt[11/35
]*x)/Sqrt[4 + 3*x^2 + x^4]])/5 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5*Sqrt[4 + 3*x^2 + x^4])
 + (9*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sq
rt[2]], 1/8])/(25*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (11*Sqrt[2]*(2 + x^2)*Sq
rt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(75*
Sqrt[4 + 3*x^2 + x^4]) + (187*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(525*Sqrt[2]*Sqrt[4 + 3*x^2
+ x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx &= -\left(\frac{1}{25}\int \frac{-8-5x^2}{\sqrt{4+3x^2+x^4}}\,dx\right)+\frac{44}{25}\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx\\ &= -\left(\frac{2}{5}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\right)-\frac{44}{75}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\frac{18}{25}\int \frac{1}{\sqrt{4+3x^2+x^4}\cdots}\\ &= \frac{x\sqrt{4+3x^2+x^4}}{5(2+x^2)}+\frac{1}{5}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)-\frac{\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2\cdots}{(2+x\cdots}}}{5\sqrt{4\cdots}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.37, size = 283, normalized size = 0.88

$$-\frac{\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(35\left(3i+\sqrt{7}\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\left(7i-35\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+88i\Pi\left(\frac{5}{14}\left(3+i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right)}{350\sqrt{2}\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2),x]
```

```
[Out] -1/350*(Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I +
Sqrt[7])]*(35*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[
7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I - 35*Sqrt[7])*EllipticF[I*
ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
+ (88*I)*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(Sqrt[2]*Sqrt[(-I)/(-3*I +
Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 386, normalized size = 1.20

method	result
default	$\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1+\cdots}}{\cdots}$

elliptic	$\frac{32\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1+\frac{3}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] 32/25/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2
+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/
2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-32/5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2
-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)
^(1/2)/(3+I*7^(1/2))*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1
/2))^(1/2))+32/5/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)
*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*Elli
pticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+44/175/(-3/8+
1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x
^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)
*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2)
)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x^2-x+2)(x^2+x+2)}}{5x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7),x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4+3\,x^2+4}}{5\,x^2+7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7),x)
```

```
[Out] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7), x)
```

3.354 $\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx$

Optimal. Leaf size=284

$$-\frac{x\sqrt{4+3x^2+x^4}}{70\,(2+x^2)}+\frac{x\sqrt{4+3x^2+x^4}}{14\,(7+5x^2)}+\frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)}{280\sqrt{385}}+\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\right.}{35\sqrt{2}\sqrt{4+3x^2+x^4}}$$

```
[Out] 51/107800*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/70*x*(x^
4+3*x^2+4)^(1/2)/(x^2+2)+1/14*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+1/70*(x^2+2)*
(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellipti
cE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)
^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-1/70*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)
^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),
1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+28
9/19600*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2
^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1240, 1211, 1117, 1209, 1230, 1720}

$$\frac{51\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{280\sqrt{385}}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{289(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{9800\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{x^4+3x^2+4}\,x}{70\,(x^2+2)}+\frac{\sqrt{x^4+3x^2+4}\,x}{14\,(5x^2+7)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] -1/70*(x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) + (x*Sqrt[4 + 3*x^2 + x^4])/(14*(
7 + 5*x^2)) + (51*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(280*Sqr
t[385]) + ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan
[x/Sqrt[2]], 1/8])/(35*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4
+ 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sqrt[2
]*Sqrt[4 + 3*x^2 + x^4]) + (289*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(9800*Sqrt[2]*Sqrt[4 + 3*x
^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1240

```
Int[Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*(Sqrt[a + b*x^2 + c*x^4]/(2*d*(d + e*x^2))), x] + (Dist[c/(2
*d*e^2), Int[(d - e*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(c*d^2 - a*
e^2)/(2*d*e^2), Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]) /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
```

```
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^2}\,dx = \frac{x\sqrt{4+3x^2+x^4}}{14\,(7+5x^2)} + \frac{1}{350}\int \frac{7-5x^2}{\sqrt{4+3x^2+x^4}}\,dx + \frac{51}{350}\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{14\,(7+5x^2)} - \frac{3}{350}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx + \frac{1}{35}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx - \frac{17}{350}\int$$

$$= -\frac{x\sqrt{4+3x^2+x^4}}{70\,(2+x^2)} + \frac{x\sqrt{4+3x^2+x^4}}{14\,(7+5x^2)} + \frac{51\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)}{280\sqrt{385}} + \frac{(2+x^2)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.42, size = 481, normalized size = 1.69

$$\frac{700\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(4+3x^2+x^4)+35\left(3i+\sqrt{7}\right)(7+5x^2)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right)-98i(7+5x^2)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-102i(7+5x^2)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\,\Pi\left(\frac{5}{14}\left(3+i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{9800\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,(7+5x^2)\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^2,x]
```

```
[Out] (700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I + Sqrt[7])*(
7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])] - EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]) - (98*I)*(7 + 5*x^2)*Sqrt[2 - ((4*
I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I
*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
 - (102*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*
I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[
(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(9800*Sqrt[(
-I)/(-3*I + Sqrt[7])]*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 410, normalized size = 1.44

method	result

risch	$\frac{x\sqrt{x^4+3x^2+4}}{70x^2+98}+\frac{16\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{}}}{4}\right.\right.}{35\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2}}$
default	$\frac{x\sqrt{x^4+3x^2+4}}{70x^2+98}+\frac{2\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6}}{4}\right.}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{x\sqrt{x^4+3x^2+4}}{70x^2+98}+\frac{2\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6}}{4}\right.}{25\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+2/25/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2
-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)
^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+16
/35/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1
/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticF(1/4*x*(
-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-16/35/(-6+2*I*7^(1/2))^(1/
2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/
(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
4*(2+6*I*7^(1/2))^(1/2))+51/2450/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*
I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2
)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1
/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}}{(5x^2 + 7)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**2,x)
```

```
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4 + 3\,x^2 + 4}}{{\left(5\,x^2 + 7\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^2,x)
```

```
[Out] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^2, x)
```

3.355 $\int \frac{\sqrt{4+3x^2+x^4}}{\left(7+5x^2\right)^3}\,dx$

Optimal. Leaf size=312

$$-\frac{139x\sqrt{4+3x^2+x^4}}{86240\left(2+x^2\right)}+\frac{x\sqrt{4+3x^2+x^4}}{28\left(7+5x^2\right)^2}+\frac{139x\sqrt{4+3x^2+x^4}}{17248\left(7+5x^2\right)}+\frac{14999\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)}{344960\sqrt{385}}+\frac{139(2\cdots}{\cdots}$$

```
[Out] 14999/132809600*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-139/
86240*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/28*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+
139/17248*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+139/86240*(x^2+2)*(cos(2*arctan(1
/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan
(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+
3*x^2+4)^(1/2)-23/5880*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2
*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))
*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+254983/7244160
0*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+
4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1242, 1237, 1710, 1728, 1209, 1722, 1117, 1720, 1230}

$$\frac{14999\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{344960\sqrt{385}}-\frac{23(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2940\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{139(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{43120\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{254983(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{36220800\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{139\sqrt{x^4+3x^2+4}\,x}{86240\left(x^2+2\right)}+\frac{139\sqrt{x^4+3x^2+4}\,x}{17248\left(5x^2+7\right)}+\frac{\sqrt{x^4+3x^2+4}\,x}{28\left(5x^2+7\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] (-139*x*Sqrt[4 + 3*x^2 + x^4])/(86240*(2 + x^2)) + (x*Sqrt[4 + 3*x^2 + x^4]
)/(28*(7 + 5*x^2)^2) + (139*x*Sqrt[4 + 3*x^2 + x^4])/(17248*(7 + 5*x^2)) +
(14999*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(344960*Sqrt[385])
+ (139*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/S
qrt[2]], 1/8])/(43120*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (23*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2940*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4]) + (254983*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(36220800*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
```

```
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{\sqrt{4+3x^2+x^4}}{(7+5x^2)^3}\,dx = \int \left(\frac{44}{25\,(7+5x^2)^3\sqrt{4+3x^2+x^4}} + \frac{1}{25\,(7+5x^2)^2\sqrt{4+3x^2+x^4}} + \frac{}{25\,(7+5x^2)}\right.$$

$$= \frac{1}{25}\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx + \frac{1}{25}\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx + \frac{44}{25}\int$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{28\,(7+5x^2)^2} + \frac{x\sqrt{4+3x^2+x^4}}{616\,(7+5x^2)} - \frac{\int \frac{12+70x^2+25x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{15400} - \frac{1}{700}\int \frac{\cdots}{(7+}$$

$$= \frac{x\sqrt{4+3x^2+x^4}}{28\,(7+5x^2)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\,(7+5x^2)} + \frac{\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)}{20\sqrt{385}} - \frac{(2+x^2)}{}$$

$$= -\frac{x\sqrt{4+3x^2+x^4}}{3080\,(2+x^2)} + \frac{x\sqrt{4+3x^2+x^4}}{28\,(7+5x^2)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\,(7+5x^2)} + \frac{\tan^{-1}\left(\frac{2\sqrt{\frac{1}{3}}}{\sqrt{4+3x}}\right.}{20\sqrt{385}}$$

$$= -\frac{139x\sqrt{4+3x^2+x^4}}{86240\,(2+x^2)} + \frac{x\sqrt{4+3x^2+x^4}}{28\,(7+5x^2)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\,(7+5x^2)} + \frac{653\tan^{-1}\left(\right.}{1232}$$

$$= -\frac{139x\sqrt{4+3x^2+x^4}}{86240\,(2+x^2)} + \frac{x\sqrt{4+3x^2+x^4}}{28\,(7+5x^2)^2} + \frac{139x\sqrt{4+3x^2+x^4}}{17248\,(7+5x^2)} + \frac{14999\tan^{-1}}{344}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.32, size = 308, normalized size = 0.99

$$\frac{\frac{700x(1589+695x^2)(4+3x^2+x^4)}{(7+5x^2)^2} + i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(4865\left(3-i\sqrt{7}\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right) + \left(-9597+4865i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right) - 29998\Pi\left(\frac{5}{14}\left(3+i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right)}{12073600\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 3*x^2 + x^4]/(7 + 5*x^2)^3,x]
```

```
[Out] ((700*x*(1589 + 695*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*I
)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(4865*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
```

```
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9597 + (4865*I)*Sqrt[7])
*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I
 + Sqrt[7])] - 29998*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*
I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(12073600*Sqrt[
4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 434, normalized size = 1.39

method	result
risch	$\frac{\sqrt{x^4+3x^2+4}\,x(695x^2+1589)}{17248(5x^2+7)^2}+\frac{139\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{}}{}\right.\right.}{2695\sqrt{-6+2i\sqrt{7}}}$
default	$\frac{x\sqrt{x^4+3x^2+4}}{28(5x^2+7)^2}+\frac{139x\sqrt{x^4+3x^2+4}}{17248(5x^2+7)}-\frac{51\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\mathrm{EllipticF}\left(\frac{x}{}\right.}{15400\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3}}$
elliptic	$\frac{x\sqrt{x^4+3x^2+4}}{28(5x^2+7)^2}+\frac{139x\sqrt{x^4+3x^2+4}}{17248(5x^2+7)}-\frac{51\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\mathrm{EllipticF}\left(\frac{x}{}\right.}{15400\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+139/17248*x*(x^4+3*x^2+4)^(1/2)/(5*x
^2+7)-51/15400/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(
1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+
2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+139/2695/(-6+2*I*7^(1/2))^(1/
2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/
(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
4*(2+6*I*7^(1/2))^(1/2))-139/2695/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x
^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(
3+I*7^(1/2))*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/
2))+14999/3018400/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^
(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-
3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(
1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x^2 - x + 2)(x^2 + x + 2)}}{(5x^2 + 7)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(1/2)/(5*x**2+7)**3,x)
[Out] Integral(sqrt((x**2 - x + 2)*(x**2 + x + 2))/(5*x**2 + 7)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(1/2)/(5*x^2+7)^3,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 3*x^2 + 4)/(5*x^2 + 7)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x^4 + 3\,x^2 + 4}}{(5\,x^2 + 7)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^3,x)
[Out] int((3*x^2 + x^4 + 4)^(1/2)/(5*x^2 + 7)^3, x)
```

3.356 $\int (7+5x^2)^4 (4+3x^2+x^4)^{3/2} \, dx$

Optimal. Leaf size=268

$$\frac{12665086x\sqrt{4+3x^2+x^4}}{2145(2+x^2)}+\frac{7x(661429+174989x^2)\sqrt{4+3x^2+x^4}}{2145}+\frac{x(452001+131080x^2)(4+3x^2+x^4)}{1287}$$

```
[Out] 1/1287*x*(131080*x^2+452001)*(x^4+3*x^2+4)^(3/2)+92150/429*x*(x^4+3*x^2+4)^
(5/2)+2250/13*x^3*(x^4+3*x^2+4)^(5/2)+125/3*x^5*(x^4+3*x^2+4)^(5/2)+1266508
6/2145*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+7/2145*x*(174989*x^2+661429)*(x^4+3*x^
2+4)^(1/2)-12665086/2145*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos
(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2
))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+2383556/429*
(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))
*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)
^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1693, 1190, 1211, 1117, 1209}

$$\frac{2383556\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{429\sqrt{x^4+3x^2+4}}-\frac{12665086\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2145\sqrt{x^4+3x^2+4}}+\frac{92150}{429}(x^4+3x^2+4)^{5/2}x+\frac{(131080x^2+452001)(x^4+3x^2+4)^{3/2}x}{1287}+\frac{7(174989x^2+661429)\sqrt{x^4+3x^2+4}\,x}{2145}+\frac{12665086\sqrt{x^4+3x^2+4}\,x}{2145(x^2+2)}+\frac{125}{3}(x^4+3x^2+4)^{5/2}x^5+\frac{2250}{13}(x^4+3x^2+4)^{5/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (12665086*x*Sqrt[4 + 3*x^2 + x^4])/(2145*(2 + x^2)) + (7*x*(661429 + 174989
*x^2)*Sqrt[4 + 3*x^2 + x^4])/2145 + (x*(452001 + 131080*x^2)*(4 + 3*x^2 + x
^4)^(3/2))/1287 + (92150*x*(4 + 3*x^2 + x^4)^(5/2))/429 + (2250*x^3*(4 + 3*
x^2 + x^4)^(5/2))/13 + (125*x^5*(4 + 3*x^2 + x^4)^(5/2))/3 - (12665086*Sqrt
[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt
[2]], 1/8])/(2145*Sqrt[4 + 3*x^2 + x^4]) + (2383556*Sqrt[2]*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(429*Sq
rt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(7+5x^2\right)^4 \left(4+3x^2+x^4\right)^{3/2} dx = \frac{125}{3}x^5\left(4+3x^2+x^4\right)^{5/2} + \frac{1}{15}\int \left(4+3x^2+x^4\right)^{3/2}(36015+102900x$$

$$= \frac{2250}{13}x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{3}x^5\left(4+3x^2+x^4\right)^{5/2} + \frac{1}{195}\int (4+3x$$

$$= \frac{92150}{429}x\left(4+3x^2+x^4\right)^{5/2} + \frac{2250}{13}x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{3}x^5\left(4+3\right.$$

$$= \frac{x(452001+131080x^2)\left(4+3x^2+x^4\right)^{3/2}}{1287} + \frac{92150}{429}x\left(4+3x^2+x^4\right)^{5/2}$$

$$= \frac{7x(661429+174989x^2)\sqrt{4+3x^2+x^4}}{2145} + \frac{x(452001+131080x^2)(4}{1287}$$

$$= \frac{7x(661429+174989x^2)\sqrt{4+3x^2+x^4}}{2145} + \frac{x(452001+131080x^2)(4}{1287}$$

$$= \frac{12665086x\sqrt{4+3x^2+x^4}}{2145\left(2+x^2\right)} + \frac{7x(661429+174989x^2)\sqrt{4+3x^2+x^4}}{2145}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.86, size = 364, normalized size = 1.36

$$\frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(180184116+391419623x^2+472235001x^4+377574349x^6+212188905x^8+83076275x^{10}+21862875x^{12}+3526875x^{14}+268125x^{16})-18997629\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+21\sqrt{2}\left(-477617i+904649\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{12870\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^4*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(180184116 + 391419623*x^2 + 472235001*x^4
 + 377574349*x^6 + 212188905*x^8 + 83076275*x^10 + 21862875*x^12 + 3526875*
x^14 + 268125*x^16) - 18997629*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7]
 - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqr
t[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7]
)/(3*I + Sqrt[7])] + 21*Sqrt[2]*(-477617*I + 904649*Sqrt[7])*Sqrt[(-3*I + S
qrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I
 + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - S
qrt[7])/(3*I + Sqrt[7])])/(12870*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2
 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 326, normalized size = 1.22

method	result
risch	$\frac{x(268125x^{12}+2722500x^{10}+12622875x^8+34317650x^6+58744455x^4+64070384x^2+45046029)\sqrt{x^4+3x^2+4}}{6435}-\frac{405282752\sqrt{}}{}$
default	$\frac{15015343x\sqrt{x^4+3x^2+4}}{2145}+\frac{356027x^5\sqrt{x^4+3x^2+4}}{39}+\frac{6863530x^7\sqrt{x^4+3x^2+4}}{1287}+\frac{841525x^9\sqrt{x^4+3x^2}}{429}$
elliptic	$\frac{15015343x\sqrt{x^4+3x^2+4}}{2145}+\frac{356027x^5\sqrt{x^4+3x^2+4}}{39}+\frac{6863530x^7\sqrt{x^4+3x^2+4}}{1287}+\frac{841525x^9\sqrt{x^4+3x^2}}{429}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 15015343/2145*x*(x^4+3*x^2+4)^(1/2)+356027/39*x^5*(x^4+3*x^2+4)^(1/2)+68635
30/1287*x^7*(x^4+3*x^2+4)^(1/2)+841525/429*x^9*(x^4+3*x^2+4)^(1/2)+5500/13*
x^11*(x^4+3*x^2+4)^(1/2)+125/3*x^13*(x^4+3*x^2+4)^(1/2)-405282752/2145/(-6+
2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2
))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^
(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1
/2),1/4*(2+6*I*7^(1/2))^(1/2)))+64070384/6435*x^3*(x^4+3*x^2+4)^(1/2)+89363
792/2145/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8
-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1
/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^{2}-x+2\right)\left(x^{2}+x+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(5\,x^2+7\right)}^4\,{\left(x^4+3\,x^2+4\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^4*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.357 $\int (7+5x^2)^3 (4+3x^2+x^4)^{3/2} \, dx$

Optimal. Leaf size=247

$$\frac{4525662x\sqrt{4+3x^2+x^4}}{5005\,(2+x^2)}+\frac{x(1653701+435441x^2)\,\sqrt{4+3x^2+x^4}}{5005}+\frac{x(53504+15365x^2)\,(4+3x^2+x^4)^{3/2}}{1001}-$$

```
[Out] 1/1001*x*(15365*x^2+53504)*(x^4+3*x^2+4)^(3/2)+3825/143*x*(x^4+3*x^2+4)^(5/
2)+125/13*x^3*(x^4+3*x^2+4)^(5/2)+4525662/5005*x*(x^4+3*x^2+4)^(1/2)/(x^2+2
)+1/5005*x*(435441*x^2+1653701)*(x^4+3*x^2+4)^(1/2)-4525662/5005*(x^2+2)*(c
os(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE
(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2
)^(1/2)/(x^4+3*x^2+4)^(1/2)+121826/143*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2
))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2
)
```

Rubi [A]
time = 0.09, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1693, 1190, 1211, 1117, 1209}

$$\frac{121826\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{143\sqrt{x^4+3x^2+4}}-\frac{4525662\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{5005\sqrt{x^4+3x^2+4}}+\frac{3825}{143}(x^4+3x^2+4)^{5/2}x+\frac{(15365x^2+53504)\,(x^4+3x^2+4)^{3/2}x}{1001}+\frac{(435441x^2+1653701)\sqrt{x^4+3x^2+4}\,x}{5005}+\frac{4525662\sqrt{x^4+3x^2+4}\,x}{5005\,(x^2+2)}+\frac{125}{13}(x^4+3x^2+4)^{5/2}x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4525662*x*Sqrt[4 + 3*x^2 + x^4])/(5005*(2 + x^2)) + (x*(1653701 + 435441*x
^2)*Sqrt[4 + 3*x^2 + x^4])/5005 + (x*(53504 + 15365*x^2)*(4 + 3*x^2 + x^4)^
(3/2))/1001 + (3825*x*(4 + 3*x^2 + x^4)^(5/2))/143 + (125*x^3*(4 + 3*x^2 +
x^4)^(5/2))/13 - (4525662*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(5005*Sqrt[4 + 3*x^2 + x^4]) + (1
21826*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Arc
Tan[x/Sqrt[2]], 1/8])/(143*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(7+5x^2\right)^3 \left(4+3x^2+x^4\right)^{3/2}\, dx = \frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{1}{13}\int \left(4+3x^2+x^4\right)^{3/2} \left(4459+8055x^2+3\right.$$

$$= \frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2} + \frac{125}{13}x^3\left(4+3x^2+x^4\right)^{5/2} + \frac{1}{143}\int (33749+$$

$$= \frac{x(53504+15365x^2)\left(4+3x^2+x^4\right)^{3/2}}{1001} + \frac{3825}{143}x\left(4+3x^2+x^4\right)^{5/2} + \frac{12}{13}$$

$$= \frac{x(1653701+435441x^2)\sqrt{4+3x^2+x^4}}{5005} + \frac{x(53504+15365x^2)\left(4+3x\right.}{1001}$$

$$= \frac{x(1653701+435441x^2)\sqrt{4+3x^2+x^4}}{5005} + \frac{x(53504+15365x^2)\left(4+3x\right.}{1001}$$

$$= \frac{4525662x\sqrt{4+3x^2+x^4}}{5005\left(2+x^2\right)} + \frac{x(1653701+435441x^2)\sqrt{4+3x^2+x^4}}{5005} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.26, size = 358, normalized size = 1.45

$$\frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x\left(19463124+36710547x^2+37166164x^4+24107711x^6+10713970x^8+3158575x^{10}+567000x^{12}+48125x^{14}\right)-2262831\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\sqrt{2}\left(-1215823i+2262831\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{10010\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(19463124 + 36710547*x^2 + 37166164*x^4 +
24107711*x^6 + 10713970*x^8 + 3158575*x^10 + 567000*x^12 + 48125*x^14) - 22
62831*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt
[2]*(-1215823*I + 2262831*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I
+ Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*A
rcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/
(10010*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 309, normalized size = 1.25

method	result

risch	$\frac{x(48125x^{10}+422625x^8+1698200x^6+3928870x^4+5528301x^2+4865781)\sqrt{x^4+3x^2+4}}{5005} - \frac{144821184\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{}}{}\right.}}{}$
default	$\frac{4865781x\sqrt{x^4+3x^2+4}}{5005}+\frac{71434x^5\sqrt{x^4+3x^2+4}}{91}+\frac{48520x^7\sqrt{x^4+3x^2+4}}{143}+\frac{12075x^9\sqrt{x^4+3x^2+}}{143}$
elliptic	$\frac{4865781x\sqrt{x^4+3x^2+4}}{5005}+\frac{71434x^5\sqrt{x^4+3x^2+4}}{91}+\frac{48520x^7\sqrt{x^4+3x^2+4}}{143}+\frac{12075x^9\sqrt{x^4+3x^2+}}{143}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4865781/5005*x*(x^4+3*x^2+4)^(1/2)+71434/91*x^5*(x^4+3*x^2+4)^(1/2)+48520/1
43*x^7*(x^4+3*x^2+4)^(1/2)+12075/143*x^9*(x^4+3*x^2+4)^(1/2)+125/13*x^11*(x
^4+3*x^2+4)^(1/2)-144821184/5005/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1
/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I
*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
)-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+552830
1/5005*x^3*(x^4+3*x^2+4)^(1/2)+32017264/5005/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/
8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4
)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^{2}-x+2\right)\left(x^{2}+x+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3*(x**4+3*x**2+4)**(3/2),x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(5\,x^2+7\right)}^3\,{\left(x^4+3\,x^2+4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.358 $\int (7+5x^2)^2 (4+3x^2+x^4)^{3/2} \, dx$

Optimal. Leaf size=226

$$\frac{175346x\sqrt{4+3x^2+x^4}}{1155(2+x^2)}+\frac{x(64533+18253x^2)\sqrt{4+3x^2+x^4}}{1155}+\frac{1}{693}x(6831+2240x^2)(4+3x^2+x^4)^{3/2}+$$

```
[Out] 1/693*x*(2240*x^2+6831)*(x^4+3*x^2+4)^(3/2)+25/11*x*(x^4+3*x^2+4)^(5/2)+175
346/1155*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/1155*x*(18253*x^2+64533)*(x^4+3*x^
2+4)^(1/2)-175346/1155*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2
*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))
*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+4628/33*(x^2+2
)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellip
ticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1
/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1190, 1211, 1117, 1209}

$$\frac{4628\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{33\sqrt{x^4+3x^2+4}}-\frac{175346\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{1155\sqrt{x^4+3x^2+4}}+\frac{25}{11}x(x^4+3x^2+4)^{5/2}+\frac{1}{693}x(2240x^2+6831)(x^4+3x^2+4)^{3/2}+\frac{x(18253x^2+64533)\sqrt{x^4+3x^2+4}}{1155}+\frac{175346x\sqrt{x^4+3x^2+4}}{1155(x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (175346*x*Sqrt[4 + 3*x^2 + x^4])/(1155*(2 + x^2)) + (x*(64533 + 18253*x^2)*
Sqrt[4 + 3*x^2 + x^4])/1155 + (x*(6831 + 2240*x^2)*(4 + 3*x^2 + x^4)^(3/2))
/693 + (25*x*(4 + 3*x^2 + x^4)^(5/2))/11 - (175346*Sqrt[2]*(2 + x^2)*Sqrt[(
4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(1155*Sq
rt[4 + 3*x^2 + x^4]) + (4628*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(33*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
```

```
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \left(7+5x^2\right)^2 \left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2}+\frac{1}{11}\int \left(439+320x^2\right)\left(4+3x^2+x^4\right)^{3/2}\,dx$$

$$= \frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x^2+x^4\right)^{3/2}+\frac{25}{11}x\left(4+3x^2+x^4\right)^{5/2}+\frac{}{2}$$

$$= \frac{x\left(64533+18253x^2\right)\sqrt{4+3x^2+x^4}}{1155}+\frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x\right.$$

$$= \frac{x\left(64533+18253x^2\right)\sqrt{4+3x^2+x^4}}{1155}+\frac{1}{693}x\left(6831+2240x^2\right)\left(4+3x\right.$$

$$= \frac{175346x\sqrt{4+3x^2+x^4}}{1155\left(2+x^2\right)}+\frac{x\left(64533+18253x^2\right)\sqrt{4+3x^2+x^4}}{1155}+\frac{1}{69}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.78, size = 354, normalized size = 1.57

$$\frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}x\left(1824876+2932753x^2+2435811x^4+1229714x^6+408480x^8+82075x^{10}+7875x^{12}\right)-263019\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+3\sqrt{2}\left(-34209i+87673\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{6930\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(1824876 + 2932753*x^2 + 2435811*x^4 + 122
9714*x^6 + 408480*x^8 + 82075*x^10 + 7875*x^12) - 263019*Sqrt[2]*(3*I + Sqr
t[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[
7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-34209*I + 87673*
Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sq
rt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
 Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(6930*Sqrt[(-I)/(-3*I + Sq
rt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 292, normalized size = 1.29

method	result
risch	$\frac{x\left(7875x^8+58450x^6+201630x^4+391024x^2+456219\right)\sqrt{x^4+3x^2+4}}{3465}-\frac{5611072\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-(}}{}$

default	$\frac{25x^9\sqrt{x^4+3x^2+4}}{11}+\frac{1670x^7\sqrt{x^4+3x^2+4}}{99}+\frac{1222x^5\sqrt{x^4+3x^2+4}}{21}+\frac{391024x^3\sqrt{x^4+3x^2+4}}{3465}+$
elliptic	$\frac{25x^9\sqrt{x^4+3x^2+4}}{11}+\frac{1670x^7\sqrt{x^4+3x^2+4}}{99}+\frac{1222x^5\sqrt{x^4+3x^2+4}}{21}+\frac{391024x^3\sqrt{x^4+3x^2+4}}{3465}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/11*x^9*(x^4+3*x^2+4)^(1/2)+1670/99*x^7*(x^4+3*x^2+4)^(1/2)+1222/21*x^5*(
x^4+3*x^2+4)^(1/2)+391024/3465*x^3*(x^4+3*x^2+4)^(1/2)+50691/385*x*(x^4+3*x
^2+4)^(1/2)+396304/385/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^
(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*
x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-5611072/1155/(-6+2*I*7^
(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2
)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))
^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2*(x**4+3*x**2+4)**(3/2),x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(5\,x^2+7\right)}^2\,{\left(x^4+3\,x^2+4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.359 $\int (7+5x^2)(4+3x^2+x^4)^{3/2}\,dx$

Optimal. Leaf size=207

$$\frac{2798x\sqrt{4+3x^2+x^4}}{105(2+x^2)}+\frac{1}{105}x(1029+289x^2)\sqrt{4+3x^2+x^4}+\frac{1}{63}x(108+35x^2)(4+3x^2+x^4)^{3/2}-\frac{2798\sqrt{2}}{}$$

```
[Out] 1/63*x*(35*x^2+108)*(x^4+3*x^2+4)^(3/2)+2798/105*x*(x^4+3*x^2+4)^(1/2)/(x^2
+2)+1/105*x*(289*x^2+1029)*(x^4+3*x^2+4)^(1/2)-2798/105*(x^2+2)*(cos(2*arct
an(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*ar
ctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)+74/3*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(
2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2)
)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1190, 1211, 1117, 1209}

$$\frac{74\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3\sqrt{x^4+3x^2+4}}-\frac{2798\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{105\sqrt{x^4+3x^2+4}}+\frac{1}{63}x(35x^2+108)(x^4+3x^2+4)^{3/2}+\frac{1}{105}x(289x^2+1029)\sqrt{x^4+3x^2+4}+\frac{2798x\sqrt{x^4+3x^2+4}}{105(x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2798*x*Sqrt[4 + 3*x^2 + x^4])/(105*(2 + x^2)) + (x*(1029 + 289*x^2)*Sqrt[4
 + 3*x^2 + x^4])/105 + (x*(108 + 35*x^2)*(4 + 3*x^2 + x^4)^(3/2))/63 - (279
8*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[
x/Sqrt[2]], 1/8])/(105*Sqrt[4 + 3*x^2 + x^4]) + (74*Sqrt[2]*(2 + x^2)*Sqrt[
(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3*Sqrt
[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
```

```
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int (7+5x^2)\left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{21}\int\left(444+289x^2\right)\sqrt{4+3x^2 \cdots}$$

$$= \frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x \cdots\right.$$

$$= \frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{63}x\left(108+35x^2\right)\left(4+3x^2+x \cdots\right.$$

$$= \frac{2798x\sqrt{4+3x^2+x^4}}{105\left(2+x^2\right)}+\frac{1}{105}x\left(1029+289x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{63}x\left(\cdots\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.01, size = 349, normalized size = 1.69

$$\frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(20988+28489x^2+19068x^4+7082x^6+1590x^8+175x^{10})-4197\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+3\sqrt{2}\left(-567i+1399\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{630\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(20988 + 28489*x^2 + 19068*x^4 + 7082*x^6
+ 1590*x^8 + 175*x^10) - 4197*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7]
- (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt
[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])
/(3*I + Sqrt[7])] + 3*Sqrt[2]*(-567*I + 1399*Sqrt[7])*Sqrt[(-3*I + Sqrt[7]
- (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt
[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])
/(3*I + Sqrt[7])])/(630*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 275, normalized size = 1.33

method	result
risch	$\frac{x\left(175x^6+1065x^4+3187x^2+5247\right)\sqrt{x^4+3x^2+4}}{315}-\frac{89536\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}}{105\sqrt{}}$
default	$\frac{5x^7\sqrt{x^4+3x^2+4}}{9}+\frac{71x^5\sqrt{x^4+3x^2+4}}{21}+\frac{3187x^3\sqrt{x^4+3x^2+4}}{315}+\frac{583x\sqrt{x^4+3x^2+4}}{35}+\frac{6352\sqrt{}}{}$
elliptic	$\frac{5x^7\sqrt{x^4+3x^2+4}}{9}+\frac{71x^5\sqrt{x^4+3x^2+4}}{21}+\frac{3187x^3\sqrt{x^4+3x^2+4}}{315}+\frac{583x\sqrt{x^4+3x^2+4}}{35}+\frac{6352\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/9*x^7*(x^4+3*x^2+4)^(1/2)+71/21*x^5*(x^4+3*x^2+4)^(1/2)+3187/315*x^3*(x^4
+3*x^2+4)^(1/2)+583/35*x*(x^4+3*x^2+4)^(1/2)+6352/35/(-6+2*I*7^(1/2))^(1/2)
*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4
+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^
(1/2))-89536/105/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*
(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(Ellip
ticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*
x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(x^{2}-x+2\right)\left(x^{2}+x+2\right)\right)^{\frac{3}{2}}\cdot\left(5x^{2}+7\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)*(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)*(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(5\,x^2+7\right)\,{\left(x^4+3\,x^2+4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2), x)
```

3.360 $\int (4+3x^2+x^4)^{3/2}\,dx$

Optimal. Leaf size=198

$$\frac{138x\sqrt{4+3x^2+x^4}}{35\,(2+x^2)}+\frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}-\frac{138\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+\cdots}{(2+x^2)^2}}}{35\sqrt{4+3x\cdots}}$$

```
[Out] 1/7*x*(x^4+3*x^2+4)^(3/2)+138/35*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+1/35*x*(9*x^
2+49)*(x^4+3*x^2+4)^(1/2)-138/35*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(
1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/
4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+4*(x
^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*E
llipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2
)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1105, 1190, 1211, 1117, 1209}

$$\frac{4\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{138\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{35\sqrt{x^4+3x^2+4}}+\frac{1}{7}x\left(x^4+3x^2+4\right)^{3/2}+\frac{1}{35}x\left(9x^2+49\right)\sqrt{x^4+3x^2+4}+\frac{138x\sqrt{x^4+3x^2+4}}{35\,(x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (138*x*Sqrt[4 + 3*x^2 + x^4])/(35*(2 + x^2)) + (x*(49 + 9*x^2)*Sqrt[4 + 3*x
^2 + x^4])/35 + (x*(4 + 3*x^2 + x^4)^(3/2))/7 - (138*Sqrt[2]*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(35*Sq
rt[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \left(4+3x^2+x^4\right)^{3/2}\,dx = \frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{3}{7}\int \left(8+3x^2\right)\sqrt{4+3x^2+x^4}\,dx$$

$$= \frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}+\frac{1}{35}\int \frac{284+138x^2}{\sqrt{4+3x^2+x^4}}$$

$$= \frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}-\frac{276}{35}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}$$

$$= \frac{138x\sqrt{4+3x^2+x^4}}{35\left(2+x^2\right)}+\frac{1}{35}x\left(49+9x^2\right)\sqrt{4+3x^2+x^4}+\frac{1}{7}x\left(4+3x^2+x^4\right)^{3/2}-$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.83, size = 343, normalized size = 1.73

$$\frac{2\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(276+303x^2+161x^4+39x^6+5x^8)-69\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\sqrt{2}\left(-77i+69\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{70\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(276 + 303*x^2 + 161*x^4 + 39*x^6 + 5*x^8)
 - 69*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqr
t[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSin
h[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt
[2]*(-77*I + 69*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])
]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqr
t[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(70*Sqrt[(
-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 258, normalized size = 1.30

method	result
risch	$\frac{x(5x^4+24x^2+69)\sqrt{x^4+3x^2+4}}{35}+\frac{1136\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{\cdots}}{\cdots}\right.}{35\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$
default	$\frac{x^5\sqrt{x^4+3x^2+4}}{7}+\frac{24x^3\sqrt{x^4+3x^2+4}}{35}+\frac{69x\sqrt{x^4+3x^2+4}}{35}+\frac{1136\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1\cdots}}{35\cdots}$
elliptic	$\frac{x^5\sqrt{x^4+3x^2+4}}{7}+\frac{24x^3\sqrt{x^4+3x^2+4}}{35}+\frac{69x\sqrt{x^4+3x^2+4}}{35}+\frac{1136\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1\cdots}}{35\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*x^5*(x^4+3*x^2+4)^(1/2)+24/35*x^3*(x^4+3*x^2+4)^(1/2)+69/35*x*(x^4+3*x^
2+4)^(1/2)+1136/35/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2
)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-
6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-4416/35/(-6+2*I*7^(1/2))^(1
/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/
```

4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^4 + 3x^2 + 4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2),x)
[Out] Integral((x**4 + 3*x**2 + 4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x^4+3\,x^2+4\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((3*x^2 + x^4 + 4)^(3/2), x)
```

3.361 $\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx$

Optimal. Leaf size=284

$$\frac{94x\sqrt{4+3x^2+x^4}}{125\,(2+x^2)}+\frac{1}{75}x(11+3x^2)\sqrt{4+3x^2+x^4}+\frac{44}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)-\frac{94\sqrt{2}\,(2+x^2)}{}$$

```
[Out] 44/4375*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+94/125*x*(x^
4+3*x^2+4)^(1/2)/(x^2+2)+1/75*x*(3*x^2+11)*(x^4+3*x^2+4)^(1/2)-94/125*(x^2+
2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Elli
pticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2
+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+54/125*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)
))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/
2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/
2)+4114/13125*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1
/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*
((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1222, 1190, 1211, 1117, 1209, 1230, 1720}

$$\frac{44}{125}\sqrt{\frac{11}{35}}\,\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)+\frac{54\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}}-\frac{94\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{125\sqrt{x^4+3x^2+4}}+\frac{4114\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}+\frac{1}{75}(3x^2+11)\sqrt{x^4+3x^2+4}\,x+\frac{94\sqrt{x^4+3x^2+4}\,x}{125\,(x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (94*x*Sqrt[4 + 3*x^2 + x^4])/(125*(2 + x^2)) + (x*(11 + 3*x^2)*Sqrt[4 + 3*x
^2 + x^4])/75 + (44*Sqrt[11/35]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x
^4]])/125 - (94*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellip
ticE[2*ArcTan[x/Sqrt[2]], 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (54*Sqrt[2]*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
 1/8])/(125*Sqrt[4 + 3*x^2 + x^4]) + (4114*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(13125*
Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{(4+3x^2+x^4)^{3/2}}{7+5x^2}\,dx = -\left(\frac{1}{25}\int \left(-8-5x^2\right)\sqrt{4+3x^2+x^4}\,dx\right)+\frac{44}{25}\int \frac{\sqrt{4+3x^2+x^4}}{7+5x^2}\,dx$$

$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}-\frac{1}{375}\int \frac{-260-150x^2}{\sqrt{4+3x^2+x^4}}\,dx-\frac{44}{625}\int \frac{-8-5}{\sqrt{4+3x^2}}$$

$$= \frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}-\frac{88}{125}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-\frac{4}{5}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+}}$$

$$= \frac{94x\sqrt{4+3x^2+x^4}}{125\left(2+x^2\right)}+\frac{1}{75}x\left(11+3x^2\right)\sqrt{4+3x^2+x^4}+\frac{44}{125}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2}{\sqrt{4-}}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.42, size = 477, normalized size = 1.68

$$\frac{350\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(44+45x^2+20x^4+3x^6)-4935\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+7\sqrt{2}\left(-241i+705\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-5808i\sqrt{2}\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\Pi\left(\frac{5}{14}\left(3+i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{26250\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2),x]
```

```
[Out] (350*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(44 + 45*x^2 + 20*x^4 + 3*x^6) - 4935*Sq
rt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*S
qrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*Sqrt[2]*(-
241*I + 705*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sq
rt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-
2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (5808*I)*Sqrt
```

```
[2]*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
 + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh
[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(26250
*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 418, normalized size = 1.47

method	result
risch	$\frac{x(3x^2+11)\sqrt{x^4+3x^2+4}}{75} - \frac{3008\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-6}}{}\right.\right.}{125\sqrt{-6+2i\sqrt{7}}\sqrt{x}}$
default	$\frac{x^3\sqrt{x^4+3x^2+4}}{25} + \frac{11x\sqrt{x^4+3x^2+4}}{75} + \frac{9424\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\text{EllipticF}\left(\frac{x}{}\right.}{1875\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x}}$
elliptic	$\frac{x^3\sqrt{x^4+3x^2+4}}{25} + \frac{11x\sqrt{x^4+3x^2+4}}{75} + \frac{9424\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\text{EllipticF}\left(\frac{x}{}\right.}{1875\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x,method=_RETURNVERBOSE)
```

```
[Out] 1/25*x^3*(x^4+3*x^2+4)^(1/2)+11/75*x*(x^4+3*x^2+4)^(1/2)+9424/1875/(-6+2*I*
7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(
1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*
(2+6*I*7^(1/2))^(1/2))-3008/125/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2
*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+
I*7^(1/2))*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)
)+3008/125/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/
8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticE(
1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1936/4375/(-3/8+1/8
*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*
7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,
-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(
1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="fricas")
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^{2} - x + 2\right)\left(x^{2} + x + 2\right)\right)^{\frac{3}{2}}}{5x^{2} + 7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7),x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7),x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^4+3\,x^2+4\right)}^{3/2}}{5\,x^2+7} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7),x)
[Out] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7), x)
```

3.362 $\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx$

Optimal. Leaf size=305

$$\frac{1}{75}x\sqrt{4+3x^2+x^4}+\frac{4x\sqrt{4+3x^2+x^4}}{175\left(2+x^2\right)}+\frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)}+\frac{13}{350}\sqrt{\frac{11}{35}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)-\frac{4\sqrt{2}}{}$$

```
[Out] 13/12250*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/75*x*(x^4
+3*x^2+4)^(1/2)+4/175*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+22/175*x*(x^4+3*x^2+4)^
(1/2)/(5*x^2+7)+2431/73500*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/c
os(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,
1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-4/
175*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/
2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^
2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+4/175*(x^2+2)*(cos(2*arctan(1/2*x
*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2
*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^
2+4)^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 372, normalized size of antiderivative = 1.22, number of steps used = 19, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1242, 1117, 1153, 1209, 1136, 1211, 1237, 1728, 1722, 1720, 1230}

$$\frac{13}{350}\sqrt{\frac{11}{35}}\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)+\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}}-\frac{4\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{175\sqrt{x^4+3x^2+4}}+\frac{187\sqrt{2}\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}+\frac{6919\left(x^2+2\right)\sqrt{\frac{x^4+3x^2+4}{\left(x^2+2\right)^2}}\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{183750\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{4\sqrt{x^4+3x^2+4}\,x}{175\left(x^2+2\right)}+\frac{22\sqrt{x^4+3x^2+4}\,x}{175\left(5x^2+7\right)}+\frac{1}{75}\sqrt{x^4+3x^2+4}\,x$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] (x*Sqrt[4 + 3*x^2 + x^4])/75 + (4*x*Sqrt[4 + 3*x^2 + x^4])/(175*(2 + x^2))
+ (22*x*Sqrt[4 + 3*x^2 + x^4])/(175*(7 + 5*x^2)) + (13*Sqrt[11/35]*ArcTan[(
2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/350 - (4*Sqrt[2]*(2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt
[4 + 3*x^2 + x^4]) + (4*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^
2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(175*Sqrt[4 + 3*x^2 + x^4]) + (6919
*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[
x/Sqrt[2]], 1/8])/(183750*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (187*Sqrt[2]*(2
+ x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sq
rt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
```

```
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^2}\,dx = \int \left(\frac{152}{625\sqrt{4+3x^2+x^4}} + \frac{16x^2}{125\sqrt{4+3x^2+x^4}} + \frac{x^4}{25\sqrt{4+3x^2+x^4}} + \frac{}{625\,(7+}\right.$$

$$= \frac{1}{25}\int \frac{x^4}{\sqrt{4+3x^2+x^4}}\,dx + \frac{16}{125}\int \frac{x^2}{\sqrt{4+3x^2+x^4}}\,dx + \frac{88}{625}\int \frac{1}{\left(7+5x^2\right)\sqrt{4+}}$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4} + \frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)} + \frac{38\sqrt{2}\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,F\Big(}{625\sqrt{4+3x^2+}}$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4} + \frac{16x\sqrt{4+3x^2+x^4}}{125\left(2+x^2\right)} + \frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)} + \frac{2}{125}\sqrt{\frac{11}{35}}\ \mathrm{ta}$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4} + \frac{4x\sqrt{4+3x^2+x^4}}{175\left(2+x^2\right)} + \frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)} + \frac{2}{125}\sqrt{\frac{11}{35}}\ \mathrm{tan}$$

$$= \frac{1}{75}x\sqrt{4+3x^2+x^4} + \frac{4x\sqrt{4+3x^2+x^4}}{175\left(2+x^2\right)} + \frac{22x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)} + \frac{13}{350}\sqrt{\frac{11}{35}}\ \mathrm{tan}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.26, size = 309, normalized size = 1.01

$$\frac{\frac{175x\left(23+7x^2\right)\left(4+3x^2+x^4\right)}{7+5x^2} - i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(105\left(3-i\sqrt{7}\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right) + 7\left(158+15i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right) + 429\Pi\left(\frac{5}{14}\left(3+i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right)}{18375\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^2,x]
```

```
[Out] ((175*x*(23 + 7*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2) - I*Sqrt[6 + (2*I)*Sqrt
[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqr
t[7])]*(105*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(158 + (15*I)*Sqrt[7])*Elliptic
F[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7
])] + 429*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(18375*Sqrt[4 + 3*x^2 + x^
4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 425, normalized size = 1.39

method	result
risch	$\frac{\sqrt{x^4+3x^2+4}\,x(7x^2+23)}{525x^2+735} - \frac{128\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-6}}{}\right.\right.}{175\sqrt{-6+2i\sqrt{7}}\,\sqrt{x}}$
default	$\frac{22x\sqrt{x^4+3x^2+4}}{175(5x^2+7)} + \frac{x\sqrt{x^4+3x^2+4}}{75} + \frac{232\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\text{EllipticF}\left(\frac{x\sqrt{}}{}\right.}{375\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2}}$
elliptic	$\frac{22x\sqrt{x^4+3x^2+4}}{175(5x^2+7)} + \frac{x\sqrt{x^4+3x^2+4}}{75} + \frac{232\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\text{EllipticF}\left(\frac{x\sqrt{}}{}\right.}{375\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 22/175*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+1/75*x*(x^4+3*x^2+4)^(1/2)+232/375/(
-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*
x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/
2),1/4*(2+6*I*7^(1/2))^(1/2))-128/175/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8
*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/
2)/(3+I*7^(1/2))*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))
^(1/2))+128/175/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*
(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*Ellip
ticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+286/6125/(-3/8
+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*
x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2
```

```
)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2
))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="fricas")
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(25*x^4 + 70*x^2 + 49), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^{2}-x+2\right)\left(x^{2}+x+2\right)\right)^{\frac{3}{2}}}{\left(5x^{2}+7\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**2,x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^2,x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^4+3\,x^2+4\right)}^{3/2}}{{\left(5\,x^2+7\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^2,x)
```

```
[Out] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^2, x)
```

3.363 $\int \frac{(4+3x^2+x^4)^{3/2}}{(7+5x^2)^3} dx$

Optimal. Leaf size=440

$$\frac{9x\sqrt{4+3x^2+x^4}}{1960(2+x^2)}+\frac{11x\sqrt{4+3x^2+x^4}}{175(7+5x^2)^2}+\frac{167x\sqrt{4+3x^2+x^4}}{9800(7+5x^2)}+\frac{1347\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{4+3x^2+x^4}}\right)}{7840\sqrt{385}}+\frac{111(2+$$

```
[Out] 1347/3018400*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+9/1960*
x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+11/175*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+167/
9800*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-9/1960*(x^2+2)*(cos(2*arctan(1/2*x*2^(
1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2
^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)
^(1/2)-3/490*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/
2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x
^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+7633/548800*(x^2+2)*(cos
(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(
sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^
(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1242, 1117, 1153, 1209, 1237, 1710, 1728, 1722, 1720, 1230}

$$\frac{1347\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}x}{\sqrt{x^4+3x^2+4}}\right)}{7840\sqrt{385}}-\frac{22\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13125\sqrt{x^4+3x^2+4}}-\frac{817(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{91875\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{6\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{875\sqrt{x^4+3x^2+4}}+\frac{111(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{24500\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{7633(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{274400\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{9\sqrt{x^4+3x^2+4}x}{1960(x^2+2)}+\frac{167\sqrt{x^4+3x^2+4}x}{9800(5x^2+7)}+\frac{11\sqrt{x^4+3x^2+4}x}{175(5x^2+7)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] (9*x*Sqrt[4 + 3*x^2 + x^4])/(1960*(2 + x^2)) + (11*x*Sqrt[4 + 3*x^2 + x^4])
/(175*(7 + 5*x^2)^2) + (167*x*Sqrt[4 + 3*x^2 + x^4])/(9800*(7 + 5*x^2)) + (
1347*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/(7840*Sqrt[385]) + (1
11*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[
2]], 1/8])/(24500*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (6*Sqrt[2]*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(875*
Sqrt[4 + 3*x^2 + x^4]) - (817*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(91875*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
 - (22*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*Ar
cTan[x/Sqrt[2]], 1/8])/(13125*Sqrt[4 + 3*x^2 + x^4]) + (7633*(2 + x^2)*Sqrt
[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8
])/(274400*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
```

```
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
```

```
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(4+3x^2+x^4\right)^{3/2}}{\left(7+5x^2\right)^3}\,dx &= \int \left(\frac{9}{625\sqrt{4+3x^2+x^4}}+\frac{x^2}{125\sqrt{4+3x^2+x^4}}+\frac{1936}{625\left(7+5x^2\right)^3\sqrt{4+3x^2+x^4}} \right.\\
&= \frac{1}{125}\int \frac{x^2}{\sqrt{4+3x^2+x^4}}\,dx+\frac{9}{625}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\frac{88}{625}\int \frac{1}{\left(7+5x^2\right)^2\sqrt{4+}} \\
&= \frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2}+\frac{x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)}+\frac{9\left(2+x^2\right)\sqrt{\frac{4+3x^2+x^4}{\left(2+x^2\right)^2}}\,F\left(2\tan^{-1}\right.}{1250\sqrt{2}\sqrt{4+3x^2+x^4}} \\
&= \frac{x\sqrt{4+3x^2+x^4}}{125\left(2+x^2\right)}+\frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2}+\frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)}+\frac{89\tan^{-1}\left(\frac{}{\sqrt{4}}\right.}{500\sqrt{}} \\
&= \frac{6x\sqrt{4+3x^2+x^4}}{875\left(2+x^2\right)}+\frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2}+\frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)}+\frac{89\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{500} \\
&= \frac{9x\sqrt{4+3x^2+x^4}}{1960\left(2+x^2\right)}+\frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2}+\frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)}+\frac{3}{175}\sqrt{\frac{11}{35}}\tan \\
&= \frac{9x\sqrt{4+3x^2+x^4}}{1960\left(2+x^2\right)}+\frac{11x\sqrt{4+3x^2+x^4}}{175\left(7+5x^2\right)^2}+\frac{167x\sqrt{4+3x^2+x^4}}{9800\left(7+5x^2\right)}+\frac{3}{175}\sqrt{\frac{11}{35}}\tan
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.31, size = 309, normalized size = 0.70

$$
\frac{\frac{140x\left(357+167x^2\right)\left(4+3x^2+x^4\right)}{\left(7+5x^2\right)^2}-i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(315\left(3-i\sqrt{7}\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+7\left(103+45i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+2694\Pi\left(\frac{5}{14}\left(3+i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right)}{274400\sqrt{4+3x^2+x^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(3/2)/(7 + 5*x^2)^3,x]
```

```
[Out] ((140*x*(357 + 167*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 - I*Sqrt[6 + (2*I)
*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
+ Sqrt[7])]*(315*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(103 + (45*I)*Sqrt[7])*Ell
ipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + S
qrt[7])] + 2694*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-
3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(274400*Sqrt[4 + 3*x
^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 434, normalized size = 0.99

method	result
risch	$\frac{\sqrt{x^4+3x^2+4}\,x(167x^2+357)}{1960(5x^2+7)^2} - \frac{36\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{}}{}\right.\right.}{245\sqrt{-6+2i\sqrt{7}}}$
default	$\frac{11x\sqrt{x^4+3x^2+4}}{175(5x^2+7)^2}+\frac{167x\sqrt{x^4+3x^2+4}}{9800(5x^2+7)}+\frac{17\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\text{EllipticF}\left(\right.}{350\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3}}$
elliptic	$\frac{11x\sqrt{x^4+3x^2+4}}{175(5x^2+7)^2}+\frac{167x\sqrt{x^4+3x^2+4}}{9800(5x^2+7)}+\frac{17\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\text{EllipticF}\left(\right.}{350\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 11/175*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+167/9800*x*(x^4+3*x^2+4)^(1/2)/(5*
x^2+7)+17/350/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1
+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2
*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-36/245/(-6+2*I*7^(1/2))^(1/2)*
(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^
4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(
2+6*I*7^(1/2))^(1/2))+36/245/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^
(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7
```

```
^(1/2))*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1
347/68600/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1
+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*
I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3
/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="maxima")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="fricas")
[Out] integral((x^4 + 3*x^2 + 4)^(3/2)/(125*x^6 + 525*x^4 + 735*x^2 + 343), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(x^{2} - x + 2\right)\left(x^{2} + x + 2\right)\right)^{\frac{3}{2}}}{\left(5x^{2} + 7\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+3*x**2+4)**(3/2)/(5*x**2+7)**3,x)
[Out] Integral(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)/(5*x**2 + 7)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+3*x^2+4)^(3/2)/(5*x^2+7)^3,x, algorithm="giac")
[Out] integrate((x^4 + 3*x^2 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(x^4+3\,x^2+4\right)^{3/2}}{\left(5\,x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^3,x)
[Out] int((3*x^2 + x^4 + 4)^(3/2)/(5*x^2 + 7)^3, x)
```

3.364 $\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=187

$$75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}-\frac{15x\sqrt{4+3x^2+x^4}}{2+x^2}+\frac{15\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\right.}{\sqrt{4+3x^2+x^4}}$$

```
[Out] 75*x*(x^4+3*x^2+4)^(1/2)+25*x^3*(x^4+3*x^2+4)^(1/2)-15*x*(x^4+3*x^2+4)^(1/2
)/(x^2+2)+13/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(
1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3
*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+15*(x^2+2)*(cos(2*arct
an(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*ar
ctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1693, 1211, 1117, 1209}

$$\frac{13(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\,\sqrt{x^4+3x^2+4}}+\frac{15\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}-\frac{15\sqrt{x^4+3x^2+4}\,x}{x^2+2}+75\sqrt{x^4+3x^2+4}\,x+25\sqrt{x^4+3x^2+4}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] 75*x*Sqrt[4 + 3*x^2 + x^4] + 25*x^3*Sqrt[4 + 3*x^2 + x^4] - (15*x*Sqrt[4 +
3*x^2 + x^4])/(2 + x^2) + (15*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 +
 x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (13*(
2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
```

```
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(7+5x^2)^3}{\sqrt{4+3x^2+x^4}}\,dx &= 25x^3\sqrt{4+3x^2+x^4}+\frac{1}{5}\int \frac{1715+2175x^2+1125x^4}{\sqrt{4+3x^2+x^4}}\,dx \\
&= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}+\frac{1}{15}\int \frac{645-225x^2}{\sqrt{4+3x^2+x^4}}\,dx \\
&= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}+13\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+30\int \frac{}{\sqrt{4-}} \\
&= 75x\sqrt{4+3x^2+x^4}+25x^3\sqrt{4+3x^2+x^4}-\frac{15x\sqrt{4+3x^2+x^4}}{2+x^2}+\frac{15\sqrt{2}\,(2+x^2)}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.27, size = 337, normalized size = 1.80

$$
\frac{100\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(12+13x^2+6x^4+x^6)+15\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-\sqrt{2}\left(131i+15\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (100*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(12 + 13*x^2 + 6*x^4 + x^6) + 15*Sqrt[2]
*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(
3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)
/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(131*I +
15*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I +
 Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*
I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(4*Sqrt[(-I)/(-3*I + Sq
rt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 241, normalized size = 1.29

method	result
risch	$25x(x^2+3)\sqrt{x^4+3x^2+4}+\frac{480\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{}}{}\right.\right.}{\sqrt{-6+2i\sqrt{7}}\sqrt{}}$
default	$25x^3\sqrt{x^4+3x^2+4}+75x\sqrt{x^4+3x^2+4}+\frac{172\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}}{\sqrt{-6+2i\sqrt{7}}\sqrt{x^4}}$

elliptic	$25x^3\sqrt{x^4+3x^2+4}+75x\sqrt{x^4+3x^2+4}+\frac{172\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25*x^3*(x^4+3*x^2+4)^(1/2)+75*x*(x^4+3*x^2+4)^(1/2)+172/(-6+2*I*7^(1/2))^(1
/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(
x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2
))^(1/2))+480/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-
(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(Elliptic
F(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(
-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^3}{\sqrt{(x^2-x+2)(x^2+x+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

[Out] Integral((5*x**2 + 7)**3/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")

[Out] integrate((5*x^2 + 7)^3/sqrt(x^4 + 3*x^2 + 4), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+7\right)}^3}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(1/2),x)

[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(1/2), x)

3.365 $\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=170

$$\frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{20x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{20\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}+\frac{167(2$$

```
[Out] 25/3*x*(x^4+3*x^2+4)^(1/2)+20*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+167/12*(x^2+2)*
(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellipti
cF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2
)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-20*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(
1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/
4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1220, 1211, 1117, 1209}

$$\frac{167(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{20\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+\frac{20\sqrt{x^4+3x^2+4}\,x}{x^2+2}+\frac{25}{3}\sqrt{x^4+3x^2+4}\,x$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (25*x*Sqrt[4 + 3*x^2 + x^4])/3 + (20*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (
20*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan
[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 + x^4] + (167*(2 + x^2)*Sqrt[(4 + 3*x^2 +
 x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(6*Sqrt[2]*Sqrt[4 +
 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
```

```
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(7+5x^2)^2}{\sqrt{4+3x^2+x^4}}\,dx &= \frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{1}{3}\int \frac{47+60x^2}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{25}{3}x\sqrt{4+3x^2+x^4}-40\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{167}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\ &= \frac{25}{3}x\sqrt{4+3x^2+x^4}+\frac{20x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{20\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\Big(2\,\mathrm{t}}{\sqrt{4+3x^2+x^4}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.24, size = 331, normalized size = 1.95

$$\frac{50\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(4+3x^2+x^4)-30\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\sqrt{2}\left(43i+30\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{6\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (50*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) - 30*Sqrt[2]*(3*I + Sqr
t[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[
7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sq
rt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(43*I + 30*Sqrt[7])*
Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (
2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])
]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(6*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt
[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 224, normalized size = 1.32

method	result
default	$\frac{25x\sqrt{x^4+3x^2+4}}{3}+\frac{188\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{}}}{4}\right.}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
risch	$\frac{25x\sqrt{x^4+3x^2+4}}{3}+\frac{188\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{}}}{4}\right.}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{25x\sqrt{x^4+3x^2+4}}{3}+\frac{188\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{}}}{4}\right.}{3\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/3*x*(x^4+3*x^2+4)^(1/2)+188/3/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1
/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*Elli
pticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-640/(-6+2*I*7
^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^
2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2)
)^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1
/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^2}{\sqrt{\left(x^2-x+2\right)\left(x^2+x+2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/sqrt(x^4 + 3*x^2 + 4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+7\right)}^2}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(1/2), x)
```

3.366 $\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=151

$$\frac{5x\sqrt{4+3x^2+x^4}}{2+x^2}-\frac{5\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}}+\frac{17(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,F}{2\sqrt{2}\sqrt{4+3x^2}}$$

```
[Out] 5*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+17/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^
2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))
),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-
5*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+
4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1211, 1117, 1209}

$$\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{5\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{x^4+3x^2+4}}+\frac{5\sqrt{x^4+3x^2+4}\,x}{x^2+2}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (5*x*Sqrt[4 + 3*x^2 + x^4])/(2 + x^2) - (5*Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^
2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/Sqrt[4 + 3*x^2 +
 x^4] + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTa
n[x/Sqrt[2]], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
```

```
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{7+5x^2}{\sqrt{4+3x^2+x^4}}\,dx = -\left(10\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx\right)+17\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{5x\sqrt{4+3x^2+x^4}}{2+x^2} - \frac{5\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{\sqrt{4+3x^2+x^4}} + \frac{17(2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 214, normalized size = 1.42

$$\frac{\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(-5\left(3i+\sqrt{7}\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\left(i+5\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right)}{2\sqrt{2}\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] (Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7]
)]*(-5*(3*I + Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (I + 5*Sqrt[7])*EllipticF[I*ArcSinh[Sq
rt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(2*Sqrt[
2]*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 209, normalized size = 1.38

method	result

default	$-\frac{160\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right.\right.}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(3+i\sqrt{7}\right)}$
elliptic	$-\frac{160\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right.\right.}{\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}\,\left(3+i\sqrt{7}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -160/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8
*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(
-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^
(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))+28/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8
+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)
^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+7}{\sqrt{(x^2-x+2)(x^2+x+2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 7)/sqrt((x**2 - x + 2)*(x**2 + x + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/sqrt(x^4 + 3*x^2 + 4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^2+7}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(1/2), x)
```

3.367 $\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=64

$$\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

```
[Out] 1/4*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/
2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^
2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1117}

$$\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] ((2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]
], 1/8])/(2*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx=\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.05, size = 142, normalized size = 2.22

$$-\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}\,x\right)|\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt[7])]
*EllipticF[I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7])]*x], (-3 - I*Sqrt[7])/(-3 + I
*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^(-1)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.02, size = 85, normalized size = 1.33

method	result	siz
default	$\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$	85
elliptic	$\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i\sqrt{7}}}{4}\right)}{\sqrt{-6+2i\sqrt{7}}\sqrt{x^4+3x^2+4}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*
7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1
/2),1/4*(2+6*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

Fricas [A]
time = 0.08, size = 38, normalized size = 0.59

$$-\frac{1}{16}\sqrt{2}\left(\sqrt{-7}+3\right)\sqrt{\sqrt{-7}-3}\,\text{ellipticF}\left(\frac{1}{4}\sqrt{2}\,x\sqrt{\sqrt{-7}-3},\frac{3}{8}\sqrt{-7}+\frac{1}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*(sqrt(-7) + 3)*sqrt(sqrt(-7) - 3)*ellipticF(1/4*sqrt(2)*x*sqr
t(sqrt(-7) - 3), 3/8*sqrt(-7) + 1/8)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^4+3x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 3*x**2 + 4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 3*x^2 + 4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + x^4 + 4)^(1/2),x)
```

```
[Out] int(1/(3*x^2 + x^4 + 4)^(1/2), x)
```

3.368 $\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=168

$$\frac{1}{4}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)-\frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{17(2+x^2)\sqrt{\frac{4+3x^2}{(2+}}}{84}$$

```
[Out] 1/308*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/12*(x^2+2)*(
cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Elliptic
F(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)
*2^(1/2)/(x^4+3*x^2+4)^(1/2)+17/168*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2
)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))
),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)
^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1230, 1117, 1720}

$$\frac{1}{4}\sqrt{\frac{5}{77}}\,\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{6\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{17(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{84\sqrt{2}\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/4 - ((2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(
6*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (17*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2
+ x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(84*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
```

```
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\right) + \frac{10}{3}\int \frac{1+\frac{x^2}{2}}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{1}{4}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right) - \frac{(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,F\left(2\tan^{-1}\right.}{6\sqrt{2}\sqrt{4+3x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 159, normalized size = 0.95

$$-\frac{i\sqrt{1-\frac{2x^2}{-3-i\sqrt{7}}}\sqrt{1-\frac{2x^2}{-3+i\sqrt{7}}}\,\Pi\left(-\frac{5}{14}\left(-3-i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2}{-3-i\sqrt{7}}}\,x\right)\middle|\frac{-3-i\sqrt{7}}{-3+i\sqrt{7}}\right)}{7\sqrt{2}\sqrt{-\frac{1}{-3-i\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] ((-1/7*I)*Sqrt[1 - (2*x^2)/(-3 - I*Sqrt[7])]*Sqrt[1 - (2*x^2)/(-3 + I*Sqrt[
7])]*EllipticPi[(-5*(-3 - I*Sqrt[7]))/14, I*ArcSinh[Sqrt[-2/(-3 - I*Sqrt[7]
)]*x], (-3 - I*Sqrt[7])/(-3 + I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-3 - I*Sqrt[7])^
(-1)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 107, normalized size = 0.64

method	result
default	$\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,x,-\frac{5}{7\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)},\frac{\sqrt{-\frac{3}{8}-\frac{i\sqrt{7}}{8}}}{\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}}\right)}{7\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,x,-\frac{5}{7\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)},\frac{\sqrt{-\frac{3}{8}-\frac{i\sqrt{7}}{8}}}{\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}}\right)}{7\sqrt{-\frac{3}{8}+\frac{i\sqrt{7}}{8}}\,\sqrt{x^4+3x^2+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x
^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1
/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8
*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^6 + 22*x^4 + 41*x^2 + 28), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2-x+2)(x^2+x+2)}\,(5x^2+7)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(1/2),x)
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(5\,x^2+7)\,\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2)),x)
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(1/2)), x)
```

3.369 $\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=286

$$-\frac{5x\sqrt{4+3x^2+x^4}}{616\,(2+x^2)}+\frac{25x\sqrt{4+3x^2+x^4}}{616\,(7+5x^2)}+\frac{37\sqrt{\frac{5}{77}}\,\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)}{2464}+\frac{5(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}}{308\sqrt{2}\,\sqrt{4+}}$$

```
[Out] 37/189728*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-5/616*x*(x
^4+3*x^2+4)^(1/2)/(x^2+2)+25/616*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+5/616*(x^2
+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ell
ipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^
2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-1/84*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2))
)^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2
))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2
)+629/103488*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/
2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*(
(x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1237, 1728, 1209, 1722, 1117, 1720}

$$\frac{37\sqrt{\frac{5}{77}}\,\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{2464}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{42\sqrt{2}\,\sqrt{x^4+3x^2+4}}+\frac{5(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{308\sqrt{2}\,\sqrt{x^4+3x^2+4}}+\frac{629(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{51744\sqrt{2}\,\sqrt{x^4+3x^2+4}}-\frac{5\sqrt{x^4+3x^2+4}\,x}{616\,(x^2+2)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{616\,(5x^2+7)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (-5*x*Sqrt[4 + 3*x^2 + x^4])/(616*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 + x^4])
/(616*(7 + 5*x^2)) + (37*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2
 + x^4]])/2464 + (5*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE
[2*ArcTan[x/Sqrt[2]], 1/8])/(308*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2
)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/
(42*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (629*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(51744*Sqrt[2]*S
qrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx = \frac{25x\sqrt{4+3x^2+x^4}}{616(7+5x^2)} - \frac{1}{616}\int \frac{12+70x^2+25x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$$

$$= \frac{25x\sqrt{4+3x^2+x^4}}{616(7+5x^2)} - \frac{\int \frac{410+425x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{3080} + \frac{5}{308}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+\cdots}}$$

$$= -\frac{5x\sqrt{4+3x^2+x^4}}{616(2+x^2)} + \frac{25x\sqrt{4+3x^2+x^4}}{616(7+5x^2)} + \frac{5(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\cdots}{308\sqrt{2}\sqrt{4+\cdots}}$$

$$= -\frac{5x\sqrt{4+3x^2+x^4}}{616(2+x^2)} + \frac{25x\sqrt{4+3x^2+x^4}}{616(7+5x^2)} + \frac{37\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{1}{3\cdots}}\cdots}{\sqrt{4+3x\cdots}}\right.\cdots}{2464}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.38, size = 481, normalized size = 1.68

$$\frac{700\sqrt{-\frac{i}{-3i+\sqrt{7}}}x(4+3x^2+x^4)+35\left(3i+\sqrt{7}\right)(7+5x^2)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right)+98i(7+5x^2)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-74i(7+5x^2)\sqrt{2-\frac{4ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\Pi\left(\frac{5}{14}\left(3+i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{17248\sqrt{-\frac{i}{-3i+\sqrt{7}}}(7+5x^2)\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (700*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(4 + 3*x^2 + x^4) + 35*(3*I + Sqrt[7])*(
7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I -
Sqrt[7])/(3*I + Sqrt[7])] - EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]) + (98*I)*(7 + 5*x^2)*Sqrt[2 - ((4*
I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I
*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
```

```
 - (74*I)*(7 + 5*x^2)*Sqrt[2 - ((4*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(17248*Sqrt[(
-I)/(-3*I + Sqrt[7])]*(7 + 5*x^2)*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 410, normalized size = 1.43

method	result
risch	$\frac{25x\sqrt{x^4+3x^2+4}}{616(5x^2+7)}+\frac{20\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\right.\right.}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2}}$
default	$\frac{25x\sqrt{x^4+3x^2+4}}{616(5x^2+7)}-\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+}}{4}\right.}{22\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$\frac{25x\sqrt{x^4+3x^2+4}}{616(5x^2+7)}-\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+}}{4}\right.}{22\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/616*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-1/22/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x
^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+
4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+
20/77/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2
+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticF(1/4*x
*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-20/77/(-6+2*I*7^(1/2))^(
1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2
)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),
1/4*(2+6*I*7^(1/2))^(1/2))+37/4312/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/
8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1
/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8
-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^8 + 145*x^6 + 359*x^4 + 427*x^2 + 196)
, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2-x+2)(x^2+x+2)}\,(5x^2+7)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(5\,x^2+7\right)}^2\,\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(1/2)), x)
```

3.370 $\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx$

Optimal. Leaf size=314

$$-\frac{555x\sqrt{4+3x^2+x^4}}{758912\,(2+x^2)}+\frac{25x\sqrt{4+3x^2+x^4}}{1232\,(7+5x^2)^2}+\frac{2775x\sqrt{4+3x^2+x^4}}{758912\,(7+5x^2)}-\frac{3285\sqrt{\frac{5}{77}}\,\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)}{3035648}$$

```
[Out] -3285/233744896*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-555/
758912*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+25/1232*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7
)^2+2775/758912*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+555/758912*(x^2+2)*(cos(2*a
rctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2
*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2
)/(x^4+3*x^2+4)^(1/2)-1/17248*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2
)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2
^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-18615/4
2499072*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2
^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1237, 1710, 1728, 1209, 1722, 1117, 1720}

$$-\frac{3285\sqrt{\frac{5}{77}}\,\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{3035648}-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{8624\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{555(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{18615(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{21249536\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{555\sqrt{x^4+3x^2+4}\,x}{758912(x^2+2)}+\frac{2775\sqrt{x^4+3x^2+4}\,x}{758912(5x^2+7)}+\frac{25\sqrt{x^4+3x^2+4}\,x}{1232(5x^2+7)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] (-555*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(2 + x^2)) + (25*x*Sqrt[4 + 3*x^2 +
x^4])/(1232*(7 + 5*x^2)^2) + (2775*x*Sqrt[4 + 3*x^2 + x^4])/(758912*(7 + 5*
x^2)) - (3285*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/3
035648 + (555*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*Arc
Tan[x/Sqrt[2]], 1/8])/(379456*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*S
qrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(86
24*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (18615*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)
/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(21249536*Sqrt[
2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^3\sqrt{4+3x^2+x^4}}\,dx = \frac{25x\sqrt{4+3x^2+x^4}}{1232\,(7+5x^2)^2} - \frac{\int \frac{-76-10x^2-25x^4}{(7+5x^2)^2\sqrt{4+3x^2+x^4}}\,dx}{1232}$$

$$= \frac{25x\sqrt{4+3x^2+x^4}}{1232\,(7+5x^2)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\,(7+5x^2)} + \frac{\int \frac{-4412-4690x^2-2775x^4}{(7+5x^2)\sqrt{4+3x^2+x^4}}}{758912}$$

$$= \frac{25x\sqrt{4+3x^2+x^4}}{1232\,(7+5x^2)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\,(7+5x^2)} + \frac{\int \frac{-60910-31775x^2}{(7+5x^2)\sqrt{4+3x^2+x^4}}}{3794560}$$

$$= -\frac{555x\sqrt{4+3x^2+x^4}}{758912\,(2+x^2)} + \frac{25x\sqrt{4+3x^2+x^4}}{1232\,(7+5x^2)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\,(7+5x^2)} +$$

$$= -\frac{555x\sqrt{4+3x^2+x^4}}{758912\,(2+x^2)} + \frac{25x\sqrt{4+3x^2+x^4}}{1232\,(7+5x^2)^2} + \frac{2775x\sqrt{4+3x^2+x^4}}{758912\,(7+5x^2)} -$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 10.44, size = 308, normalized size = 0.98

$$\frac{\frac{700x(1393+555x^2)(4+3x^2+x^4)}{(7+5x^2)^2} + i\sqrt{6+2i\sqrt{7}}\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(3885\left(3-i\sqrt{7}\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\left(-9401+3885i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+6570\Pi\left(\frac{5}{14}\left(3+i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right)}{21249536\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*Sqrt[4 + 3*x^2 + x^4]),x]
```

```
[Out] ((700*x*(1393 + 555*x^2)*(4 + 3*x^2 + x^4))/(7 + 5*x^2)^2 + I*Sqrt[6 + (2*I
)*Sqrt[7]]*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I
 + Sqrt[7])]*(3885*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I +
Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (-9401 + (3885*I)*Sqrt[7])
*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I
 + Sqrt[7])] + 6570*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I
)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(21249536*Sqrt[4
 + 3*x^2 + x^4])
```

<u>**Maple**</u> [C] Result contains complex when optimal does not.
time = 0.13, size = 434, normalized size = 1.38

method	result

risch	$\frac{25\sqrt{x^4+3x^2+4}\,x(555x^2+1393)}{758912(5x^2+7)^2}+\frac{555\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\text{EllipticF}\left(\right.\right.}{23716\sqrt{-6+2i\sqrt{}}}$
default	$\frac{25x\sqrt{x^4+3x^2+4}}{1232(5x^2+7)^2}+\frac{2775x\sqrt{x^4+3x^2+4}}{758912(5x^2+7)}-\frac{23\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\text{EllipticF}\left(\right.}{27104\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+}}$
elliptic	$\frac{25x\sqrt{x^4+3x^2+4}}{1232(5x^2+7)^2}+\frac{2775x\sqrt{x^4+3x^2+4}}{758912(5x^2+7)}-\frac{23\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\text{EllipticF}\left(\right.}{27104\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 25/1232*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+2775/758912*x*(x^4+3*x^2+4)^(1/2)
/(5*x^2+7)-23/27104/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1
/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x
*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+555/23716/(-6+2*I*7^(1/2
))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^
(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1
/2),1/4*(2+6*I*7^(1/2))^(1/2))-555/23716/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-
1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^
(1/2)/(3+I*7^(1/2))*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/
2))^(1/2))-3285/5312384/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(
1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipti
cPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1
/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^10 + 900*x^8 + 2810*x^6 + 4648*x^4 +
3969*x^2 + 1372), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2-x+2)(x^2+x+2)}\,(5x^2+7)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 - x + 2)*(x**2 + x + 2))*(5*x**2 + 7)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 4)*(5*x^2 + 7)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(5\,x^2+7\right)}^3\,\sqrt{x^4+3\,x^2+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(1/2)), x)
```

3.371 $\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=219

$$\frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{5000}{3}x\sqrt{4+3x^2+x^4}+625x^3\sqrt{4+3x^2+x^4}-\frac{220779x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}+\frac{220779(2-$$

```
[Out] 1/28*x*(45779*x^2+99493)/(x^4+3*x^2+4)^(1/2)+5000/3*x*(x^4+3*x^2+4)^(1/2)+6
25*x^3*(x^4+3*x^2+4)^(1/2)-220779/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+220779/2
8*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+
4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-130729/24*(x^2+2)*(cos(2*arctan(1/2
*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1
/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*
x^2+4)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1219, 1693, 1211, 1117, 1209}

$$-\frac{130729(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{220779(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{220779\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}+\frac{5000}{3}\sqrt{x^4+3x^2+4}\,x+\frac{(45779x^2+99493)\,x}{28\sqrt{x^4+3x^2+4}}+625\sqrt{x^4+3x^2+4}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(99493 + 45779*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (5000*x*Sqrt[4 + 3*x^2
 + x^4])/3 + 625*x^3*Sqrt[4 + 3*x^2 + x^4] - (220779*x*Sqrt[4 + 3*x^2 + x^4
])/(28*(2 + x^2)) + (220779*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*E
llipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (1
30729*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sq
rt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
```

```
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \frac{(7+5x^2)^5}{(4+3x^2+x^4)^{3/2}}\,dx = \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{1}{28}\int \frac{18156+269221x^2+350000x^4+87500x^6}{\sqrt{4+3x^2+x^4}}\,dx$$
$$= \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}} + 625x^3\sqrt{4+3x^2+x^4} + \frac{1}{140}\int \frac{90780+296105x^2+7000}{\sqrt{4+3x^2+x^4}}$$
$$= \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{5000}{3}x\sqrt{4+3x^2+x^4} + 625x^3\sqrt{4+3x^2+x^4} + \frac{1}{420}\int$$
$$= \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{5000}{3}x\sqrt{4+3x^2+x^4} + 625x^3\sqrt{4+3x^2+x^4} + \frac{220779}{14}$$
$$= \frac{x(99493+45779x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{5000}{3}x\sqrt{4+3x^2+x^4} + 625x^3\sqrt{4+3x^2+x^4} - \frac{220779}{2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.26, size = 339, normalized size = 1.55

$$\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(858479+767337x^2+297500x^4+52500x^6)+662337\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-\sqrt{2}\left(975947i+662337\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{336\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^5/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(858479 + 767337*x^2 + 297500*x^4 + 52500*
x^6) + 662337*Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3
*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[
I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])
] - Sqrt[2]*(975947*I + 662337*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(
-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*Elliptic
F[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7
])])/(336*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 379, normalized size = 1.73

method	result
risch	$\frac{x(52500x^6+297500x^4+767337x^2+858479)}{84\sqrt{x^4+3x^2+4}} + \frac{1766232\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\left(\text{Elli}\right.}{7\sqrt{-6+2}}$

elliptic	$-\frac{2\left(-\frac{45779}{56}x^3-\frac{99493}{56}x\right)}{\sqrt{x^4+3x^2+4}}+625x^3\sqrt{x^4+3x^2+4}+\frac{5000x\sqrt{x^4+3x^2+4}}{3}+\frac{1766232\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)}}{}$
default	$-\frac{6250\left(\frac{31}{14}x^3+\frac{18}{7}x\right)}{\sqrt{x^4+3x^2+4}}+625x^3\sqrt{x^4+3x^2+4}+\frac{5000x\sqrt{x^4+3x^2+4}}{3}-\frac{505532\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6250*(31/14*x^3+18/7*x)/(x^4+3*x^2+4)^(1/2)+625*x^3*(x^4+3*x^2+4)^(1/2)+50
00/3*x*(x^4+3*x^2+4)^(1/2)-505532/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*
7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*
EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+1766232/7
/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7
^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2
*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2
))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-43750*(-9/14*x^3+2/7*x)/(x^4+3*x^2+4)^
(1/2)-122500*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)-171500*(3/14*x^3+4/7*x)/
(x^4+3*x^2+4)^(1/2)-120050*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-33614*(1/5
6*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2 + 7)^5}{((x^2 - x + 2)(x^2 + x + 2))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**5/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**5/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^5/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^5/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(5\,x^2 + 7)^5}{(x^4 + 3\,x^2 + 4)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^5/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.372 $\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=200

$$\frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{625}{3}x\sqrt{4+3x^2+x^4}+\frac{14523x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}-\frac{14523(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\Big(2\tan}{14\sqrt{2}\,\sqrt{4+3x^2+x^4}}$$

```
[Out] 1/28*x*(-4023*x^2+2719)/(x^4+3*x^2+4)^(1/2)+625/3*x*(x^4+3*x^2+4)^(1/2)+145
23/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-14523/28*(x^2+2)*(cos(2*arctan(1/2*x*2^
(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*
2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4
)^(1/2)+4243/24*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan
(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+
3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1219, 1693, 1211, 1117, 1209}

$$\frac{4243(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\,\sqrt{x^4+3x^2+4}}-\frac{14523(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\,\sqrt{x^4+3x^2+4}}+\frac{14523\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}+\frac{625}{3}\sqrt{x^4+3x^2+4}\,x+\frac{(2719-4023x^2)\,x}{28\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(2719 - 4023*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) + (625*x*Sqrt[4 + 3*x^2 +
x^4])/3 + (14523*x*Sqrt[4 + 3*x^2 + x^4])/(28*(2 + x^2)) - (14523*(2 + x^2)
*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(
14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (4243*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/
(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[2]*Sqrt[4 + 3*x^
2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
```

```
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(7+5x^2)^4}{(4+3x^2+x^4)^{3/2}}\,dx &= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{1}{28}\int \frac{14088+49523x^2+17500x^4}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{625}{3}x\sqrt{4+3x^2+x^4} + \frac{1}{84}\int \frac{-27736+43569x^2}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{625}{3}x\sqrt{4+3x^2+x^4} + \frac{4243}{6}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx - \frac{14523}{14} \\
&= \frac{x(2719-4023x^2)}{28\sqrt{4+3x^2+x^4}} + \frac{625}{3}x\sqrt{4+3x^2+x^4} + \frac{14523x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)} - \frac{14523(2+}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.25, size = 333, normalized size = 1.66

$$
\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(78157+40431x^2+17500x^4)-43569\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\sqrt{2}\left(186179i+43569\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{336\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^4/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(78157 + 40431*x^2 + 17500*x^4) - 43569*Sq
rt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*S
qrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(186
179*I + 43569*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*
Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[
(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(336*Sqrt[(-
I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 339, normalized size = 1.70

method	result
risch	$\frac{x\left(17500x^4+40431x^2+78157\right)}{84\sqrt{x^4+3x^2+4}} - \frac{116184\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-6}}{}\right.\right.}{7\sqrt{-6+2i\sqrt{7}}\sqrt{x^4}}$

elliptic	$-\frac{2\left(\frac{4023}{56}x^3-\frac{2719}{56}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{625x\sqrt{x^4+3x^2+4}}{3}-\frac{116184\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)}}{7\sqrt{}}$
default	$-\frac{1250\left(-\frac{9}{14}x^3+\frac{2}{7}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{625x\sqrt{x^4+3x^2+4}}{3}-\frac{27736\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x}}{21\sqrt{-6+2i\sqrt{7}}\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1250*(-9/14*x^3+2/7*x)/(x^4+3*x^2+4)^(1/2)+625/3*x*(x^4+3*x^2+4)^(1/2)-277
36/21/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/
8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2)
)^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-116184/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8
+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)
^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(
1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/
2)))-7000*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)-14700*(3/14*x^3+4/7*x)/(x^4
+3*x^2+4)^(1/2)-13720*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-4802*(1/56*x+3/
56*x^3)/(x^4+3*x^2+4)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^4}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**4/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**4/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^4/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^4/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(5\,x^2+7)^4}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^4/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.373 $\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{4449x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}-\frac{4449(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{973(\cdots}{\cdots}$$

```
[Out] -1/28*x*(949*x^2+2323)/(x^4+3*x^2+4)^(1/2)+4449/28*x*(x^4+3*x^2+4)^(1/2)/(x
^2+2)-4449/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1
/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*
((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+973/8*(x^2+2)*(cos(2*ar
ctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*
arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)
/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1219, 1211, 1117, 1209}

$$\frac{973(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{4449(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{4449\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}-\frac{(949x^2+2323)\,x}{28\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/28*(x*(2323 + 949*x^2))/Sqrt[4 + 3*x^2 + x^4] + (4449*x*Sqrt[4 + 3*x^2 +
 x^4])/(28*(2 + x^2)) - (4449*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]
*EllipticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) +
(973*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqr
t[2]], 1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
```

```
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(7+5x^2)^3}{(4+3x^2+x^4)^{3/2}}\,dx &= -\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int \frac{4724+4449x^2}{\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}}-\frac{4449}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{973}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\
&= -\frac{x(2323+949x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{4449x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}-\frac{4449(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\Big(}{14\sqrt{2}\,\sqrt{4+3x^2-}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.24, size = 328, normalized size = 1.81

$$\frac{-4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(2323+949x^2)-4449\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\sqrt{2}\left(3899i+4449\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{112\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^3/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (-4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(2323 + 949*x^2) - 4449*Sqrt[2]*(3*I + Sq
rt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt
[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + S
qrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(3899*I + 4449*Sqrt
[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7
] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqr
t[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7]
)]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 301, normalized size = 1.66

method	result
risch	$-\frac{x(949x^2+2323)}{28\sqrt{x^4+3x^2+4}}-\frac{35592\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-6+}}{4}\right.\right.}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+}}$
elliptic	$-\frac{2\left(\frac{949}{56}x^3+\frac{2323}{56}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{35592\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-6+2}}{4}\right.\right.}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3}}$
default	$-\frac{250\left(-\frac{1}{14}x^3-\frac{6}{7}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{4724\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-6+2i}}{4}\right.}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -250*(-1/14*x^3-6/7*x)/(x^4+3*x^2+4)^(1/2)+4724/7/(-6+2*I*7^(1/2))^(1/2)*(1
-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*
x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/
2))-35592/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-
3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(
1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6
+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-1050*(3/14*x^3+4/7*x)/(x^4+
3*x^2+4)^(1/2)-1470*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-686*(1/56*x+3/56*
x^3)/(x^4+3*x^2+4)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(5x^2+7)^3}{((x^2-x+2)(x^2+x+2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral((5*x**2 + 7)**3/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate((5*x^2 + 7)^3/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(5\,x^2+7)^3}{(x^4+3\,x^2+4)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(3/2),x)
[Out] int((5*x^2 + 7)^3/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.374 $\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}}-\frac{113x\sqrt{4+3x^2+x^4}}{28(2+x^2)}+\frac{113(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{4+3x^2+x^4}}+\frac{9(2+x$$

```
[Out] -1/28*x*(-113*x^2+9)/(x^4+3*x^2+4)^(1/2)-113/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+
2)+113/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x
*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^
4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+9/8*(x^2+2)*(cos(2*arctan(1
/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan
(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+
3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1219, 1211, 1117, 1209}

$$\frac{9(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{113(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{113\sqrt{x^4+3x^2+4}\,x}{28(x^2+2)}-\frac{(9-113x^2)x}{28\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/28*(x*(9 - 113*x^2))/Sqrt[4 + 3*x^2 + x^4] - (113*x*Sqrt[4 + 3*x^2 + x^4
])/(28*(2 + x^2)) + (113*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Elli
pticE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (9*(2
 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]],
1/8])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
```

```
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1219

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^4)^(p + 1)*((a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(7+5x^2)^2}{(4+3x^2+x^4)^{3/2}}\,dx &= -\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int \frac{352-113x^2}{\sqrt{4+3x^2+x^4}}\,dx \\
&= -\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{9}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx+\frac{113}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx \\
&= -\frac{x(9-113x^2)}{28\sqrt{4+3x^2+x^4}}-\frac{113x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}+\frac{113(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\Big(2\mathrm{t}}{14\sqrt{2}\,\sqrt{4+3x^2+}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.23, size = 329, normalized size = 1.82

$$\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(-9+113x^2)+113\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-\sqrt{2}\left(1043i+113\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{112\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)^2/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(-9 + 113*x^2) + 113*Sqrt[2]*(3*I + Sqrt[7
])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7]
+ (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[
7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(1043*I + 113*Sqrt[7])*
Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (
2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])
]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sq
rt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 278, normalized size = 1.54

method	result
risch	$\frac{x(113x^2-9)}{28\sqrt{x^4+3x^2+4}}+\frac{904\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{}}}{4}\right.\right.}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x}}$
elliptic	$-\frac{2\left(-\frac{113}{56}x^3+\frac{9}{56}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{904\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{}}}{4}\right.\right.}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x}}$
default	$-\frac{50\left(\frac{3}{14}x^3+\frac{4}{7}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{352\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{}}}{4}\right.}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -50*(3/14*x^3+4/7*x)/(x^4+3*x^2+4)^(1/2)+352/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-
3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2
+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))
+904/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1
/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x
*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*
7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-140*(-1/7*x^3-3/14*x)/(x^4+3*x^2
+4)^(1/2)-98*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(5x^2+7\right)^2}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)**2/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)^2/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(5\,x^2+7\right)}^2}{{\left(x^4+3\,x^2+4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)^2/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.375 $\int \frac{7+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=181

$$\frac{x(53+19x^2)}{28\sqrt{4+3x^2+x^4}}-\frac{19x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}+\frac{19(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\,\sqrt{4+3x^2+x^4}}-\frac{3(2+x^2)\sqrt{}}{}$$

```
[Out] 1/28*x*(19*x^2+53)/(x^4+3*x^2+4)^(1/2)-19/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+
19/28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(
1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*
x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)-3/8*(x^2+2)*(cos(2*arctan(1/2*x
*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2
*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^
2+4)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1192, 1211, 1117, 1209}

$$-\frac{3(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\,\sqrt{x^4+3x^2+4}}+\frac{19(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\,\sqrt{x^4+3x^2+4}}-\frac{19\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}+\frac{(19x^2+53)\,x}{28\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (x*(53 + 19*x^2))/(28*Sqrt[4 + 3*x^2 + x^4]) - (19*x*Sqrt[4 + 3*x^2 + x^4])
/(28*(2 + x^2)) + (19*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*Ellipti
cE[2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3*(2 +
x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8
])/(4*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
```

```
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{7+5x^2}{\left(4+3x^2+x^4\right)^{3/2}}\,dx &= \frac{x(53+19x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int \frac{-4-19x^2}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{x(53+19x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{19}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx-\frac{3}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx \\
&= \frac{x(53+19x^2)}{28\sqrt{4+3x^2+x^4}}-\frac{19x\sqrt{4+3x^2+x^4}}{28\left(2+x^2\right)}+\frac{19(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\Big(2\tan^{-}}{14\sqrt{2}\sqrt{4+3x^2+x^4}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.23, size = 329, normalized size = 1.82

$$\frac{4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(53+19x^2)+19\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-\sqrt{2}\left(49i+19\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{112\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x^2)/(4 + 3*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(53 + 19*x^2) + 19*Sqrt[2]*(3*I + Sqrt[7])
*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] +
(2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7]
)]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - Sqrt[2]*(49*I + 19*Sqrt[7])*Sqrt[
(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*
x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x],
 (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4
+ 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 255, normalized size = 1.41

method	result
risch	$\frac{x(19x^2+53)}{28\sqrt{x^4+3x^2+4}}+\frac{152\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\right.\right.}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x}}$
elliptic	$-\frac{2\left(-\frac{19}{56}x^3-\frac{53}{56}x\right)}{\sqrt{x^4+3x^2+4}}+\frac{152\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\right.\right.}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x}}$
default	$-\frac{10\left(-\frac{1}{7}x^3-\frac{3}{14}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{4\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\right.}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -10*(-1/7*x^3-3/14*x)/(x^4+3*x^2+4)^(1/2)-4/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3
/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+
4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+
152/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/
8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*
(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7
^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))-14*(1/56*x+3/56*x^3)/(x^4+3*x^2+4
)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{5x^2+7}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 7)/((x**2 - x + 2)*(x**2 + x + 2))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 7)/(x^4 + 3*x^2 + 4)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{5\,x^2+7}{{\left(x^4+3\,x^2+4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int((5*x^2 + 7)/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.376 $\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=181

$$-\frac{x(1+3x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{3x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}-\frac{3(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\,\sqrt{4+3x^2+x^4}}+\frac{(2+x^2)\sqrt{\cdot}}{}$$

```
[Out] -1/28*x*(3*x^2+1)/(x^4+3*x^2+4)^(1/2)+3/28*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-3/
28*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2
)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2
+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)+1/8*(x^2+2)*(cos(2*arctan(1/2*x*2^
(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*
2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4
)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1106, 1211, 1117, 1209}

$$\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{4\sqrt{2}\,\sqrt{x^4+3x^2+4}}-\frac{3(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{14\sqrt{2}\,\sqrt{x^4+3x^2+4}}+\frac{3\sqrt{x^4+3x^2+4}\,x}{28\,(x^2+2)}-\frac{(3x^2+1)\,x}{28\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] -1/28*(x*(1 + 3*x^2))/Sqrt[4 + 3*x^2 + x^4] + (3*x*Sqrt[4 + 3*x^2 + x^4])/(
28*(2 + x^2)) - (3*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[
2*ArcTan[x/Sqrt[2]], 1/8])/(14*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + ((2 + x^2)*
Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(4
*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(4+3x^2+x^4)^{3/2}}\,dx &= -\frac{x(1+3x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{1}{28}\int \frac{8+3x^2}{\sqrt{4+3x^2+x^4}}\,dx\\ &= -\frac{x(1+3x^2)}{28\sqrt{4+3x^2+x^4}}-\frac{3}{14}\int \frac{1-\frac{x^2}{2}}{\sqrt{4+3x^2+x^4}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{4+3x^2+x^4}}\,dx\\ &= -\frac{x(1+3x^2)}{28\sqrt{4+3x^2+x^4}}+\frac{3x\sqrt{4+3x^2+x^4}}{28\,(2+x^2)}-\frac{3(2+x^2)\sqrt{\frac{4+3x^2+x^4}{(2+x^2)^2}}\,E\Big(2\tan^{-1}}{14\sqrt{2}\,\sqrt{4+3x^2+x^4}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.24, size = 328, normalized size = 1.81

$$\frac{-4\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x(1+3x^2)-3\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\sqrt{2}\left(-7i+3\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{112\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^2 + x^4)^(-3/2),x]
```

```
[Out] (-4*Sqrt[(-I)/(-3*I + Sqrt[7])]*x*(1 + 3*x^2) - 3*Sqrt[2]*(3*I + Sqrt[7])*S
qrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2
```

```
*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]
*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(-7*I + 3*Sqrt[7])*Sqrt[(-3
*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2
)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3
*I - Sqrt[7])/(3*I + Sqrt[7])])/(112*Sqrt[(-I)/(-3*I + Sqrt[7])]*Sqrt[4 + 3
*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 232, normalized size = 1.28

method	result
risch	$-\frac{x(3x^2+1)}{28\sqrt{x^4+3x^2+4}}+\frac{8\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{\cdots}}}{4}\cdots\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}\cdots$
default	$-\frac{2\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}+\frac{8\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\cdots\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}\cdots$
elliptic	$-\frac{2\left(\frac{1}{56}x+\frac{3}{56}x^3\right)}{\sqrt{x^4+3x^2+4}}+\frac{8\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4}\cdots\right)}{7\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1/56*x+3/56*x^3)/(x^4+3*x^2+4)^(1/2)+8/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/
8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4
)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-2
4/7/(-6+2*I*7^(1/2))^(1/2)*(1-(-3/8+1/8*I*7^(1/2))*x^2)^(1/2)*(1-(-3/8-1/8*
I*7^(1/2))*x^2)^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*(EllipticF(1/4*x*(-
6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))-EllipticE(1/4*x*(-6+2*I*7^(
1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{4}+3x^{2}+4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral((x**4 + 3*x**2 + 4)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 3*x^2 + 4)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(x^4+3\,x^2+4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 + x^4 + 4)^(3/2),x)
```

```
[Out] int(1/(3*x^2 + x^4 + 4)^(3/2), x)
```

3.377 $\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=284

$$-\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}}+\frac{x\sqrt{4+3x^2+x^4}}{77(2+x^2)}+\frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)-\frac{\sqrt{2}\,(2+x^2)\sqrt{\frac{4+3x^2+}{(2+x^2)}}}{77\sqrt{4+}}$$

```
[Out] 25/13552*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)-1/308*x*(4*
x^2+13)/(x^4+3*x^2+4)^(1/2)+1/77*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)-1/24*(x^2+2)
*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*Ellipt
icF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/
2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)+425/7392*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)
))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1
/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^
2+4)^(1/2)-1/77*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan
(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2
)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1235, 1192, 1211, 1117, 1209, 1230, 1720}

$$\frac{25}{176}\sqrt{\frac{5}{77}}\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)-\frac{(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{12\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{\sqrt{2}(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{77\sqrt{x^4+3x^2+4}}+\frac{425(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{3696\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{\sqrt{x^4+3x^2+4}\,x}{77(x^2+2)}-\frac{(4x^2+13)x}{308\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/308*(x*(13 + 4*x^2))/Sqrt[4 + 3*x^2 + x^4] + (x*Sqrt[4 + 3*x^2 + x^4])/(
77*(2 + x^2)) + (25*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^
4]])/176 - (Sqrt[2]*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE
[2*ArcTan[x/Sqrt[2]], 1/8])/(77*Sqrt[4 + 3*x^2 + x^4]) - ((2 + x^2)*Sqrt[(4
 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(12*Sqrt[
2]*Sqrt[4 + 3*x^2 + x^4]) + (425*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)
^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(3696*Sqrt[2]*Sqrt[4 + 3*
x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)(4+3x^2+x^4)^{3/2}}\,dx = \frac{1}{44}\int \frac{-8-5x^2}{(4+3x^2+x^4)^{3/2}}\,dx + \frac{25}{44}\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx$$

$$= -\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}} + \frac{\int \frac{-4+16x^2}{\sqrt{4+3x^2+x^4}}\,dx}{1232} - \frac{25}{132}\int \frac{1}{\sqrt{4+3x^2+x^4}}$$

$$= -\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}} + \frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right) - \frac{25(2+}{}$$

$$= -\frac{x(13+4x^2)}{308\sqrt{4+3x^2+x^4}} + \frac{x\sqrt{4+3x^2+x^4}}{77(2+x^2)} + \frac{25}{176}\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{2\sqrt{}}{\sqrt{4+3}}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.33, size = 483, normalized size = 1.70

$$\frac{-26\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x-8\sqrt{-\frac{i}{-3i+\sqrt{7}}}\,x^3-2\sqrt{2}\left(3i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+\sqrt{2}\left(7i+2\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-25i\sqrt{2}\sqrt{\frac{-3i+\sqrt{7}-2ix^2}{-3i+\sqrt{7}}}\sqrt{\frac{3i+\sqrt{7}+2ix^2}{3i+\sqrt{7}}}\,\Pi\left(\frac{5}{14}\left(3+i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)}{616\sqrt{-\frac{i}{-3i+\sqrt{7}}}\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-26*Sqrt[(-I)/(-3*I + Sqrt[7])]*x - 8*Sqrt[(-I)/(-3*I + Sqrt[7])]*x^3 - 2*
Sqrt[2]*(3*I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]
*Sqrt[(3*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticE[I*ArcSinh[Sqrt
[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + Sqrt[2]*(7
```

```
*I + 2*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3
*I + Sqrt[7] + (2*I)*x^2)/(3*I + Sqrt[7])]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/
(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] - (25*I)*Sqrt[2]*Sqr
t[(-3*I + Sqrt[7] - (2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[(3*I + Sqrt[7] + (2*I
)*x^2)/(3*I + Sqrt[7])]*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(
-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])])/(616*Sqrt[(-I
)/(-3*I + Sqrt[7])]*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 409, normalized size = 1.44

method	result
risch	$-\frac{x(4x^2+13)}{308\sqrt{x^4+3x^2+4}}-\frac{32\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i}}{4}\right.\right.}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3}}$
default	$-\frac{2\left(\frac{1}{154}x^3+\frac{13}{616}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i}}{4}\right.}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$
elliptic	$-\frac{2\left(\frac{1}{154}x^3+\frac{13}{616}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-6+2i\sqrt{7}}}{4},\frac{\sqrt{2+6i}}{4}\right.}{77\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2+4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1/154*x^3+13/616*x)/(x^4+3*x^2+4)^(1/2)-1/77/(-6+2*I*7^(1/2))^(1/2)*(1+
3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3
*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1
/2))-32/77/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/
8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticF(
1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+32/77/(-6+2*I*7^(1/
2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))
^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2))+25/308/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^
2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4
)^(1/2)*EllipticPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(
-3/8-1/8*I*7^(1/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(5*x^10 + 37*x^8 + 127*x^6 + 239*x^4 + 248*x
^2 + 112), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^{2}-x+2\right)\left(x^{2}+x+2\right)\right)^{\frac{3}{2}} \cdot\left(5x^{2}+7\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(5\,x^2+7\right)\,{\left(x^4+3\,x^2+4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)*(3*x^2 + x^4 + 4)^(3/2)), x)
```

3.378 $\int \frac{1}{(7+5x^2)^2 (4+3x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=312

$$\frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} - \frac{199x\sqrt{4+3x^2+x^4}}{27104\,(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{27104\,(7+5x^2)} + \frac{575\sqrt{\frac{5}{77}}\,\tan^{-1}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{4+3x^2+x^4}}\right)}{108416} +$$

```
[Out] 575/8348032*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+1/13552*
x*(37*x^2+24)/(x^4+3*x^2+4)^(1/2)-199/27104*x*(x^4+3*x^2+4)^(1/2)/(x^2+2)+6
25/27104*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+199/27104*(x^2+2)*(cos(2*arctan(1/
2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(
1/2*x*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3
*x^2+4)^(1/2)+9775/4553472*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/c
os(2*arctan(1/2*x*2^(1/2)))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,
1/4*2^(1/2))*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-2/
231*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/
2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((x^4+3*x^2+4)/(x^
2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1242, 1192, 1211, 1117, 1209, 1237, 1728, 1722, 1720, 1230}

$$\frac{575\sqrt{\frac{5}{77}}\,\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{108416} - \frac{2\sqrt{2}\,(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{231\sqrt{x^4+3x^2+4}} + \frac{199(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{13552\sqrt{2}\,\sqrt{x^4+3x^2+4}} + \frac{9775(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{2276736\sqrt{2}\,\sqrt{x^4+3x^2+4}} - \frac{199\sqrt{x^4+3x^2+4}\,x}{27104\,(x^2+2)} + \frac{625\sqrt{x^4+3x^2+4}\,x}{27104\,(5x^2+7)} + \frac{(37x^2+24)\,x}{13552\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (x*(24 + 37*x^2))/(13552*Sqrt[4 + 3*x^2 + x^4]) - (199*x*Sqrt[4 + 3*x^2 + x
^4])/(27104*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(27104*(7 + 5*x^2))
+ (575*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/108416 +
 (199*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sq
rt[2]], 1/8])/(13552*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (2*Sqrt[2]*(2 + x^2)*
Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(2
31*Sqrt[4 + 3*x^2 + x^4]) + (9775*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2
)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(2276736*Sqrt[2]*Sqrt[4
+ 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
```

```
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^2 (4+3x^2+x^4)^{3/2}}\,dx = \int \left(\frac{-36+5x^2}{1936\,(4+3x^2+x^4)^{3/2}} + \frac{25}{44\,(7+5x^2)^2 \sqrt{4+3x^2+x^4}} - \frac{}{1936\,(7}\right.$$

$$= \frac{\int \frac{-36+5x^2}{(4+3x^2+x^4)^{3/2}}\,dx}{1936} - \frac{25\int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{1936} + \frac{25}{44}\int \frac{}{(7+5x^2)^2}$$

$$= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{27104\,(7+5x^2)} + \frac{\int \frac{-348-148x^2}{\sqrt{4+3x^2+x^4}}\,dx}{54208}$$

$$= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{27104\,(7+5x^2)} - \frac{25\sqrt{\frac{5}{77}}\tan^{-1}\left(\frac{}{\sqrt{4}}\right.}{7744}$$

$$= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} - \frac{199x\sqrt{4+3x^2+x^4}}{27104\,(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{27104\,(7+5x^2)}$$

$$= \frac{x(24+37x^2)}{13552\sqrt{4+3x^2+x^4}} - \frac{199x\sqrt{4+3x^2+x^4}}{27104\,(2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{27104\,(7+5x^2)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.27, size = 311, normalized size = 1.00

$$\frac{28x(2836+2633x^2+995x^4)+i\sqrt{6+2i\sqrt{7}}\,(7+5x^2)\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(1393\left(3-i\sqrt{7}\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+7\left(101+199i\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)-1150\Pi\left(\frac{5}{14}\left(3+i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)\middle|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right)}{758912\,(7+5x^2)\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^2*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (28*x*(2836 + 2633*x^2 + 995*x^4) + I*Sqrt[6 + (2*I)*Sqrt[7]]*(7 + 5*x^2)*S
qrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 + ((2*I)*x^2)/(3*I + Sqrt[7])]
*(1393*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 7*(101 + (199*I)*Sqrt[7])*EllipticF[I*
ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]
- 1150*EllipticPi[(5*(3 + I*Sqrt[7]))/14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqr
t[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])]))/(758912*(7 + 5*x^2)*Sqrt[4 +
3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 433, normalized size = 1.39

method	result
risch	$\frac{x(995x^4+2633x^2+2836)}{27104(5x^2+7)\sqrt{x^4+3x^2+4}}+\frac{199\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\,\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-\cdots}}{}\right.\right.}{847\sqrt{-6+2i\sqrt{7}}\,\sqrt{\cdots}}$ …
default	$\frac{625x\sqrt{x^4+3x^2+4}}{27104(5x^2+7)}-\frac{2\left(-\frac{37}{27104}x^3-\frac{3}{3388}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{349\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{\cdots}}{}\right.}{6776\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2\cdots}}$ …
elliptic	$\frac{625x\sqrt{x^4+3x^2+4}}{27104(5x^2+7)}-\frac{2\left(-\frac{37}{27104}x^3-\frac{3}{3388}x\right)}{\sqrt{x^4+3x^2+4}}-\frac{349\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\,\sqrt{1+\frac{3x^2}{8}+\frac{ix^2\sqrt{7}}{8}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{\cdots}}{}\right.}{6776\sqrt{-6+2i\sqrt{7}}\,\sqrt{x^4+3x^2\cdots}}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 625/27104*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)-2*(-37/27104*x^3-3/3388*x)/(x^4+3
*x^2+4)^(1/2)-349/6776/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))
^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/
4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+199/847/(-6+2*I*7^(1/
2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))
^(1/2)/(x^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(
1/2),1/4*(2+6*I*7^(1/2))^(1/2))-199/847/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1
/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(
1/2)/(3+I*7^(1/2))*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2
))^(1/2))+575/189728/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2
))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticPi
((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1/2)
)^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(25*x^12 + 220*x^10 + 894*x^8 + 2084*x^6 + 2
913*x^4 + 2296*x^2 + 784), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^{2}-x+2\right)\left(x^{2}+x+2\right)\right)^{\frac{3}{2}}\left(5x^{2}+7\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**2/(x**4+3*x**2+4)**(3/2),x)
```

```
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^2/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(5\,x^2+7\right)}^2\,{\left(x^4+3\,x^2+4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2)),x)
```

```
[Out] int(1/((5*x^2 + 7)^2*(3*x^2 + x^4 + 4)^(3/2)), x)
```

3.379 $\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=340

$$\frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}}-\frac{18159x\sqrt{4+3x^2+x^4}}{33392128\,(2+x^2)}+\frac{625x\sqrt{4+3x^2+x^4}}{54208\,(7+5x^2)^2}+\frac{51875x\sqrt{4+3x^2+x^4}}{33392128\,(7+5x^2)}-\frac{529425\sqrt{\ }}{}$$

```
[Out] -529425/10284775424*arctan(2/35*x*385^(1/2)/(x^4+3*x^2+4)^(1/2))*385^(1/2)+
1/596288*x*(139*x^2+548)/(x^4+3*x^2+4)^(1/2)-18159/33392128*x*(x^4+3*x^2+4)
^(1/2)/(x^2+2)+625/54208*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+51875/33392128*x
*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)+18159/33392128*(x^2+2)*(cos(2*arctan(1/2*x*2
^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)))*EllipticE(sin(2*arctan(1/2*x
*2^(1/2))),1/4*2^(1/2))*2^(1/2)*((x^4+3*x^2+4)/(x^2+2)^2)^(1/2)/(x^4+3*x^2+
4)^(1/2)+843/758912*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*ar
ctan(1/2*x*2^(1/2)))*EllipticF(sin(2*arctan(1/2*x*2^(1/2))),1/4*2^(1/2))*((
x^4+3*x^2+4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)-3000075/186995916
8*(x^2+2)*(cos(2*arctan(1/2*x*2^(1/2)))^2)^(1/2)/cos(2*arctan(1/2*x*2^(1/2)
))*EllipticPi(sin(2*arctan(1/2*x*2^(1/2))),-9/280,1/4*2^(1/2))*((x^4+3*x^2+
4)/(x^2+2)^2)^(1/2)*2^(1/2)/(x^4+3*x^2+4)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {1242, 1192, 1211, 1117, 1209, 1237, 1710, 1728, 1722, 1720, 1230}

$$-\frac{529425\sqrt{\frac{5}{77}}\,\text{ArcTan}\left(\frac{2\sqrt{\frac{11}{35}}\,x}{\sqrt{x^4+3x^2+4}}\right)}{133568512}+\frac{843(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{379456\sqrt{2}\sqrt{x^4+3x^2+4}}+\frac{18159(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{16696064\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{3000075(x^2+2)\sqrt{\frac{x^4+3x^2+4}{(x^2+2)^2}}\,\Pi\left(-\frac{9}{280};2\text{ArcTan}\left(\frac{x}{\sqrt{2}}\right)|\frac{1}{8}\right)}{934979584\sqrt{2}\sqrt{x^4+3x^2+4}}-\frac{18159\sqrt{x^4+3x^2+4}\,x}{33392128\,(x^2+2)}+\frac{51875\sqrt{x^4+3x^2+4}\,x}{33392128\,(5x^2+7)}+\frac{625\sqrt{x^4+3x^2+4}\,x}{54208\,(5x^2+7)^2}+\frac{(139x^2+548)\,x}{596288\sqrt{x^4+3x^2+4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (x*(548 + 139*x^2))/(596288*Sqrt[4 + 3*x^2 + x^4]) - (18159*x*Sqrt[4 + 3*x^
2 + x^4])/(33392128*(2 + x^2)) + (625*x*Sqrt[4 + 3*x^2 + x^4])/(54208*(7 +
5*x^2)^2) + (51875*x*Sqrt[4 + 3*x^2 + x^4])/(33392128*(7 + 5*x^2)) - (52942
5*Sqrt[5/77]*ArcTan[(2*Sqrt[11/35]*x)/Sqrt[4 + 3*x^2 + x^4]])/133568512 + (
18159*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticE[2*ArcTan[x/Sq
rt[2]], 1/8])/(16696064*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) + (843*(2 + x^2)*Sqr
t[(4 + 3*x^2 + x^4)/(2 + x^2)^2]*EllipticF[2*ArcTan[x/Sqrt[2]], 1/8])/(3794
56*Sqrt[2]*Sqrt[4 + 3*x^2 + x^4]) - (3000075*(2 + x^2)*Sqrt[(4 + 3*x^2 + x^
4)/(2 + x^2)^2]*EllipticPi[-9/280, 2*ArcTan[x/Sqrt[2]], 1/8])/(934979584*Sq
rt[2]*Sqrt[4 + 3*x^2 + x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
```

```
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1242

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Module[{aa, bb, cc}, Int[ExpandIntegrand[1/Sqrt[aa + bb*x^2 + c
c*x^4], (d + e*x^2)^q*(aa + bb*x^2 + cc*x^4)^(p + 1/2), x] /. {aa -> a, bb
-> b, cc -> c}, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, 0] && IntegerQ[p + 1/2]
```

Rule 1710

```
Int[((P4x_)*((d_) + (e_.)*(x_)^2)^(q_))/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x
_)^4], x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = C
oeff[P4x, x, 4]}, Simp[(-(C*d^2 - B*d*e + A*e^2))*x*(d + e*x^2)^(q + 1)*(Sq
rt[a + b*x^2 + c*x^4]/(2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(
2*d*(q + 1)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x
^2 + c*x^4])*Simp[a*d*(C*d - B*e) + A*(a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1)) - 2*((B*d - A*e)*(b*e*(q + 2) - c*d*(q + 1)) - C*d*(b*d + a*e*(q + 1
)))*x^2 + c*(C*d^2 - B*d*e + A*e^2)*(2*q + 5)*x^4, x], x], x]] /; FreeQ[{a,
 b, c, d, e}, x] && PolyQ[P4x, x^2] && LeQ[Expon[P4x, x], 4] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{(7+5x^2)^3 (4+3x^2+x^4)^{3/2}}\,dx = \int \left(\frac{388+215x^2}{85184 (4+3x^2+x^4)^{3/2}} + \frac{25}{44 (7+5x^2)^3 \sqrt{4+3x^2+x^4}} - \frac{}{1936 (7}\right.$$

$$= \frac{\int \frac{388+215x^2}{(4+3x^2+x^4)^{3/2}}\,dx}{85184} - \frac{1075 \int \frac{1}{(7+5x^2)\sqrt{4+3x^2+x^4}}\,dx}{85184} - \frac{25 \int \frac{}{(7+5x^2)^2 \sqrt{4}}}{19}$$

$$= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{54208 (7+5x^2)^2} - \frac{625x\sqrt{4+3x^2+x^4}}{1192576 (7+5x^2)}$$

$$= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} + \frac{625x\sqrt{4+3x^2+x^4}}{54208 (7+5x^2)^2} + \frac{51875x\sqrt{4+3x^2+x}}{33392128 (7+5x^2)}$$

$$= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{153x\sqrt{4+3x^2+x^4}}{1192576 (2+x^2)} + \frac{625x\sqrt{4+3x^2+x^4}}{54208 (7+5x^2)^2}$$

$$= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{18159x\sqrt{4+3x^2+x^4}}{33392128 (2+x^2)} + \frac{625x\sqrt{4+3x^2+x}}{54208 (7+5x^2)^2}$$

$$= \frac{x(548+139x^2)}{596288\sqrt{4+3x^2+x^4}} - \frac{18159x\sqrt{4+3x^2+x^4}}{33392128 (2+x^2)} + \frac{625x\sqrt{4+3x^2+x}}{54208 (7+5x^2)^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.36, size = 320, normalized size = 0.94

$$\frac{28x(4496212+5811451x^2+2838330x^4+453975x^6)+3i\sqrt{6+2i\sqrt{7}}\,(7+5x^2)^2\sqrt{1-\frac{2ix^2}{-3i+\sqrt{7}}}\sqrt{1+\frac{2ix^2}{3i+\sqrt{7}}}\left(42371\left(3-i\sqrt{7}\right)E\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+7i\left(23633i+6053\sqrt{7}\right)F\left(i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)+352950\Pi\left(\frac{5}{14}\left(3+i\sqrt{7}\right);i\sinh^{-1}\left(\sqrt{-\frac{2i}{-3i+\sqrt{7}}}\,x\right)|\frac{3i-\sqrt{7}}{3i+\sqrt{7}}\right)\right)}{934979584\,(7+5x^2)^2\sqrt{4+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((7 + 5*x^2)^3*(4 + 3*x^2 + x^4)^(3/2)),x]
```

```
[Out] (28*x*(4496212 + 5811451*x^2 + 2838330*x^4 + 453975*x^6) + (3*I)*Sqrt[6 + (
2*I)*Sqrt[7]]*(7 + 5*x^2)^2*Sqrt[1 - ((2*I)*x^2)/(-3*I + Sqrt[7])]*Sqrt[1 +
```

```
 ((2*I)*x^2)/(3*I + Sqrt[7])]*(42371*(3 - I*Sqrt[7])*EllipticE[I*ArcSinh[Sq
rt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt[7])] + (7*I)*(2
3633*I + 6053*Sqrt[7])*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x]
, (3*I - Sqrt[7])/(3*I + Sqrt[7])] + 352950*EllipticPi[(5*(3 + I*Sqrt[7]))/
14, I*ArcSinh[Sqrt[(-2*I)/(-3*I + Sqrt[7])]*x], (3*I - Sqrt[7])/(3*I + Sqrt
[7])]))/(934979584*(7 + 5*x^2)^2*Sqrt[4 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 457, normalized size = 1.34

method	result
risch	$\frac{x(453975x^6+2838330x^4+5811451x^2+4496212)}{33392128(5x^2+7)^2\sqrt{x^4+3x^2+4}}+\frac{18159\sqrt{1-\left(-\frac{3}{8}+\frac{i\sqrt{7}}{8}\right)x^2}\sqrt{1-\left(-\frac{3}{8}-\frac{i\sqrt{7}}{8}\right)x^2}\left(\mathrm{E}\right.}{1043504\sqrt{-}}$
default	$\frac{625x\sqrt{x^4+3x^2+4}}{54208(5x^2+7)^2}+\frac{51875x\sqrt{x^4+3x^2+4}}{33392128(5x^2+7)}-\frac{2(-\frac{139}{1192576}x^3-\frac{137}{298144}x)}{\sqrt{x^4+3x^2+4}}+\frac{1173\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{}}{119}$
elliptic	$\frac{625x\sqrt{x^4+3x^2+4}}{54208(5x^2+7)^2}+\frac{51875x\sqrt{x^4+3x^2+4}}{33392128(5x^2+7)}-\frac{2(-\frac{139}{1192576}x^3-\frac{137}{298144}x)}{\sqrt{x^4+3x^2+4}}+\frac{1173\sqrt{1+\frac{3x^2}{8}-\frac{ix^2\sqrt{7}}{8}}\sqrt{}}{119}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 625/54208*x*(x^4+3*x^2+4)^(1/2)/(5*x^2+7)^2+51875/33392128*x*(x^4+3*x^2+4)^
(1/2)/(5*x^2+7)-2*(-139/1192576*x^3-137/298144*x)/(x^4+3*x^2+4)^(1/2)+1173/
1192576/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x
^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*EllipticF(1/4*x*(-6+2*I*7^(
1/2))^(1/2),1/4*(2+6*I*7^(1/2))^(1/2))+18159/1043504/(-6+2*I*7^(1/2))^(1/2)
*(1+3/8*x^2-1/8*I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x
^4+3*x^2+4)^(1/2)/(3+I*7^(1/2))*EllipticF(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*
(2+6*I*7^(1/2))^(1/2))-18159/1043504/(-6+2*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*
I*x^2*7^(1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2
)/(3+I*7^(1/2))*EllipticE(1/4*x*(-6+2*I*7^(1/2))^(1/2),1/4*(2+6*I*7^(1/2))^
(1/2))-529425/233744896/(-3/8+1/8*I*7^(1/2))^(1/2)*(1+3/8*x^2-1/8*I*x^2*7^(
1/2))^(1/2)*(1+3/8*x^2+1/8*I*x^2*7^(1/2))^(1/2)/(x^4+3*x^2+4)^(1/2)*Ellipti
cPi((-3/8+1/8*I*7^(1/2))^(1/2)*x,-5/7/(-3/8+1/8*I*7^(1/2)),(-3/8-1/8*I*7^(1
/2))^(1/2)/(-3/8+1/8*I*7^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 3*x^2 + 4)/(125*x^14 + 1275*x^12 + 6010*x^10 + 16678*x^
8 + 29153*x^6 + 31871*x^4 + 19992*x^2 + 5488), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x^2-x+2\right)\left(x^2+x+2\right)\right)^{\frac{3}{2}}\left(5x^2+7\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+7)**3/(x**4+3*x**2+4)**(3/2),x)
[Out] Integral(1/(((x**2 - x + 2)*(x**2 + x + 2))**(3/2)*(5*x**2 + 7)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+7)^3/(x^4+3*x^2+4)^(3/2),x, algorithm="giac")
[Out] integrate(1/((x^4 + 3*x^2 + 4)^(3/2)*(5*x^2 + 7)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(5\,x^2+7\right)^3\,\left(x^4+3\,x^2+4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2)),x)
[Out] int(1/((5*x^2 + 7)^3*(3*x^2 + x^4 + 4)^(3/2)), x)
```

3.380 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=467

$$\frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2}+\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}+\frac{e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))\,x\sqrt{a+bx^2\cdot}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

[Out] 1/15*e^2*(-4*b*e+15*c*d)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/5*e^3*x^3*(c*x^4+b*x
^2+a)^(1/2)/c+1/15*e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d))*x*(c*x^4+b
*x^2+a)^(1/2)/c^(5/2)/(a^(1/2)+x^2*c^(1/2))-1/15*a^(1/4)*e*(45*c^2*d^2+8*b^
2*e^2-3*c*e*(3*a*e+10*b*d))*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
(2-b/a^(1/2)/c^(1/2))^(1/2))(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/
2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)+1/30*a^(1/4)*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellip
ticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(
1/2)+x^2*c^(1/2))*(e*(45*c^2*d^2+8*b^2*e^2-3*c*e*(3*a*e+10*b*d))+(4*a*b*e^3
-15*a*c*d*e^2+15*c^2*d^3)*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^
(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)

Rubi [A]
time = 0.27, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1220, 1693, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2)\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(e(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2)+\frac{\sqrt{c}\,(4abe^3-15acde^2+15c^2d^3)}{\sqrt{a}}\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{11/4}\sqrt{a+bx^2+cx^4}}+\frac{ex\sqrt{a+bx^2+cx^4}\,(-3ce(3ae+10bd)+8b^2e^2+45c^2d^2)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{e^2x\sqrt{a+bx^2+cx^4}\,(15cd-4be)}{15c^2}+\frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]

[Out] (e^2*(15*c*d - 4*b*e)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (e^3*x^3*Sqrt[a
 + b*x^2 + c*x^4])/(5*c) + (e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a
*e))*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(
1/4)*e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e))*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTa
n[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(11/4)*Sqrt[a +
 b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*(15*c^2*d^3 - 15*a*c*d*e^2 + 4*a*b*e^
3))/Sqrt[a] + e*(45*c^2*d^2 + 8*b^2*e^2 - 3*c*e*(10*b*d + 3*a*e)))*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4
)*Sqrt[a + b*x^2 + c*x^4])

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\int \frac{5cd^3+3e(5cd^2-ae^2)x^2+e^2(15cd-4be)x^4}{\sqrt{a+bx^2+cx^4}}\,dx}{5c}$$

$$= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\int \frac{15c^2d^3-15acde^2+4abe^3+}{\sqrt{a}}}{}$$

$$= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{(\sqrt{a}\,e(45c^2d^2+8b^2e}{}$$

$$= \frac{e^2(15cd-4be)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{e^3x^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{e(45c^2d^2+8b^2e^2-3}{15c^5}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.78, size = 584, normalized size = 1.25

$$\frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,e^2x(a+bx^2+cx^4)(-4be+3c(5d+ex^2))+i\left(-b+\sqrt{b^2-4ac}\right)e(45c^2d^2+8b^2e^2-3ce(10bd+3ae))\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(30c^3d^3+8b^2\left(-b+\sqrt{b^2-4ac}\right)e^3+15c^2de\left(-3bd+3\sqrt{b^2-4ac}\,d-2ae\right)+ce^2\left(30b^2d-30b\sqrt{b^2-4ac}\,d+17abe-9a\sqrt{b^2-4ac}\,e\right)\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{60c^3\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4)*(-4*b*e + 3*
c*(5*d + e*x^2)) + I*(-b + Sqrt[b^2 - 4*a*c])*e*(45*c^2*d^2 + 8*b^2*e^2 - 3
*c*e*(10*b*d + 3*a*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + S
qrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(30*c^3*d^3 + 8*b^2*(-b + Sq
rt[b^2 - 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*Sqrt[b^2 - 4*a*c]*d - 2*a*e)
+ c*e^2*(30*b^2*d - 30*b*Sqrt[b^2 - 4*a*c]*d + 17*a*b*e - 9*a*Sqrt[b^2 - 4*
a*c]*e))*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sq
rt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
a + b*x^2 + c*x^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(455) = 910$.
time = 0.15, size = 1186, normalized size = 2.54 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] e^3*(1/5/c*x^3*(c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(c*x^4+b*x^2+a)^(1/2)+1/1
5*a*b/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*
b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5*a/c+8/15*b^2/c^2)*a*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)
(4+2(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+
b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+
3*d*e^2*(1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-1/12*a/c*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+
1/3*b/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))))-3/2*d^2*e*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-
2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(
1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/4*d^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x^2*e + d)^3/sqrt(c*x^4 + b*x^2 + a), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^3/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^3}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^3/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.381 $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=356

$$\frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}+\frac{2e(3cd-be)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2\sqrt[4]{a}\,e(3cd-be)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{3c^{7/4}\sqrt{a+bx^2+}}$$

```
[Out] 1/3*e^2*x*(c*x^4+b*x^2+a)^(1/2)/c+2/3*e*(-b*e+3*c*d)*x*(c*x^4+b*x^2+a)^(1/2
)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))-2/3*a^(1/4)*e*(-b*e+3*c*d)*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^
2+a)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b
/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*e*(-b*e+3*c*d)+(-a*e^2+3*
c*d^2)*c^(1/2)/a^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(
7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1220, 1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,(3cd^2-ae^2)}{\sqrt{a}}+2e(3cd-be)\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}(3cd-be)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2ex\sqrt{a+bx^2+cx^4}(3cd-be)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) + (2*e*(3*c*d - b*e)*x*Sqrt[a + b*x^2
 + c*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*a^(1/4)*e*(3*c*d - b*e)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3
*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*e*(3*c*d - b*e) + (Sqrt[c]*
(3*c*d^2 - a*e^2))/Sqrt[a])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}+\frac{\int \frac{3cd^2-ae^2+2e(3cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c}$$

$$= \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}-\frac{\left(2\sqrt{a}\,e(3cd-be)\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}}+\frac{\Big(3cd^2-ae^2}{}$$

$$= \frac{e^2x\sqrt{a+bx^2+cx^4}}{3c}+\frac{2e(3cd-be)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{2\sqrt[4]{a}\,e(3cd-be)\left(\sqrt{a}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.99, size = 488, normalized size = 1.37

$$\frac{2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,e^2x(a+bx^2+cx^4)-i\left(-b+\sqrt{b^2-4ac}\right)e(-3cd+be)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+i\left(-3c^2d^2+b\left(-b+\sqrt{b^2-4ac}\right)e^2+ce\left(3bd-3\sqrt{b^2-4ac}\,d+ae\right)\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{6c^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*e^2*x*(a + b*x^2 + c*x^4) - I*(-b + Sq
rt[b^2 - 4*a*c])*e*(-3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]
*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-3*c^2*d^2 + b*(
-b + Sqrt[b^2 - 4*a*c])*e^2 + c*e*(3*b*d - 3*Sqrt[b^2 - 4*a*c]*d + a*e))*Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2
*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])])/(6*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 +
c*x^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(348) = 696.
time = 0.12, size = 756, normalized size = 2.12

method	result
elliptic	$\frac{e^2x\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\left(d^2-\frac{a\,e^2}{3c}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \mathrm{Elli}}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c}}-\frac{\left(2e^2b-6cde\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}}{}$
risch	$\frac{e^2x\sqrt{c\,x^4+b\,x^2+a}}{3c}-$

default	$e^2\left(\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\ \mathrm{EllipticF}}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-1/12*a/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*
b/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2))))-d*e*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elli
pticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2)))+1/4*d^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex^2\right)^2}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^2}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.382 $\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=283

$$\frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}+$$

```
[Out] e*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x
^4+b*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))
*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (e*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+bx^2+cx^4}}\,dx = -\frac{\left(\sqrt{a}\,e\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}} + \left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{ex\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.14, size = 302, normalized size = 1.07

$$\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(\left(-b+\sqrt{b^2-4ac}\right)eE\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{2\sqrt{2}\,c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] + (-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.02, size = 362, normalized size = 1.28

method	result
default	$-\frac{ea\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\left(\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac}}{a}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{}}$
elliptic	$-\frac{ea\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\left(\text{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac}}{a}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2)))+1/4*d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{\sqrt{c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.383 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=401

$$\frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\,\sqrt{cd^2-bde+ae^2}}+\frac{\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2\right.\right.}{2\sqrt[4]{a}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] 1/2*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2
))*e^(1/2)/d^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(
1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(-e*a^(1/2)+d
*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)-1/4*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)
))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4
)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*(2-b/a^
(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((c*x^4
+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(c*x^4+b*
x^2+a)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1230, 1117, 1720}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,de};2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{4\sqrt[4]{c}\,d\sqrt{a+bx^2+cx^4}\,\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\,\sqrt{ae^2-bde+cd^2}}+\frac{\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\,\sqrt{a+bx^2+cx^4}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b
*x^2 + c*x^4])])/(2*Sqrt[d]*Sqrt[c*d^2 - b*d*e + a*e^2]) + (c^(1/4)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)
*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[a + b*x^2 + c*x^4]) - (a^(3/4)*((Sqrt[c]*d)/S
qrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d
*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*c^(1/4
)*d*(c*d^2 - a*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{\sqrt{c}\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e} - \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e}$$

$$= \frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{d}\,\sqrt{cd^2-bde+ae^2}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx}{(\sqrt{a}+}}}{2\sqrt[4]{a}\,(\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.14, size = 214, normalized size = 0.53

$$-\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*
a*c])]*d*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.12, size = 200, normalized size = 0.50

method	result
default	$\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^2}{2a}-\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\sqrt{1+\frac{b\,x^2}{2a}+\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\mathrm{EllipticPi}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\right.}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
elliptic	$\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^2}{2a}-\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\sqrt{1+\frac{b\,x^2}{2a}+\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\mathrm{EllipticPi}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\right.}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-
4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/
(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(c*d*x^4 + b*d*x^2 + a*d + (c*x^6 + b*x^4
+ a*x^2)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.384 $\int \frac{1}{(d+ex^2)^2 \sqrt{a+bx^2+cx^4}} \, dx$

Optimal. Leaf size=718

$$-\frac{\sqrt{c}\, ex\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}+\frac{\sqrt{e}\left(3cd^2-e(2bd-ae)\right)\tan^{-1}\Big(}{4d^{3/2}\left(cd^2-bde\right.}$$

```
[Out] 1/4*(3*c*d^2-e*(-a*e+2*b*d))*arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(
1/2)/(c*x^4+b*x^2+a)^(1/2))*e^(1/2)/d^(3/2)/(a*e^2-b*d*e+c*d^2)^(3/2)+1/2*e
^2*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1/2*e*x*c^(1/2)*
(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(a^(1/2)+x^2*c^(1/2))+1/2*a^(1/
4)*c^(1/4)*e*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c
^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))
^2)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)+1/2*c^(1/4)*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
F(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/d/(-
e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)-1/8*(3*c*d^2-e*(-a*e+2*b*d))*(co
s(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e
/a^(1/2)/c^(1/2),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(e*a^(1/2)+d*c^(1/2))*(a^
(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/
c^(1/4)/d^2/(a*e^2-b*d*e+c*d^2)/(-e*a^(1/2)+d*c^(1/2))/(c*x^4+b*x^2+a)^(1/2
)
```

Rubi [A]

time = 0.68, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1237, 1728, 1209, 1722, 1117, 1720}

$$\frac{\sqrt[4]{a}\sqrt[4]{c}\, e\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\, x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2d\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\left(\sqrt{a}\, e+\sqrt{c}\, d\right)\left(3cd^2-e(2bd-ae)\right)\Pi\left(-\frac{\left(\sqrt{c}\, d-\sqrt{a}\, e\right)^2}{4\sqrt{a}\sqrt{c}\, de};2\text{ArcTan}\left(\frac{\sqrt[4]{c}\, x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{8\sqrt[4]{a}\sqrt[4]{c}\, d^2\sqrt{a+bx^2+cx^4}\left(\sqrt{c}\, d-\sqrt{a}\, e\right)\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{e}\left(3cd^2-e(2bd-ae)\right)\text{ArcTan}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{4d^{3/2}\left(ae^2-bde+cd^2\right)^{3/2}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\, F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\, x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\, d\sqrt{a+bx^2+cx^4}\left(\sqrt{c}\, d-\sqrt{a}\, e\right)}+\frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\left(d+ex^2\right)\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\, ex\sqrt{a+bx^2+cx^4}}{2d\left(\sqrt{a}+\sqrt{c}\, x^2\right)\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*(Sqrt[c]*e*x*Sqrt[a + b*x^2 + c*x^4])/(d*(c*d^2 - b*d*e + a*e^2)*(Sqrt
[a] + Sqrt[c]*x^2)) + (e^2*x*Sqrt[a + b*x^2 + c*x^4])/(2*d*(c*d^2 - b*d*e +
 a*e^2)*(d + e*x^2)) + (Sqrt[e]*(3*c*d^2 - e*(2*b*d - a*e))*ArcTan[(Sqrt[c*
d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^(3
/2)*(c*d^2 - b*d*e + a*e^2)^(3/2)) + (a^(1/4)*c^(1/4)*e*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
```

```
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*d*(c*d^2 - b*d*e +
a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*d*(Sqrt[c]*d - Sqrt[a]*e
)*Sqrt[a + b*x^2 + c*x^4]) - ((Sqrt[c]*d + Sqrt[a]*e)*(3*c*d^2 - e*(2*b*d -
 a*e))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*Sqrt[c]*d*e), 2*
ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(8*a^(1/4)*c^(1/
4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
```

```
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2+cx^4}}\,dx = \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\,(cd^2-bde+ae^2)\,(d+ex^2)} - \frac{\int \frac{-2cd^2+e(2bd-ae)+2cdex^2+ce^2x^4}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx}{2d\,(cd^2-bde+ae^2)}$$

$$= \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\,(cd^2-bde+ae^2)\,(d+ex^2)} - \frac{\int \frac{\sqrt{a}\,c^{3/2}de^2+ce\left(-2cd^2+e(2bd-ae)\right)+\Big(2c^2de \ldots}{(d+ex^2)\sqrt{a+bx^2+\ldots}}}{2cde\,(cd^2-bde+\ldots}$$

$$= -\frac{\sqrt{c}\,ex\sqrt{a+bx^2+cx^4}}{2d\,(cd^2-bde+ae^2)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\,(cd^2-bde+ae^2)\,(d+ex^2 \ldots}$$

$$= -\frac{\sqrt{c}\,ex\sqrt{a+bx^2+cx^4}}{2d\,(cd^2-bde+ae^2)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2x\sqrt{a+bx^2+cx^4}}{2d\,(cd^2-bde+ae^2)\,(d+ex^2 \ldots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 11.09, size = 1069, normalized size = 1.49

$$\frac{4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de^2x(a+bx^2+cx^4)+i\sqrt{2}\left(b-\sqrt{b^2-4ac}\right)de\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}(d+ex^2)\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)+2i\sqrt{2}cd^2\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}(d+ex^2)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-6i\sqrt{2}cd^2\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}(d+ex^2)\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+4i\sqrt{2}bde\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}(d+ex^2)\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-2i\sqrt{2}ae^2\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}(d+ex^2)\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{8\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}d(cd^3+de(-bd+ae))(d+ex^2)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e^2*x*(a + b*x^2 + c*x^4) + I*Sqrt[2]*
(b - Sqrt[b^2 - 4*a*c])*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*(E
llipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])
]) + (2*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(d + e*x^2)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - (6*I)*Sqrt[2]*c*d^2*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])] + (4*I)*Sqrt[2]*b*d*e*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*(d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh
[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - S
qrt[b^2 - 4*a*c])] - (2*I)*Sqrt[2]*a*e^2*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(
d + e*x^2)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])])/(8*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*(c*d^3 + d*e*(-(b*d) + a*
e))*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.12, size = 1279, normalized size = 1.78 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^2*x*(c*x^4+b*x^2+a)^(1/2)/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)-1/8*c/(a*e^
2-b*d*e+c*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x
^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/
2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/d/(a*e^2-b*d
*e+c*d^2)*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/
a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/
(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
```

```
1/4*c*e/d/(a*e^2-b*d*e+c*d^2)*a*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)
*(4+2*b*x^2/a-2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(-4*a*
c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*EllipticE(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))+1/2/(a*e^2-b*d*e+c*d^2)/d^2*e^2*2^(1/2)/(-b/a+1/a*(-4*a
*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+
1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellip
ticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(
1/2))*a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2))*a-1/(a*e^2-b*d*e+c*d^2)/d*e*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2
)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*
x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*
a*e/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2))*b+3/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))
^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/
2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/d,(-
1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2))*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex^{2}\right)^{2} \sqrt{a + bx^{2} + cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,x^2+d\right)^2\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.385 $\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=553

$$-\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}-\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e(45c^2d^2 -}{}$$

```
[Out] -1/15*e^2*(4*b*e+15*c*d)*x*(-c*x^4+b*x^2+a)^(1/2)/c^2-1/5*e^3*x^3*(-c*x^4+b
*x^2+a)^(1/2)/c-1/60*e*(45*c^2*d^2+8*b^2*e^2+3*c*e*(3*a*e+10*b*d))*Elliptic
E(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(
4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(
1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^
(1/2)/c^(7/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/60*EllipticF(x*2^(1/2)*c^(1/
2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2))
)^(1/2))*(e*(45*c^2*d^2+8*b^2*e^2+3*c*e*(3*a*e+10*b*d))+2*c*(4*a*b*e^3+15*a
*c*d*e^2+15*c^2*d^3)/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^
2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4
*a*c+b^2)^(1/2)))^(1/2)/c^(7/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.85, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1220, 1693, 1216, 538, 435, 430}

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(3ce(3ae+10bd)+8b^2e^2+45c^2d^2\right)E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}+\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(\frac{2c(4abe^3+15acde^2+15c^2d^3)}{b-\sqrt{4ac+b^2}}+e(3ce(3ae+10bd)+8b^2e^2+45c^2d^2)\right)F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{30\sqrt{2}c^{7/2}\sqrt{a+bx^2-cx^4}}-\frac{e^2x\sqrt{a+bx^2-cx^4}(4be+15cd)}{15c^2}-\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/15*(e^2*(15*c*d + 4*b*e)*x*Sqrt[a + b*x^2 - c*x^4])/c^2 - (e^3*x^3*Sqrt[
a + b*x^2 - c*x^4])/(5*c) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*e*(45*c^2*d^2 + 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*Sqrt[1 - (2*c*x^2
)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Elli
pticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^
2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 -
c*x^4]) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*((2*c*(15*c^
2*d^3 + 15*a*c*d*e^2 + 4*a*b*e^3))/(b - Sqrt[b^2 + 4*a*c]) + e*(45*c^2*d^2
+ 8*b^2*e^2 + 3*c*e*(10*b*d + 3*a*e)))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sq
rt[b^2 + 4*a*c])])/(30*Sqrt[2]*c^(7/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
```

```
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && !LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\int \frac{-5cd^3-3e(5cd^2+ae^2)x^2-e^2(15cd+4be)x^4}{\sqrt{a+bx^2-cx^4}}\,dx}{5c}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} + \frac{\int \frac{15c^2d^3+15acde^2+4abe}{\sqrt{}}}{}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} + \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+}}}\right.}{}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\left(\left(b-\sqrt{b^2+4ac}\right.\right.}{}$$

$$= -\frac{e^2(15cd+4be)x\sqrt{a+bx^2-cx^4}}{15c^2} - \frac{e^3x^3\sqrt{a+bx^2-cx^4}}{5c} - \frac{\left(b-\sqrt{b^2+4ac}\right)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.53, size = 596, normalized size = 1.08

$$\frac{-4c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}e^2x(a+bx^2-cx^4)(4be+3c(5d+ex^2)) - i\sqrt{2}\left(-b+\sqrt{b^2+4ac}\right)e(45c^2d^2+8b^2e^2+3ce(10bd+3ae))\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2cx^2}{-b+\sqrt{b^2+4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right) + i\sqrt{2}\left(-30c^3d^3+8b^2\left(-b+\sqrt{b^2+4ac}\right)e^3+15c^2de\left(-3bd+3\sqrt{b^2+4ac}d-2ae\right)+ce^2\left(-30b^2d+30b\sqrt{b^2+4ac}d-17abe+9a\sqrt{b^2+4ac}e\right)\right)\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2cx^2}{-b+\sqrt{b^2+4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{60c^3\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-4*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(a + b*x^2 - c*x^4)*(4*b*e +
 3*c*(5*d + e*x^2)) - I*Sqrt[2]*(-b + Sqrt[b^2 + 4*a*c])*e*(45*c^2*d^2 + 8*
b^2*e^2 + 3*c*e*(10*b*d + 3*a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b
 + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b
^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))
```

```
]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-30*c^3
*d^3 + 8*b^2*(-b + Sqrt[b^2 + 4*a*c])*e^3 + 15*c^2*d*e*(-3*b*d + 3*Sqrt[b^2
 + 4*a*c]*d - 2*a*e) + c*e^2*(-30*b^2*d + 30*b*Sqrt[b^2 + 4*a*c]*d - 17*a*b
*e + 9*a*Sqrt[b^2 + 4*a*c]*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2
+ 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x
], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(60*c^3*Sqrt[-(c/(b +
Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. $2(477) = 954$.
time = 0.12, size = 1195, normalized size = 2.16 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(-1/5/c*x^3*(-c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(-c*x^4+b*x^2+a)^(1/2)+
1/15*a*b/c^2*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a
)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*
b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(3/5*a/c+8/15*b^2/c^2)*a*2^(1/2)/((
-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+
2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^
(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*
b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))))+3*d*e^2*(
-1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c+1/12*a/c*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b/c*a*2
^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*
a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))))-
3/2*d^2*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)
^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2
))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2
^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a
/c)^(1/2)))+1/4*d^3*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+
b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^3}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(-c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d + e*x**2)**3/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)^3/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^3}{\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(a + b*x^2 - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^3/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.386 $\int \frac{(d+ex^2)^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=454

$$-\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}-\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e(3cd+be)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2}{b+\sqrt{}}}}{3\sqrt{2}\,c^{5/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -1/3*e^2*x*(-c*x^4+b*x^2+a)^(1/2)/c-1/6*e*(b*e+3*c*d)*EllipticE(x*2^(1/2)*c
^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1
/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*
(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(5/2)
*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/6*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b
^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(3*c^
2*d^2+b*e^2*(b-(4*a*c+b^2)^(1/2))+c*e*(3*b*d+a*e-3*d*(4*a*c+b^2)^(1/2)))*(1
-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^
2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(5/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1220, 1216, 538, 435, 430}

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}(be+3cd)E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}+\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(ce\left(-3d\sqrt{4ac+b^2}+ae+3bd\right)+be^2\left(b-\sqrt{4ac+b^2}\right)+3c^2d^2\right)F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/3*(e^2*x*Sqrt[a + b*x^2 - c*x^4])/c - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b +
Sqrt[b^2 + 4*a*c]]*e*(3*c*d + b*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c
])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b
^2 + 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4]) + (Sqrt[b + Sqrt
[b^2 + 4*a*c]]*(3*c^2*d^2 + b*(b - Sqrt[b^2 + 4*a*c])*e^2 + c*e*(3*b*d - 3*
Sqrt[b^2 + 4*a*c]*d + a*e))*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqr
t[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*
a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x^4])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\int \frac{-3cd^2-ae^2-2e(3cd+be)x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{3c}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{\cdots}{\sqrt{1\cdots}}}{3c\sqrt{a+bx^2-c\cdots}}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(\left(b-\sqrt{b^2+4ac}\right)e(3cd+be)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{\cdots}\right.}{3c^2\sqrt{a+bx^2\cdots}}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{3c} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e(3cd+be)\sqrt{1-\frac{\cdots}{b\cdots}}}{\cdots}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 10.83, size = 503, normalized size = 1.11

$$\frac{2c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}e^2x(-a-bx^2+cx^4)-i\sqrt{2}\left(-b+\sqrt{b^2+4ac}\right)e(3cd+be)\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2cx^2}{-b+\sqrt{b^2+4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+i\sqrt{2}\left(-3c^2d^2+b\left(-b+\sqrt{b^2+4ac}\right)e^2-ce\left(3bd-3\sqrt{b^2+4ac}\,d+ae\right)\right)\sqrt{\frac{b+\sqrt{b^2+4ac}-2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{\frac{-b+\sqrt{b^2+4ac}+2cx^2}{-b+\sqrt{b^2+4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{6c^2\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*e^2*x*(-a - b*x^2 + c*x^4) - I*Sqrt
[2]*(-b + Sqrt[b^2 + 4*a*c])*e*(3*c*d + b*e)*Sqrt[(b + Sqrt[b^2 + 4*a*c] -
2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(
-b + Sqrt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2
 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[
2]*(-3*c^2*d^2 + b*(-b + Sqrt[b^2 + 4*a*c])*e^2 - c*e*(3*b*d - 3*Sqrt[b^2 +
 4*a*c]*d + a*e))*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 + 4*
a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*El
lipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[
b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*c^2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a
*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [**A**]
time = 0.12, size = 761, normalized size = 1.68

method	result
elliptic	$-\frac{e^{2}x\sqrt{-c\,x^{4}+b\,x^{2}+a}}{3c}+\frac{\left(d^{2}+\frac{a\,e^{2}}{3c}\right)\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{E}}{4\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}\,\sqrt{-}}$
risch	$-\frac{e^{2}x\sqrt{-c\,x^{4}+b\,x^{2}+a}}{3c}+\frac{-\frac{\left(2e^{2}b+6cde\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}}{}}{}$
default	$e^{2}\left(-\frac{x\sqrt{-c\,x^{4}+b\,x^{2}+a}}{3c}+\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}}{12c\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}\,\sqrt{-c}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(-1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c+1/12*a/c*2^(1/2)/((-b+(4*a*c+b^2)^(1/2
))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*
c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b/
c*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(
b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*(
(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2
))))-d*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^
(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^
(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/
c)^(1/2)))+1/4*d^2*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b
*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
```

```
(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d + ex^2)^2}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^2}{\sqrt{-c\,x^4+b\,x^2+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*x^2 - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)^2/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.387 $\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=385

$$-\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\,\sqrt{}}{\sqrt{b+\sqrt{b^2}}}\right.\right.}{2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] 1/4*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)
^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(2*c*d+e*(b-(4*a*c+b^2)^(1/2)))*(1-2*
c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(
b+(4*a*c+b^2)^(1/2)))^(1/2)/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)-1/4*e*El
lipticE(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2)
)/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+
b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1
/2)))^(1/2)/c^(3/2)*2^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1216, 538, 435, 430}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -1/2*((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x
^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*El
lipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[
b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c
*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*S
qrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2
 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]
]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sq
rt[a + b*x^2 - c*x^4])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{d+ex^2}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\cdots}}}{\sqrt{a+bx^2-cx^4}}$$

$$= -\frac{\left(\left(b-\sqrt{b^2+4ac}\right)\left(-\frac{2cd}{b-\sqrt{b^2+4ac}}-e\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{\cdots}{b+\cdots}}\right.}{2c\sqrt{a+bx^2-c\cdots}}$$

$$= -\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+\cdots}}}}{2\sqrt{2}\,c^{3/2}\sqrt{a+bx^2-cx^4}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.15, size = 293, normalized size = 0.76

$$-\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\left(\left(-b+\sqrt{b^2+4ac}\right)eE\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)\right)}{2\sqrt{2}c\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(
b + Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sq
rt[b^2 + 4*a*c])] + (2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b
- Sqrt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[
a + b*x^2 - c*x^4])
```

Maple [A]
time = 0.03, size = 364, normalized size = 0.95

method	result
default	$-\frac{ea\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2}, \cdots\right.\right.}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\sqrt{-c\,x^4 \cdots}}$
elliptic	$-\frac{ea\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2}, \cdots\right.\right.}{2\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}\sqrt{-c\,x^4 \cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1
/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1
/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)
^(1/2)))+1/4*d*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2
+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-
2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2*e + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d + e*x**2)/sqrt(a + b*x**2 - c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)/sqrt(-c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{\sqrt{-c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*x^2 - c*x^4)^(1/2),x)
[Out] int((d + e*x^2)/(a + b*x^2 - c*x^4)^(1/2), x)
```

3.388 $\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=197

$$\frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\ldots}{\sqrt{b+\sqrt{b^2-\ldots}}}\right.\right.}{\sqrt{2}\sqrt{c}\,d\sqrt{a+bx^2-cx^4}}$$

```
[Out] 1/2*EllipticPi(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),-1/2*e*(b+(4*a
*c+b^2)^(1/2))/c/d,((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(1-
2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2
/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$,
Rules used = {1234, 551}

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\,d\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sq
rt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2 +
4*a*c])*e)/(c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]],
(b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[a
 + b*x^2 - c*x^4])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1234

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/((d + e*x^2)*Sq
```

```
rt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx = \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4}}}}}{\sqrt{a+bx^2-cx^4}}$$

$$= \frac{\sqrt{b+\sqrt{b^2+4ac}}\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\Pi\Bigg(-}{\sqrt{2}\sqrt{c}\,d\sqrt{a+b}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.15, size = 205, normalized size = 1.04

$$-\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right)|-\frac{b+\sqrt{b^2+4ac}}{-b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}\,d\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2 + 4*a*c])*e)/(c*d), I*Ar
cSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*
c])/(-b + Sqrt[b^2 + 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]
*d*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A]
time = 0.12, size = 201, normalized size = 1.02

method	result
default	$\frac{\sqrt{2}\sqrt{1+\frac{bx^2}{2a}-\frac{x^2\sqrt{4ac+b^2}}{2a}}\sqrt{1+\frac{bx^2}{2a}+\frac{x^2\sqrt{4ac+b^2}}{2a}}\,\mathrm{EllipticPi}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{4ac+b^2}}{a}}}{2},-\right.}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{4ac+b^2}}{a}}\sqrt{-cx^4+bx^2+a}}$

elliptic	$\frac{\sqrt{2}\,\sqrt{1+\frac{b\,x^{2}}{2a}-\frac{x^{2}\sqrt{4ac+b^{2}}}{2a}}\,\sqrt{1+\frac{b\,x^{2}}{2a}+\frac{x^{2}\sqrt{4ac+b^{2}}}{2a}}\,\mathrm{EllipticPi}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{4ac+b^{2}}}{a}}}{2},-\frac{}{\left(-b\right.}\right.}{d\sqrt{-\frac{b}{a}+\frac{\sqrt{4ac+b^{2}}}{a}}\,\sqrt{-c\,x^{4}+b\,x^{2}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(4*
a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)/(-c
*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/
2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2^(
1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(x^2*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^{2})\sqrt{a+bx^{2}-cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(a + b*x**2 - c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(x^2*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(e\,x^2+d\right)\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 - c*x^4)^(1/2)),x)
[Out] int(1/((d + e*x^2)*(a + b*x^2 - c*x^4)^(1/2)), x)
```

3.389 $\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=718

$$-\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(cd^2+bde-ae^2\right)\left(d+ex^2\right)}+\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\,\sqrt{1-\frac{2}{b+\sqrt{}}}}{4\sqrt{2}\,\sqrt{c}\,d\left(cd^2+e(bd-ae)\right)}$$

```
[Out] -1/2*e^2*x*(-c*x^4+b*x^2+a)^(1/2)/d/(-a*e^2+b*d*e+c*d^2)/(e*x^2+d)+1/4*(3*c
*d^2+e*(-a*e+2*b*d))*EllipticPi(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/
2),-1/2*e*(b+(4*a*c+b^2)^(1/2))/c/d,((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(
1/2)))^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(4*a*c+b^2)^(1/2))
^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d^2/(c*d^2+e*(-a*e+b*d))*2^(
1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)-1/8*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a
*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(
2*c*d+e*(b-(4*a*c+b^2)^(1/2)))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(b+(
4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d/(c*d^2+e*
(-a*e+b*d))*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)+1/8*e*EllipticE(x*2^(1/2
)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2),((b+(4*a*c+b^2)^(1/2))/(b-(4*a*c+b^2)
^(1/2)))^(1/2))*(b-(4*a*c+b^2)^(1/2))*(1-2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/
2)*(b+(4*a*c+b^2)^(1/2))^(1/2)*(1-2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d/(c
*d^2+e*(-a*e+b*d))*2^(1/2)/c^(1/2)/(-c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.66, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1237, 1730, 1216, 538, 435, 430, 1234, 551}

$$-\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e\left(b-\sqrt{4ac+b^2}\right)+2cd\right)F\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}+\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{4\sqrt{2}\sqrt{c}d\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}+\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(e(2bd-ae)+3cd^2\right)\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}\sqrt{c}d^2\sqrt{a+bx^2-cx^4}\left(e(bd-ae)+cd^2\right)}-\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\left(d+ex^2\right)\left(e(bd-ae)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -1/2*(e^2*x*Sqrt[a + b*x^2 - c*x^4])/(d*(c*d^2 + e*(b*d - a*e))*(d + e*x^2)
) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*e*Sqrt[1 - (2*c*x^
2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Ell
ipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b
^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d
 - a*e))*Sqrt[a + b*x^2 - c*x^4]) - (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(2*c*d + (
b - Sqrt[b^2 + 4*a*c])*e)*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[
1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*
c])])/(4*Sqrt[2]*Sqrt[c]*d*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4])
```

```
 + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(3*c*d^2 + e*(2*b*d - a*e))*Sqrt[1 - (2*c*x
^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*El
lipticPi[-1/2*((b + Sqrt[b^2 + 4*a*c])*e)/(c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*
c])])/(2*Sqrt[2]*Sqrt[c]*d^2*(c*d^2 + e*(b*d - a*e))*Sqrt[a + b*x^2 - c*x^4
])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1234

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/((d + e*x^2)*Sq
rt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx = -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\,(cd^2+e(bd-ae))\,(d+ex^2)} + \frac{\int \frac{2cd^2+e(2bd-ae)-2cdex^2-ce^2x^4}{(d+ex^2)\sqrt{a+bx^2-cx^4}}\,dx}{2d\,(cd^2+e(bd-ae))}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\,(cd^2+e(bd-ae))\,(d+ex^2)} - \frac{\int \frac{cde^2+ce^3x^2}{\sqrt{a+bx^2-cx^4}}\,dx}{2de^2\,(cd^2+e(bd-ae))} + \frac{(3cd^2+\cdots}{\cdots}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\,(cd^2+e(bd-ae))\,(d+ex^2)} - \frac{\left(\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\,\sqrt{1-\frac{\cdots}{b-\cdots}}\cdots\right.}{2de\cdots}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\,(cd^2+e(bd-ae))\,(d+ex^2)} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\,(3cd^2+e(2bd-\cdots}{\cdots}$$

$$= -\frac{e^2x\sqrt{a+bx^2-cx^4}}{2d\,(cd^2+e(bd-ae))\,(d+ex^2)} + \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4a\cdots}}}{\cdots}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 13.34, size = 464, normalized size = 0.65

$$-\frac{\sqrt{a+bx^2-cx^4}\left(4de^2x+\frac{i\sqrt{2+\frac{4cx^2}{-b+\sqrt{b^2+4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2+4ac}}}(d+ex^2)\left(\left(-b+\sqrt{b^2+4ac}\right)deE\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+d\left(2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+2(-3cd^2+e(-2bd+ae))\Pi\left(-\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)\right)}{\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}(-a-bx^2+cx^4)}\right)}{8d^2\,(cd^2+e(bd-ae))\,(d+ex^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -1/8*(Sqrt[a + b*x^2 - c*x^4]*(4*d*e^2*x + (I*Sqrt[2 + (4*c*x^2)/(-b + Sqrt
[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(d + e*x^2)*((-
b + Sqrt[b^2 + 4*a*c])*d*e*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b
^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + d*(2*
c*d + (b - Sqrt[b^2 + 4*a*c])*e)*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b +
Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] +
 2*(-3*c*d^2 + e*(-2*b*d + a*e))*EllipticPi[-1/2*((b + Sqrt[b^2 + 4*a*c])*e
)/(c*d), I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt
[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c])
)]*(-a - b*x^2 + c*x^4))))/(d^2*(c*d^2 + e*(b*d - a*e))*(d + e*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1292 vs. $2(613) = 1226$.
time = 0.13, size = 1293, normalized size = 1.80 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^2/(a*e^2-b*d*e-c*d^2)/d*x*(-c*x^4+b*x^2+a)^(1/2)/(e*x^2+d)+1/8*c/(a*e
^2-b*d*e-c*d^2)*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x
^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)
/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*c*e/(a*e^2-b*d*e-c*
d^2)/d*a*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a-2*x^2/a*(4
*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^
4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/4*c*e/
(a*e^2-b*d*e-c*d^2)/d*a*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x
^2/a-2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(4+2*b*x^2/a+2*x^2/a*(4*a*c+b^2)^(1/2
))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*EllipticE(1/2*x*2^(1/
2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^
(1/2))+1/2/(a*e^2-b*d*e-c*d^2)/d^2*e^2*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))
^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2
*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(
1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,(-1/2
*(b+(4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*a
-1/(a*e^2-b*d*e-c*d^2)/d*e*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/
2*b*x^2/a-1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(4*a*
c+b^2)^(1/2))^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4
*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(4*a*c+b
^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*b-3/2/(a*e^2-
b*d*e-c*d^2)*2^(1/2)/(-b/a+1/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/2*
x^2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(4*a*c+b^2)^(1/2))^
(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2
))/a)^(1/2),-2/(-b+(4*a*c+b^2)^(1/2))*a*e/d,(-1/2*(b+(4*a*c+b^2)^(1/2))/a)^
(1/2)*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2))*c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(x^2*e + d)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^2\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(-c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(1/((d + e*x**2)**2*sqrt(a + b*x**2 - c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(-c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*(x^2*e + d)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x^2+d)^2\,\sqrt{-c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^2*(a + b*x^2 - c*x^4)^(1/2)),x)
[Out] int(1/((d + e*x^2)^2*(a + b*x^2 - c*x^4)^(1/2)), x)
```

3.390 $\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=479

$$\frac{\left(b-\sqrt{b^2+4ac}\right)ex\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)}{2c\sqrt{-a+bx^2+cx^4}}-\frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4ac}}\,e\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)E\Bigg(}{2\sqrt{2}\,c^{3/2}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\ \sqrt{}}$$

```
[Out] 1/2*e*x*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))/c/(c*x^4+b*
x^2-a)^(1/2)+1/2*d*(1/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2/(
b+(4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2
))^(1/2)/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/(b-(
4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b+(4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*a*c+b^
2)^(1/2)))/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)-1/4*e*(1/(1+2*c*x^2/(b+
(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2)*Elliptic
E(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(b+(4*a*c+b^2)^(
1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/(b-(4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2
/(b-(4*a*c+b^2)^(1/2)))*(b-(4*a*c+b^2)^(1/2))*(b+(4*a*c+b^2)^(1/2))^(1/2)/c
^(3/2)*2^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))/(1+
2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1216, 545, 429, 506, 422}

$$-\frac{e\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)E\left(\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}+\frac{d\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)F\left(\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}+\frac{ex\left(b-\sqrt{4ac+b^2}\right)\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)}{2c\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

```
[Out] ((b - Sqrt[b^2 + 4*a*c])*e*x*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])))/(2*c*
Sqrt[-a + b*x^2 + c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*
a*c]]*e*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*EllipticE[ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^
2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]
))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sq
rt[b + Sqrt[b^2 + 4*a*c]]*d*(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))*Ellipti
```

```
cF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (-2*Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b -
 Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[-a + b*x
^2 + c*x^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{d+ex^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\cdots}}}{\sqrt{-a+bx^2+cx^4}}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1\cdots}}}{\sqrt{-a+bx^2+cx^4}}$$

$$= \frac{\left(b-\sqrt{b^2+4ac}\right)ex\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)}{2c\sqrt{-a+bx^2+cx^4}} + \frac{\sqrt{b+\sqrt{b^2+4ac}}\,d\left(1+\frac{2cx^2}{b-\sqrt{b^2+\cdots}}\right.}{\sqrt{2}\sqrt{c}\sqrt{\frac{1\cdots}{1\cdots}}}$$

$$= \frac{\left(b-\sqrt{b^2+4ac}\right)ex\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)}{2c\sqrt{-a+bx^2+cx^4}} - \frac{\left(b-\sqrt{b^2+4ac}\right)\sqrt{b+\sqrt{b^2+4a\cdots}}}{2\sqrt{\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.18, size = 304, normalized size = 0.63

$$\frac{i\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\left(\left(-b+\sqrt{b^2+4ac}\right)eE\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)+\left(-2cd+\left(b-\sqrt{b^2+4ac}\right)e\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)\right)}{2\sqrt{2}\,c\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*((-b + Sqrt[b^2 + 4*a*c])*e*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 + 4*
a*c])/(b - Sqrt[b^2 + 4*a*c])] + (-2*c*d + (b - Sqrt[b^2 + 4*a*c])*e)*Ellip
ticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt[b^2 +
4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 + 4*a*c]
)]*Sqrt[-a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.03, size = 355, normalized size = 0.74

method	result
default	$\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\left(\operatorname{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{2},\sqrt{-\right.\right.\right.}{\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+}}$
elliptic	$\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\left(\operatorname{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{2},\sqrt{-\right.\right.\right.}{\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*a/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)/(b+(4*a
*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*(-2*(-b+(4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/2*d/(-2*(-
b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2
*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*EllipticF(1/2*x*(
-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(
1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)/sqrt(c*x^4 + b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(b*x^2 - a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(b*x^2 - a + c*x^4)^(1/2), x)
```

3.391 $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=204

$$\frac{\sqrt{-b+\sqrt{b^2+4ac}}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\Pi\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd};\sin^{-1}\left(\frac{\sqrt{2}\ \cdots}{\sqrt{-b+\sqrt{\cdots}}}\right.\right.}{\sqrt{2}\,\sqrt{c}\,d\sqrt{-a+bx^2+cx^4}}$$

```
[Out] 1/2*EllipticPi(x*2^(1/2)*c^(1/2)/(-b+(4*a*c+b^2)^(1/2))^(1/2),1/2*e*(b-(4*a
*c+b^2)^(1/2))/c/d,((b-(4*a*c+b^2)^(1/2))/(b+(4*a*c+b^2)^(1/2)))^(1/2))*(1+
2*c*x^2/(b-(4*a*c+b^2)^(1/2)))^(1/2)*(-b+(4*a*c+b^2)^(1/2))^(1/2)*(1+2*c*x^
2/(b+(4*a*c+b^2)^(1/2)))^(1/2)/d*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1234, 551}

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\,\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,\Pi\left(\frac{\left(b-\sqrt{b^2+4ac}\right)e}{2cd};\text{ArcSin}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)\middle|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\,\sqrt{c}\,d\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticPi[((b - Sqrt[b^2 + 4*a*
c])*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (
b - Sqrt[b^2 + 4*a*c])/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*Sqrt[-a
 + b*x^2 + c*x^4])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 1234

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/((d + e*x^2)*Sq
```

```
rt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\right)\int \frac{}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+}}}}}{\sqrt{-a+bx^2+cx^4}}$$

$$= \frac{\sqrt{-b+\sqrt{b^2+4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}\,\Pi\Big(}{\sqrt{2}\sqrt{c}\,d\sqrt{-a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.14, size = 216, normalized size = 1.06

$$-\frac{i\sqrt{\frac{b+\sqrt{b^2+4ac}+2cx^2}{b+\sqrt{b^2+4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}\,\Pi\left(\frac{\left(b+\sqrt{b^2+4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\,x\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2+4ac}}}\,d\sqrt{-a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*EllipticPi[((b + Sqrt[b^2 + 4*a*c])*
e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*a*c])]*x], (b + Sqrt
[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 + 4*
a*c])]*d*Sqrt[-a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.12, size = 198, normalized size = 0.97

method	result
default	$\frac{\sqrt{1-\frac{b x^2}{2a}+\frac{x^2\sqrt{4ac+b^2}}{2a}}\sqrt{1-\frac{b x^2}{2a}-\frac{x^2\sqrt{4ac+b^2}}{2a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-b+\sqrt{4ac+b^2}}{2a}}\,x,\frac{2ae}{\left(-b+\sqrt{4ac+\right.}}\right.}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{4ac+b^2}}{2a}}\sqrt{c x^4+b x^2-a}}$

elliptic	$\frac{\sqrt{1-\frac{b x^2}{2a}+\frac{x^2\sqrt{4ac+b^2}}{2a}}\sqrt{1-\frac{b x^2}{2a}-\frac{x^2\sqrt{4ac+b^2}}{2a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-b+\sqrt{4ac+b^2}}{2a}}\,x,\frac{2ae}{\left(-b+\sqrt{4ac\right.}}\right.}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{4ac+b^2}}{2a}}\sqrt{c x^4+b x^2-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(1/2*b/a-1/2/a*(4*a*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a+1/2*x^2/a*(4*a*c
+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a-1/2*x^2/a*(4*a*c+b^2)^(1/2))^(1/2)/(c*x^4
+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x,2/(-b+(4
*a*c+b^2)^(1/2))*a*e/d,1/2*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)^(1/2)/(-1/2*(-
b+(4*a*c+b^2)^(1/2))/a)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(x^2*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 - a)/(c*d*x^4 + b*d*x^2 - a*d + (c*x^6 + b*x^4
- a*x^2)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 - a)*(x^2*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(b*x^2 - a + c*x^4)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)*(b*x^2 - a + c*x^4)^(1/2)), x)
```

3.392 $\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=293

$$-\frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2+\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{-a+bx^2-cx^4}}$$

```
[Out] -e*x*(-c*x^4+b*x^2-a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e*(cos(2*
arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellipt
icE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1
/2)+x^2*c^(1/2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-
c*x^4+b*x^2-a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/
2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(-c*x^4+b*x^2-a
)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{2c^{3/4}\sqrt{-a+bx^2-cx^4}}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\sqrt{c}}+2\right)\right)}{c^{3/4}\sqrt{-a+bx^2-cx^4}}-\frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

```
[Out] -((e*x*Sqrt[-a + b*x^2 - c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2))) - (a^(1
/4)*e*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4
])/(c^(3/4)*Sqrt[-a + b*x^2 - c*x^4]) + (a^(1/4)*((Sqrt[c]*d)/Sqrt[a] + e)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*
c^(3/4)*Sqrt[-a + b*x^2 - c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx &= -\frac{\left(\sqrt{a}\,e\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}}+\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx \\ &= -\frac{ex\sqrt{-a+bx^2-cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,e\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{c^{3/4}\sqrt{-a+bx^2-cx^4}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.18, size = 295, normalized size = 1.01

$$-\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\left(\left(-b+\sqrt{b^2-4ac}\right)eE\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+\left(2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{2\sqrt{2}\,c\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{-a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[-a + b*x^2 - c*x^4],x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*((-b + Sqrt[b^2 - 4*a*c])*e*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])] + (2*c*d + (b - Sqrt[b^2 - 4*a*c])*e)*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[
-a + b*x^2 - c*x^4])
```

Maple [A]
time = 0.03, size = 357, normalized size = 1.22

method	result
default	$\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}}{2}\right.\right.}{\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-}}$
elliptic	$\frac{ea\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}}{2}\right.\right.}{\sqrt{-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)}{a}}\,\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*a/(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)/(b+
(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*(-2*(-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))+1/2
*d/(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2-a)^(1/2)*Elli
pticF(1/2*x*(-2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{\sqrt{-a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt(-a + b*x**2 - c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)/sqrt(-c*x^4 + b*x^2 - a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{\sqrt{-c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(b*x^2 - a - c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x^2)/(b*x^2 - a - c*x^4)^(1/2), x)
```

3.393 $\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=412

$$\frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{-cd^2-e(bd+ae)}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\,\sqrt{-cd^2-e(bd+ae)}}+\frac{\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\right.}{2\sqrt[4]{a}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\sqrt{-a+bx^2-cx^4}}$$

```
[Out] 1/2*arctan(x*(-a*e^2-b*d*e-c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(-c*x^4+b*x^2-a)^(1
/2))*e^(1/2)/d^(1/2)/(-a*e^2-b*d*e-c*d^2)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2+b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*
c^(1/2))*((c*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(-e*a^(1/2
)+d*c^(1/2))/(-c*x^4+b*x^2-a)^(1/2)-1/4*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^
(1/4)*x/a^(1/4))),-1/4*(-e*a^(1/2)+d*c^(1/2))^2/d/e/a^(1/2)/c^(1/2),1/2*(2+
b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e+d*c^(1/2)/a^(1/2))^2*((c
*x^4-b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(1/4)/d/(-a*e^2+c*d^2)/(-c*x
^4+b*x^2-a)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1230, 1117, 1720}

$$-\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)^2\Pi\left(-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)^2}{4\sqrt{a}\,\sqrt{c}\,de};2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\,\sqrt{c}}+2\right)\right)}{4\sqrt[4]{c}\,d\sqrt{-a+bx^2-cx^4}\,\left(cd^2-ae^2\right)}+\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{x\sqrt{-e(ae+bd)-cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\,\sqrt{-e(ae+bd)-cd^2}}+\frac{\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a-bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(\frac{b}{\sqrt{a}\,\sqrt{c}}+2\right)\right)}{2\sqrt[4]{a}\,\sqrt{-a+bx^2-cx^4}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

```
[Out] (Sqrt[e]*ArcTan[(Sqrt[-(c*d^2) - e*(b*d + a*e)]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[-a
 + b*x^2 - c*x^4])])/(2*Sqrt[d]*Sqrt[-(c*d^2) - e*(b*d + a*e)]) + (c^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(2*
a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[-a + b*x^2 - c*x^4]) - (a^(3/4)*((Sqrt
[c]*d)/Sqrt[a] + e)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a - b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticPi[-1/4*(Sqrt[c]*d - Sqrt[a]*e)^2/(Sqrt[a]*S
qrt[c]*d*e), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 + b/(Sqrt[a]*Sqrt[c]))/4])/(
4*c^(1/4)*d*(c*d^2 - a*e^2)*Sqrt[-a + b*x^2 - c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx = \frac{\sqrt{c}\int \frac{1}{\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e} - \frac{\left(\sqrt{a}\,e\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx}{\sqrt{c}\,d-\sqrt{a}\,e}$$

$$= \frac{\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{-cd^2-e(bd+ae)}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{-a+bx^2-cx^4}}\right)}{2\sqrt{d}\,\sqrt{-cd^2-e(bd+ae)}} + \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a}{(\sqrt{\ldots}}}\ldots}{2\sqrt[4]{a}\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 207, normalized size = 0.50

$$-\frac{i\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,\Pi\left(-\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|-\frac{b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2-4ac}}}\,d\sqrt{-a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[-a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*EllipticPi[-1/2*((b + Sqrt[b^2 - 4*a*c])*e)/(c*d), I*Ar
cSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]*x], -((b + Sqrt[b^2 - 4*a*
c])/(-b + Sqrt[b^2 - 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 - 4*a*c]))]
*d*Sqrt[-a + b*x^2 - c*x^4])
```

Maple [A]
time = 0.11, size = 199, normalized size = 0.48

method	result
default	$\frac{\sqrt{1-\frac{b x^2}{2a}+\frac{x^2\sqrt{-4ac+b^2}}{2a}}\sqrt{1-\frac{b x^2}{2a}-\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-b+\sqrt{-4ac+b^2}}{2a}}\,x, \frac{}{\left(-b+\right.}\right.}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{-4ac+b^2}}{2a}}\sqrt{-c x^4+b x^2-a}}$
elliptic	$\frac{\sqrt{1-\frac{b x^2}{2a}+\frac{x^2\sqrt{-4ac+b^2}}{2a}}\sqrt{1-\frac{b x^2}{2a}-\frac{x^2\sqrt{-4ac+b^2}}{2a}}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{-b+\sqrt{-4ac+b^2}}{2a}}\,x, \frac{}{\left(-b+\right.}\right.}{d\sqrt{\frac{b}{2a}-\frac{\sqrt{-4ac+b^2}}{2a}}\sqrt{-c x^4+b x^2-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/(1/2*b/a-1/2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a+1/2*x^2/a*(-4*a
*c+b^2)^(1/2))^(1/2)*(1-1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(-c
*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*x,2/(
-b+(-4*a*c+b^2)^(1/2))*a*e/d,1/2*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(
-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(x^2*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\sqrt{-a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(-c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/((d + e*x**2)*sqrt(-a + b*x**2 - c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(-c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 - a)*(x^2*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x^2+d)\,\sqrt{-c\,x^4+b\,x^2-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(b*x^2 - a - c*x^4)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)*(b*x^2 - a - c*x^4)^(1/2)), x)
```

3.394 $\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=229

$$\frac{3e(5d^2-10de+6e^2)\,x(2+x^2)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4}+\frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}-\frac{3\sqrt{2}\,e(5d^2-10de}{}$$

```
[Out] 3/5*e*(5*d^2-10*d*e+6*e^2)*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/10*(5*d^3-10*d*e
^2+8*e^3)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(
1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-3/5*e*(5*d^2-10*d
*e+6*e^2)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(
1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)+1/5*(5*d-4*e)*e^2
*x*(x^4+3*x^2+2)^(1/2)+1/5*e^3*x^3*(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1693, 1203, 1113, 1149}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(5d^3-10de^2+8e^3)\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{5\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{3\sqrt{2}\,e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(5d^2-10de+6e^2)\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{5\sqrt{x^4+3x^2+2}}+\frac{3e(x^2+2)\,x(5d^2-10de+6e^2)}{5\sqrt{x^4+3x^2+2}}+\frac{1}{5}e^2\sqrt{x^4+3x^2+2}\,x(5d-4e)+\frac{1}{5}e^3\sqrt{x^4+3x^2+2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (3*e*(5*d^2 - 10*d*e + 6*e^2)*x*(2 + x^2))/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*
d - 4*e)*e^2*x*Sqrt[2 + 3*x^2 + x^4])/5 + (e^3*x^3*Sqrt[2 + 3*x^2 + x^4])/5
 - (3*Sqrt[2]*e*(5*d^2 - 10*d*e + 6*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)
]*EllipticE[ArcTan[x], 1/2])/(5*Sqrt[2 + 3*x^2 + x^4]) + ((5*d^3 - 10*d*e^2
 + 8*e^3)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(5
*Sqrt[2]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
```

```
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex^2)^3}{\sqrt{2+3x^2+x^4}}\,dx &= \frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}+\frac{1}{5}\int \frac{5d^3+3e(5d^2-2e^2)\,x^2+3(5d-4e)e^2x^4}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4}+\frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}+\frac{1}{15}\int \frac{3(5d^3-10de^2+8}{\sqrt{}} \\
&= \frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4}+\frac{1}{5}e^3x^3\sqrt{2+3x^2+x^4}+\frac{1}{5}\big(3e\big(5d^2-10de+6e^2\big) \\
&= \frac{3e(5d^2-10de+6e^2)\,x(2+x^2)}{5\sqrt{2+3x^2+x^4}}+\frac{1}{5}(5d-4e)e^2x\sqrt{2+3x^2+x^4}+\frac{1}{5}e^3x^3\sqrt{2+3x
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.12, size = 154, normalized size = 0.67

$$
\frac{e^2x(2+3x^2+x^4)\,(5d+e(-4+x^2))-3ie(5d^2-10de+6e^2)\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)-5i(d^3-3d^2e+4de^2-2e^3)\,\sqrt{1+x^2}\,\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{5\sqrt{2+3x^2+x^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e^2*x*(2 + 3*x^2 + x^4)*(5*d + e*(-4 + x^2)) - (3*I)*e*(5*d^2 - 10*d*e + 6
*e^2)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (5*I
)*(d^3 - 3*d^2*e + 4*d*e^2 - 2*e^3)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I
*ArcSinh[x/Sqrt[2]], 2])/(5*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 380, normalized size = 1.66

method	result
elliptic	$\frac{e^3x^3\sqrt{x^4+3x^2+2}}{5}+\left(d\,e^2-\frac{4}{5}e^3\right)x\sqrt{x^4+3x^2+2}-\frac{i\left(d^3-2d\,e^2+\frac{8}{5}e^3\right)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{Elli}}{2\sqrt{x^4+3x^2+2}}$
risch	$\frac{x\,e^2\left(e\,x^2+5d-4e\right)\sqrt{x^4+3x^2+2}}{5}+\frac{i\left(15d^2e-30d\,e^2+18e^3\right)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right.}{10\sqrt{x^4+3x^2+2}}$
default	$e^3\left(\frac{x^3\sqrt{x^4+3x^2+2}}{5}-\frac{4x\sqrt{x^4+3x^2+2}}{5}-\frac{4i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{5\sqrt{x^4+3x^2+2}}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^3*(1/5*x^3*(x^4+3*x^2+2)^(1/2)-4/5*x*(x^4+3*x^2+2)^(1/2)-4/5*I*2^(1/2)*(2
*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2
^(1/2))+9/5*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(El
lipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2))))+3*d*e
^2*(1/3*x*(x^4+3*x^2+2)^(1/2)+1/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(
x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-I*2^(1/2)*(2*x^2+4)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-
EllipticE(1/2*I*2^(1/2)*x,2^(1/2))))+3/2*I*d^2*e*2^(1/2)*(2*x^2+4)^(1/2)*(x
^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-Ellipti
cE(1/2*I*2^(1/2)*x,2^(1/2)))-1/2*I*d^3*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2
)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex^2\right)^3}{\sqrt{\left(x^2 + 1\right)\left(x^2 + 2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**3/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^3/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^3}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^3/(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((d + e*x^2)^3/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.395 $\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=168

$$\frac{2(d-e)ex(2+x^2)}{\sqrt{2+3x^2+x^4}}+\frac{1}{3}e^2x\sqrt{2+3x^2+x^4}-\frac{2\sqrt{2}\,(d-e)e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{(3d^2-2e^2)\,(1\cdots}{3\sqrt{\cdots}}$$

```
[Out] 2*(d-e)*e*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/6*(3*d^2-2*e^2)*(x^2+1)^(3/2)*(1/
(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/
2)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-2*(d-e)*e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*El
lipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3
*x^2+2)^(1/2)+1/3*e^2*x*(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1220, 1203, 1113, 1149}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(3d^2-2e^2)\,F\left(\text{ArcTan}(x)\left|\frac{1}{2}\right.\right)}{3\sqrt{2}\,\sqrt{x^4+3x^2+2}}-\frac{2\sqrt{2}\,e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(d-e)E\left(\text{ArcTan}(x)\left|\frac{1}{2}\right.\right)}{\sqrt{x^4+3x^2+2}}+\frac{2ex(x^2+2)\,(d-e)}{\sqrt{x^4+3x^2+2}}+\frac{1}{3}e^2x\sqrt{x^4+3x^2+2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (2*(d - e)*e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] + (e^2*x*Sqrt[2 + 3*x^2 + x
^4])/3 - (2*Sqrt[2]*(d - e)*e*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE
[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + ((3*d^2 - 2*e^2)*(1 + x^2)*Sqrt[(
2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(3*Sqrt[2]*Sqrt[2 + 3*x^2 +
x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
```

```
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{\sqrt{2+3x^2+x^4}}\,dx = \frac{1}{3}e^2x\sqrt{2+3x^2+x^4} + \frac{1}{3}\int \frac{3d^2-2e^2+6(d-e)ex^2}{\sqrt{2+3x^2+x^4}}\,dx$$

$$= \frac{1}{3}e^2x\sqrt{2+3x^2+x^4} + (2(d-e)e)\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx + \frac{1}{3}(3d^2-2e^2)\int \frac{}{\sqrt{2+}}$$

$$= \frac{2(d-e)ex(2+x^2)}{\sqrt{2+3x^2+x^4}} + \frac{1}{3}e^2x\sqrt{2+3x^2+x^4} - \frac{2\sqrt{2}(d-e)e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E(\mathrm{t}}{\sqrt{2+3x^2+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 127, normalized size = 0.76

$$\frac{e^2x(2+3x^2+x^4) - 6i(d-e)e\sqrt{1+x^2}\sqrt{2+x^2}\,E\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - i(3d^2-6de+4e^2)\sqrt{1+x^2}\sqrt{2+x^2}\,F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{3\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[2 + 3*x^2 + x^4],x]
```

[Out] (e^2*x*(2 + 3*x^2 + x^4) - (6*I)*(d - e)*e*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - I*(3*d^2 - 6*d*e + 4*e^2)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x/Sqrt[2]], 2])/(3*Sqrt[2 + 3*x^2 + x^4])

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 235, normalized size = 1.40

method	result
elliptic	$\frac{e^2 x\sqrt{x^4+3x^2+2}}{3}-\frac{i\left(d^2-\frac{2e^2}{3}\right)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{x^4+3x^2+2}}+\frac{i(2de-2e^2)\sqrt{2}\,\sqrt{2x^2}}{}$
risch	$\frac{e^2 x\sqrt{x^4+3x^2+2}}{3}+\frac{i(6de-6e^2)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right.}{6\sqrt{x^4+3x^2+2}}$
default	$e^2\left(\frac{x\sqrt{x^4+3x^2+2}}{3}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{3\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)

[Out] e^2*(1/3*x*(x^4+3*x^2+2)^(1/2)+1/3*I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-I*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2))))+I*d*e*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-1/2*I*d^2*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate((x^2*e + d)^2/sqrt(x^4 + 3*x^2 + 2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex^{2}\right)^{2}}{\sqrt{\left(x^{2} + 1\right)\left(x^{2} + 2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^2/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,x^2+d\right)}^2}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(3*x^2 + x^4 + 2)^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.396 $\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=122

$$\frac{ex(2+x^2)}{\sqrt{2+3x^2+x^4}}-\frac{\sqrt{2}\,e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}}+\frac{d(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2+3x^2+x^4}}$$

```
[Out] e*x*(x^2+2)/(x^4+3*x^2+2)^(1/2)+1/2*d*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*Ellip
ticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)*2^(1/2)/(x^4+3*x^
2+2)^(1/2)-e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*
2^(1/2))*2^(1/2)*((x^2+2)/(x^2+1))^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1203, 1113, 1149}

$$\frac{d(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4+3x^2+2}}-\frac{\sqrt{2}\,e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{x^4+3x^2+2}}+\frac{ex(x^2+2)}{\sqrt{x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] (e*x*(2 + x^2))/Sqrt[2 + 3*x^2 + x^4] - (Sqrt[2]*e*(1 + x^2)*Sqrt[(2 + x^2)
/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/Sqrt[2 + 3*x^2 + x^4] + (d*(1 + x^2)
*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[2 + 3*x
^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
```

```
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^2}{\sqrt{2+3x^2+x^4}}\,dx &= d\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx + e\int \frac{x^2}{\sqrt{2+3x^2+x^4}}\,dx \\
&= \frac{ex(2+x^2)}{\sqrt{2+3x^2+x^4}} - \frac{\sqrt{2}\,e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2+3x^2+x^4}} + \frac{d(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}}{\sqrt{2}\,\sqrt{2+3x}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 73, normalized size = 0.60

$$
-\frac{i\sqrt{1+x^2}\,\sqrt{2+x^2}\left(eE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+(d-e)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{\sqrt{2+3x^2+x^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[2 + 3*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*(e*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + (
d - e)*EllipticF[I*ArcSinh[x/Sqrt[2]], 2]))/Sqrt[2 + 3*x^2 + x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 108, normalized size = 0.89

method	result
default	$\frac{ie\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}} - \frac{id\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2}}{2\sqrt{x^4+}}$
elliptic	$\frac{ie\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\left(\text{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)-\text{EllipticE}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)\right)}{2\sqrt{x^4+3x^2+2}} - \frac{id\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2}}{2\sqrt{x^4+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*e*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(Elliptic
F(1/2*I*2^(1/2)*x,2^(1/2))-EllipticE(1/2*I*2^(1/2)*x,2^(1/2)))-1/2*I*d*2^(1
/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/
2)*x,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{\sqrt{(x^2 + 1)(x^2 + 2)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/sqrt((x**2 + 1)*(x**2 + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)/sqrt(x^4 + 3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(3*x^2 + x^4 + 2)^(1/2),x)
[Out] int((d + e*x^2)/(3*x^2 + x^4 + 2)^(1/2), x)
```

3.397 $\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx$

Optimal. Leaf size=124

$$\frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,(d-e)\sqrt{2+3x^2+x^4}}-\frac{e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,\Pi\left(1-\frac{e}{d};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,d(d-e)\sqrt{2+3x^2+x^4}}$$

```
[Out] 1/2*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
((x^2+2)/(x^2+1))^(1/2)/(d-e)*2^(1/2)/(x^4+3*x^2+2)^(1/2)-1/2*e*(x^2+1)^(3/
2)*(1/(x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-e/d,1/2*2^(1/2))*((x^2+2)
/(x^2+1))^(1/2)/d/(d-e)*2^(1/2)/(x^4+3*x^2+2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1228, 1113, 1470, 553}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4+3x^2+2}\,(d-e)}-\frac{e(x^2+2)\,\Pi\left(1-\frac{e}{d};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\,d\sqrt{\frac{x^2+2}{x^2+1}}\,\sqrt{x^4+3x^2+2}\,(d-e)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*(d
 - e)*Sqrt[2 + 3*x^2 + x^4]) - (e*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x],
1/2])/(Sqrt[2]*d*(d - e)*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
```

```
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx = \frac{\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{d-e} - \frac{e\int \frac{2+2x^2}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx}{2(d-e)}$$

$$= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,(d-e)\sqrt{2+3x^2+x^4}} - \frac{\left(e\sqrt{1+\frac{x^2}{2}}\,\sqrt{2+2x^2}\right)\int \frac{\sqrt{2}}{\sqrt{1+}}}{2(d-e)\sqrt{2+3x^2+x^4}}$$

$$= \frac{(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,(d-e)\sqrt{2+3x^2+x^4}} - \frac{e(2+x^2)\,\Pi\left(1-\frac{e}{d};\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,d(d-e)\sqrt{\frac{2+x^2}{1+x^2}}\,\sqrt{2+3x^2+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.08, size = 59, normalized size = 0.48

$$-\frac{i\sqrt{1+x^2}\,\sqrt{2+x^2}\,\Pi\left(\frac{2e}{d};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)}{d\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticPi[(2*e)/d, I*ArcSinh[x/Sqrt[2]],
 2])/(d*Sqrt[2 + 3*x^2 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 55, normalized size = 0.44

method	result	size
default	$-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\mathrm{EllipticPi}\left(\frac{i\sqrt{2}\,x}{2},\frac{2e}{d},\sqrt{2}\right)}{d\sqrt{x^4+3x^2+2}}$	55
elliptic	$-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\mathrm{EllipticPi}\left(\frac{i\sqrt{2}\,x}{2},\frac{2e}{d},\sqrt{2}\right)}{d\sqrt{x^4+3x^2+2}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I/d*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi
(1/2*I*2^(1/2)*x,2*e/d,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(x^2*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 3*x^2 + 2)/(d*x^4 + 3*d*x^2 + (x^6 + 3*x^4 + 2*x^2)*e +
 2*d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2+1)(x^2+2)}\,(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(x^2*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(e\,x^2+d)\,\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(3*x^2 + x^4 + 2)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.398 $\int \frac{1}{(d+ex^2)^2 \sqrt{2+3x^2+x^4}} \, dx$

Optimal. Leaf size=316

$$-\frac{ex(2+x^2)}{2d(d^2-3de+2e^2)\sqrt{2+3x^2+x^4}}+\frac{e^2x\sqrt{2+3x^2+x^4}}{2d(d^2-3de+2e^2)(d+ex^2)}+\frac{e(1+x^2)\sqrt{\frac{2+x^2}{1+x^2}}\,E\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\,d(d-2e)(d-e)\sqrt{2+3x^2+x^4}}$$

```
[Out] -1/2*e*x*(x^2+2)/d/(d^2-3*d*e+2*e^2)/(x^4+3*x^2+2)^(1/2)-1/4*e*(3*d^2-6*d*e
+2*e^2)*(x^2+2)*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticPi(x/(x^2+1)^(1/2),
1-e/d,1/2*2^(1/2))/d^2/(d-2*e)/(d-e)^2*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)/(x^4
+3*x^2+2)^(1/2)+1/2*e*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(
1/2),1/2*2^(1/2))*((x^2+2)/(x^2+1))^(1/2)/d/(d-2*e)/(d-e)*2^(1/2)/(x^4+3*x^
2+2)^(1/2)+1/2*(2*d-e)*(x^2+1)^(3/2)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^
(1/2),1/2*2^(1/2))*((x^2+2)/(2*x^2+2))^(1/2)/d/(d-e)^2/(x^4+3*x^2+2)^(1/2)+
1/2*e^2*x*(x^4+3*x^2+2)^(1/2)/d/(d^2-3*d*e+2*e^2)/(e*x^2+d)
```

Rubi [A]

time = 0.21, antiderivative size = 399, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1237, 1730, 1203, 1113, 1149, 1228, 1470, 553}

$$\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,(3d^2-6de+2e^2)\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{2\sqrt{2}\,d\sqrt{x^4+3x^2+2}\,(d-2e)(d-e)^2}-\frac{e(x^2+2)\,(3d^2-6de+2e^2)\,\Pi\left(1-\frac{e}{d};\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{2\sqrt{2}\,d^2\sqrt{\frac{x^2+2}{x^2+1}}\,\sqrt{x^4+3x^2+2}\,(d-2e)(d-e)^2}-\frac{(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,F\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{2\sqrt{2}\,\sqrt{x^4+3x^2+2}\,(d-2e)(d-e)}+\frac{e(x^2+1)\sqrt{\frac{x^2+2}{x^2+1}}\,E\left(\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{\sqrt{2}\,d\sqrt{x^4+3x^2+2}\,(d-2e)(d-e)}+\frac{e^2x\sqrt{x^4+3x^2+2}}{2d(d^2-3de+2e^2)(d+ex^2)}-\frac{ex(x^2+2)}{2d\sqrt{x^4+3x^2+2}\,(d^2-3de+2e^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] -1/2*(e*x*(2 + x^2))/(d*(d^2 - 3*d*e + 2*e^2)*Sqrt[2 + 3*x^2 + x^4]) + (e^2
*x*Sqrt[2 + 3*x^2 + x^4])/(2*d*(d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (e*(1 +
 x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*EllipticE[ArcTan[x], 1/2])/(Sqrt[2]*d*(d -
2*e)*(d - e)*Sqrt[2 + 3*x^2 + x^4]) - ((1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*
EllipticF[ArcTan[x], 1/2])/(2*Sqrt[2]*(d - 2*e)*(d - e)*Sqrt[2 + 3*x^2 + x^
4]) + ((3*d^2 - 6*d*e + 2*e^2)*(1 + x^2)*Sqrt[(2 + x^2)/(1 + x^2)]*Elliptic
F[ArcTan[x], 1/2])/(2*Sqrt[2]*d*(d - 2*e)*(d - e)^2*Sqrt[2 + 3*x^2 + x^4])
- (e*(3*d^2 - 6*d*e + 2*e^2)*(2 + x^2)*EllipticPi[1 - e/d, ArcTan[x], 1/2])
/(2*Sqrt[2]*d^2*(d - 2*e)*(d - e)^2*Sqrt[(2 + x^2)/(1 + x^2)]*Sqrt[2 + 3*x^
2 + x^4])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
```

```
[d/c]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1228

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/(2*c*d - e*(b - q))), In
t[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[e/(2*c*d - e*(b - q)), Int[(b -
q + 2*c*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] &&  !LtQ[c, 0]
```

Rule 1237

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> Simp[(-e^2)*x*(d + e*x^2)^(q + 1)*(Sqrt[a + b*x^2 + c*x^4]/(2*d*
(q + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e
+ a*e^2)), Int[((d + e*x^2)^(q + 1)/Sqrt[a + b*x^2 + c*x^4])*Simp[a*e^2*(2*
q + 3) + 2*d*(c*d - b*e)*(q + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*
e^2*(2*q + 5)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[q, -1]
```

Rule 1470

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((f_) + (g_.)*(x_)^(n_))^(r_.)*((a_) + (
b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x
^(2*n))^FracPart[p]/((d + e*x^n)^FracPart[p]*(a/d + (c*x^n)/e)^FracPart[p])
, Int[(d + e*x^n)^(p + q)*(f + g*x^n)^r*(a/d + (c/e)*x^n)^p, x], x] /; Free
Q[{a, b, c, d, e, f, g, n, p, q, r}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p]
```

Rule 1730

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[
P4x, x, 4]}, Dist[-(e^2)^(-1), Int[(C*d - B*e - C*e*x^2)/Sqrt[a + b*x^2 + c
*x^4], x], x] + Dist[(C*d^2 - B*d*e + A*e^2)/e^2, Int[1/((d + e*x^2)*Sqrt[a
 + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[P4x, x^2,
 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 -
a*e^2, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2\sqrt{2+3x^2+x^4}}\,dx = \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)(d+ex^2)} - \frac{\int \frac{-2\left(d^2-3de+e^2\right)+2dex^2+e^2x^4}{(d+ex^2)\sqrt{2+3x^2+x^4}}\,dx}{2d(d-2e)(d-e)}$$

$$= \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)(d+ex^2)} + \frac{\int \frac{-de^2-e^3x^2}{\sqrt{2+3x^2+x^4}}\,dx}{2d(d-2e)(d-e)e^2} + \frac{(3d^2-6de+2e\cdots}{2d\cdots}$$

$$= \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)(d+ex^2)} - \frac{\int \frac{1}{\sqrt{2+3x^2+x^4}}\,dx}{2(d-2e)(d-e)} + \frac{(3d^2-6de+2e\cdots}{2d(d\cdots}$$

$$= -\frac{ex(2+x^2)}{2d\left(d^2-3de+2e^2\right)\sqrt{2+3x^2+x^4}} + \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)(d+ex^2)} + \cdots$$

$$= -\frac{ex(2+x^2)}{2d\left(d^2-3de+2e^2\right)\sqrt{2+3x^2+x^4}} + \frac{e^2x\sqrt{2+3x^2+x^4}}{2d\left(d^2-3de+2e^2\right)(d+ex^2)} + \cdots$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.32, size = 175, normalized size = 0.55

$$\frac{\frac{e^2x(2+3x^2+x^4)}{(d^2-3de+2e^2)(d+ex^2)}+\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\left(deE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+d(d-e)F\left(i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+(-3d^2+6de-2e^2)\Pi\left(\frac{2e}{d};i\sinh^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)\right)}{d(d-2e)(d-e)}}{2d\sqrt{2+3x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[2 + 3*x^2 + x^4]),x]
```

```
[Out] ((e^2*x*(2 + 3*x^2 + x^4))/((d^2 - 3*d*e + 2*e^2)*(d + e*x^2)) + (I*Sqrt[1
+ x^2]*Sqrt[2 + x^2]*(d*e*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] + d*(d - e)*El
lipticF[I*ArcSinh[x/Sqrt[2]], 2] + (-3*d^2 + 6*d*e - 2*e^2)*EllipticPi[(2*e
)/d, I*ArcSinh[x/Sqrt[2]], 2]))/(d*(d - 2*e)*(d - e)))/(2*d*Sqrt[2 + 3*x^2
+ x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 443, normalized size = 1.40

method	result
default	$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d(d^2-3de+2e^2)(e\,x^2+d)}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4(d^2-3de+2e^2)\sqrt{x^4+3x^2+2}}-\frac{ie\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}}{4(d^2-3de+2e^2)d\sqrt{x}}$
elliptic	$\frac{e^2x\sqrt{x^4+3x^2+2}}{2d(d^2-3de+2e^2)(e\,x^2+d)}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\mathrm{EllipticF}\left(\frac{i\sqrt{2}\,x}{2},\sqrt{2}\right)}{4(d^2-3de+2e^2)\sqrt{x^4+3x^2+2}}-\frac{ie\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}}{4(d^2-3de+2e^2)d\sqrt{x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^2*x*(x^4+3*x^2+2)^(1/2)/d/(d^2-3*d*e+2*e^2)/(e*x^2+d)+1/4*I/(d^2-3*d*
e+2*e^2)*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*Elliptic
F(1/2*I*2^(1/2)*x,2^(1/2))-1/4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1
/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*2^(1/2)*x,2^(1/2))+1/
4*I*e/(d^2-3*d*e+2*e^2)/d*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+
2)^(1/2)*EllipticE(1/2*I*2^(1/2)*x,2^(1/2))-3/2*I/(d^2-3*d*e+2*e^2)*2^(1/2)
*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/
2)*x,2*e/d,2^(1/2))+3*I/(d^2-3*d*e+2*e^2)/d*e*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^
2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,2*e/d,2^(1/2))-I/
(d^2-3*d*e+2*e^2)/d^2*e^2*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^
2+2)^(1/2)*EllipticPi(1/2*I*2^(1/2)*x,2*e/d,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(x^2*e + d)^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x^2+1)(x^2+2)}\,(d+ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x**2 + 1)*(x**2 + 2))*(d + e*x**2)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(x^4+3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 3*x^2 + 2)*(x^2*e + d)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x^2+d)^2\,\sqrt{x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^2*(3*x^2 + x^4 + 2)^(1/2)),x)
```

```
[Out] int(1/((d + e*x^2)^2*(3*x^2 + x^4 + 2)^(1/2)), x)
```

3.399 $\int (c + ex^2)^q (a + cx^2 + bx^4)^p \, dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\left(c + ex^2\right)^q \left(a + cx^2 + bx^4\right)^p, x\right)$$

```
[Out] Unintegrable((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(c + ex^2\right)^q \left(a + cx^2 + bx^4\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] Defer[Int][(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

Rubi steps

$$\int \left(c + ex^2\right)^q \left(a + cx^2 + bx^4\right)^p \, dx = \int \left(c + ex^2\right)^q \left(a + cx^2 + bx^4\right)^p \, dx$$

Mathematica [A]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(c + ex^2\right)^q \left(a + cx^2 + bx^4\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] Integrate[(c + e*x^2)^q*(a + c*x^2 + b*x^4)^p, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(e\,x^2 + c\right)^q \left(b\,x^4 + c\,x^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
[Out] int((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^4 + c*x^2 + a)^p*(x^2*e + c)^q, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^4 + c*x^2 + a)^p*(x^2*e + c)^q, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**q*(b*x**4+c*x**2+a)**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^q*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^4 + c*x^2 + a)^p*(x^2*e + c)^q, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(e\,x^2+c\right)^q\,\left(b\,x^4+c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + e*x^2)^q*(a + b*x^4 + c*x^2)^p,x)
[Out] int((c + e*x^2)^q*(a + b*x^4 + c*x^2)^p, x)
```

3.400 $\int (c+ex^2)^3 (a+cx^2+bx^4)^p \, dx$

Optimal. Leaf size=498

$$\frac{ce^2(21b-5e+12bp-2ep)x(a+cx^2+bx^4)^{1+p}}{b^2(5+4p)(7+4p)}+\frac{e^3x^3(a+cx^2+bx^4)^{1+p}}{b(7+4p)}+\frac{c(ae^3(5+2p)-3abe^2(7+4p)+}{}$$

```
[Out] -c*e^2*(e*(5+2*p)-3*b*(7+4*p))*x*(b*x^4+c*x^2+a)^(1+p)/b^2/(16*p^2+48*p+35)
+e^3*x^3*(b*x^4+c*x^2+a)^(1+p)/b/(7+4*p)+c*(a*e^3*(5+2*p)-3*a*b*e^2*(7+4*p)
+b^2*c^2*(16*p^2+48*p+35))*x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*
x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/b^2/(5+4*p)/(7+
4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c^2)^(1/
2)))^p)+1/3*e*(c^2*e^2*(4*p^2+16*p+15)+3*b^2*c^2*(16*p^2+48*p+35)-3*b*e*(a*
e*(5+4*p)+c^2*(8*p^2+26*p+21)))*x^3*(b*x^4+c*x^2+a)^p*AppellF1(3/2,-p,-p,5/
2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/b^2/(5+4
*p)/(7+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c
^2)^(1/2)))^p)
```

Rubi [A]
time = 0.52, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1220, 1693, 1217, 1119, 440, 1155, 524}

$$\frac{ex^3\left(-3be\left(ae(4p+5)+c^2(8p^2+26p+21)\right)+3b^2c^2(16p^2+48p+35)+c^2e^2(4p^2+16p+15)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}(a+bx^4+cx^2)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)}{3b^2(4p+5)(4p+7)}+\frac{cx\left(-3abe^2(4p+7)+ae^3(2p+5)+b^2c^2(16p^2+48p+35)\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}(a+bx^4+cx^2)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)}{b^2(4p+5)(4p+7)}+\frac{ce^2x(12bp+21b-2ep-5e)(a+bx^4+cx^2)^{p+1}}{b^2(4p+5)(4p+7)}+\frac{e^3x^3(a+bx^4+cx^2)^{p+1}}{b(4p+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (c*e^2*(21*b - 5*e + 12*b*p - 2*e*p)*x*(a + c*x^2 + b*x^4)^(1 + p))/(b^2*(5
 + 4*p)*(7 + 4*p)) + (e^3*x^3*(a + c*x^2 + b*x^4)^(1 + p))/(b*(7 + 4*p)) +
(c*(a*e^3*(5 + 2*p) - 3*a*b*e^2*(7 + 4*p) + b^2*c^2*(35 + 48*p + 16*p^2))*x
*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4*a
*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(b^2*(5 + 4*p)*(7 + 4*p)*
(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b
+ c^2]))^p) + (e*(c^2*e^2*(15 + 16*p + 4*p^2) + 3*b^2*c^2*(35 + 48*p + 16*p
^2) - 3*b*e*(a*e*(5 + 4*p) + c^2*(21 + 26*p + 8*p^2)))*x^3*(a + c*x^2 + b*x
^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b
*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*b^2*(5 + 4*p)*(7 + 4*p)*(1 + (2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
```

```
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1119

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p])), Int[(1
+ 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1217

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 1693

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q =
 Expon[Pq, x^2], e = Coeff[Pq, x^2, Expon[Pq, x^2]]}, Simp[e*x^(2*q - 3)*((
```

```
a + b*x^2 + c*x^4)^(p + 1)/(c*(2*q + 4*p + 1))), x] + Dist[1/(c*(2*q + 4*p
+ 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandToSum[c*(2*q + 4*p + 1)*Pq - a*e*(2*
q - 3)*x^(2*q - 4) - b*e*(2*q + 2*p - 1)*x^(2*q - 2) - c*e*(2*q + 4*p + 1)*
x^(2*q), x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && Expon[P
q, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] &&  !LtQ[p, -1]
```

Rubi steps

$$\int \left(c+ex^2\right)^3\left(a+cx^2+bx^4\right)^p\,dx=\frac{e^3x^3\left(a+cx^2+bx^4\right)^{1+p}}{b(7+4p)}+\frac{\int\left(a+cx^2+bx^4\right)^p\left(bc^3(7+4p)-3e(ae^2-\right.}{b}$$

$$=\frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)}+\frac{e^3x^3\left(a+cx^2+bx^4\right.}{b(7+4p)}$$

$$=\frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)}+\frac{e^3x^3\left(a+cx^2+bx^4\right.}{b(7+4p)}$$

$$=\frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)}+\frac{e^3x^3\left(a+cx^2+bx^4\right.}{b(7+4p)}$$

$$=\frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)}+\frac{e^3x^3\left(a+cx^2+bx^4\right.}{b(7+4p)}$$

$$=\frac{ce^2(21b-5e+12bp-2ep)x\left(a+cx^2+bx^4\right)^{1+p}}{b^2(5+4p)(7+4p)}+\frac{e^3x^3\left(a+cx^2+bx^4\right.}{b(7+4p)}$$

Mathematica [A]
time = 0.55, size = 373, normalized size = 0.75

$$\frac{1}{35}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\left(35c^3F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+ex^2\left(35c^2F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+ex^2\left(21cF_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+5ex^2F_1\left(\frac{7}{2};-p,-p;\frac{9}{2};-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)^3*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(35*c^3*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c +
 Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*(35*c^2*
AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(
-c + Sqrt[-4*a*b + c^2])] + e*x^2*(21*c*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^
2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + 5*e*x^2
*AppellF1[7/2, -p, -p, 9/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/
(-c + Sqrt[-4*a*b + c^2])]))))/(35*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c -
 Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*
b + c^2]))^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+c\right)^{3} \left(b\,x^{4}+c\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
[Out] int((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((x^2*e + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((x^6*e^3 + 3*c*x^4*e^2 + 3*c^2*x^2*e + c^3)*(b*x^4 + c*x^2 + a)^p,
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**3*(b*x**4+c*x**2+a)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^3*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2*e + c)^3*(b*x^4 + c*x^2 + a)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x^2+c\right)^3\,\left(b\,x^4+c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + e*x^2)^3*(a + b*x^4 + c*x^2)^p,x)
```

```
[Out] int((c + e*x^2)^3*(a + b*x^4 + c*x^2)^p, x)
```

3.401 $\int (c+ex^2)^2 (a+cx^2+bx^4)^p \, dx$

Optimal. Leaf size=358

$$\frac{e^2x(a+cx^2+bx^4)^{1+p}}{b(5+4p)} - \frac{(ae^2-bc^2(5+4p))\,x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+}{b(5+4p)}$$

```
[Out] e^2*x*(b*x^4+c*x^2+a)^(1+p)/b/(5+4*p)-(a*e^2-b*c^2*(5+4*p))*x*(b*x^4+c*x^2+
a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4
*a*b+c^2)^(1/2)))/b/(5+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*
x^2/(c+(-4*a*b+c^2)^(1/2)))^p)+1/3*c*e*(8*b*p-2*e*p+10*b-3*e)*x^3*(b*x^4+c*
x^2+a)^p*AppellF1(3/2,-p,-p,5/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c
+(-4*a*b+c^2)^(1/2)))/b/(5+4*p)/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+
2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)
```

Rubi [A]
time = 0.22, antiderivative size = 345, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1217, 1119, 440, 1155, 524}

$$x\left(c^2-\frac{ae^2}{4bp+5b}\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}(a+bx^4+cx^2)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)+\frac{1}{3}cex^3\left(2-\frac{e(2p+3)}{b(4p+5)}\right)\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}(a+bx^4+cx^2)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)+\frac{e^2x(a+bx^4+cx^2)^{p+1}}{b(4p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (e^2*x*(a + c*x^2 + b*x^4)^(1 + p))/(b*(5 + 4*p)) + ((c^2 - (a*e^2)/(5*b +
4*b*p))*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c -
Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/
(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (
c*e*(2 - (e*(3 + 2*p))/(b*(5 + 4*p)))*x^3*(a + c*x^2 + b*x^4)^p*AppellF1[3/
2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-
4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2
)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1119

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p])), Int[(1
+ 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1217

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 1220

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[e^q*x^(2*q - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(4*p + 2*q
 + 1))), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rubi steps

$$\int \left(c+ex^2\right)^2 \left(a+cx^2+bx^4\right)^p \, dx = \frac{e^2x(a+cx^2+bx^4)^{1+p}}{b(5+4p)} + \frac{\int \left(-ae^2+bc^2(5+4p)+ce(10b-3e+8bp-\right.}{b(5+4p)}$$

$$= \frac{e^2x(a+cx^2+bx^4)^{1+p}}{b(5+4p)} + \frac{\int \left(-ae^2\left(1-\frac{bc^2(5+4p)}{ae^2}\right)(a+cx^2+bx^4)^p - ce(\right.}{b(5}$$

$$= \frac{e^2x(a+cx^2+bx^4)^{1+p}}{b(5+4p)} + \left(ce\left(2-\frac{e(3+2p)}{b(5+4p)}\right)\right)\int x^2\left(a+cx^2+bx^4\right)^p$$

$$= \frac{e^2x(a+cx^2+bx^4)^{1+p}}{b(5+4p)} + \left(ce\left(2-\frac{e(3+2p)}{b(5+4p)}\right)\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right.\right.$$

$$= \frac{e^2x(a+cx^2+bx^4)^{1+p}}{b(5+4p)} + \left(c^2-\frac{ae^2}{5b+4bp}\right)x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}$$

Mathematica [A]
time = 0.47, size = 303, normalized size = 0.85

$$\frac{1}{15}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p\left(15c^2F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+ex^2\left(10cF_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+3ex^2F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + e*x^2)^2*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(15*c^2*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c +
 Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*(10*c*Ap
pellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c
 + Sqrt[-4*a*b + c^2])] + 3*e*x^2*AppellF1[5/2, -p, -p, 7/2, (-2*b*x^2)/(c
+ Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])))/(15*((c - Sq
rt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b +
 c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+c\right)^2\left(b\,x^4+c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((x^2*e + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
[Out] integral((x^4*e^2 + 2*c*x^2*e + c^2)*(b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)**2*(b*x**4+c*x**2+a)**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)^2*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((x^2*e + c)^2*(b*x^4 + c*x^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,x^2+c\right)}^2\,{\left(b\,x^4+c\,x^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + e*x^2)^2*(a + b*x^4 + c*x^2)^p,x)
[Out] int((c + e*x^2)^2*(a + b*x^4 + c*x^2)^p, x)
```

3.402 $\int (c+ex^2)(a+cx^2+bx^4)^p\,dx$

Optimal. Leaf size=274

$$cx\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{-4ab+}}\right.$$

```
[Out] c*x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)
),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/(
(1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)+1/3*e*x^3*(b*x^4+c*x^2+a)^p*AppellF1(
3/2,-p,-p,5/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),-2*b*x^2/(c+(-4*a*b+c^2)^(1/2
)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1+2*b*x^2/(c+(-4*a*b+c^2)^(1/2
)))^p)
```

Rubi [A]
time = 0.13, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1217, 1119, 440, 1155, 524}

$$\frac{1}{3}ex^3\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}(a+bx^4+cx^2)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)+cx\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}(a+bx^4+cx^2)^p\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (c*x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[
-4*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c -
Sqrt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p) + (e*x^3
*(a + c*x^2 + b*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*b*x^2)/(c - Sqrt[-4*a
*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/(3*(1 + (2*b*x^2)/(c - Sq
rt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1119

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p])), Int[(1
+ 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1217

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)*(a + b*x^2 + c*x^4)^p, x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rubi steps

$$\begin{aligned}\int \left(c+ex^2\right)\left(a+cx^2+bx^4\right)^p\,dx &= \int \left(c\left(a+cx^2+bx^4\right)^p+ex^2\left(a+cx^2+bx^4\right)^p\right)\,dx\\ &= c\int \left(a+cx^2+bx^4\right)^p\,dx+e\int x^2\left(a+cx^2+bx^4\right)^p\,dx\\ &= \left(c\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+b\right.\right.\\ &= cx\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx\right.\end{aligned}$$

Mathematica [A]
time = 0.40, size = 232, normalized size = 0.85

$$\frac{1}{3}x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\left(3cF_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)+ex^2F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + e*x^2)*(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*(3*c*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sq
rt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])] + e*x^2*AppellF1[3/
2, -p, -p, 5/2, (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-
4*a*b + c^2])]))/(3*((c - Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c - Sqrt[-4*a*b +
c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+c\right) \left(b\,x^{4}+c\,x^{2}+a\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((e*x^2+c)*(b*x^4+c*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + c)*(b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((x^2*e + c)*(b*x^4 + c*x^2 + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + ex^{2}\right) \left(a + bx^{4} + cx^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+c)*(b*x**4+c*x**2+a)**p,x)
```

```
[Out] Integral((c + e*x**2)*(a + b*x**4 + c*x**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+c)*(b*x^4+c*x^2+a)^p,x, algorithm="giac")
[Out] integrate((x^2*e + c)*(b*x^4 + c*x^2 + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x^2+c\right)\,{\left(b\,x^4+c\,x^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + e*x^2)*(a + b*x^4 + c*x^2)^p,x)
[Out] int((c + e*x^2)*(a + b*x^4 + c*x^2)^p, x)
```

3.403 $\int (a+cx^2+bx^4)^p \, dx$

Optimal. Leaf size=133

$$x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}(a+cx^2+bx^4)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right.$$

```
[Out] x*(b*x^4+c*x^2+a)^p*AppellF1(1/2,-p,-p,3/2,-2*b*x^2/(c-(-4*a*b+c^2)^(1/2)),
-2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))/((1+2*b*x^2/(c-(-4*a*b+c^2)^(1/2)))^p)/((1
+2*b*x^2/(c+(-4*a*b+c^2)^(1/2)))^p)
```

Rubi [A]
time = 0.03, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1119, 440}

$$x\left(\frac{2bx^2}{c-\sqrt{c^2-4ab}}+1\right)^{-p}\left(\frac{2bx^2}{\sqrt{c^2-4ab}+c}+1\right)^{-p}(a+bx^4+cx^2)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c-\sqrt{c^2-4ab}},-\frac{2bx^2}{c+\sqrt{c^2-4ab}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c - Sqrt[-4
*a*b + c^2]), (-2*b*x^2)/(c + Sqrt[-4*a*b + c^2])])/((1 + (2*b*x^2)/(c - Sq
rt[-4*a*b + c^2]))^p*(1 + (2*b*x^2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1119

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*
c*(x^2/(b + q)))^FracPart[p]*(1 + 2*c*(x^2/(b - q)))^FracPart[p])), Int[(1
+ 2*c*(x^2/(b + q)))^p*(1 + 2*c*(x^2/(b - q)))^p, x], x]] /; FreeQ[{a, b, c
, p}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a+cx^2+bx^4\right)^p\,dx = \left(\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p\right)\int \Bigg($$
$$= x\left(1+\frac{2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(1+\frac{2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p F_1\Bigg(\frac{1}{2};$$

Mathematica [A]
time = 0.10, size = 161, normalized size = 1.21

$$x\left(\frac{c-\sqrt{-4ab+c^2}+2bx^2}{c-\sqrt{-4ab+c^2}}\right)^{-p}\left(\frac{c+\sqrt{-4ab+c^2}+2bx^2}{c+\sqrt{-4ab+c^2}}\right)^{-p}\left(a+cx^2+bx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2bx^2}{c+\sqrt{-4ab+c^2}},\frac{2bx^2}{-c+\sqrt{-4ab+c^2}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + c*x^2 + b*x^4)^p,x]
```

```
[Out] (x*(a + c*x^2 + b*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*b*x^2)/(c + Sqrt[-4
*a*b + c^2]), (2*b*x^2)/(-c + Sqrt[-4*a*b + c^2])])/(((c - Sqrt[-4*a*b + c^
2] + 2*b*x^2)/(c - Sqrt[-4*a*b + c^2]))^p*((c + Sqrt[-4*a*b + c^2] + 2*b*x^
2)/(c + Sqrt[-4*a*b + c^2]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b\,x^4+c\,x^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+c*x^2+a)^p,x)
```

```
[Out] int((b*x^4+c*x^2+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^4 + cx^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+c*x**2+a)**p,x)
```

```
[Out] Integral((a + b*x**4 + c*x**2)**p, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,x^4+c\,x^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4 + c*x^2)^p,x)
```

```
[Out] int((a + b*x^4 + c*x^2)^p, x)
```

3.404 $\int \frac{(a+cx^2+bx^4)^p}{c+ex^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(a+cx^2+bx^4)^p}{c+ex^2},x\right)$$

```
[Out] Unintegrable((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]
[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]
```

Rubi steps

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx = \int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$$

Mathematica [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(a+cx^2+bx^4)^p}{c+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2),x]
[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4+c\,x^2+a)^p}{e\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

```
[Out] int((b*x^4+c*x^2+a)^p/(e*x^2+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(x^2*e + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p/(x^2*e + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+c*x**2+a)**p/(e*x**2+c),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(x^2*e + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(b\,x^4+c\,x^2+a)^p}{e\,x^2+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4 + c*x^2)^p/(c + e*x^2),x)
```

```
[Out] int((a + b*x^4 + c*x^2)^p/(c + e*x^2), x)
```

3.405 $\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2},x\right)$$

[Out] Unintegrable((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

Verification is not applicable to the result.

[In] Int[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]

[Out] Defer[Int][(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]

Rubi steps

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx = \int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

Mathematica [A]
time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{(a+cx^2+bx^4)^p}{(c+ex^2)^2} dx$$

Verification is not applicable to the result.

[In] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2,x]

[Out] Integrate[(a + c*x^2 + b*x^4)^p/(c + e*x^2)^2, x]

Maple [A]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^4+c\,x^2+a)^p}{(e\,x^2+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

```
[Out] int((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(x^2*e + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + c*x^2 + a)^p/(x^4*e^2 + 2*c*x^2*e + c^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+c*x**2+a)**p/(e*x**2+c)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+c*x^2+a)^p/(e*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + c*x^2 + a)^p/(x^2*e + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b\,x^4+c\,x^2+a\right)^p}{{\left(e\,x^2+c\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4 + c*x^2)^p/(c + e*x^2)^2,x)
[Out] int((a + b*x^4 + c*x^2)^p/(c + e*x^2)^2, x)
```

3.406 $\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=446

$$\frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}}-\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}}+\frac{\left(\sqrt{c}\,df+\sqrt{a}\,eg\right)\left(\sqrt{a}\right.}{2\sqrt[4]{c}}$$

```
[Out] 1/2*(-d*g+e*f)*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c
*d^4)^(1/2)-1/2*(-d*g+e*f)*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c
*x^4+a)^(1/2))/(a*e^4+c*d^4)^(1/2)-1/4*(-d*g+e*f)*(cos(2*arctan(c^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2)
,1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/e/(e^2*a^(1/2)+d^2*c^(1/2))
/(c*x^4+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(e*g*a^(1/2)+d*f*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*
c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 446, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1756, 12, 1262, 739, 212, 1723, 226, 1721}

$$\frac{(ef-dg)\text{ArcTan}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}}+\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{a}\,eg+\sqrt{c}\,df\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)(ef-dg)\Pi\left(\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\sqrt{a}\,\sqrt{c}\,d^2e^2};2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\,\sqrt[4]{c}\,de\sqrt{a+cx^4}\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}-\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*S
qrt[-(c*d^4) - a*e^4]) - ((e*f - d*g)*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d
^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + ((Sqrt[c]*d*f + Sq
rt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*(Sqrt
[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f
 - d*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*Ar
cTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c]*d^2 + Sqrt
[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1756

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{a+cx^4}}\,dx = \int \frac{(-ef+dg)x}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx + \int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}\,de(ef-dg)\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{\sqrt{c}\,d^2+\sqrt{a}\,e^2} + (-ef+dg)\int \frac{x}{(d^2-e^2x^2)\sqrt{a+c}}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} + \frac{\left(\sqrt{c}\,df+\sqrt{a}\,eg\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{}{(\sqrt{a}}}}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{c}\,d^2+\sqrt{}\right.}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} + \frac{\left(\sqrt{c}\,df+\sqrt{a}\,eg\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{}{(\sqrt{a}}}}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\left(\sqrt{c}\,d^2+\sqrt{}\right.}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\,\sqrt{a+cx^4}}\right.}{2\sqrt{cd^4+ae^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.66, size = 270, normalized size = 0.61

$$\frac{-\frac{ig\sqrt{1+\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}} + \frac{(ef-dg)\left(\sqrt[4]{c}\,de\sqrt{a+cx^4}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d^2-e^2x^2)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)-\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt{-cd^4-ae^4}\,\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)\right)}{\sqrt[4]{c}\,d\sqrt{-cd^4-ae^4}}}{e\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (((-I)*g*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*
x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + ((e*f - d*g)*(c^(1/4)*d*e*Sqrt[a + c*x
^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) -
a*e^4]] - (-1)^(1/4)*a^(1/4)*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]*Ell
ipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4
)], -1]))/(c^(1/4)*d*Sqrt[-(c*d^4) - a*e^4]))/(e*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 251, normalized size = 0.56

method	result
default	$\frac{g\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{(-dg+ef)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^{2}d^{2}}{e^{2}}+2a}{2\sqrt{\frac{c\,d^{4}}{e^{4}}+a}\,\sqrt{c\,x^{4}+a}}\right)}{2\sqrt{\frac{c\,d^{4}}{e^{4}}+a}}+\cdots\right.}{\cdots}$
elliptic	$\frac{g\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\operatorname{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}-\frac{(dg-ef)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^{2}d^{2}}{e^{2}}+2a}{2\sqrt{\frac{c\,d^{4}}{e^{4}}+a}\,\sqrt{c\,x^{4}+a}}\right)}{2\sqrt{\frac{c\,d^{4}}{e^{4}}+a}}+\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g/e/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*
c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)
/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*
EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/
2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx}{\sqrt{a + cx^4}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+a)**(1/2),x)
[Out] Integral((f + g*x)/(sqrt(a + c*x**4)*(d + e*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)/(sqrt(c*x^4 + a)*(x*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f + g\,x}{\sqrt{c\,x^4 + a}\,(d + e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((a + c*x^4)^(1/2)*(d + e*x)),x)
[Out] int((f + g*x)/((a + c*x^4)^(1/2)*(d + e*x)), x)
```

3.407 $\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx$

Optimal. Leaf size=218

$$\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cd^4-ae^4}\sqrt{-a+cx^4}}\right)}{2\sqrt{cd^4-ae^4}}+\frac{\sqrt[4]{a}\,g\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,e\sqrt{-a+cx^4}}+\frac{\sqrt[4]{a}\,(ef-dg)}{}$$

```
[Out] 1/2*(-d*g+e*f)*arctanh((-c*d^2*x^2+a*e^2)/(-a*e^4+c*d^4)^(1/2)/(c*x^4-a)^(1
/2))/(-a*e^4+c*d^4)^(1/2)+a^(1/4)*g*EllipticF(c^(1/4)*x/a^(1/4),I)*(1-c*x^4
/a)^(1/2)/c^(1/4)/e/(c*x^4-a)^(1/2)+a^(1/4)*(-d*g+e*f)*EllipticPi(c^(1/4)*x
/a^(1/4),e^2*a^(1/2)/d^2/c^(1/2),I)*(1-c*x^4/a)^(1/2)/c^(1/4)/d/e/(c*x^4-a)
^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1756, 12, 1262, 739, 212, 1725, 230, 227, 1233, 1232}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{cx^4}{a}}(ef-dg)\Pi\left(\frac{\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,de\sqrt{cx^4-a}}+\frac{\sqrt[4]{a}\,g\sqrt{1-\frac{cx^4}{a}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,e\sqrt{cx^4-a}}+\frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cx^4-a}\sqrt{cd^4-ae^4}}\right)}{2\sqrt{cd^4-ae^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTanh[(a*e^2 - c*d^2*x^2)/(Sqrt[c*d^4 - a*e^4]*Sqrt[-a + c*x
^4])])/(2*Sqrt[c*d^4 - a*e^4]) + (a^(1/4)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[A
rcSin[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*e*Sqrt[-a + c*x^4]) + (a^(1/4)*(e
*f - d*g)*Sqrt[1 - (c*x^4)/a]*EllipticPi[(Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSi
n[(c^(1/4)*x)/a^(1/4)], -1])/(c^(1/4)*d*e*Sqrt[-a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
```

```
b/a] && GtQ[a, 0]
```

Rule 230

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1233

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1725

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + c*x^4], x], x] + Dist[(e*A - d*B)/
e, Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e, A, B},
 x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 1756

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
```

```
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+cx^4}}\,dx = \int \frac{(-ef+dg)x}{(d^2-e^2x^2)\sqrt{-a+cx^4}}\,dx + \int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{-a+cx^4}}\,dx$$

$$= \frac{g\int \frac{1}{\sqrt{-a+cx^4}}\,dx}{e} + \frac{(d(ef-dg))\int \frac{1}{(d^2-e^2x^2)\sqrt{-a+cx^4}}\,dx}{e} + (-ef+dg)$$

$$= \frac{1}{2}(-ef+dg)\text{Subst}\left(\int \frac{1}{(d^2-e^2x)\sqrt{-a+cx^2}}\,dx, x, x^2\right) + \frac{\left(g\sqrt{1-\frac{cx^4}{a}}\right)}{e\sqrt{-a}}$$

$$= \frac{\sqrt[4]{a}\,g\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt[4]{c}\,e\sqrt{-a+cx^4}} + \frac{\sqrt[4]{a}\,(ef-dg)\sqrt{1-\frac{cx^4}{a}}\,\Pi\left(\frac{\sqrt{c}}{\sqrt{c}}\right.}{\sqrt[4]{c}\,de\sqrt{-a}}$$

$$= \frac{(ef-dg)\tanh^{-1}\left(\frac{ae^2-cd^2x^2}{\sqrt{cd^4-ae^4}\sqrt{-a+cx^4}}\right)}{2\sqrt{cd^4-ae^4}} + \frac{\sqrt[4]{a}\,g\sqrt{1-\frac{cx^4}{a}}\,F\left(\sin^{-1}\left(\right.\right.}{\sqrt[4]{c}\,e\sqrt{-a+cx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.00, size = 719, normalized size = 3.30

$$\frac{-\frac{ig\sqrt{1-\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,e}+\frac{if\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2\sqrt{-\frac{(1-i)\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{i\sqrt[4]{a}+\sqrt[4]{c}x}}\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+i\sqrt[4]{c}x\right)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2}}\left(\left(-\sqrt[4]{c}d+\sqrt[4]{a}e\right)F\left(\sin^{-1}\left(\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{2i\sqrt[4]{a}+2\sqrt[4]{c}x}}\right)\middle|2\right)-(1-i)\sqrt[4]{a}\,e\,\Pi\left(\frac{(1-i)\left(\sqrt[4]{c}d-i\sqrt[4]{a}e\right)}{\sqrt[4]{c}d-\sqrt[4]{a}e};\sin^{-1}\left(\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{2i\sqrt[4]{a}+2\sqrt[4]{c}x}}\right)\middle|2\right)\right)}{\sqrt[4]{a}\left(-\sqrt[4]{c}d+\sqrt[4]{a}e\right)\left(i\sqrt[4]{c}d+\sqrt[4]{a}e\right)}+\frac{dg\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2\sqrt{-\frac{(1-i)\left(\sqrt[4]{a}-\sqrt[4]{c}x\right)}{i\sqrt[4]{a}+\sqrt[4]{c}x}}\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+i\sqrt[4]{c}x\right)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{\left(\sqrt[4]{a}-i\sqrt[4]{c}x\right)^2}}\left(i\left(\sqrt[4]{c}d-\sqrt[4]{a}e\right)F\left(\sin^{-1}\left(\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{2i\sqrt[4]{a}+2\sqrt[4]{c}x}}\right)\middle|2\right)+(1+i)\sqrt[4]{a}\,e\,\Pi\left(\frac{(1-i)\left(\sqrt[4]{c}d-i\sqrt[4]{a}e\right)}{\sqrt[4]{c}d-\sqrt[4]{a}e};\sin^{-1}\left(\sqrt{\frac{(1+i)\left(\sqrt[4]{a}+\sqrt[4]{c}x\right)}{2i\sqrt[4]{a}+2\sqrt[4]{c}x}}\right)\middle|2\right)\right)}{\sqrt[4]{a}\,e\left(-\sqrt[4]{c}d+\sqrt[4]{a}e\right)\left(i\sqrt[4]{c}d+\sqrt[4]{a}e\right)}}{\sqrt{-a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + c*x^4]),x]
```

```
[Out] (((-I)*g*Sqrt[1 - (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[c]/Sqrt[a])]*x
], -1])/(Sqrt[-(Sqrt[c]/Sqrt[a])]*e) + (I*f*(a^(1/4) - I*c^(1/4)*x)^2*Sqrt[
((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/4)*x)]*Sqrt[((1 + I)*(a^
(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1/4)*x)^2]*((-(
c^(1/4)*d) + a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x
))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] - (1 - I)*a^(1/4)*e*EllipticPi[((1 -
 I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin[Sqrt[((1 + I
)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]))/(a^(1/4)*(-(
c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e)) + (d*g*(a^(1/4) - I*c^(1
/4)*x)^2*Sqrt[((-1 + I)*(a^(1/4) - c^(1/4)*x))/(I*a^(1/4) + c^(1/4)*x)]*Sqr
t[((1 + I)*(a^(1/4) + I*c^(1/4)*x)*(a^(1/4) + c^(1/4)*x))/(a^(1/4) - I*c^(1
```

```
/4)*x)^2]*(I*(c^(1/4)*d - a^(1/4)*e)*EllipticF[ArcSin[Sqrt[((1 + I)*(a^(1/4
) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2] + (1 + I)*a^(1/4)*e*Ell
ipticPi[((1 - I)*(c^(1/4)*d - I*a^(1/4)*e))/(c^(1/4)*d - a^(1/4)*e), ArcSin
[Sqrt[((1 + I)*(a^(1/4) + c^(1/4)*x))/((2*I)*a^(1/4) + 2*c^(1/4)*x)]], 2]))
/(a^(1/4)*e*(-(c^(1/4)*d) + a^(1/4)*e)*(I*c^(1/4)*d + a^(1/4)*e)))/Sqrt[-a
+ c*x^4]
```

Maple [A]
time = 0.14, size = 247, normalized size = 1.13

method	result
default	$\frac{g\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}+\frac{(-dg+ef)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}-2a}{2\sqrt{\frac{c\,d^4}{e^4}-a}\,\sqrt{c\,x^4-a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}-a}}+ \right.}{}$
elliptic	$\frac{g\sqrt{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}},i\right)}{e\sqrt{-\frac{\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4-a}}-\frac{(dg-ef)\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}-2a}{2\sqrt{\frac{c\,d^4}{e^4}-a}\,\sqrt{c\,x^4-a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}-a}}+ \right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] g/e/(-1/a^(1/2)*c^(1/2))^(1/2)*(1+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/2)*EllipticF(x*(-1/a^(1/2)*c^(1/2))^(1/2),
I)+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4-a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2-2*
a)/(c*d^4/e^4-a)^(1/2)/(c*x^4-a)^(1/2))+1/(-1/a^(1/2)*c^(1/2))^(1/2)/d*e*(1
+1/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1-1/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4-a)^(1/
2)*EllipticPi(x*(-1/a^(1/2)*c^(1/2))^(1/2),-e^2*a^(1/2)/d^2/c^(1/2),(1/a^(1
/2)*c^(1/2))^(1/2)/(-1/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)/(sqrt(c*x^4 - a)*(x*e + d)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 - a)*(g*x + f)/(c*d*x^4 - a*d + (c*x^5 - a*x)*e), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx}{\sqrt{-a + cx^4}\,(d + ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4-a)**(1/2),x)
[Out] Integral((f + g*x)/(sqrt(-a + c*x**4)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4-a)^(1/2),x, algorithm="giac")
[Out] integrate((g*x + f)/(sqrt(c*x^4 - a)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f + g\,x}{\sqrt{c\,x^4 - a}\,(d + e\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((c*x^4 - a)^(1/2)*(d + e*x)),x)
[Out] int((f + g*x)/((c*x^4 - a)^(1/2)*(d + e*x)), x)
```

3.408 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\right)$$

```
[Out] 1/3*arctanh((1+x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-4+x^4+4*3^(1/2)*x^2)^(1/
2))*(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1754, 213}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tanh^{-1}\left(\frac{\left(x-\sqrt{3}+1\right)^2}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{x^4+4\sqrt{3}\,x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + x)^2/(Sqrt[3*(-3 + 2*Sqrt[3])]
*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^4])])/3
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-4+4\sqrt{3}\,x^2+x^4}}\,dx = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{3\left(1-\sqrt{3}\right)^4+6\left(1-\sqrt{3}\right)\right.\right.$$

$$= \frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+x\right)^2}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{-4+4\sqrt{3}\,\right.$$

Mathematica [A]
time = 7.75, size = 77, normalized size = 1.18

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{9+6\sqrt{3}}\sqrt{-4+4\sqrt{3}\,x^2+x^4}}{2+\left(-2-2\sqrt{3}\right)x+\left(2+\sqrt{3}\right)x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-4 + 4*Sqrt[3]*x^2 + x^
4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[9 + 6*Sqrt[3]]*Sqrt[-4 + 4*Sqrt[3]*x^2
+ x^4])/(2 + (-2 - 2*Sqrt[3])*x + (2 + Sqrt[3])*x^2)])/3
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.38, size = 327, normalized size = 5.03

method	result
default	$\frac{\sqrt{1-\left(\frac{\sqrt{3}}{2}-1\right)x^2}\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(x\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right),i\sqrt{1+4\sqrt{3}\,\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{-4+x^4+4x^2\sqrt{3}}} -$
elliptic	$\frac{\sqrt{1-\left(\frac{\sqrt{3}}{2}-1\right)x^2}\sqrt{1-\left(1+\frac{\sqrt{3}}{2}\right)x^2}\,\text{EllipticF}\left(x\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right),i\sqrt{1+4\sqrt{3}\,\left(1+\frac{\sqrt{3}}{2}\right)}\right)}{\left(\frac{i\sqrt{3}}{2}-\frac{i}{2}\right)\sqrt{-4+x^4+4x^2\sqrt{3}}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*x^2*3^(1/2))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(1/2*I*3^(1/2)-1/2*I)*(1-(1/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^
2)^(1/2)/(-4+x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*I*3^(1/2)-1/2*I),I*(
1+4*3^(1/2)*(1+1/2*3^(1/2)))^(1/2))-2*3^(1/2)*(-1/2/((-1-3^(1/2))^4+4*3^(1/
2)*(-1-3^(1/2))^2-4)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1-3^(1/2))^2-8+4*x^2*3^
(1/2)+2*x^2*(-1-3^(1/2))^2)/((-1-3^(1/2))^4+4*3^(1/2)*(-1-3^(1/2))^2-4)^(1/
2)/(-4+x^4+4*x^2*3^(1/2))^(1/2))-1/(1/2*3^(1/2)-1)^(1/2)/(-1-3^(1/2))*(1-(1
/2*3^(1/2)-1)*x^2)^(1/2)*(1-(1+1/2*3^(1/2))*x^2)^(1/2)/(-4+x^4+4*x^2*3^(1/2
))^(1/2)*EllipticPi((1/2*3^(1/2)-1)^(1/2)*x,1/(1/2*3^(1/2)-1)/(-1-3^(1/2))^
2,(1+1/2*3^(1/2))^(1/2)/(1/2*3^(1/2)-1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(47) = 94$.
time = 0.50, size = 323, normalized size = 4.97

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{37x^{12}-204x^{11}+804x^{10}-2408x^{9}+3708x^{8}-5472x^{7}+6432x^{6}+10944x^{5}+14832x^{4}+19264x^{3}+12864x^{2}+\left(54x^{10}-300x^{9}+1026x^{8}-2232x^{7}+3024x^{6}-3024x^{5}-1008x^{4}-2016x^{3}-2592x^{2}+\sqrt{3}(31x^{10}-176x^{9}+576x^{8}-1320x^{7}+1848x^{6}-1008x^{5}+1344x^{4}+1632x^{3}+1008x^{2}+832x+256)-1152x-480\right)\sqrt{x^4+4\sqrt{3}x^2-4}\sqrt{2\sqrt{3}-3}+3\sqrt{3}(7x^{12}-40x^{11}+160x^{10}-400x^{9}+924x^{8}-960x^{7}-1920x^{5}-3696x^{4}-3200x^{3}-2560x^{2}-1280x-448)+6528x+2368}{x^{12}+12x^{11}+48x^{10}+40x^{9}-180x^{8}-288x^{7}+384x^{6}+576x^{5}-720x^{4}-320x^{3}+768x^{2}-384x+64}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(37*x^12 - 204*x^11 + 804*x^10 - 2408*x^9 + 3
708*x^8 - 5472*x^7 + 6432*x^6 + 10944*x^5 + 14832*x^4 + 19264*x^3 + 12864*x
^2 + (54*x^10 - 300*x^9 + 1026*x^8 - 2232*x^7 + 3024*x^6 - 3024*x^5 - 1008*
x^4 - 2016*x^3 - 2592*x^2 + sqrt(3)*(31*x^10 - 176*x^9 + 576*x^8 - 1320*x^7
 + 1848*x^6 - 1008*x^5 + 1344*x^4 + 1632*x^3 + 1008*x^2 + 832*x + 256) - 11
52*x - 480)*sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(
7*x^12 - 40*x^11 + 160*x^10 - 400*x^9 + 924*x^8 - 960*x^7 - 1920*x^5 - 3696
```

```
*x^4 - 3200*x^3 - 2560*x^2 - 1280*x - 448) + 6528*x + 2368)/(x^12 + 12*x^11
 + 48*x^10 + 40*x^9 - 180*x^8 - 288*x^7 + 384*x^6 + 576*x^5 - 720*x^4 - 320
*x^3 + 768*x^2 - 384*x + 64))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\left(x+1+\sqrt{3}\right)\sqrt{x^4+4\sqrt{3}\,x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-4+x**4+4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/((x + 1 + sqrt(3))*sqrt(x**4 + 4*sqrt(3)*x**2 -
4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-4+x^4+4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^4 + 4*sqrt(3)*x^2 - 4)*(x + sqrt(3) + 1
)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-\sqrt{3}+1}{\left(x+\sqrt{3}+1\right)\,\sqrt{x^4+4\,\sqrt{3}\,x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),x
)
```

```
[Out] int((x - 3^(1/2) + 1)/((x + 3^(1/2) + 1)*(4*3^(1/2)*x^2 + x^4 - 4)^(1/2)),
x)
```

3.409 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx$

Optimal. Leaf size=63

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\,\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\right)$$

```
[Out] -1/3*arctan((1+x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-4+x^4-4*3^(1/2)*x^2)^(1/2
))*(3+2*3^(1/2))^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1754, 209}

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\text{ArcTan}\left(\frac{\left(x+\sqrt{3}+1\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\,\sqrt{x^4-4\sqrt{3}\,x^2-4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + x)^2/(Sqrt[3*(3 + 2*Sqrt[3]
)]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4])])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-4-4\sqrt{3}\,x^2+x^4}}\,dx = -\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+3\right.\right.$$

$$= -\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}+x\right)^2}{\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-4-4\sqrt{3}\,x^2}\right.$$

Mathematica [A]
time = 7.73, size = 77, normalized size = 1.22

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{-9+6\sqrt{3}}\sqrt{-4-4\sqrt{3}\,x^2+x^4}}{-2+\left(2-2\sqrt{3}\right)x+\left(-2+\sqrt{3}\right)x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-9 + 6*Sqrt[3]]*Sqrt[-4 - 4*Sqrt[3]*x^2 + x^4])/(-2 + (2 - 2*Sqrt[3])*x + (-2 + Sqrt[3])*x^2)])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.39, size = 311, normalized size = 4.94

method	result
default	$\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{3}}{2}+1\right)x^2}\,\text{EllipticF}\left(x\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right),i\sqrt{1-4\sqrt{3}}\left(-\frac{\sqrt{3}}{2}+1\right.\right.}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-4+x^4-4x^2\sqrt{3}}}$
elliptic	$\frac{\sqrt{1-\left(-1-\frac{\sqrt{3}}{2}\right)x^2}\sqrt{1-\left(-\frac{\sqrt{3}}{2}+1\right)x^2}\,\text{EllipticF}\left(x\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right),i\sqrt{1-4\sqrt{3}}\left(-\frac{\sqrt{3}}{2}+1\right.\right.}{\left(\frac{i}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{-4+x^4-4x^2\sqrt{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*x^2*3^(1/2))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/(1/2*I+1/2*I*3^(1/2))*(1-(-1-1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*
x^2)^(1/2)/(-4+x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(1/2*I+1/2*I*3^(1/2)),I
*(1-4*3^(1/2)*(-1/2*3^(1/2)+1))^(1/2))+2*3^(1/2)*(-1/2/((3^(1/2)-1)^4-4*3^(
1/2)*(3^(1/2)-1)^2-4)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(3^(1/2)-1)^2-8-4*x^2*3
^(1/2)+2*x^2*(3^(1/2)-1)^2)/((3^(1/2)-1)^4-4*3^(1/2)*(3^(1/2)-1)^2-4)^(1/2)
/(-4+x^4-4*x^2*3^(1/2))^(1/2))-1/(-1-1/2*3^(1/2))^(1/2)/(3^(1/2)-1)*(1-(-1-
1/2*3^(1/2))*x^2)^(1/2)*(1-(-1/2*3^(1/2)+1)*x^2)^(1/2)/(-4+x^4-4*x^2*3^(1/2
))^(1/2)*EllipticPi((-1-1/2*3^(1/2))^(1/2)*x,1/(-1-1/2*3^(1/2))/(3^(1/2)-1)
^2,(-1/2*3^(1/2)+1)^(1/2)/(-1-1/2*3^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(45) = 90$.
time = 0.58, size = 112, normalized size = 1.78

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\arctan\left(-\frac{\left(9x^4-30x^3+18x^2-2\sqrt{3}\left(2x^4-10x^3+3x^2-10x+2\right)+24\right)\sqrt{x^4-4\sqrt{3}x^2-4}\sqrt{2\sqrt{3}+3}}{11x^6-42x^5+66x^4-176x^3-132x^2-168x-88}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(9*x^4 - 30*x^3 + 18*x^2 - 2*sqrt(3)*(2*x^4
 - 10*x^3 + 3*x^2 - 10*x + 2) + 24)*sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*sqrt(2*sq
rt(3) + 3)/(11*x^6 - 42*x^5 + 66*x^4 - 176*x^3 - 132*x^2 - 168*x - 88))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1+\sqrt{3}}{\left(x-\sqrt{3}+1\right)\sqrt{x^4-4\sqrt{3}x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-4+x**4-4*3**(1/2)*x**2)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/((x - sqrt(3) + 1)*sqrt(x**4 - 4*sqrt(3)*x**2 -
4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-4+x^4-4*3^(1/2)*x^2)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^4 - 4*sqrt(3)*x^2 - 4)*(x - sqrt(3) + 1
)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x+\sqrt{3}+1}{\sqrt{x^4-4\,\sqrt{3}\,x^2-4}\,\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),x
)
```

```
[Out] int((x + 3^(1/2) + 1)/((x^4 - 4*3^(1/2)*x^2 - 4)^(1/2)*(x - 3^(1/2) + 1)),
x)
```

3.410 $\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}\,x^2+4x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+2x\right)^2}{2\sqrt{3\left(-3+2\sqrt{3}\right)}\,\sqrt{-1+4\sqrt{3}\,x^2+4x^4}}\right)$$

```
[Out] 1/3*arctanh(1/2*(1+2*x-3^(1/2))^2/(-9+6*3^(1/2))^(1/2)/(-1+4*x^4+4*3^(1/2)*
x^2)^(1/2))*(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1754, 213}

$$\frac{1}{3}\sqrt{2\sqrt{3}-3}\,\tanh^{-1}\left(\frac{\left(2x-\sqrt{3}+1\right)^2}{2\sqrt{3\left(2\sqrt{3}-3\right)}\,\sqrt{4x^4+4\sqrt{3}\,x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^
4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(1 - Sqrt[3] + 2*x)^2/(2*Sqrt[3*(-3 + 2*Sqrt[
3])]*Sqrt[-1 + 4*Sqrt[3]*x^2 + 4*x^4])])/3
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+2x}{\left(1+\sqrt{3}+2x\right)\sqrt{-1+4\sqrt{3}\,x^2+4x^4}}\,dx = -\left(\left(4\left(2-\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{6\left(1-\sqrt{3}\right)^4+12\left(1-\sqrt{\right.}\right.}\right.\right.$$

$$= \frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\left(1-\sqrt{3}+2x\right)^2}{2\sqrt{3\left(-3+2\sqrt{3}\right)}\,\sqrt{-1+4\sqrt{\right.}}}\right.$$

Mathematica [A]
time = 7.82, size = 81, normalized size = 1.12

$$\frac{1}{3}\sqrt{-3+2\sqrt{3}}\,\tanh^{-1}\left(\frac{\sqrt{9+6\sqrt{3}}\,\sqrt{-1+4\sqrt{3}\,x^2+4x^4}}{1+\left(-2-2\sqrt{3}\right)x+\left(4+2\sqrt{3}\right)x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + 2*x)/((1 + Sqrt[3] + 2*x)*Sqrt[-1 + 4*Sqrt[3]*x^2
+ 4*x^4]),x]
```

```
[Out] (Sqrt[-3 + 2*Sqrt[3]]*ArcTanh[(Sqrt[9 + 6*Sqrt[3]]*Sqrt[-1 + 4*Sqrt[3]*x^2
+ 4*x^4])/(1 + (-2 - 2*Sqrt[3])*x + (4 + 2*Sqrt[3])*x^2)])/3
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.35, size = 336, normalized size = 4.67

method	result
default	$\frac{\sqrt{1-\left(2\sqrt{3}-4\right)x^2}\,\sqrt{1-\left(4+2\sqrt{3}\right)x^2}\,\text{EllipticF}\left(x\left(i\sqrt{3}-i\right),i\sqrt{1+\sqrt{3}\,\left(4+2\sqrt{3}\right)}\right)}{\left(i\sqrt{3}-i\right)\sqrt{-1+4x^4+4x^2\sqrt{3}}}-2\sqrt{}$

elliptic	$\frac{\sqrt{1-\left(2\sqrt{3}-4\right)x^2}\,\sqrt{1-\left(4+2\sqrt{3}\right)x^2}\,\operatorname{EllipticF}\left(x\left(i\sqrt{3}-i\right),i\sqrt{1+\sqrt{3}\,\left(4+2\sqrt{3}\right)}\right)}{\left(i\sqrt{3}-i\right)\sqrt{-1+4x^4+4x^2\sqrt{3}}}-\sqrt{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*x^2*3^(1/2))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/(I*3^(1/2)-I)*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(4+2*3^(1/2))*x^2)^(1/2)/(-1
+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I*3^(1/2)-I),I*(1+3^(1/2)*(4+2*3^(
1/2)))^(1/2))-2*3^(1/2)*(-1/4/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2*3
^(1/2))^2-1)^(1/2)*arctanh(1/2*(4*3^(1/2)*(-1/2-1/2*3^(1/2))^2-2+4*x^2*3^(1
/2)+8*x^2*(-1/2-1/2*3^(1/2))^2)/(4*(-1/2-1/2*3^(1/2))^4+4*3^(1/2)*(-1/2-1/2
*3^(1/2))^2-1)^(1/2)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2))-1/2/(2*3^(1/2)-4)^(1/2
)/(-1/2-1/2*3^(1/2))*(1-(2*3^(1/2)-4)*x^2)^(1/2)*(1-(4+2*3^(1/2))*x^2)^(1/2
)/(-1+4*x^4+4*x^2*3^(1/2))^(1/2)*EllipticPi((2*3^(1/2)-4)^(1/2)*x,1/(2*3^(1
/2)-4)/(-1/2-1/2*3^(1/2))^2,(4+2*3^(1/2))^(1/2)/(2*3^(1/2)-4)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(
3) + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(52) = 104$.
time = 0.57, size = 328, normalized size = 4.56

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\log\left(-\frac{2368x^{12}-6528x^{11}+12864x^{10}-19264x^9+14832x^8-10944x^7+6432x^6+5472x^5+3708x^4+2408x^3+804x^2+\left(1728x^{10}-4800x^9+8208x^8-8928x^7+6048x^6-3024x^5-504x^4-504x^3-324x^2+2\sqrt{3}\left(496x^{10}-1408x^9+2304x^8-2640x^7+1848x^6-504x^5+336x^4+204x^3+63x^2+26x+4\right)-72x-15\right)\sqrt{4x^4+4\sqrt{3}x^2-1}\sqrt{2\sqrt{3}-3}+3\sqrt{3}\left(448x^{12}-1280x^{11}+2560x^{10}-3200x^9+3696x^8-1920x^7-960x^5-924x^4-400x^3-160x^2-40x-7\right)+204x+37}{64x^{12}+384x^{11}+768x^{10}+320x^9-720x^8-576x^7+384x^6+288x^5-180x^4-40x^3+48x^2-12x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log(-(2368*x^12 - 6528*x^11 + 12864*x^10 - 19264*x
^9 + 14832*x^8 - 10944*x^7 + 6432*x^6 + 5472*x^5 + 3708*x^4 + 2408*x^3 + 80
4*x^2 + (1728*x^10 - 4800*x^9 + 8208*x^8 - 8928*x^7 + 6048*x^6 - 3024*x^5 -
 504*x^4 - 504*x^3 - 324*x^2 + 2*sqrt(3)*(496*x^10 - 1408*x^9 + 2304*x^8 -
2640*x^7 + 1848*x^6 - 504*x^5 + 336*x^4 + 204*x^3 + 63*x^2 + 26*x + 4) - 72
*x - 15)*sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*sqrt(2*sqrt(3) - 3) + 3*sqrt(3)*(4
48*x^12 - 1280*x^11 + 2560*x^10 - 3200*x^9 + 3696*x^8 - 1920*x^7 - 960*x^5
- 924*x^4 - 400*x^3 - 160*x^2 - 40*x - 7) + 204*x + 37)/(64*x^12 + 384*x^11
 + 768*x^10 + 320*x^9 - 720*x^8 - 576*x^7 + 384*x^6 + 288*x^5 - 180*x^4 - 4
0*x^3 + 48*x^2 - 12*x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x - \sqrt{3} + 1}{\left(2x + 1 + \sqrt{3}\right)\sqrt{4x^4 + 4\sqrt{3}\,x^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3**(1/2))/(1+2*x+3**(1/2))/(-1+4*x**4+4*3**(1/2)*x**2)**(1
/2),x)
```

```
[Out] Integral((2*x - sqrt(3) + 1)/((2*x + 1 + sqrt(3))*sqrt(4*x**4 + 4*sqrt(3)*x
**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x-3^(1/2))/(1+2*x+3^(1/2))/(-1+4*x^4+4*3^(1/2)*x^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((2*x - sqrt(3) + 1)/(sqrt(4*x^4 + 4*sqrt(3)*x^2 - 1)*(2*x + sqrt(
3) + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x - \sqrt{3} + 1}{\sqrt{4\,x^4 + 4\,\sqrt{3}\,x^2 - 1}\,\left(2\,x + \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 3^(1/2) + 1)/((4*3^(1/2)*x^2 + 4*x^4 - 1)^(1/2)*(2*x + 3^(1/2) +
 1)),x)
```

```
[Out] int((2*x - 3^(1/2) + 1)/((4*3^(1/2)*x^2 + 4*x^4 - 1)^(1/2)*(2*x + 3^(1/2) +
 1)), x)
```

3.411 $\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}+2x\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}\right)$$

```
[Out] -1/3*arctan(1/2*(1+2*x+3^(1/2))^2/(9+6*3^(1/2))^(1/2)/(-1+4*x^4-4*3^(1/2)*x
^2)^(1/2))*(3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1754, 209}

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\text{ArcTan}\left(\frac{\left(2x+\sqrt{3}+1\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{4x^4-4\sqrt{3}\,x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^
4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(1 + Sqrt[3] + 2*x)^2/(2*Sqrt[3*(3 + 2*Sqr
t[3])]*Sqrt[-1 - 4*Sqrt[3]*x^2 + 4*x^4])])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1754

```
Int[((A_) + (B_.)*(x_))/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> Dist[(-A^2)*((B*d + A*e)/e), Subst[Int[1/(6*A^3*B
*d + 3*A^4*e - a*e*x^2), x], x, (A + B*x)^2/Sqrt[a + b*x^2 + c*x^4]], x] /;
 FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[B*d - A*e, 0] && EqQ[c^2*d^6 + a*e^
4*(13*c*d^2 + b*e^2), 0] && EqQ[b^2*e^4 - 12*c*d^2*(c*d^2 - b*e^2), 0] && E
qQ[4*A*c*d*e + B*(2*c*d^2 - b*e^2), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+2x}{\left(1-\sqrt{3}+2x\right)\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}\,dx = -\left(\left(4\left(2+\sqrt{3}\right)\right)\text{Subst}\left(\int \frac{1}{12\left(1-\sqrt{3}\right)\left(1+\sqrt{3}\right)^3+\right.\right.\right.$$

$$= -\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\left(1+\sqrt{3}+2x\right)^2}{2\sqrt{3\left(3+2\sqrt{3}\right)}\sqrt{-1-4\sqrt{3}\,x\right.}}\right.$$

Mathematica [**A**]
time = 7.83, size = 81, normalized size = 1.16

$$-\frac{1}{3}\sqrt{3+2\sqrt{3}}\,\tan^{-1}\left(\frac{\sqrt{-9+6\sqrt{3}}\sqrt{-1-4\sqrt{3}\,x^2+4x^4}}{-1+\left(2-2\sqrt{3}\right)x+\left(-4+2\sqrt{3}\right)x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + 2*x)/((1 - Sqrt[3] + 2*x)*Sqrt[-1 - 4*Sqrt[3]*x^2
+ 4*x^4]),x]
```

```
[Out] -1/3*(Sqrt[3 + 2*Sqrt[3]]*ArcTan[(Sqrt[-9 + 6*Sqrt[3]]*Sqrt[-1 - 4*Sqrt[3]*
x^2 + 4*x^4])/(-1 + (2 - 2*Sqrt[3])*x + (-4 + 2*Sqrt[3])*x^2)])
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.39, size = 337, normalized size = 4.81

method	result
elliptic	$\frac{\sqrt{1-\left(-4-2\sqrt{3}\right)x^2}\sqrt{1-\left(-2\sqrt{3}+4\right)x^2}\,\text{EllipticF}\left(x\left(i+i\sqrt{3}\right),i\sqrt{1-\sqrt{3}\,\left(-2\sqrt{3}+4\right)}\right)}{\left(i+i\sqrt{3}\right)\sqrt{-1+4x^4-4x^2\sqrt{3}}}+$

default	$\frac{\sqrt{1-\left(-4-2\sqrt{3}\right)x^{2}}\,\sqrt{1-\left(-2\sqrt{3}+4\right)x^{2}}\,\mathrm{EllipticF}\left(x\left(i+i\sqrt{3}\right),i\sqrt{1-\sqrt{3}\,\left(-2\sqrt{3}+4\right)}\right)}{\left(i+i\sqrt{3}\right)\sqrt{-1+4x^{4}-4x^{2}\sqrt{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*x^2*3^(1/2))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/(I+I*3^(1/2))*(1-(-4-2*3^(1/2))*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(1/2)/(
-1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticF(x*(I+I*3^(1/2)),I*(1-3^(1/2)*(-2*3^
(1/2)+4))^(1/2))+2*3^(1/2)*(-1/4/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1
/2)-1/2)^2-1)^(1/2)*arctanh(1/2*(-4*3^(1/2)*(1/2*3^(1/2)-1/2)^2-2-4*x^2*3^(
1/2)+8*x^2*(1/2*3^(1/2)-1/2)^2)/(4*(1/2*3^(1/2)-1/2)^4-4*3^(1/2)*(1/2*3^(1/
2)-1/2)^2-1)^(1/2)/(-1+4*x^4-4*x^2*3^(1/2))^(1/2))-1/2/(-4-2*3^(1/2))^(1/2)
/(1/2*3^(1/2)-1/2)*(1-(-4-2*3^(1/2))*x^2)^(1/2)*(1-(-2*3^(1/2)+4)*x^2)^(1/2
)/(-1+4*x^4-4*x^2*3^(1/2))^(1/2)*EllipticPi((-4-2*3^(1/2))^(1/2)*x,1/(-4-2*
3^(1/2))/(1/2*3^(1/2)-1/2)^2,(-2*3^(1/2)+4)^(1/2)/(-4-2*3^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(
3) + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(50) = 100$.
time = 0.52, size = 114, normalized size = 1.63

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\,\arctan\left(-\frac{\left(36\,x^{4}-60\,x^{3}+18\,x^{2}-\sqrt{3}\,(16\,x^{4}-40\,x^{3}+6\,x^{2}-10\,x+1)+6\right)\sqrt{4\,x^{4}-4\,\sqrt{3}\,x^{2}-1}\,\sqrt{2\,\sqrt{3}+3}}{88\,x^{6}-168\,x^{5}+132\,x^{4}-176\,x^{3}-66\,x^{2}-42\,x-11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2),x,
 algorithm="fricas")
```

```
[Out] 1/6*sqrt(2*sqrt(3) + 3)*arctan(-(36*x^4 - 60*x^3 + 18*x^2 - sqrt(3)*(16*x^4
 - 40*x^3 + 6*x^2 - 10*x + 1) + 6)*sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*sqrt(2*s
qrt(3) + 3)/(88*x^6 - 168*x^5 + 132*x^4 - 176*x^3 - 66*x^2 - 42*x - 11))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1+\sqrt{3}}{\left(2x-\sqrt{3}+1\right)\sqrt{4x^4-4\sqrt{3}\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3**(1/2))/(1+2*x-3**(1/2))/(-1+4*x**4-4*3**(1/2)*x**2)**(1
/2),x)
```

```
[Out] Integral((2*x + 1 + sqrt(3))/((2*x - sqrt(3) + 1)*sqrt(4*x**4 - 4*sqrt(3)*x
**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x+3^(1/2))/(1+2*x-3^(1/2))/(-1+4*x^4-4*3^(1/2)*x^2)^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate((2*x + sqrt(3) + 1)/(sqrt(4*x^4 - 4*sqrt(3)*x^2 - 1)*(2*x - sqrt(
3) + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x+\sqrt{3}+1}{\sqrt{4\,x^4-4\,\sqrt{3}\,x^2-1}\,\left(2\,x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 3^(1/2) + 1)/((4*x^4 - 4*3^(1/2)*x^2 - 1)^(1/2)*(2*x - 3^(1/2) +
 1)),x)
```

```
[Out] int((2*x + 3^(1/2) + 1)/((4*x^4 - 4*3^(1/2)*x^2 - 1)^(1/2)*(2*x - 3^(1/2) +
 1)), x)
```

3.412 $\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=560

$$\frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2\left(bd^2+ae^2\right)}}-\frac{(ef-dg)\tanh^{-1}\left(\frac{bd^2+2ae^2+\left(2cd^2+be^2\right)x^2}{2\sqrt{cd^4+bd^2e^2+ae^4}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2+ae^4}}+$$

```
[Out] 1/2*(-d*g+e*f)*arctan(x*(-a*e^4-b*d^2*e^2-c*d^4)^(1/2)/d/e/(c*x^4+b*x^2+a)^
(1/2))/(-a*e^4-b*d^2*e^2-c*d^4)^(1/2)-1/2*(-d*g+e*f)*arctanh(1/2*(b*d^2+2*a
*e^2+(b*e^2+2*c*d^2)*x^2)/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2+a)^(1/
2))/(a*e^4+b*d^2*e^2+c*d^4)^(1/2)-1/4*(-d*g+e*f)*(cos(2*arctan(c^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c
^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),
1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^
(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/e/
(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*x
/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan
(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(e*g*a^(1/2)+d*f*c^(1
/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
a^(1/4)/c^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 560, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1755, 12, 1261, 738, 212, 1722, 1117, 1720}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)\left(\sqrt{a}eg+\sqrt{c}df\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)(ef-dg)\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\text{ArcTan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx^2+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}+\frac{(ef-dg)\text{ArcTan}\left(\frac{x\sqrt{-ae^4-bd^2e^2-cd^4}}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-e^2\left(ae^2+bd^2\right)-cd^4}}-\frac{(ef-dg)\tanh^{-1}\left(\frac{2ae^2+x^2\left(be^2+2cd^2\right)+bd^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{ae^4+bd^2e^2+cd^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((e*f - d*g)*ArcTan[(Sqrt[-(c*d^4) - b*d^2*e^2 - a*e^4]*x)/(d*e*Sqrt[a + b*
x^2 + c*x^4])])/(2*Sqrt[-(c*d^4) - e^2*(b*d^2 + a*e^2)]) - ((e*f - d*g)*Arc
Tanh[(b*d^2 + 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt[c*d^4 + b*d^2*e^2 +
a*e^4]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c*d^4 + b*d^2*e^2 + a*e^4]) + ((S
qrt[c]*d*f + Sqrt[a]*e*g)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[
a + b*x^2 + c*x^4]) - ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(e*f - d*g)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[
(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4
```

```
)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)*c^(1/4)*d*e*(Sqrt[c
]*d^2 + Sqrt[a]*e^2)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1755

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^2+cx^4}}\,dx = \int \frac{(-ef+dg)x}{(d^2-e^2x^2)\sqrt{a+bx^2+cx^4}}\,dx + \int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{a+bx^2+cx^4}}\,dx$$

$$= \frac{\left(\sqrt{a}\,de(ef-dg)\right)\int \frac{1+\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{(d^2-e^2x^2)\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}\,d^2+\sqrt{a}\,e^2} + (-ef+dg)\int \frac{}{(d^2-}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2(bd^2+ae^2)}} + \frac{\left(\sqrt{c}\,df+\sqrt{a}\,eg\right)\left(\sqrt{a}\,+\right.}{}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2(bd^2+ae^2)}} + \frac{\left(\sqrt{c}\,df+\sqrt{a}\,eg\right)\left(\sqrt{a}\,+\right.}{}$$

$$= \frac{(ef-dg)\tan^{-1}\left(\frac{\sqrt{-cd^4-bd^2e^2-ae^4}\,x}{de\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{-cd^4-e^2(bd^2+ae^2)}} - \frac{(ef-dg)\tanh^{-1}\left(\frac{}{2\sqrt{cd^4}}\right.}{2\sqrt{cd^4}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 17.34, size = 3652, normalized size = 6.52

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2
- 4*a*c]))]*x], (-b - Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(Sqrt[2
]*Sqrt[-(c/(-b - Sqrt[b^2 - 4*a*c]))]*e*Sqrt[a + b*x^2 + c*x^4]) + (2*(Sqrt
[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]
/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)^2*Sqrt[(Sqr
t[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]
) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b
^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x)
)]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c)
 + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 -
 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b -
 Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - S
qrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]
+ 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/
c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt[b
^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^2
- 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2] - Sqrt[2]*Sqrt[(-b - Sq
rt[b^2 - 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[
2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d + (Sqrt[-(b/c) - Sqrt[b
^2 - 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
 + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2
 - 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] -
 Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c
] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c]
)/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt
[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^
2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2]))/(Sqrt[(-b - Sqrt[b^
2 - 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) +
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/
Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*Sqrt[a + b*x^
2 + c*x^4]) - (2*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c)
+ Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*d*g*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/S
qrt[2]) + x)^2*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt
```

```
[b^2 - 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 -
 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b
/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - S
qrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]
 - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])
/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*
c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b
/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - S
qrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b
- Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(
-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2*x
))]], (Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])
^2/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^2]
 - Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sq
rt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d
 + (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[
b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(d -
 (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b -
 Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 - 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt
[(-b + Sqrt[b^2 - 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - 2
*x))]], (Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c
])^2/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c])^
2]))/(Sqrt[(-b - Sqrt[b^2 - 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/
Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*e*(-d - (Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]*e)/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]
*e)/Sqrt[2])*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]

time = 0.12, size = 437, normalized size = 0.78

method	result
default	$\frac{g\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2}\right.}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$

elliptic	$\frac{g\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}}{2}\right.}{4e\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*g/e*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+b*d^2/e^
2+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+b*d^2/e^2+b*x^2+2*a)/(c*d^4/e^4+b*d
^2/e^2+a)^(1/2)/(c*x^4+b*x^2+a)^(1/2))+2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)/d*e*(1-1/2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1+1/2*(b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),2/(-b+(-4*a*c+b^2)^(1/2))*a/d^2*e^2,(-1/2*(b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f+gx}{(d+ex)\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f + g*x)/((d + e*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 + a)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f+g\,x}{\left(d+e\,x\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((f + g*x)/((d + e*x)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.413 $\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=527

$$-\frac{(ef-dg)\tanh^{-1}\left(\frac{bd^2-2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2-ae^4}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2-ae^4}}+\frac{\sqrt{b+\sqrt{b^2+4ac}}\,g\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)F}{\sqrt{2}\sqrt{c}\,e\sqrt{\frac{1+\frac{}{b-\sqrt{}}}{1+\frac{}{b+\sqrt{}}}}}$$

```
[Out] -1/2*(-d*g+e*f)*arctanh(1/2*(b*d^2-2*a*e^2+(b*e^2+2*c*d^2)*x^2)/(-a*e^4+b*d
^2*e^2+c*d^4)^(1/2)/(c*x^4+b*x^2-a)^(1/2))/(-a*e^4+b*d^2*e^2+c*d^4)^(1/2)+1
/2*g*(1/(1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^2)^
(1/2)))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)/(b+(4*a*c+b^2)^(1/2))^(1/2)/(1+2*
c*x^2/(b+(4*a*c+b^2)^(1/2)))^(1/2),(-2*(4*a*c+b^2)^(1/2)/(b-(4*a*c+b^2)^(1/
2)))^(1/2))*(1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))*(b+(4*a*c+b^2)^(1/2))^(1/2)/e
*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)/((1+2*c*x^2/(b-(4*a*c+b^2)^(1/2)))/(
1+2*c*x^2/(b+(4*a*c+b^2)^(1/2))))^(1/2)+1/2*(-d*g+e*f)*EllipticPi(x*2^(1/2)
*c^(1/2)/(-b+(4*a*c+b^2)^(1/2))^(1/2),-1/2*e^2*(b-(4*a*c+b^2)^(1/2))/c/d^2,
((b-(4*a*c+b^2)^(1/2))/(b+(4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(4*a*c+b
^2)^(1/2)))^(1/2)*(-b+(4*a*c+b^2)^(1/2))^(1/2)*(1+2*c*x^2/(b+(4*a*c+b^2)^(1
/2)))^(1/2)/d/e*2^(1/2)/c^(1/2)/(c*x^4+b*x^2-a)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$,
Rules used = {1755, 12, 1261, 738, 212, 1724, 1118, 429, 1234, 551}

$$\frac{\sqrt{\sqrt{4ac+b^2}-b}\sqrt{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}\,(ef-dg)\Pi\left(-\frac{\left(b-\sqrt{b^2+4ac}\right)e^2}{2cd^2};\text{ArcSin}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2+4ac}-b}}\right)|\frac{b-\sqrt{b^2+4ac}}{b+\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\,de\sqrt{-a+bx^2+cx^4}}+\frac{g\sqrt{\sqrt{4ac+b^2}+b}\left(\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1\right)F\left(\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|-\frac{2\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\,e\sqrt{\frac{\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}{\frac{2cx^2}{\sqrt{4ac+b^2}+b}+1}}\sqrt{-a+bx^2+cx^4}}-\frac{(ef-dg)\tanh^{-1}\left(\frac{-2ae^2+x^2(be^2+2cd^2)+bd^2}{2\sqrt{-a+bx^2+cx^4}\sqrt{-ae^4+bd^2e^2+cd^4}}\right)}{2\sqrt{-ae^4+bd^2e^2+cd^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*((e*f - d*g)*ArcTanh[(b*d^2 - 2*a*e^2 + (2*c*d^2 + b*e^2)*x^2)/(2*Sqrt
[c*d^4 + b*d^2*e^2 - a*e^4]*Sqrt[-a + b*x^2 + c*x^4]])])/Sqrt[c*d^4 + b*d^2*
e^2 - a*e^4] + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*g*(1 + (2*c*x^2)/(b - Sqrt[b^2
+ 4*a*c]))*EllipticF[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]
], (-2*Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*e*Sqrt
[(1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 + 4*a
*c]))]*Sqrt[-a + b*x^2 + c*x^4]) + (Sqrt[-b + Sqrt[b^2 + 4*a*c]]*(e*f - d*g
)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
```

```
b^2 + 4*a*c])]*EllipticPi[-1/2*((b - Sqrt[b^2 + 4*a*c])*e^2)/(c*d^2), ArcSi
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 + 4*a*c]]], (b - Sqrt[b^2 + 4*a*c]
)/(b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*d*e*Sqrt[-a + b*x^2 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1118

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2
*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1234

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(
Sqrt[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/((d + e*x^2)*Sq
rt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1724

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> Dist[B/e, Int[1/Sqrt[a + b*x^2 + c*x^4], x],
 x] + Dist[(e*A - d*B)/e, Int[1/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x],
x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

Rule 1755

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x
_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x
, 2], D = Coeff[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e
^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4
)/((d^2 - e^2*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]] /; FreeQ[{a, b, c, d, e},
x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*d^4 + b*d^2*e^2 + a*e^4
, 0]
```

Rubi steps

$$\int \frac{f+gx}{(d+ex)\sqrt{-a+bx^2+cx^4}}\,dx = \int \frac{(-ef+dg)x}{(d^2-e^2x^2)\sqrt{-a+bx^2+cx^4}}\,dx + \int \frac{df-egx^2}{(d^2-e^2x^2)\sqrt{-a+bx^2+cx^4}}$$

$$= \frac{g\int \frac{1}{\sqrt{-a+bx^2+cx^4}}\,dx}{e} + \frac{(d(ef-dg))\int \frac{1}{(d^2-e^2x^2)\sqrt{-a+bx^2+cx^4}}}{e}$$

$$= \frac{1}{2}(-ef+dg)\text{Subst}\left(\int \frac{1}{(d^2-e^2x)\sqrt{-a+bx+cx^2}}\,dx, x, x^2\right) + \frac{\left(g\sqrt{}\right.}{}$$

$$= \frac{\sqrt{b+\sqrt{b^2+4ac}}\,g\left(1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}\right)F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2+4ac}}}\right.\right.}{\sqrt{2}\sqrt{c}\,e\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2+4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2+4ac}}}}\sqrt{-a+bx^2+cx^4}}$$

$$= -\frac{(ef-dg)\tanh^{-1}\left(\frac{bd^2-2ae^2+(2cd^2+be^2)x^2}{2\sqrt{cd^4+bd^2e^2-ae^4}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{cd^4+bd^2e^2-ae^4}} + \frac{\sqrt{b}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.33, size = 3658, normalized size = 6.94

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)/((d + e*x)*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-I)*g*Sqrt[1 - (2*c*x^2)/(-b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(-b
 + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(-b - Sqrt[b^2
+ 4*a*c]))]*x], (-b - Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c])])/(Sqrt[2
]*Sqrt[-(c/(-b - Sqrt[b^2 + 4*a*c]))]*e*Sqrt[-a + b*x^2 + c*x^4]) + (2*(Sqr
t[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c
]/Sqrt[2])*f*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x)^2*Sqrt[(Sq
rt[(-b - Sqrt[b^2 + 4*a*c])/c]*(-(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2
]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[
```

```
b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x
))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]
/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c
) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqr
t[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2
+ 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b
- Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(
-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]
/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] -
Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]
 + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])
/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqrt[
b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b^2
 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2] - Sqrt[2]*Sqrt[(-b - S
qrt[b^2 + 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt
[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d + (Sqrt[-(b/c) - Sqrt[
b^2 + 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]
) + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^
2 + 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]
- Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/
c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c
])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]], (Sqrt[(-b - Sqr
t[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2/(Sqrt[(-b - Sqrt[b
^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^2]))/(Sqrt[(-b - Sqrt[b
^2 + 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) +
 Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)
/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*Sqrt[-a + b*
x^2 + c*x^4]) - (2*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c
) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*d*g*(-(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]
/Sqrt[2]) + x)^2*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(-(Sqrt[-(b/c) + Sq
rt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]/Sqrt
[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2
 + 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-
(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + x))/((Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c
]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) -
 Sqrt[b^2 + 4*a*c]/c]/Sqrt[2]) + x))]*Sqrt[((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/
c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c
])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*
a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2*x))]*((-d + (Sqrt[-
(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*EllipticF[ArcSin[Sqrt[((Sqrt[(-b -
 Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-
b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt
[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - 2
*x))]], (Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c
])^2/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])^
```

```
2] - Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*e*EllipticPi[((Sqrt[-(b/c) -
Sqrt[b^2 + 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*
(d + (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2]))/((-(Sqrt[-(b/c) - Sqr
t[b^2 + 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*(d
 - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])), ArcSin[Sqrt[((Sqrt[(-b
 - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[
(-b - Sqrt[b^2 + 4*a*c])/c] + 2*x))/((Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sq
rt[(-b + Sqrt[b^2 + 4*a*c])/c])*(Sqrt[2]*Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] -
 2*x))]], (Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] + Sqrt[(-b + Sqrt[b^2 + 4*a*c])
/c])^2/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c] - Sqrt[(-b + Sqrt[b^2 + 4*a*c])/c]
)^2]))/(Sqrt[(-b - Sqrt[b^2 + 4*a*c])/c]*(Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/c
]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 + 4*a*c]/c]/Sqrt[2])*e*(-d - (Sqrt[-(b/c
) - Sqrt[b^2 + 4*a*c]/c]*e)/Sqrt[2])*(d - (Sqrt[-(b/c) - Sqrt[b^2 + 4*a*c]/
c]*e)/Sqrt[2])*Sqrt[-a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.11, size = 439, normalized size = 0.83

method	result
default	$\frac{g\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{2},\sqrt{-4\right.}{2e\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+b\,x^2-a}}$
elliptic	$\frac{g\sqrt{4+\frac{2\left(-b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\sqrt{4-\frac{2\left(b+\sqrt{4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}}{2},\sqrt{-4\right.}{2e\sqrt{-\frac{2\left(-b+\sqrt{4ac+b^2}\right)}{a}}\,\sqrt{c\,x^4+b\,x^2-a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*g/e/(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*(-b+(4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4-2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2-a)^(1/2)*Ell
ipticF(1/2*x*(-2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)
^(1/2))/a/c)^(1/2))+(-d*g+e*f)/e^2*(-1/2/(c*d^4/e^4+b*d^2/e^2-a)^(1/2)*arct
```

```
anh(1/2*(2*c*x^2*d^2/e^2+b*d^2/e^2+b*x^2-2*a)/(c*d^4/e^4+b*d^2/e^2-a)^(1/2)
/(c*x^4+b*x^2-a)^(1/2))+1/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)/d*e*(1+1/2*
(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(1-1/2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(c*x^4+b*x^2-a)^(1/2)*EllipticPi((-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*x
,-2/(-b+(4*a*c+b^2)^(1/2))*a/d^2*e^2,1/2*2^(1/2)*((b+(4*a*c+b^2)^(1/2))/a)^
(1/2)/(-1/2*(-b+(4*a*c+b^2)^(1/2))/a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{f + gx}{(d + ex)\sqrt{-a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x**4+b*x**2-a)**(1/2),x)
[Out] Integral((f + g*x)/((d + e*x)*sqrt(-a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(e*x+d)/(c*x^4+b*x^2-a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(sqrt(c*x^4 + b*x^2 - a)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{f + g\,x}{(d + e\,x)\,\sqrt{c\,x^4 + b\,x^2 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f + g*x)/((d + e*x)*(b*x^2 - a + c*x^4)^(1/2)),x)
[Out] int((f + g*x)/((d + e*x)*(b*x^2 - a + c*x^4)^(1/2)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):  #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```